" boxing the compass," which takes young people so long to become familiar with, will be entirely superseded, and I think the sooner such method becomes obsolete the better it will be for the interests of the mariner, for, together with other advantages, the tedious operation of a " day's work " will be divested of half the usual trouble.

When giving a course to the " quarter-master," or " man at the wheel," no mistake, so liable to be the case at present, can well occur; it will merely be necessary to direct him to steer, for instance, " north five points east," or more briefly, " north five east," " south six west," &c. &c.

I recollect an instance of a vessel steering N.W. by N. $\frac{1}{2}$ N., instead of W. by N. $\frac{1}{2}$ N. during thick weather in the Bristol Channel, thus running into danger from the similarity of sound between the courses alluded to.

The practical application of the decimal card would not materially affect the charts previously published, which could have printed compasses containing thirty-six points pasted over the others. Such might be sold by any chart-seller.

III. "On the Theory of the Electric Telegraph." By Professor William Thomson, F.R.S. Received May 3, 1855.

The following investigation was commenced in consequence of a letter received by the author from Prof. Stokes, dated Oct. 16, 1854. It is now communicated to the Royal Society, although only in an incomplete form, as it may serve to indicate some important practical applications of the theory, especially in estimating the dimensions of telegraph wires and cables required for long distances; and the author reserves a more complete development and illustration of the mathematical parts of the investigation for a paper on the conduction of Electricity and Heat through solids, which he intends to lay before the Royal Society on another occasion.

Extract from a letter to Prof. Stokes, *dated Largs, Oct.* 28, 1854.

" Let c be the electro-statical capacity per unit of length of the wire; that is, let c be such that clv is the quantity of electricity required to charge a length l of the wire up to potential v. In a note communicated as an addition to a paper in the last June Number of the Philosophical Magazine, and I believe at present in the Editor's hands for publication, I proved that the value of c is $\frac{I}{2\log\frac{R'}{R}}$, if I denote the specific inductive capacity of the gutta percha, and R, R′ the radii of its inner and outer cylindrical surfaces.

"Let k denote the galvanic resistance of the wire in absolute electro-statical measure (see a paper 'On the application of the Principle of Mechanical Effect to the Measurement of Electromotive Forces and Galvanic Resistances,' Phil. Mag. Dec. 1851).

"Let γ denote the strength at the time t, of the current (also in electro-statical measure) at a point P of the wire at a distance x from one end which may be called O. Let v denote the potential at the same point P, at the time t.

"The potential at the outside of the gutta percha may be taken as at each instant rigorously zero (the resistance of the water, if the wire be extended as in a submarine telegraph, being certainly incapable of preventing the inductive action from being completed instantaneously round each point of the wire. If the wire be closely coiled, the resistance of the water may possibly produce sensible effects).

"Hence, at the time t, the quantity of electricity on a length dx of the wire at P will be $vcdx$.

"The quantity that leaves it in the time dt will be

$$dt\frac{d\gamma}{dx}dx.$$

"Hence we must have

$$-cdx\frac{dv}{dt}dt=dt\frac{d\gamma}{dx}dx \quad \ldots\ldots\ldots\ldots\ldots \quad (1).$$

"But the electromotive force, in electro-static units, at the point P, is

$$-\frac{dv}{dx},$$

and therefore at each instant

$$k\gamma=-\frac{dv}{dx} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2).$$

"Eliminating γ from (1) by means of this, we have

$$ck\frac{dv}{dt}=\frac{d^2v}{dx^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3),$$

which is the equation of electrical excitation in a submarine telegraph-wire, perfectly insulated by its gutta percha covering.

