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Regresión Lineal

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REGRESIÓN LINEAL EN R

DATOS: FORMAS Y TAMAÑOS DE 3 VARIEDADES DE TRIGO MEDIDAS CON RAYOS-X

Los DATOS aquí examinados son clases de granos de Trigo pertenecientes a tres variedades diferentes: Kama, Rosa y Canadian, con 70 elementos cada una, seleccionadas al azar para el experimento. Se utilizaron "rayos X suaves" para tener visualización de alta calidad de la estructura interna del núcleo. Las imágenes se registraron en placas KODAK de rayos X de 13x18 cm. Los estudios se llevaron a cabo utilizando granos de trigo cosechados con cosechadora provenientes de campos experimentales, explorados en el Instituto de Agrofísica de la Academia de Ciencias de Polonia en Lublin.

Estos datos fueron donados el 29 de septiembre de 2012 al repositorio de Machine Learning "UCI"

Instalamos Librerías

```
install.packages('GGally')  
install.packages('dplyr')  
install.packages('statsr')  
install.packages('ggfortify')  
install.packages('tidyverse')  
install.packages('olsrr')
```

```
Installing package into '/usr/local/lib/R/site-library'  
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(as 'lib' is unspecified)

Cargamos los datos

```
df<-read.csv("seeds_dataset.csv", header = FALSE, sep = ";", dec =  
".")  
colnames(df) <- c("Area", "Perimeter", "Compactness",  
"Length_Kernel","Width_Kernel","Asymmetry_Coeff","Length_Groove","Vari  
ety_Wheat")  
df <- subset(df, select =  
c(Area,Perimeter,Compactness,Length_Kernel,Width_Kernel,Asymmetry_Coef  
f,Length_Groove,Variety_Wheat))  
df
```

	Area	Perimeter	Compactness	Length_Kernel	Width_Kernel	Asymmetry_Coeff
1	15.26	14.84	0.8710	5.763	3.312	2.2210
2	14.88	14.57	0.8811	5.554	3.333	1.0180
3	14.29	14.09	0.9050	5.291	3.337	2.6990
4	13.84	13.94	0.8955	5.324	3.379	2.2590
5	16.14	14.99	0.9034	5.658	3.562	1.3550
6	14.38	14.21	0.8951	5.386	3.312	2.4620
7	14.69	14.49	0.8799	5.563	3.259	3.5860
8	14.11	14.10	0.8911	5.420	3.302	2.7000
9	16.63	15.46	0.8747	6.053	3.465	2.0400
10	16.44	15.25	0.8880	5.884	3.505	1.9690
11	15.26	14.85	0.8696	5.714	3.242	4.5430
12	14.03	14.16	0.8796	5.438	3.201	1.7170
13	13.89	14.02	0.8880	5.439	3.199	3.9860
14	13.78	14.06	0.8759	5.479	3.156	3.1360
15	13.74	14.05	0.8744	5.482	3.114	2.9320

16	14.59	14.28	0.8993	5.351	3.333	4.1850
17	13.99	13.83	0.9183	5.119	3.383	5.2340
18	15.69	14.75	0.9058	5.527	3.514	1.5990
19	14.70	14.21	0.9153	5.205	3.466	1.7670
20	12.72	13.57	0.8686	5.226	3.049	4.1020
21	14.16	14.40	0.8584	5.658	3.129	3.0720
22	14.11	14.26	0.8722	5.520	3.168	2.6880
23	15.88	14.90	0.8988	5.618	3.507	0.7651
24	12.08	13.23	0.8664	5.099	2.936	1.4150
25	15.01	14.76	0.8657	5.789	3.245	1.7910
26	16.19	15.16	0.8849	5.833	3.421	0.9030
27	13.02	13.76	0.8641	5.395	3.026	3.3730
28	12.74	13.67	0.8564	5.395	2.956	2.5040
29	14.11	14.18	0.8820	5.541	3.221	2.7540
30	13.45	14.02	0.8604	5.516	3.065	3.5310
:	:	:	:	:	:	:
181	11.41	12.95	0.8560	5.090	2.775	4.957
182	12.46	13.41	0.8706	5.236	3.017	4.987
183	12.19	13.36	0.8579	5.240	2.909	4.857
