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Mathematical modeling and simulation of cryogenic tunnel freezers

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Abstract

Modern food industries employ either mechanical or cryogenic methods for freezing products. A wealth of literature is available on design, implementation and optimization of mechanical freezing systems in the food industry. Cryogenic freezing is a relatively new technology for the food industry and there is a need for developing mathematical models to characterize this technology. Our focus here is to develop analytical and numerical models for describing the dynamics of the cryogenic freezing tunnel system. Two models for sizing and rating of the tunnel freezer have been developed. A composite model combining the freezer and the food freezing dynamics using a two step finite difference methods has been proposed for sizing the tunnel freezer. The error in prediction the temperature profile of the food material and the tunnel freezer is reduced to less than 5%, consequently reducing cryogen consumption by up to 30%. The proposed model can be useful for minimizing the operating costs of tunnel freezers by deriving suitable control strategies and provide insights for improvements in their design. A dynamic tank model has been developed for rating the tunnel freezer. The tank model guarantees stability of the system, is accurate, and can be readily extended to complex designs and applications where there are multiple zones with nozzles and fans.

Keywords: Cryogenic tunnel freezer; Mathematical modeling

1. Introduction

Cryogenic freezing is an attractive option for food-processing industries because of the several benefits it offers over mechanical freezing; namely increased production capacity, better quality of food in terms of texture, taste and appearance, reduced losses due to dehydration and drip, and longer shelf life of the processed food (Awonorin, 1997; Miller & Roberts, 2001). This form of freezing is particularly attractive for low volume/high variety food production because of the process flexibility it offers. However, high operating costs are a major drawback.

The setup cost of a cryogenic freezing system is approximately one-fourth of the cost of its mechanical counterpart; however, the operating costs are almost eight times as much (Ramakrishnan, Wysk, & Prabhu, 2004). Current industrial controllers use programmable logic controllers (PLCs) for regulating the belt speed of the tunnel freezers, which leads to conservative set-points and consequently significant operational cost and frequent over-freezing. It is therefore imperative that formal models be developed for understanding and predicting the system behavior for efficient sizing and rating of the tunnel freezers.

The sizing problem is encountered during the design and implementation stages wherein the objective is to determine the operating parameters for an expected heat load and space restrictions. Rating is the evaluation of a systems performance in handling a heat load and the variance therein. Different models are required for each of these problems. The purpose of this work is to understand the dynamics of the cryogenic tunnel freezer and develop analytical and numerical models to address the sizing and the rating problem. Amongst other things, these models will act as enablers for

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Nomenclature

A_i	cross sectional area of tank i, (m^2)
Bi	Biot number (hR/k)
C_p	heat capacity (J/kg K)
D_e	equivalent diameter for fish shape (m)
d	sample thickness (m)
f_0	heat transfer area of the body (m^2)
h	surface heat transfer coefficient (W/m^2 K)
h_i	height of liquid in tank i
H_f	enthalpy during freezing step (J/kg)
H_v	volumetric specific enthalpy (J/m^3)
H_{belt}	enthalpy associated with belt
H_{sur}	enthalpy associated with heat leak
l	length
l_i	constant for equation
L	latent heat of evaporation of water (J/kg)
P, Q	geometric factor in plank equation
q	volumetric flow rate (m^3/s)
R	characteristic dimension (m)
R_i	constants in equations
S	surface area
T	temperature (K)
T_i	initial temperature (K)
u	conveyor belt speed
v	volumetric flow rate of LIN
V_o	volume of the food (m^3)
z	heat to be removed from the product in a zone

Greek

ρ	density (kg/m^3)
τ_i	constants defined in equations
θ	freezing time
θ_i	freezing time in zone i
Δ	change operator
λ	thermal conductivity (W/m K)

Subscripts

a	ambient
ex	exhaust
f	freezing
g	gas zone/tank
i	zone/tank
in	input
l	liquid zone/tank
leak	loss to the atmosphere
s	frozen state
L	unfrozen state
Plank	related to Planks model

1. Determination of the length of the tunnel freezer, and the temperature profile therein for a given throughput and heat load.
2. The use of model based control strategies, specifically model predictive control (MPC), for changing the residence time and temperature profile to deal with the variance in the heat load.

2. System description

The cryogenic freezer is a heat transfer device wherein the cooling capacity of the cryogen is used to extract heat from the product. Two designs are common for commercial freezers – the tunnel and spiral belt equipment (Awonorin, 1997) and carbon dioxide (CO_2) and liquid nitrogen (LIN) are the popular cryogens used therein. The models developed in this work are for tunnel freezers using LIN as the cryogen.

The tunnel freezer is made up of a food conveyor, an insulated enclosure around the conveyor, usually shaped in the form of a rectangular duct, method of introducing the cryogenic fluid into the system, and in some cases, a controller to regulate the flow of the cryogen (liquid and vaporizing cold gases) within the system (Wagner, 1971). Some examples of commercially available tunnel freezers are the Modular Tunnel Freezers[1] and the Cryomaster Tunnel Freezer[2] marketed by BOC, and the CRYO-QUICK™[3] marketed by Air Products.

Low temperatures are reached as LIN boils at approximately 77 K at which it extracts approximately 360 kJ/kg. High latent heat of vaporization and large temperature gradients between the cryogen and food products cause rapid freezing. Proportioning the exposure time of the product in the gaseous and liquid nitrogen zones to obtain optimum utilization of the total refrigerant available is therefore an important design criterion. The length of the tunnel increases as the portion of freezing achieved in the vapor zone is increased. The throughput decreases if the dwell time of food in the tunnel, i.e., in either of the zones is increased. Similarly, the operating cost increases if the length of the liquid zone or, the proportion of freezing achieved in the liquid zone is increased. In practice, about half of the heat is removed as the liquid expands and becomes a gas while the vapor removes the other half. The specifics of the tunnel freezer used in the study are presented in Table 1.