"This equation agrees with the well-known equation of the linear motion of heat in a solid conductor; and various forms of solution which Fourier has given are perfectly adapted for answering practical questions regarding the use of the telegraph-wire. Thus first, suppose the wire infinitely long and communicating with the earth at its infinitely distant end: let the end O be suddenly raised to the potential V (by being put in communication with the positive pole of a galvanic battery of which the negative pole is in communication with the ground, the resistance of the battery being small, say not more than a few yards of the wire); let it be kept at that potential for a time T; and lastly, let it be put in communication with the ground (*i. e.* suddenly reduced to, and ever afterwards kept at, the zero of potential). An elementary expression for the solution of the equation in this case is

$$v=\frac{V}{\pi}\int_0^\infty dn\epsilon^{-zn^{\frac{1}{2}}}\frac{\sin\left[2nt-zn^{\frac{1}{2}}\right]-\sin\left[(t-T)2n-zn^{\frac{1}{2}}\right]}{n} \quad . \quad . \quad (4),$$

where for brevity

$$z=x\sqrt{kc} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)."$$

That this expresses truly the solution with the stated conditions is proved by observing,—1st, that the second member of the equation, (4), is convergent for all positive values of z and vanishes when z is infinitely great; 2ndly, that it fulfils the differential equation (3); and 3rdly, that when $z=0$ it vanishes except for values of t between 0 and T, and for these it is equal to V. It is curious to remark, that we may conclude, by considering the physical circumstances of the problem, that the value of the definite integral in the second member of (4) is zero for all negative values of t, and positive values of z.

"This solution may be put under the following form,

$$v=\frac{2V}{\pi}\int_{t-T}^{t}d\theta\int_0^\infty dn\epsilon^{-zn^{\frac{1}{2}}}\cos(2n\theta-zn^{\frac{1}{2}}) \quad . \quad . \quad . \quad . \quad (6),"$$

which is in fact the primary solution as derived from the elementary type $\cos\left(2\pi\frac{it}{T}-z\sqrt{\frac{\pi i}{T}}\right)\epsilon^{-z\sqrt{\frac{\pi i}{T}}}$ given by Fourier in his investigation of periodic variations of terrestrial temperature.

"This, if T be infinitely small, becomes

$$v=\frac{2V}{\pi}T\int_0^\infty dn\epsilon^{-zn^{\frac{1}{2}}}\cos(2nt-zn^{\frac{1}{2}})\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$

which expresses the effect of putting the end O of the wire for an infinitely short time in communication with the battery and immediately after with the ground. It may be reduced at once to finite terms by the evaluation of the integral, which stands as follows :—

when t is positive, $\int_0^\infty dn\epsilon^{-zn^{\frac{1}{2}}}\cos(2nt-zn^{\frac{1}{2}})=\frac{\pi^{\frac{1}{2}}z}{4t^{\frac{3}{2}}}\epsilon^{-\frac{z^2}{4t}}$,

and when t is negative, $\qquad\qquad =0.$

And so we have

$$v=T\frac{Vz}{4\pi^{\frac{1}{2}}t^{\frac{3}{2}}}\epsilon^{-\frac{z^2}{4t}}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8),$$

or by (6), when t is not infinitely small,

$$v=\frac{Vz}{2\pi^{\frac{1}{2}}}\int_{t-T}^{t}\frac{d\theta}{\theta^{\frac{3}{2}}}\epsilon^{-\frac{z^2}{4\theta}}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9),$$

or which is the same,

$$v=\frac{Vz}{2\pi^{\frac{1}{2}}}\int_0^{T}\frac{d\theta}{(t-\theta)^{\frac{3}{2}}}\epsilon^{-\frac{z^2}{4(t-\theta)}}\quad . \quad . \quad . \quad . \quad (10).$$

It is to be remarked that in (9) and (10) the limits of the integral must be taken 0 to t (instead of $t-T$ to t, or 0 to T), if it be desired to express the potential at any time t between 0 and T, since the quantity multiplied by $d\theta$ in the second number of (6) vanishes for all negative values of θ.