184	11.65	13.07	0.8575	5.108	2.850	5.209
185	12.89	13.77	0.8541	5.495	3.026	6.185
186	11.56	13.31	0.8198	5.363	2.683	4.062
187	11.81	13.45	0.8198	5.413	2.716	4.898
188	10.91	12.80	0.8372	5.088	2.675	4.179

189	11.23	12.82	0.8594	5.089	2.821	7.524
190	10.59	12.41	0.8648	4.899	2.787	4.975
191	10.93	12.80	0.8390	5.046	2.717	5.398
192	11.27	12.86	0.8563	5.091	2.804	3.985
193	11.87	13.02	0.8795	5.132	2.953	3.597
194	10.82	12.83	0.8256	5.180	2.630	4.853
195	12.11	13.27	0.8639	5.236	2.975	4.132
196	12.80	13.47	0.8860	5.160	3.126	4.873
197	12.79	13.53	0.8786	5.224	3.054	5.483
198	13.37	13.78	0.8849	5.320	3.128	4.670
199	12.62	13.67	0.8481	5.410	2.911	3.306
200	12.76	13.38	0.8964	5.073	3.155	2.828
201	12.38	13.44	0.8609	5.219	2.989	5.472
202	12.67	13.32	0.8977	4.984	3.135	2.300
203	11.18	12.72	0.8680	5.009	2.810	4.051
204	12.70	13.41	0.8874	5.183	3.091	8.456
205	12.37	13.47	0.8567	5.204	2.960	3.919
206	12.19	13.20	0.8783	5.137	2.981	3.631
207	11.23	12.88	0.8511	5.140	2.795	4.325
208	13.20	13.66	0.8883	5.236	3.232	8.315
209	11.84	13.21	0.8521	5.175	2.836	3.598
210	12.30	13.34	0.8684	5.243	2.974	5.637

	Length_Groove	Variety_Wheat
1	5.220	1
2	4.956	1

3	4.825	1
4	4.805	1
5	5.175	1
6	4.956	1
7	5.219	1
8	5.000	1
9	5.877	1
10	5.533	1
11	5.314	1
12	5.001	1
13	4.738	1
14	4.872	1
15	4.825	1
16	4.781	1
17	4.781	1
18	5.046	1
19	4.649	1
20	4.914	1
21	5.176	1
22	5.219	1
23	5.091	1
24	4.961	1
25	5.001	1
26	5.307	1
27	4.825	1
28	4.869	1
29	5.038	1
30	5.097	1
:	:	:
181	4.825	3
182	5.147	3
183	5.158	3
184	5.135	3
185	5.316	3
186	5.182	3
187	5.352	3
188	4.956	3
189	4.957	3
190	4.794	3
191	5.045	3
192	5.001	3
193	5.132	3
194	5.089	3
195	5.012	3
196	4.914	3
197	4.958	3
198	5.091	3
199	5.231	3
200	4.830	3
201	5.045	3

202	4.745	3
203	4.828	3
204	5.000	3
205	5.001	3
206	4.870	3
207	5.003	3
208	5.056	3
209	5.044	3
210	5.063	3

Análisis Exploratorio de los Datos

head(df)

	Area	Perimeter	Compactness	Length_Kernel	Width_Kernel	Asymmetry_Coeff
1	15.26	14.84	0.8710	5.763	3.312	2.221
2	14.88	14.57	0.8811	5.554	3.333	1.018
3	14.29	14.09	0.9050	5.291	3.337	2.699
4	13.84	13.94	0.8955	5.324	3.379	2.259
5	16.14	14.99	0.9034	5.658	3.562	1.355
6	14.38	14.21	0.8951	5.386	3.312	2.462

	Length_Groove	Variety_Wheat
1	5.220	1
2	4.956	1
3	4.825	1
4	4.805	1
5	5.175	1
6	4.956	1

summary(df)

Area	Perimeter	Compactness	Length_Kernel
Min. :10.59	Min. :12.41	Min. :0.8081	Min. :4.899
1st Qu.:12.27	1st Qu.:13.45	1st Qu.:0.8569	1st Qu.:5.262
Median :14.36	Median :14.32	Median :0.8734	Median :5.524
Mean :14.85	Mean :14.56	Mean :0.8710	Mean :5.629
3rd Qu.:17.30	3rd Qu.:15.71	3rd Qu.:0.8878	3rd Qu.:5.980
Max. :21.18	Max. :17.25	Max. :0.9183	Max. :6.675
Width_Kernel	Asymmetry_Coeff	Length_Groove	Variety_Wheat
Min. :2.630	Min. :0.7651	Min. :4.519	Min. :1
1st Qu.:2.944	1st Qu.:2.5615	1st Qu.:5.045	1st Qu.:1
Median :3.237	Median :3.5990	Median :5.223	Median :2
Mean :3.259	Mean :3.7002	Mean :5.408	Mean :2

```

3rd Qu.:3.562    3rd Qu.:4.7687    3rd Qu.:5.877    3rd Qu.:3
Max.      :4.033    Max.      :8.4560    Max.      :6.550    Max.      :3

```