[1] http://www.boc.com/markets/food_and_drink/gases/modular_tunnel_freezer.asp.

[2] http://www.boc.com/markets/food_and_drink/gases/cryomaster_tunnel_freezer.asp.

[3] http://www.airproducts.co.uk/food/freezing.htm.

Table 1
System characteristics

Characteristics freezer (LIN based tunnel freezer)		Characteristics food (meat patties)	
Length (m)	12	Heat transfer area (cm^2)	400
Width (m)	1	Thickness d (mm)	12
Height (m)	1	Initial temperature T_i (K)	303
Product residence time (min)	12–18	Freezing point T_f (K)	272
Throughput (kg/h)	450	Temperature final T_o (K)	255

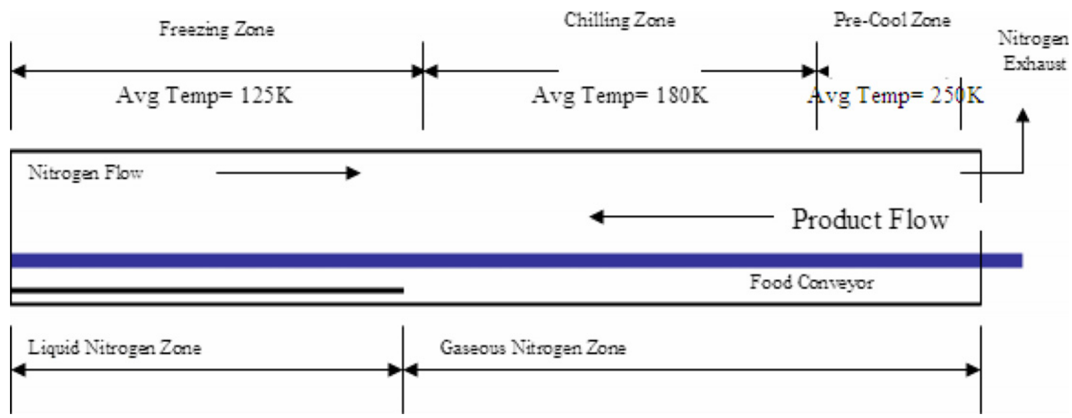

Fig. 1. Setup of a freezer tunnel.

The non-linear nature of the freezing curve of food dictates that the LIN flow be counter-current to the direction in which the food travels. Fig. 1 gives a general schematic of a counter-current cryogenic tunnel freezer.

Food (meat patties) was assumed to enter at a temperature of 303 K and the desired end temperature was 255 K. The average heat transfer coefficients in the precooling section were taken as 35 W/m^2/K. These values are about 7 W/m^2/K in the freezing section, and about 3 W/m^2/K in the subcooling section. The temperature profile of the tunnel is assumed to be fixed and maintained at 133–123 K during freezing. These values are based on the parameters determined by Awonorin (1997) for the cryogenic tunnel freezers and the operating conditions prevalent in the meat industry.

3. Background

A brief review of the models available for predicting the freezing time and dynamics of food, the freezer dynamics, and the control of the tunnel freezer system is presented in this section.

3.1. Freezing dynamics

Food freezing has three distinct stages (Delgado & Sun, 2001):

1. Precooling and chilling phase, in which the food is cooled from its initial temperature to the freezing point temperature.
2. Phase change period, which represents the crystallization of most of the water content of the food.
3. Tempering/equilibrating phase in which the product reaches the final established temperature.

The freezing process is therefore a typical heat conduction process accompanied with phase change. Its characteristic feature is the coexistence of phase change, absorption and the release of the latent heat, and movement of the phase change interface (Zhongjie, He, & Ma, 2003).

Heat transfer with phase change is a highly non-linear problem. The models for describing the freezing process of foods vary from analytical equations that are based on approximations and suppositions, to exact numerical methods. A detailed review of the literature available in this field has been presented by Delgado and Sun (2001). One of the simplest and frequently used approaches for prediction of freezing times is based on the Plank equation or modifications of the Plank equation. The Plank equation is derived from Newton's law of cooling and assumes that the heat released by the food (during freezing) takes place at a constant temperature (no precooling or tempering period). It also assumes homogeneity of the product, i.e., isotropic and regular shape, and that the heat transfer coefficient is constant in space and time. Detailed information on simple models of Planks type can be found in the literature (Cleland, 1985) and Planks equation itself can be expressed as follows:

$$\theta_{\mathrm{Plank}} = \frac{\rho_s \Delta H_f}{T_f - T_a}\left[P\frac{d}{h} + Q\frac{d^2}{\lambda}\right] \tag{1}$$

The equation indicates that the freezing time depends on two groups of factors. One group characterizing the product, i.e., the latent heat of freezing H_f, density of the product in its frozen state (ρ), thermal conductivity of the product λ, its characteristic dimension d and shape defined by shape factors P and Q. The other group characterizing the process and the factors comprises of the temperature difference between the cryogen and the products temperature ($T_f - T_a$), and the surface heat transfer coefficient (h). The unmodified Plank equation however under-predicts freezing times by 10–40%. Further, Planks equation presented an analytical solution for the phase change period only.