"These last forms may be obtained synthetically from the following solution, also one of Fourier's elementary solutions :—

$$v=\frac{\epsilon^{-\frac{z^2}{4t}}}{t^{\frac{1}{2}}}\cdot\frac{Q}{\pi^{\frac{1}{2}}}\cdot\sqrt{\frac{k}{c}}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11),$$

which expresses the potential in the wire consequent upon instantaneously communicating a quantity Q of electricity to it at O, and leaving this end insulated. For if we suppose the wire to be continued to an infinite distance on each side of O, and its infinitely distant ends to be in communication with the earth, the same equation will express the consequence of instantly communicating 2Q to the wire at O. Now suppose at the same instant a quantity $-2Q$ to be communicated at the point O′ at a distance $\frac{\alpha}{\sqrt{kc}}$ on the negative side of O, the consequent potential at any time t, at a distance $\frac{z}{\sqrt{kc}}$ along the wire from O, will be

$$v=\frac{Q}{\pi^{\frac{1}{2}}}\left\{\frac{\epsilon^{-\frac{z^2}{4t}}}{t^{\frac{1}{2}}}-\frac{\epsilon^{-\frac{(z+\alpha)^2}{4t}}}{t^{\frac{1}{2}}}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12);$$

and if α be infinitely small, this becomes

$$v=\frac{Q\alpha}{2\pi^{\frac{1}{2}}}\cdot\frac{z\epsilon^{-\frac{z^2}{4t}}}{t^{\frac{3}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13),$$

which with positive values of z, expresses obviously the effect of communicating the point O with the positive pole for an infinitely short time, and then instantly with the ground.

"The strength of the current at any point of the wire, being equal to $-\frac{1}{k}\cdot\frac{dv}{dx}$, as shown above, in equation (2), will vary proportionally to $\frac{dv}{dx}$ or to $\frac{dv}{dz}$. The time of the maximum electrodynamic effect of impulses such as those expressed by (11) or (13) will be found by determining t, in each case, to make $\frac{dv}{dz}$ a maximum. Thus we find

$$t=\frac{z^2}{6}=\frac{kcx^2}{6}*,$$

as the time at which the maximum electrodynamic effect of connecting the battery for an instant at O, and then leaving this point insulated, is experienced at a distance x.

"In these cases there is no regular 'velocity of transmission.' But, on the other hand, if the potential at O be made to vary regularly according to the simple harmonic law ($\sin 2nt$), the phases are propagated regularly at the rate $2\sqrt{\frac{n}{kc}}$, as is shown by the well-known solution

$$v=\epsilon^{-zn^{\frac{1}{2}}}\sin(2nt-zn^{\frac{1}{2}}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (14).$$

* We may infer that the retardations of signals are proportional to the squares of the distances, and not to the distances simply; and hence different observers, believing they have found a "velocity of electric propagation," may well have obtained widely discrepant results; and the apparent velocity would, *cæteris paribus*, be the less, the greater the length of wire used in the observation.

The effects of pulses at one end, when the other is in connexion with the ground, and the length finite, will be most conveniently investigated by considering a wire of double length, with equal positive and negative agencies applied at its two extremities. The synthetical method founded on the use of the solution (11) appears perfectly adapted for answering all the practical questions that can be proposed.

"To take into account the effect of imperfect insulation (which appears to have been very sensible in Faraday's experiments), we may assume the gutta-percha to be uniform, and the flow of electricity across it to be proportional to the difference of potential at its outer and inner surfaces. The equation of electrical excitation will then become

$$kc\frac{dv}{dt}=\frac{d^2v}{dx^2}-hv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15),$$

and if we assume

$$v=\epsilon^{-\frac{h}{kc}t}\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16),$$

we have

$$kc\frac{d\phi}{dt}=\frac{d^2\phi}{dx^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17),$$

an equation, to the treatment of which the preceding investigations are applicable."

Extract from Letter to Prof. Stokes, *dated Largs, Oct.* 30, 1854.

"An application of the theory of the transmission of electricity along a submarine telegraph-wire, shows how the question recently raised as to the practicability of sending distinct signals along such a length as the 2000 or 3000 miles of wire that would be required for America, may be answered. The general investigation will show exactly how much the sharpness of the signals will be worn down* and will show what maximum strength of current through the apparatus, in America, would be produced by a specified battery action on the end in England, with wire of given dimensions, &c.