```
dim(df)
```

```
[1] 210    8
```

```
names(df)
```

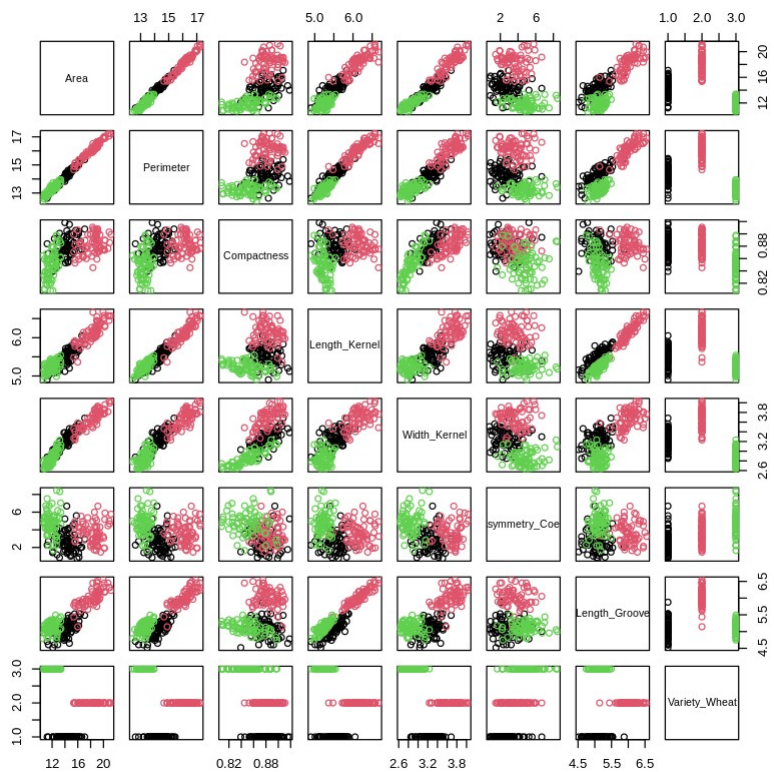
```

[1] "Area"           "Perimeter"      "Compactness"
[2] "Length_Kernel"
[3] "Width_Kernel"   "Asymmetry_Coeff" "Length_Groove"
[4] "Variety_Wheat"

```

VISUALIZACION

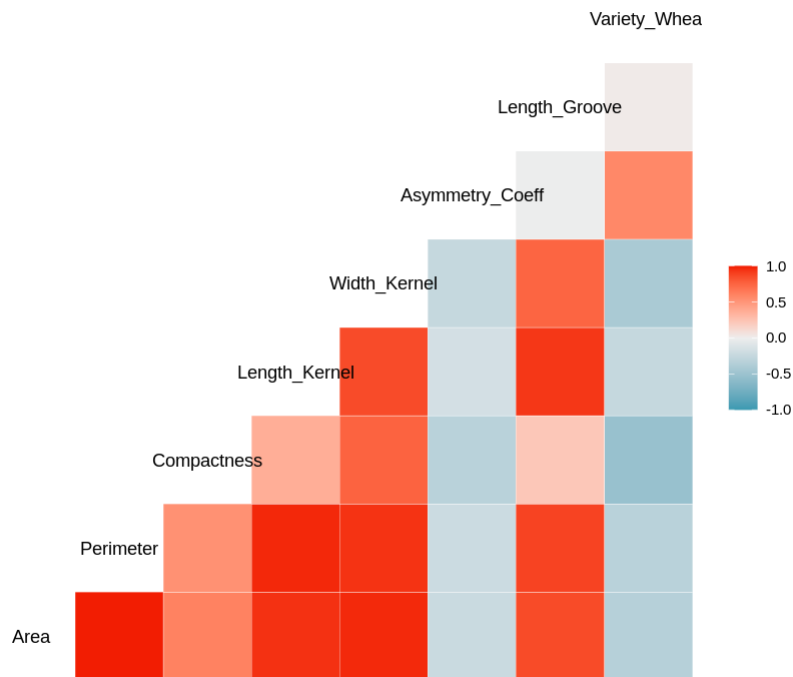
```
plot(df, col=df$Variety_Wheat)
```



```

library('GGally')
ggcorr(df, method=c("everything", "pearson"))

```



```
library('dplyr')
df<- df %>% mutate(Nombre =
  case_when(Variety_Wheat ==1 ~ "Kama",
            Variety_Wheat ==2~ "Rosa",
            Variety_Wheat ==3~ "Canadian")
)
```

Attaching package: 'dplyr'

The following object is masked from 'package:MASS':

select

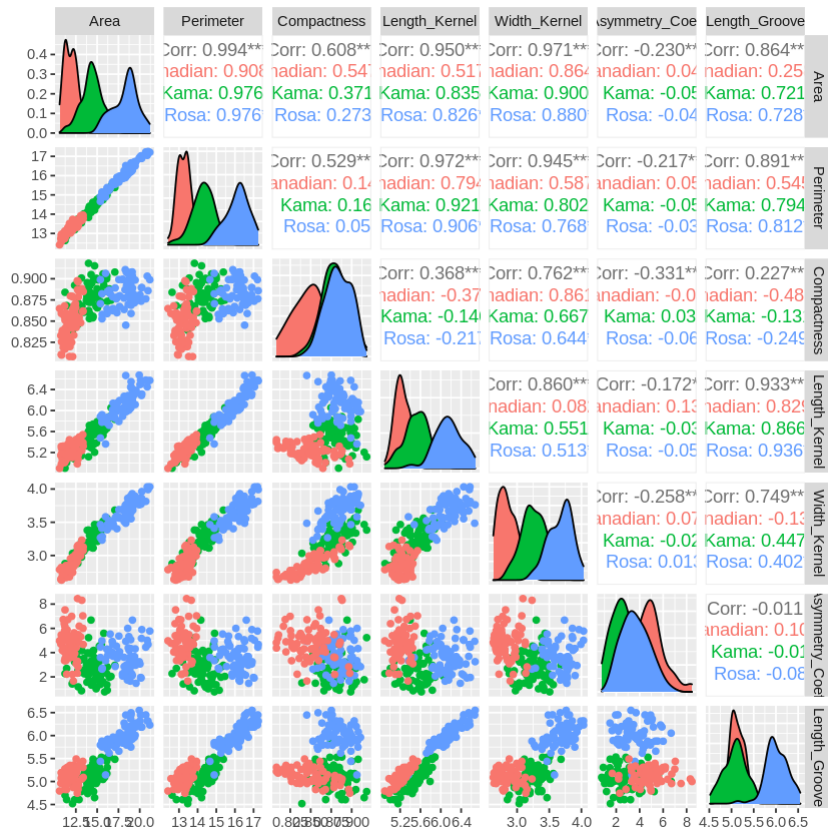
The following objects are masked from 'package:stats':

filter, lag

The following objects are masked from 'package:base':

intersect, setdiff, setequal, union