Nagaoka, Takaji, and Hohani (1955) presented a modified version of Plank's basic equation for calculating freezing time by combining the precooling and the freezing times, and can be expressed as

$$\theta_{\text{subcooling+freezing}} = (1 + 0.008T_i)\frac{\rho \Delta H_f}{(T_f - T_a)}\left(\frac{D_e^2}{16\lambda} + \frac{D_e}{4\alpha}\right) \quad (2)$$

Pham (1984) proposed further modifications to the Plank equation by introducing similar expressions for the precooling and subcooling periods, giving the following set of equations for the three stages of cooling and freezing:

$$\theta_{\text{precooling}} = \frac{Q_{\text{pre}}}{hf_0 \Delta T_{m1}}\left(1 + \frac{Bi_1}{6}\right) \quad (3)$$

$$\theta = \frac{Q}{hf_0(T_f - T_a)}\left(1 + \frac{Bi_s}{4}\right) \quad (4)$$

$$\theta_{\text{subcooling}} = \frac{Q_{\text{sub}}}{hf_0 \Delta T_{m3}}\left(1 + \frac{Bi_3}{6}\right) \quad (5)$$

Here ΔT_{m1} and ΔT_{m3} are the logarithmic mean temperature differences during the precooling and subcooling periods. In a later paper Pham (1986) introduced some approximations resulting in a simplified version of the original set of equations.

$$\theta_{\text{eff}} = \frac{V_0}{hf_0}\left(\frac{\Delta H_1}{\Delta T_1} + \frac{\Delta H_2}{\Delta T_2}\right)\left(1 + \frac{Bi_s}{4}\right) \quad (6)$$

where $\Delta H_1 = C_{pL}(T_i - T_{fm})$, $\Delta H_2 = \Delta H_f + C_{pS}(T_{fm} - T_e)$, $\Delta T_1 = (T_i - T_{fm})/2 - T_a$.

Further, Lovatt, Pham, Cleland, and Loeffen (1993) developed a method to predict the way in which the heat load placed upon a refrigeration system by cooling food products varies with time. The method utilizes ODE models of both chilling and phase change processes. By considering only conduction heat transfer mechanisms with phase-change, Sanz, Ramos, and Aguirre-Puente (1999) developed a strictly exact method to derive analytical solutions common to these types of substances. Different characteristic parameters such as the movement of the interface boundary and the freezing time were included in their model. However, these models get more and more complicated and unusable in practice. The equations presented by Pham (1984, 1986), i.e., Eqs. (3)–(6) have been described as "the most recommended equations for predicting the freezing times of one dimensional foodstuffs", by Delgado and Sun (2001), and will be used in the model development and analysis in this paper.

3.2. Freezer dynamics

The methods described in the previous section enable the calculation of heat load under the assumption that the surrounding temperature is constant (Pham, 1985). However, in case of a tunnel freezer, this assumption may be invalid. The inlet temperature of the cryogen (liquid phase) and the temperature at the outlet (gaseous phase) are significantly different in case of the tunnel freezers, and a temperature gradient exists along the length of the tunnel freezer.

Agnelli and Mascheroni (2001) had developed a model for heat transfer in a cryo-mechanical freezer – a batch operation. The freezer was modeled as a constant low temperature zone, and the dynamics of the freezing process and the freezer were combined for the special case of constant external temperature. For the case of tunnel freezer, wherein the freezing process is continuous, approach similar to that adopted by Narayanan and Venkatarathnam (1999) for shell and tube heat exchangers is required; the tunnel freezer can be modeled as a single pass shell and tube heat exchanger with the cryogen and the food moving counter-current to each other.

The time taken for food to travel a differential length (δx) can be considered as the instantaneous residence time (δt) in that zone, and the relation between instantaneous residence mass ($\tilde{m}$) and the instantaneous mass flow rate becomes ($\tilde{m} = \dot{m}\delta t$) where $u_h = \delta x/\delta t$ is the instantaneous conveyor belt velocity and $\dot{m}$ is the mass of the food entering the region (see Fig. 2). Since the belt speed is generally kept a constant, i.e., $u_h = \delta x/\delta t$ is fixed, we have, from Smith (1997),

$$\tilde{m}_h = \frac{\dot{m}_h \delta x}{u_h} \quad (7)$$

Similarly for the cryogen

$$\tilde{m}_c = \frac{\dot{m}_c \delta x}{u_c} \quad (8)$$

where u_c is the flow rate of the cryogen. Developing an energy balances for food and the cryogen, we get

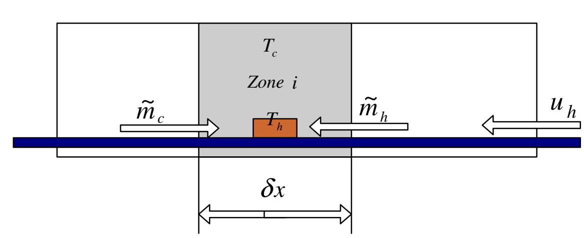

Fig. 2. Counter-current heat exchanger.

$$\text{Food}\frac{\partial T_{\mathrm{h}}}{\partial \theta}+u_{\mathrm{h}}\frac{\partial T_{\mathrm{h}}}{\partial x}=-\frac{\alpha_{\mathrm{h}}S}{\dot{m}_{\mathrm{h}}C_{\mathrm{h}}}\cdot\frac{u_{\mathrm{h}}}{L}(T_{\mathrm{h}}-T_{\mathrm{c}}) \tag{9}$$

$$\text{Cryogen}\frac{\partial T_{\mathrm{c}}}{\partial \theta}-u_{\mathrm{c}}\frac{\partial T_{\mathrm{c}}}{\partial x}=+\frac{\alpha_{\mathrm{c}}S}{\dot{m}_{\mathrm{c}}C_{\mathrm{c}}}\cdot\frac{u_{\mathrm{c}}}{L}(T_{\mathrm{h}}-T_{\mathrm{c}}) \tag{10}$$