"The following form of solution of the general equation

$$kc\frac{dv}{dt}=\frac{d^2v}{dx^2}-hv,$$

which is the first given by Fourier, enables us to compare the times

* See the diagram of curves given below.

until a given strength of current shall be obtained, with different dimensions, &c. of wire;—

$$v=\epsilon^{-\frac{ht}{kc}} \,.\, \Sigma A_i \sin\left(\pi\frac{ix}{l}\right) \,.\, \epsilon^{-\frac{i^2\pi^2 t}{kcl^2}} \,.$$

If l denote the length of the wire, and V the potential at the end communicating with the battery, the final distribution of potential in the wire will be expressed by the equation

$$v=V\frac{\epsilon^{(l-x)\sqrt{h}}-\epsilon^{-(l-x)\sqrt{h}}}{\epsilon^{l\sqrt{h}}-\epsilon^{-l\sqrt{h}}},$$

which, when $h=0$, becomes reduced to

$$v=V\left(1-\frac{x}{l}\right),$$

corresponding to the case of perfect insulation. The final maximum strength of current at the remote end is expressed by

$$\gamma=\frac{V}{k\,l}\cdot\frac{2l\sqrt{h}}{\epsilon^{l\sqrt{h}}-\epsilon^{-l\sqrt{h}}},$$

or, when $h=0$, $\qquad \gamma=\frac{v}{kl}\cdot$

Hence if we determine A_i so that

$$\Sigma A_i \sin\left(\pi\frac{i\,x}{l}\right)=-V\frac{\epsilon^{(l-x)\sqrt{h}}-\epsilon^{-(l-x)\sqrt{h}}}{\epsilon^{l\sqrt{h}}-\epsilon^{-l\sqrt{h}}} \text{ when } x>0 \text{ and } x<l,$$

the equation

$$v=V\frac{\epsilon^{(l-x)\sqrt{h}}-\epsilon^{-(l-x)\sqrt{h}}}{\epsilon^{l\sqrt{h}}-\epsilon^{-l\sqrt{h}}}+\epsilon^{-\frac{ht}{kc}}\,\Sigma A_i \sin\left(\pi\frac{ix}{l}\right)\epsilon^{-\frac{i^2\pi^2 t}{kcl^2}}$$

will express the actual condition of the wire at any time t after one end is put in connexion with the battery, the other being kept in connexion with the ground.

" We may infer that the time required to reach a stated fraction of the maximum strength of current at the remote end will be proportional to kcl^2. We may be *sure* beforehand that the American telegraph will succeed, with a battery sufficient to give a sensible current at the remote end, when kept long enough in action; but the time required for each deflection will be sixteen times as long as would be with a wire a quarter of the length, such, for instance, as in the French submarine telegraph to Sardinia and Africa. One very important result is, that by increasing the diameter of the wire and of the gutta-percha covering in proportion to the whole length, the distinctness of utterance will be kept constant; for n varies inversely as the square of the diameter, and c (the electro-statical capacity of the unit of length) is unchanged when the diameters of the wire and the covering are altered in the same proportion.

"Hence when the French submarine telegraph is fairly tested, we may make sure of the same degree of success in an American telegraph by increasing all the dimensions of the wire in the ratio of the greatest distance to which it is to extend, to that for which the French one has been tried." It will be an economical problem, easily solved by the ordinary analytical method of maxima and minima, to determine the dimensions of wire and covering which, with stated prices of copper, gutta percha, and iron, will give a stated rapidity of action with the smallest initial expense.

"The solution derived from the type $\frac{\epsilon^{-\frac{x^2}{4t}}}{t^{\frac{1}{2}}}$ may be applied to give the condition of the wire, when one end, E, is kept connected with the ground, and the other, O, is operated on so that its potential may be kept varying according to a given arbitrary function of the time: only this, which I omitted to mention in my last letter, must be attended to: instead of merely considering sources (so to speak) at O and O′ (the latter in an imaginary continuation of the wire), we must suppose sources at O, O_1, O_2, &c., and at O', O_1', O_2', &c. arranged according to the general principle of successive images, so that the potential at E may be zero, and that at O may be uninfluenced by all other sources except the source at O itself. Taking .... $O_2, O_1, O, O', O_1', O_2'$.... equidistant, we have only to suppose equal sources, each represented by the type