```
ggpairs(df, columns=1:7, ggplot2::aes(colour=Nombre))
```



Del anterior Análisis Exploratorio podemos decir 2 cosas:

Por un lado, el dataset contiene variables que pueden ser en la practica más sencillas que medir que otras, por este motivo es de gran interes poder predecir el valor de esas variables que son más extrañas, más difíciles de medir o más costosas de conseguir. Por lo tanto optaremos por escoger un modelo de Regresión Lineal que pueda explicar alguna de estas variables (variable Y). Estas variables son: 'Assymetry Coefficient', 'Compactness' y 'Length of Groove Kernel'.

Por otro lado, Visualizando las graficas de Dispersion y distribucion de los datos, junto con la matriz de correlacion podemos ver A SIMPLE VISTA vemos varios modelos lineales interesantes:

- Width Kernel vs Compactness
- Length Kernel vs Compactness

Sin embargo es de gran interes poder predecir el "Coeficiente de Simetria". Por tanto, tomando el mayor valor de la matriz de correlación tenemos !?

- Length of Kernel vs Assymetry Coefficient

Regresión Lineal para predecir el coeficiente de simetría

```
library(statsr)  
plot_ss(x =Asymmetry_Coeff, y = Length_Kernel, data = df)
```

Click two points to make a line.

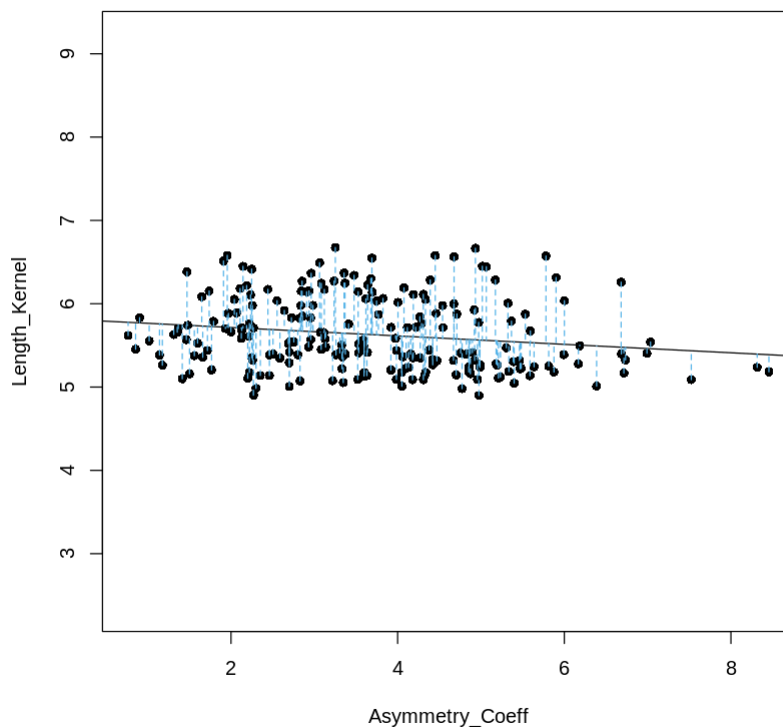
Call:

```
lm(formula = y ~ x, data = pts)
```

Coefficients:

(Intercept)	x
5.81560	-0.05056

Sum of Squares: 39.82



Modelo 1

```
lm1<- lm(Asymmetry_Coeff~Length_Kernel, data=df)  
summary(lm1)
```

Call:

```
lm(formula = Asymmetry_Coeff ~ Length_Kernel, data = df)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-2.947	-1.157	-0.019	0.977	4.496