When no transients are present, but longitudinal conduction is important, the equations reduce to

$$u_{\mathrm{h}}\frac{\partial T_{\mathrm{h}}}{\partial x}=-\frac{\alpha_{\mathrm{h}}S}{\dot{m}_{\mathrm{h}}C_{\mathrm{h}}}\cdot\frac{u_{\mathrm{h}}}{L}(T_{\mathrm{h}}-T_{\mathrm{c}}) \tag{11}$$

$$-u_{\mathrm{c}}\frac{\partial T_{\mathrm{c}}}{\partial x}=+\frac{\alpha_{\mathrm{c}}S}{\dot{m}_{\mathrm{c}}C_{\mathrm{c}}}\cdot\frac{u_{\mathrm{c}}}{L}(T_{\mathrm{h}}-T_{\mathrm{c}}) \tag{12}$$

Irrespective of the method employed for the estimation of the required freezing time, the strategy used for controlling the tunnel freezer is also critical. A simulation based feed-forward strategy has been proposed by Ramakrishnan et al. (2004). However, a model predictive control strategy that integrates feed-forward and feedback strategies has been shown to be more effective (Shaikh & Prabhu, 2002). Similar results have also been found in control of drying process in industrial dryers (Platt, Palazoglu, & Rumsey, 1992; Whitfield, 1986). A combined model with emphasis on the control variables, viz., the residence time and the temperature profile is required for the MPC based strategy and is presented in Section 4.2.

4. Mathematical models

There is very little literature available that discusses the combination of the freezer dynamics with freezing dynamics in the food products (Awonorin, 1989a, 1989b, 1997). The mathematical models presented in this section combine the freezing dynamics in the food with the tunnel dynamics for efficient sizing and rating.

4.1. Composite model for sizing the tunnel freezer

Solution to the sizing problem is equivalent to customizing the designs of the tunnel freezers for each individual application. The final design involves decisions regarding the length of the tunnel freezer, proportioning of the liquid and gaseous zones in the tunnel, determining the ideal temperature profile that needs to be maintained for the design load and the corresponding throughput. Since sizing assumes that the design load is known, the end temperatures for the food product and the throughput are fixed by the process requirements. The required throughput determines the conveyor belt velocity, which in turn is assumed to be a constant for the sizing problem (for the rating problem, the conveyor velocity is a variable). Space considerations usually constraints the length to the tunnel freezers. The residence time of the food product in the tunnel is therefore dependent upon the production requirements and constraints. With the residence time fixed, the temperature profile and the LIN consumption can be determined. However, this independent strategy is generally conservative. There usually is an over-design that causes over consumption of LIN; combining the two models, viz. the freezer and the freezing dynamics therefore have a lot of potential.

We therefore propose an iterative process for determining the temperature profiles. The iterative method comprises of alternatively solving the transient equations for the counter-current heat exchanger to obtain the temperature profile inside the tunnel and the Pham's equation to determine the temperature profile inside the food material as it travels through the length of the tunnel freezer. It may be noted that Pham's equations have been selected for illustration purposes. Any set of equations that fit the requirements better for some particular food type can be taken for the combined model. The initiating step comprises of fixing the temperature profile inside the tunnel at a constant. Using this as the coolant temperature, the temperature profile of the food material is calculated at each discrete step as it moves along the length of the tunnel freezer. Eqs. (13)–(15) are used for determining this profile. The method can be summarized as follows:

Step 1: Initialization of tunnel profile.
Step 2: Determination of the food dynamics – solution to Eqs. (13)–(15).
Step 3: Determination of the tunnel profile – solution to Eq. (16).
Step 4: Repetition of steps 2 and 3 until convergence is achieved.

$$Q_i=\frac{\Delta t h f_0 \Delta T_{\mathrm{h},i}}{\left(1+\frac{Bi_1}{6}\right)} \quad \forall i \subset \text{pre-freezing region} \tag{13}$$

$$Q_i=\frac{\Delta t h f_0 (T_{\mathrm{h,f}}-T_{\mathrm{h},i})}{\left(1+\frac{Bi_s}{4}\right)} \quad \forall i \subset \text{freezing region} \tag{14}$$

$$Q_i=\frac{\Delta t h f_0 \Delta T_{\mathrm{h},i}}{\left(1+\frac{Bi_3}{6}\right)} \quad \forall i \subset \text{sub-cooling region} \tag{15}$$

$$-u_{\mathrm{c}}\frac{T_{\mathrm{c},i}-T_{\mathrm{c},i-1}}{\Delta x}=\frac{\alpha_{\mathrm{c}}S}{\dot{m}_{\mathrm{c}}C_{\mathrm{c}}}\cdot\frac{u_{\mathrm{c}}}{L}(T_{\mathrm{h},i}-T_{\mathrm{c},i}) \tag{16}$$

The assumption made while using this approach is that a controller (with the control objective of maintaining the temperature profile constant) is used in the freezing tunnel, which in general is true (Ramakrishnan et al., 2004). The heat lost by the food product as it travels through the tunnel freezer is given by Eq. (17) and the residence time is given by Eq. (18).