$$\frac{x\epsilon^{-\frac{x^2}{4t}}}{t^{\frac{3}{2}}},$$

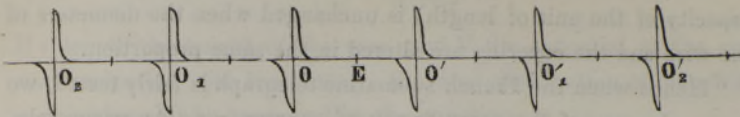
to be placed at these points. For the effects of O_1 and O' will balance one another as far as regards the potential at O.

"So will those of O_2 and O'_1.

" " O_3 and O'_2.

&c. &c.

And again, O and O′ would alone keep the potential at E, zero.
So would O_1 and O'_1.
,, O_2 and O'_2.
&c. &c.



Hence if we denote $2lkc$ by a, for brevity, the general solution is

$$v=\frac{1}{2\pi^{\frac{1}{2}}}\int_0^t \frac{d\theta F(\theta)}{(t-\theta)^{\frac{3}{2}}}\left\{\ldots(z+2a)\epsilon^{-\frac{(z+2a)^2}{4(t-\theta)}}+(z+a)\epsilon^{-\frac{(z+a)^2}{4(t-\theta)}}+z\,\epsilon^{-\frac{z^2}{4(t-\theta)}}\right.$$
$$\left.+(z-a)\epsilon^{-\frac{(z-a)^2}{4(t-\theta)}}+(z-2a)\epsilon^{-\frac{(z-2a)^2}{4(t-\theta)}}+\ldots\ldots\right\},$$

where $F(\theta)$ is an arbitrary function such that $F(t)$ expresses the potential sustained at O by the battery.

"The corresponding solution of the equation

$$kc\frac{dv}{dt}=\frac{d^2v}{dx^2}-hv$$

is

$$v=\frac{1}{2\pi^{\frac{1}{2}}}\epsilon^{-\frac{ht}{kc}}\int_0^t \frac{d\theta\epsilon^{\frac{h\theta}{kc}}F\theta}{(t-\theta)^{\frac{3}{2}}}\,\Sigma_{-\infty}^{\infty}\left\{(z-ia)\epsilon^{-\frac{(z-ia)^2}{4(t-\theta)}}\right\},$$

by which the effect of imperfect insulation may be taken into account."

Extract of Letter from Prof. Stokes to Prof. W. Thomson (dated Nov. 1854).

"In working out for myself various forms of the solution of the equation $\frac{dv}{dt}=\frac{d^2v}{dx^2}$ under the conditions $v=0$ when $t=0$ from $x=0$ to $x=\infty$; $v=f(t)$, when $x=0$ from $t=0$ to $t=\infty$, I found that the solution with a single integral only (and there must necessarily be this one) was got out most easily thus :—

"Let v be expanded in a definite integral of the form

$$v=\int_0^\infty \varpi\,(t,\alpha)\sin\alpha x d\alpha,$$

which we know is possible.

"Since v does not vanish when $x=0$, $\frac{d^2v}{dx^2}$ is not obtained by differentiating under the integral sign, but the term $\frac{2}{\pi}\alpha\, v_{x=0}$ must be supplied*, so that (observing that $v_{x=0}=f(t)$ by one of the equations of condition) we have

$$\frac{d^2v}{dx^2}=\int_0^\infty\left\{\frac{2}{\pi}\alpha f(t)-\alpha^2\varpi.\right\}\sin\alpha x dx.$$