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	6.9772	1.3088	5.331	2.53e-07 ***
Length_Kernel	-0.5822	0.2318	-2.512	0.0128 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.485 on 208 degrees of freedom

Multiple R-squared: 0.02943, Adjusted R-squared: 0.02477

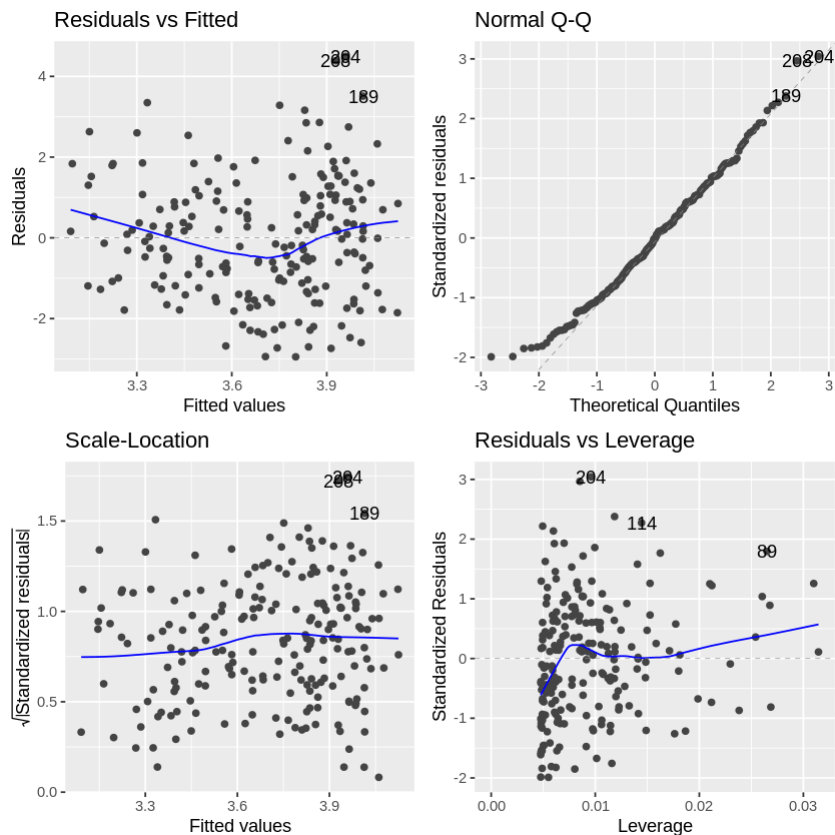
F-statistic: 6.308 on 1 and 208 DF, p-value: 0.01278

confint(lm1)

	2.5 %	97.5 %
(Intercept)	4.396997	9.5573285
Length_Kernel	-1.039206	-0.1252041

library(ggfortify)

autoplot(lm1)



library(MASS)

AIC(lm1)

```
[1] 765.9681
```

Viendo Otros Modelos

```
lm2 <- lm(Asymmetry_Coeff~Area, data=df)
```

```
lm3 <- lm(Asymmetry_Coeff~Compactness, data=df)
```

```
lm4 <- lm(Asymmetry_Coeff~Compactness*Length_Kernel, data=df)
```

```
lm5 <- lm(Asymmetry_Coeff~Compactness*Length_Kernel*Area, data=df)
```

```
summary(lm2)
```

Call:

```
lm(formula = Asymmetry_Coeff ~ Area, data = df)
```

Residuals:

Min	1Q	Median	3Q	Max
-3.0453	-1.0670	-0.0326	0.9476	4.5010

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	5.46155	0.52757	10.352	< 2e-16	***
Area	-0.11863	0.03487	-3.402	0.000803	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.467 on 208 degrees of freedom

Multiple R-squared: 0.0527, Adjusted R-squared: 0.04815

F-statistic: 11.57 on 1 and 208 DF, p-value: 0.0008028

```
summary(lm3)
```

Call:

```
lm(formula = Asymmetry_Coeff ~ Compactness, data = df)
```

Residuals:

Min	1Q	Median	3Q	Max
-2.9463	-0.9196	-0.0655	0.8393	5.1017

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	22.071	3.627	6.085	5.51e-09	***
Compactness	-21.092	4.163	-5.067	8.90e-07	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.422 on 208 degrees of freedom
Multiple R-squared: 0.1099, Adjusted R-squared: 0.1056
F-statistic: 25.67 on 1 and 208 DF, p-value: 8.903e-07

summary(lm4)

Call:

lm(formula = Asymmetry_Coeff ~ Compactness * Length_Kernel, data = df)

Residuals:

Min	1Q	Median	3Q	Max
-3.1292	-0.9251	-0.1255	0.8454	5.1483

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	145.09	60.55	2.396	0.0175 *
Compactness	-160.31	69.17	-2.318	0.0214 *
Length_Kernel	-22.75	11.08	-2.054	0.0413 *
Compactness:Length_Kernel	25.74	12.64	2.036	0.0430 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.412 on 206 degrees of freedom
Multiple R-squared: 0.1302, Adjusted R-squared: 0.1176
F-statistic: 10.28 on 3 and 206 DF, p-value: 2.454e-06

summary(lm5)

