Eigenvalue branches near $\omega_0 = 1.414\pi, \alpha = 0.25$