Hence

$$\frac{dv}{dt}-\frac{d^2v}{dx^2}=\int_0^\infty\left\{\frac{d\varpi}{dt}+\alpha^2\varpi-\frac{2}{\pi}\alpha f(t)\right\}\sin\alpha x dx,$$

and the second member of the equation being the direct development of the first, which is equal to zero, we must have

$$\frac{d\varpi}{dt}+\alpha^2\varpi-\frac{2}{\pi}\alpha f(t)=0,$$

whence

$$\varpi=\epsilon^{-\alpha^2 t}\int^t\frac{2}{\pi}\alpha\, f(t)\,\epsilon^{\alpha^2 t}\,dt,$$

the inferior limit being an arbitrary function of α. But the other equation of condition gives

$$\varpi=\epsilon^{-\alpha^2 t}\int_0^t\frac{2}{\pi}\alpha f(t)\,\epsilon^{\alpha^2 t}\,dt=\left(\frac{\pi}{2}\right)^{-1}\alpha\int_0^t\epsilon^{-\alpha^2\overline{t-t'}}f(t')dt',$$

therefore

$$v=\left(\frac{\pi}{2}\right)^{-1}\int_0^\infty\int_0^t f(t')\alpha\,\epsilon^{-\alpha^2\overline{t-t'}}\sin\alpha x\, d\alpha\, dt'.$$

$$\text{But}\int_0^\infty\epsilon^{-a\alpha^2}\cos b\alpha\, d\alpha=\frac{1}{2}\left(\frac{\pi}{a}\right)^{\frac{1}{2}}\epsilon^{-\frac{b^2}{4a}},$$

therefore

$$\int_0^\infty\epsilon^{-a\alpha^2}\sin b\,a\,.\,\alpha d\alpha=-\frac{d}{db}\left\{\frac{1}{2}\left(\frac{\pi}{a}\right)^{\frac{1}{2}}\epsilon^{-\frac{b^2}{4a}}\right\}$$

$$=\frac{\pi^{\frac{1}{2}}b}{4a^{\frac{3}{2}}}\epsilon^{-\frac{b^2}{4a}},$$

whence writing $t-t'$, x, for a, b, and substituting, we have

$$v=\frac{x}{2\pi^{\frac{1}{2}}}\int_0^t(t-t')^{-\frac{3}{2}}\,\epsilon^{\frac{x^2}{4(t-t')}}f(t')\,dt'.$$

"Your conclusion as to the American wire follows from the dif-

*According to the method explained in a paper "On the Critical Values of the Sums of Periodic Series," Camb. Phil. Trans. vol. viii. p. 533.

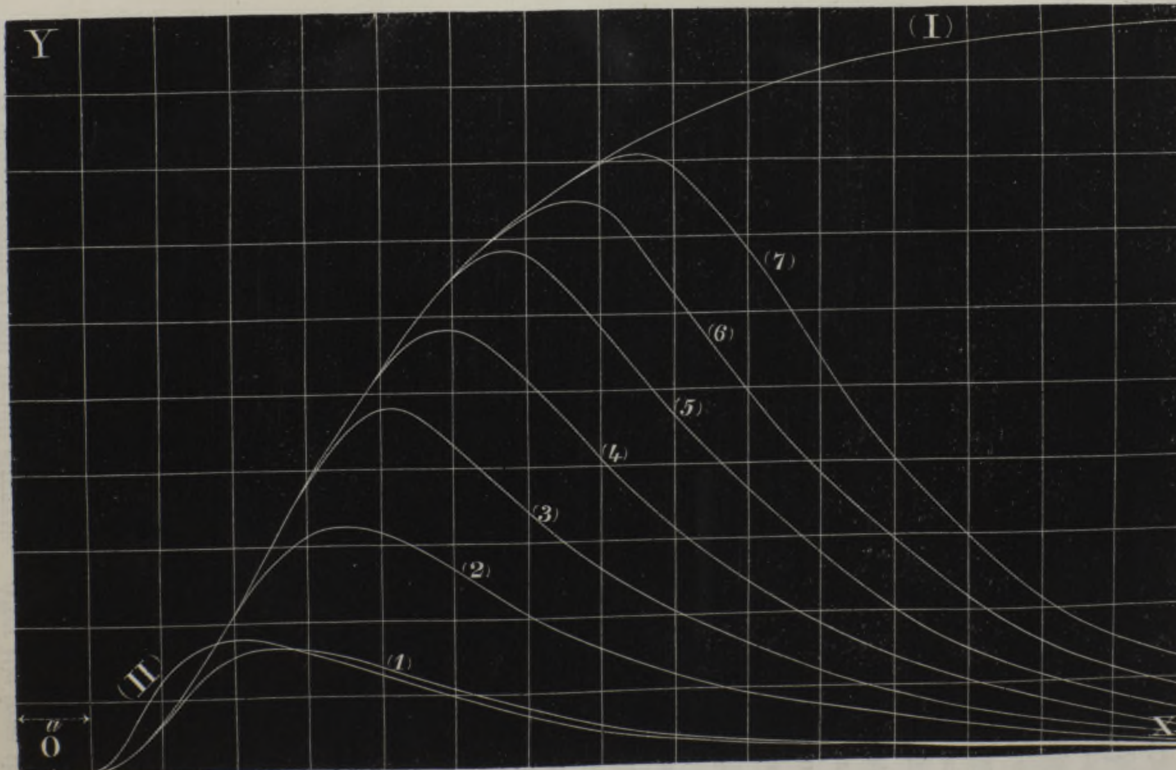
ferential equation itself which you have obtained. For the equation $kc\frac{dv}{dt}=\frac{d^2v}{dx^2}$ shows that two submarine wires will be similar, provided the squares of the lengths x, measured to similarly situated points, and therefore of course those of the whole lengths l, vary as the times divided by ck; or the time of any electrical operation is proportional to kcl^2.