Call:

lm(formula = Asymmetry_Coeff ~ Compactness * Length_Kernel *
Area, data = df)

Residuals:

Min	1Q	Median	3Q	Max
-2.9037	-0.9179	-0.1585	0.8510	4.8715

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-458.273	423.963	-1.081	0.281
Compactness	559.235	486.218	1.150	0.251
Length_Kernel	100.619	79.143	1.271	0.205
Area	20.826	26.133	0.797	0.426
Compactness:Length_Kernel	-120.791	90.769	-1.331	0.185
Compactness:Area	-25.530	29.848	-0.855	0.393
Length_Kernel:Area	-4.731	4.651	-1.017	0.310
Compactness:Length_Kernel:Area	5.723	5.318	1.076	0.283

Residual standard error: 1.388 on 202 degrees of freedom

Multiple R-squared: 0.1767, Adjusted R-squared: 0.1481
F-statistic: 6.192 on 7 and 202 DF, p-value: 1.41e-06

```
AIC(lm1)
```

```
[1] 765.9681
```

```
AIC(lm2)
```

```
[1] 760.8719
```

```
AIC(lm3)
```

```
[1] 747.7998
```

```
AIC(lm4)
```

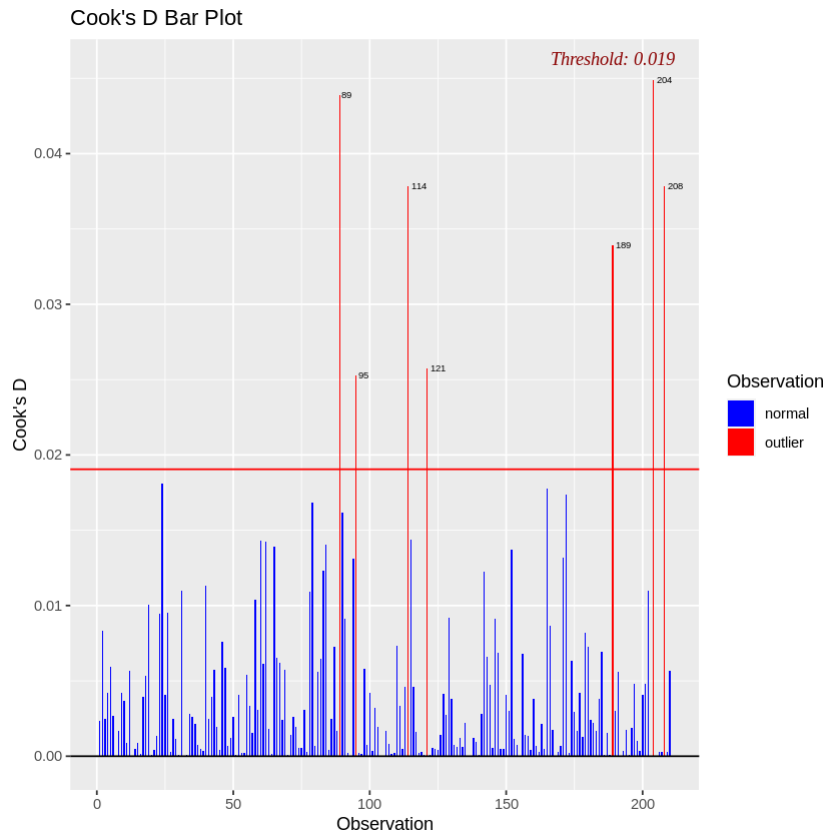
```
[1] 746.9416
```

```
AIC(lm5)
```

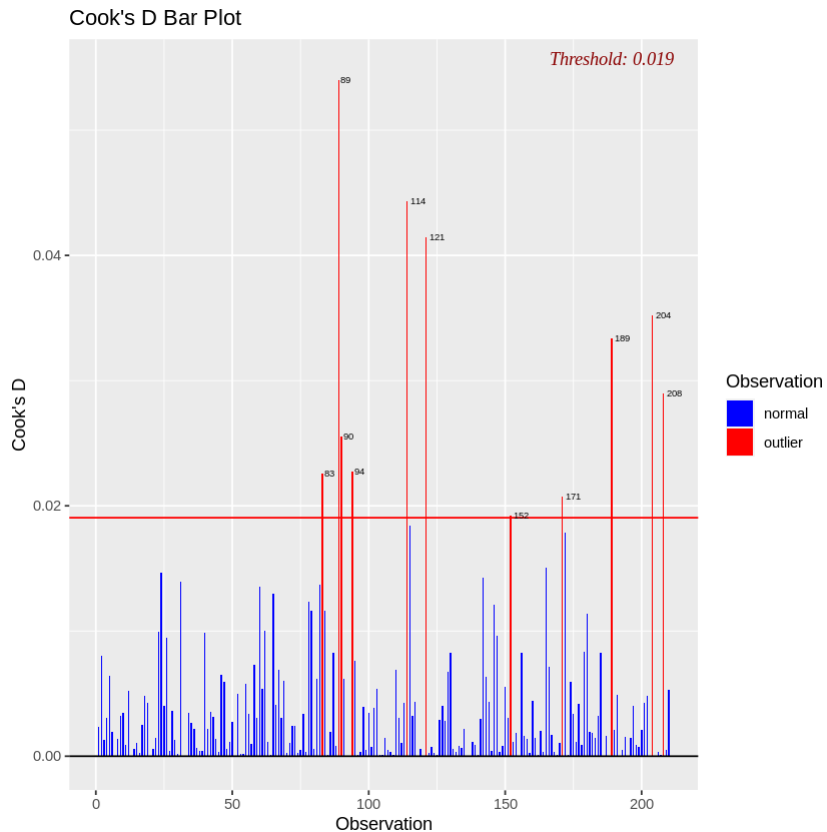
```
[1] 743.4211
```