$$\begin{aligned}Q_{\mathrm{food}}=&\sum_{\text{pre-freezing}}Q_i+\sum_{\text{freezing vapourph}}Q_i\\&+\sum_{\text{freezing liquidph}}Q_i+\sum_{\text{subcooling}}Q_i=m_{\mathrm{c}}C_{\mathrm{c}}(T_{\mathrm{c},1}-T_{\mathrm{c},n})+m_{\mathrm{c}}h_{\mathrm{c}}\end{aligned} \tag{17}$$

$$\begin{aligned}t_{\mathrm{food}}=&\sum_{\text{pre-freezing}}\Delta t_i+\sum_{\text{freezing vapourph}}\Delta t_i\\&+\sum_{\text{freezing liquidph}}\Delta t_i+\sum_{\text{subcooling}}\Delta t_i=\frac{L}{V}\end{aligned} \tag{18}$$

4.2. Dynamic model for rating of a tunnel freezer

The models that were developed in the previous section are suitable during the design and sizing stage. However, to manipulate the flow rate of LIN to maintain the temperature profile constant, different modeling strategies capturing the system dynamics are required. The approach that we propose is to draw a one-to-one correspondence of the true dynamics of the system with the dynamics of other existing systems for which the modeling as well as control strategies are well developed. For this purpose, the numerical model is first transformed into the enthalpy form. The equation can be expressed as

$$\rho \frac{\partial H_{\mathrm{v}}}{\partial t} = h(T_{\mathrm{s}} - T) - H_{\mathrm{belt}} - H_{\mathrm{sur}} \tag{19}$$

where H_{belt} is the heat entering in a zone with the conveyor belt and H_{sur} is the heat entering from the surroundings in the zone. ∂H_{v} can be seen as the change in enthalpy that is required in a given zone. This model is similar to the interacting tank model shown in Fig. 4. The height of the liquid in tank 1 corresponds to the existing cooling capacity of the cryogen in the given zone. The height of the liquid in tank 2 corresponds to the cryogen that is used up to cool the food in the given zone. The q terms refer to the flow rates. q_{leak} refers to the heat entering the zone through the surrounding, equivalent to H_{sur}. q_{ex} refers to the cryogen flowing to the next zone. q_{f} is equivalent to $h(T_{\mathrm{s}} - T)$ (shown in Fig. 3). The tank model for the tunnel freezer can therefore be developed as follows:

$$A_1 \frac{\mathrm{d}h_1}{\mathrm{d}t} = q_{\mathrm{in}} - q_{\mathrm{f}} - q_{\mathrm{leak}} - q_{\mathrm{ex}} \tag{20}$$

$$A_2 \frac{\mathrm{d}h_2}{\mathrm{d}t} = q_{\mathrm{f}} - q_d \tag{21}$$

Also, the specific flow rates and the valve characteristics can be introduced as

$$q_{\mathrm{f}} = \frac{h_1 - h_2}{R_1}, \quad q_{\mathrm{leak}} = l_1 h_1, \quad q_{\mathrm{ex}} = e_1 h_1 \quad \text{and} \quad q_d = \frac{h_2}{R_2} \tag{22}$$

These transform to

$$A_1 R_1 \frac{\mathrm{d}h_1}{\mathrm{d}t} = q_{\mathrm{in}} R_1 - h_1(l_1 R_1 + e_1 R_1 + 1) + h_2 \tag{23}$$

$$A_2 R_2 \frac{\mathrm{d}h_2}{\mathrm{d}t} = h_1 \frac{R_2}{R_1} - h_2 \left(1 + \frac{R_2}{R_1}\right) \tag{24}$$

Subtracting and after introducing the deviation variables, the Laplace transform gives us

$$[A_1 R_1 s + (1 + l_1 R_1 + e_1 R_1)]\bar{h}_1'(s) - \bar{h}_2'(s) = \bar{q}_{\mathrm{in}}'(s) R_1 \tag{25}$$

$$\left[A_2 R_2 s + \left(1 + \frac{R_2}{R_1}\right)\right]\bar{h}_2'(s) = \frac{R_2}{R_1}\bar{h}_1'(s) \tag{26}$$

Solving the equations algebraically, we get

$$\bar{h}_2'(s) = \frac{R_2}{R_1(\tau_2 s + k_2)}\bar{h}_1'(s) \tag{27}$$

$$\bar{h}_1'(s) = \frac{R_1 \tau_2 s + (R_1 + R_2)}{\tau_1 \tau_2 s^2 + (\tau_1 k_2 + \tau_2 k_1)s + \left(k_1 k_2 - \frac{R_2}{R_1}\right)}\bar{q}_1'(s) \tag{28}$$

where

$$\tau_1 = A_1 R_1, \quad \tau_2 = A_2 R_2,$$
$$k_1 = (1 + l_1 R_1 + e_1 R_1), \quad k_2 = \left(1 + \frac{R_2}{R_1}\right) \tag{29}$$

The advantage that the tank model offers is that the dynamics of the system can be captured. The dwell time and the temperature profile inside the tunnel are the manipulated variables for the system from an operational point