Podemos ver que el AIC es coherente con el coeficiente de correlación de pearson y dicen que el mejor modelo sera el 1. Pero si vemos los resultados estadisticos de las pruebas de hipotesis es mucho mejor el modelo 3.

```
library(olsrr)  
ols_plot_cooksd_bar(lm1)
```



```
ols_plot_cooksd_bar(lm2)
```



Viendo los resultados de una regresion Lineal más "ideal" :

Width_Kernel vs Compactness

```
plot_ss(x =Asymmetry_Coeff, y = Width_Kernel, data = df)
```

Click two points to make a line.

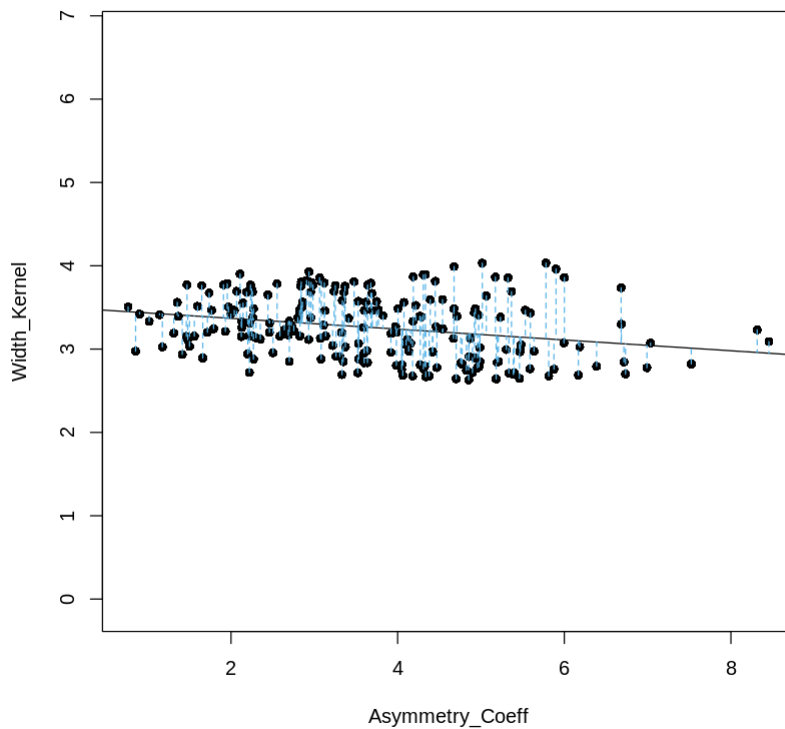
Call:

```
lm(formula = y ~ x, data = pts)
```

Coefficients:

(Intercept)	x
3.49846	-0.06482

Sum of Squares: 27.832



```
lmZ<- lm(Asymmetry_Coeff~Length_Kernel, data=df)
summary(lmZ)
```

```
Call:
lm(formula = Asymmetry_Coeff ~ Length_Kernel, data = df)
```

```
Residuals:
```

Min	1Q	Median	3Q	Max
-2.947	-1.157	-0.019	0.977	4.496

```
Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	6.9772	1.3088	5.331	2.53e-07 ***
Length_Kernel	-0.5822	0.2318	-2.512	0.0128 *

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 1.485 on 208 degrees of freedom
```