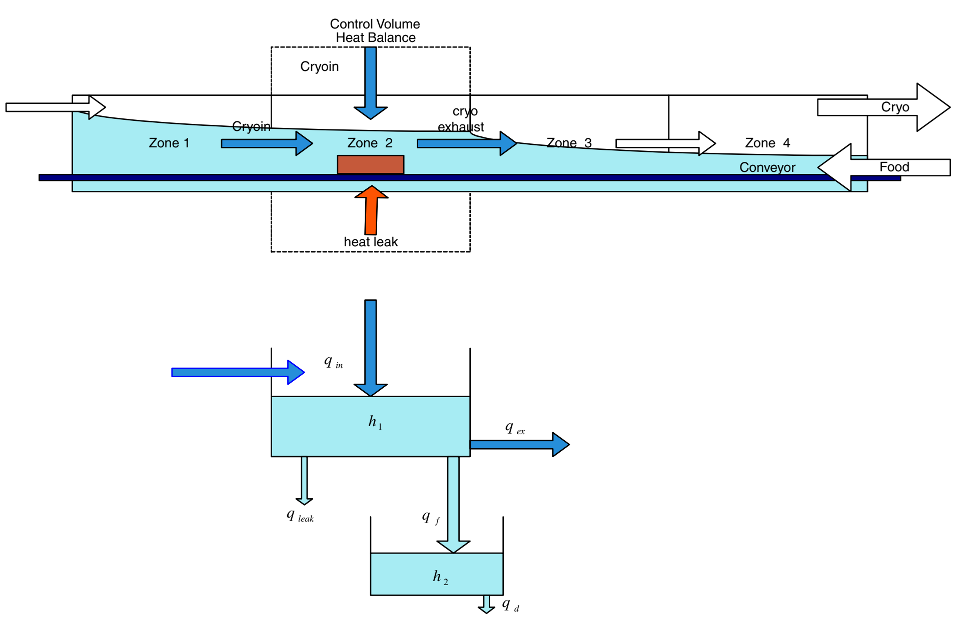

Fig. 3. Interacting tank model.

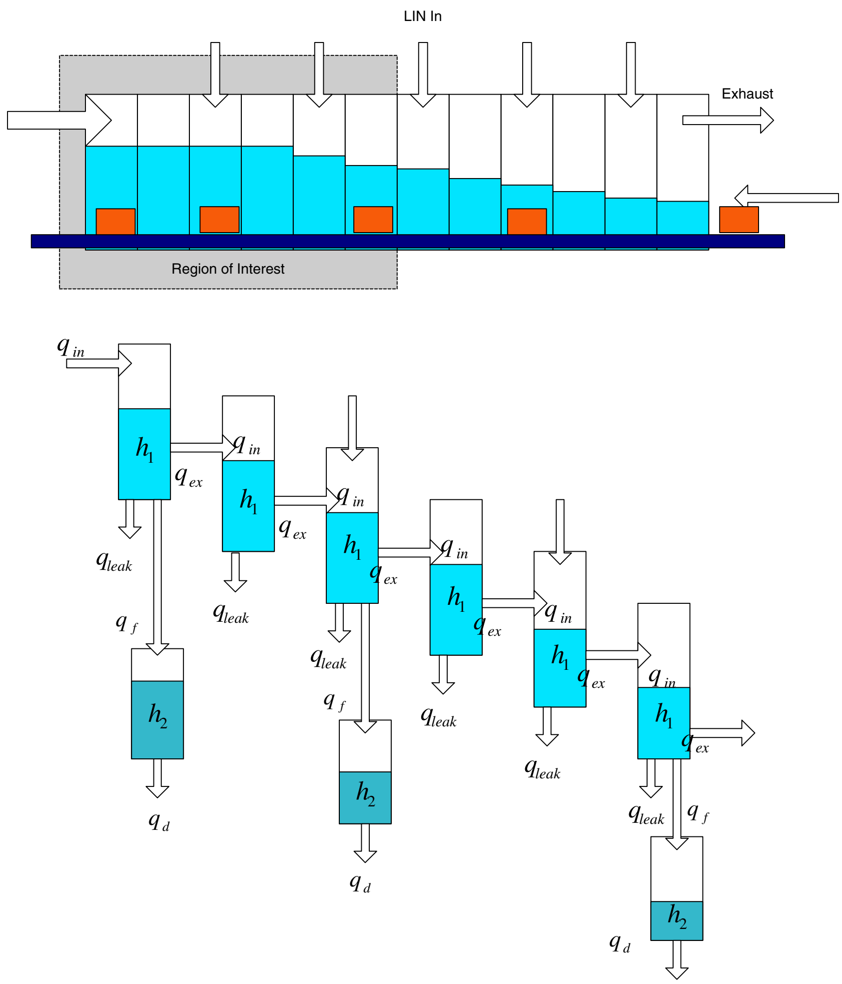

Fig. 4. Composite tank model.

of view. These in turn determine the rating of the tunnel freezer when the design and dimensions are fixed. From the model dynamics, the response time for the temperature profile to change can be calculated. Based on this information, the conveyor velocity and the LIN flow can be manipulated to achieve optimal throughput.

5. Simulation and results for the sizing problem

5.1. Description of the simulation model and analysis

For the purpose of simulations, the tunnel length was divided into 40 zones of equal length. The conveyor belt speed was assumed constant and so to proportion the exposure time between the liquid and the gaseous phases, zones 1 through 20 units were assumed to have a gaseous state whereas zones 21 through 38 were assumed to hold the cryogen in liquid phase. The parameters that were used are presented in Table 2. The entrance for food has been taken as the origin and its path as it moves along the length of the tunnel is traced in the plot.

The prediction using Planks equation, Nagaoka's equation, and Pham's equation when the food material travels along the length of the tunnel freezer is determined. The Pham's equation is then combined with the composite model, and the profile is determined again. The difference and the potential are determined.

5.2. Results and discussion

The prediction using Planks equation, Nagaoka's equation, and Pham's equation when the food material travels along the length of the tunnel freezer is presented in Fig. 5. The temperature profile of the tunnel was assumed to be a constant (fixed at the average value). The following was observed.