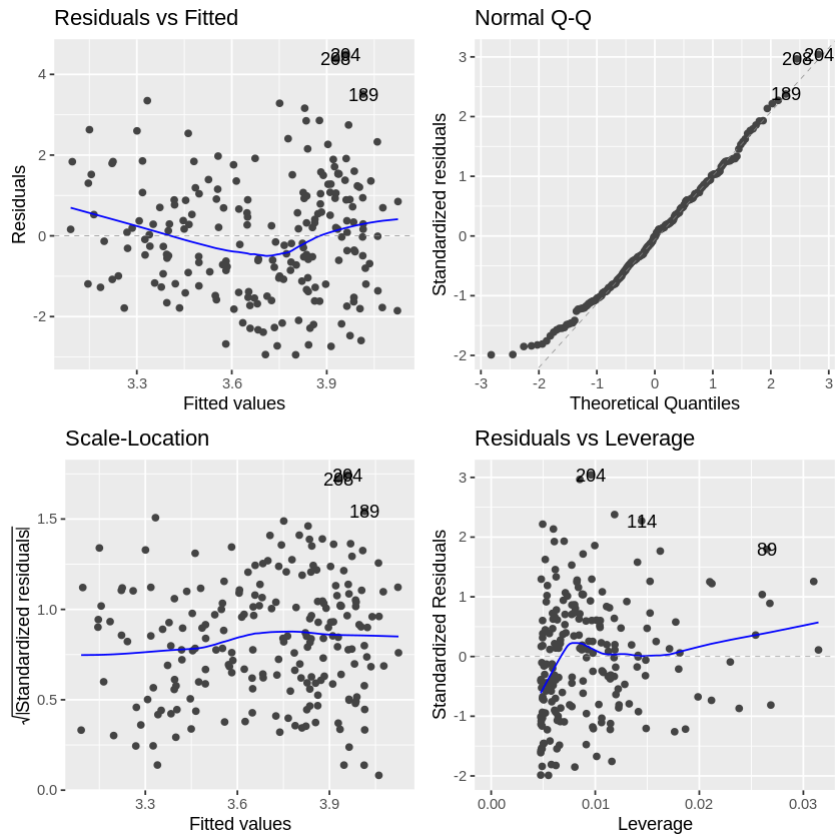
```
Multiple R-squared:  0.02943,    Adjusted R-squared:  0.02477
```

```
F-statistic: 6.308 on 1 and 208 DF,  p-value: 0.01278
```

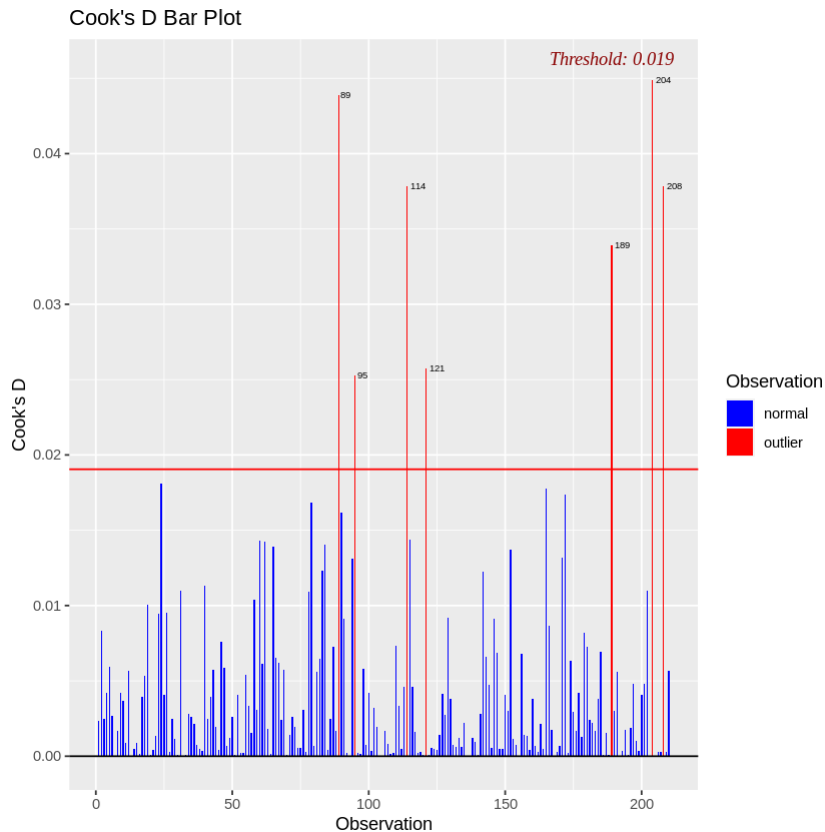
```
confint(lmZ)
```

	2.5 %	97.5 %
(Intercept)	4.396997	9.5573285
Length_Kernel	-1.039206	-0.1252041

autoplot(lmZ)



ols_plot_cooksd_bar(lmZ)



AIC (lmZ)

[1] 765.9681

Conclusion

Podemos decir que se ajusto un modelo lineal a unos datos que no tenian estructura lineal evidente, pero que era de gran valor predecir la variable Y; aquí obtuvimos 1 modelo que tenía mejor cCoeficientes de Pearson y AIC, en contraste encontré un modelop que era mejor con las pruebas de hipotesis en general excepto en los residuales, en general se escogería el modelo con mejor AIC.

Ademas ajustamos un modelo a unos datos con grafico de dispersión lineal y coeficiente de person muy alto, le idea era tener una idea general del comportamiento de las pruebas de hipotesis, graficas, AIC, entre ambos casos descritos, pero no se encontro ningun patron evidente.

William Andrés Gómez Roa