Table 2
System characteristics for simulation model

Characteristics freezer (LIN based tunnel freezer)	
Thermal conductivity of liquid λ_L (W/m °C)	0.50
Thermal conductivity of food λ_S (W/m °C)	1.50
Specific heat of liquid C_{pL} (J/m^3 °C)	3.5E+6
Specific heat of food C_{pS} (J/m^3 °C)	2.0E+6
Heat flux (Q/A W m^{-2})	20,000

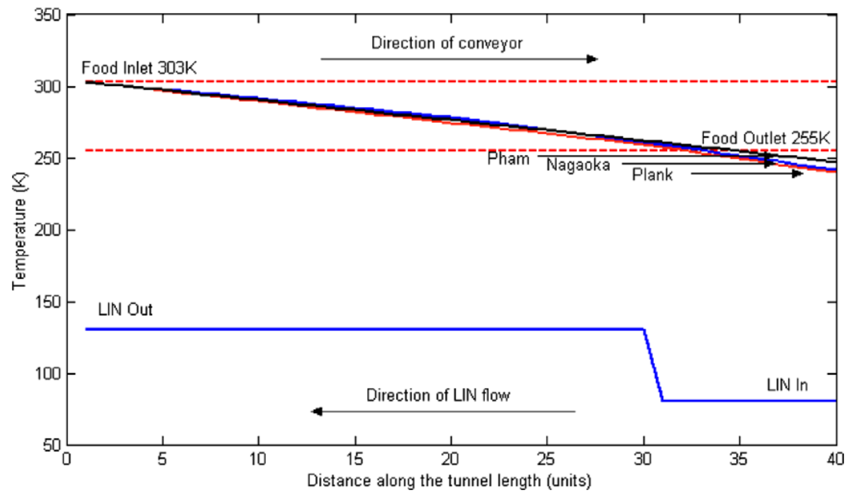

Fig. 5. Performance of the Planks, Nagaoka, and Pham models.

1. For the Planks equation, the temperature of 255 K is reached when the food has traveled only 70% of the tunnel length. This model therefore under-predicts the freezing time.
2. For the Nagaoka model, the predicted temperature profile of the food is closer to the actual temperature profile that occurs in the tunnel freezer. The profile takes a sharp turn when food moves from the gaseous cryogen zone to the liquid cryogen zone. As was observed in the case of predictions from Planks equation, the temperature of 255 K is reached when the food has traveled approximately 80% of the tunnel length.
3. For the Pham's model, the predicted temperature profile of the food temperature is similar to that predicted by the Nagaoka's equation. The profile takes a sharp turn when food moves from the gaseous cryogen zone to the liquid cryogen zone. As was observed in the case of predictions from Planks equation, the temperature of 255 K is reached when the food has traveled approximately 85% of the tunnel length.

If these equations are used for sizing, the tunnel length will be insufficient to bring in the required temperature change if it maintains the stated temperature profiles. This leads to either an increase in LIN consumption for maintaining the same throughput, or a decrease in throughput if the LIN flow is maintained at the same level. As costs are associated with both throughput and LIN consumption, there is reduction in profitability in either of the cases. The freezing costs can increase by as much as 4–5 c/kg due to this error in sizing of the tunnel. Nagaoka's model is therefore an improvement over Planks model, however, it under-predicts the freezing time, which can increase by 2–3 c/kg due to this error in sizing.

The data for the composite model is plotted in Fig. 6. This model predicts the temperature and the dynamics of the food more accurately. The temperature of 255 K is attained when the food enters or leaves the 38th zone. Fig. 6 shows that there are significant improvements in the models prediction of the temperature profile in food along its passage through the tunnel freezer. The error in

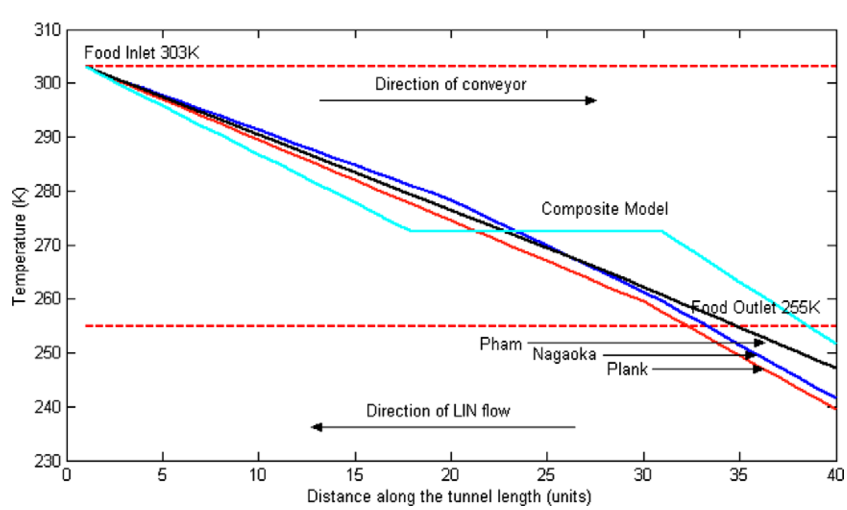

Fig. 6. Performance of the Planks, Nagaoka, and Pham models compared to that of the composite model.

prediction is reduced to less than 5% and the variance in freezing cost can be kept below 1 c/kg.

6. Simulation and results for the rating problem

The underlying freezer model and the heat transfer data used for the simulation is the same as that described in Section 5.1 and presented in Tables 1 and 2. The tunnel freezer was divided into 38 zones with each of these 38 zones is considered as tanks and the temperatures in the zones are mapped to the liquid level in the tank as shown in Fig. 5. LIN is assumed to exist in a gaseous phase in zones 1 through 20 whereas zones 21 through 38 are assumed to hold LIN in liquid phase.

We look into three properties of the model and its performance; improved control strategy translates to increased stability, accuracy and robustness of the process. This has been shown in the following sections:

6.1. Stability

The tank systems have been well studied; some of the concepts developed in that field of research can be interpolated for the tunnel freezer systems as well. The stability of the freezer system when the LIN flow and the belt speed are manipulated can be determined easily. The response of the tank system is always over-damped as the roots of the denominator of Eq. (28) are real.

$$(\tau_1 k_2 + \tau_2 k_1)^2 - 4\tau_1\tau_2\left(k_1 k_2 - \frac{R_2}{R_1}\right) = (\tau_1 k_2 - \tau_2 k_1)^2 + 4\tau_1\tau_2\frac{R_2}{R_1} \tag{30}$$

and

$$(\tau_1 k_2 - \tau_2 k_1)^2 + 4\tau_1\tau_2\frac{R_2}{R_1} \geqslant 0 \tag{31}$$

Therefore stability is guaranteed if the tank model is substituted for the tunnel model in model predictive controllers for the tunnel freezing systems.

6.2. Accuracy

To determine the accuracy of the proposed control algorithm, the system was simulated using ARENA, a commercial simulation software package. A batch of 100 food items was created and an enthalpy between 33 and 38 was assigned to them randomly. Fig. 7 gives the enthalpy distribution of the incoming foods. The enthalpy values are not the absolute enthalpy but the change in enthalpy that is required to reach the desired set point. Thus the enthalpy of the food items should reach zero on exit from

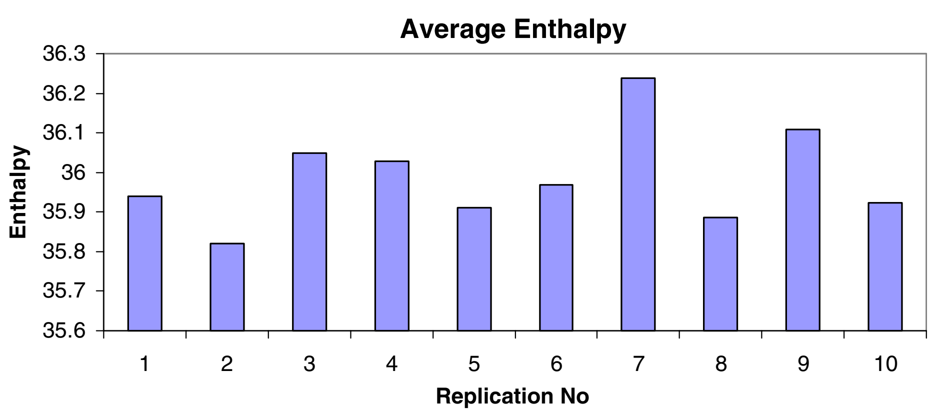

Fig. 7. Average enthalpy change required (in the food) at the entrance of the tunnel.

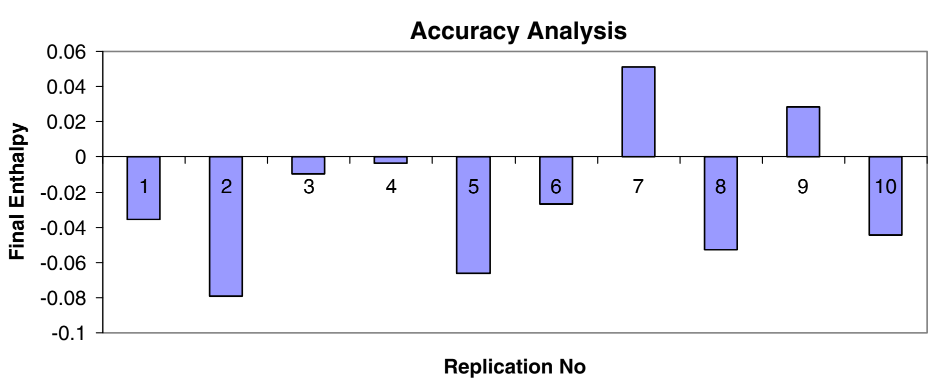

Fig. 8. Average enthalpy change required (in the food) at the exit of the tunnel to reach the desired set point.

the tunnel freezer. Fig. 8 gives the distribution of enthalpy at the exit.

The average error was observed to be -0.0357, with the outlet enthalpy close to zero for most of the entities. The minimum enthalpy at the exit was observed to be 0.575 units below the set point of zero and the maximum was 0.375 units above the set point for the food product.

7. Conclusions

In this paper, models for food freezing and heat exchange processes have been developed and a combined model based on two step finite difference methods has been proposed for sizing the tunnel freezer. The error in prediction is reduced to less than 5% and the variance in freezing costs can be kept below 1 c/kg. This is crucial in the setup phase for the plants as the additional costs are above the planned costs and can make all the difference between a profitable and a non-profitable venture.

The tank model developed for rating can also be used for MPC (Shaikh & Prabhu, 2002). The advantage of using this model is that the dynamics of the system can be captured and the model can also be extended to more complex cases such as when the zones have nozzles and fans. The accuracy and stability analysis of the proposed rating model has also been presented. The tank model structure guarantees stability for all feasible parameter values, and for an input enthalpy change requirements ranging from $(33, 38)$, the output change requirement is transformed to $(-0.0375, 0.575)$. The integration of the tank model with the MPC architecture has been presented in the companion paper by Shaikh and Prabhu (2002).

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