"The equation $kc\frac{dv}{dt}=\frac{d^2v}{dx^2}-hv$ gives $h \propto l^{-2}$ for the additional condition of similarity of leakage."

The accompanying set of curves represents the strength of the current through the instrument at the remote end of a wire as it gradually rises, or gradually rises and falls, after the end operated on is put in connexion with one pole of a battery, and either kept so permanently, or detached and put in connexion with the ground after various short intervals of time.

The abscissas, measured on OX, represent the time reckoned from the first application of the battery, and the ordinates, measured parallel to OY, the strength of the current.

The time corresponding to a is equal to $\frac{kcl^2}{\pi^2}\log_\epsilon\left(\frac{4}{3}\right)$, if l be the length of the wire in feet, k its "resistance" per foot, in electro-statical units, and c its electro-statical capacity per foot (which is equal to $\frac{\mathrm{I}}{2\log\frac{\mathrm{R}'}{\mathrm{R}}}$, if I be the electro-statical inductive power of the gutta percha, probably about 2, and R, R′ the radii of its outer and inner surfaces). The principal curve (I.) represents the rise of the current in the remote instrument, when the end operated on is kept permanently in connexion with the battery. It so nearly coincides with the line of abscissas at first as to indicate no sensible current until the interval of time corresponding to a has elapsed; although, strictly speaking, the effect at the remote end is instantaneous (*i.e.* according to data limited as regards knowledge of electricity, to such as those assumed in hydrodynamics when water is treated as if incompressible, or the velocity of sound in it considered infinitely great, which requires instantaneous effects to be propagated through the whole mass of the water, on a disturbance being made in any part


O
a
Y
X
(I)
(II)
(1)
(2)
(3)
(4)
(5)
(6)
(7)

of it). After the interval a, the current very rapidly rises, and after about $4a$ more, attains to half its full strength. After $10a$ from the commencement, it has attained so nearly its full strength, that the farther increase would be probably insensible. The full strength is theoretically reached only after an infinite time has passed. The first (1) of the smaller curves represents the rise and fall of the current in the remote instrument when the end operated on is put in connexion with the ground after having been for a time a in connexion with the battery; the second (2) represents similarly the effect of the battery for a time $2a$; the third (3) for a time $3a$ and so on. The curve (II) derived from the primary curve (I) by differentiation (exhibiting in fact the steepness of the primary curve at its different points, as regards the line of abscissas), represents the strength of current at different times through the remote end of the wire, consequent upon putting a very intense battery in communication with the end from which the signal is sent, for a very short time, and then instantly putting this end in communication with the ground. Thus, relatively to one another, the curves (1) and (II) may be considered as representing the relative effects of putting a certain battery in communication for the time a, and a battery of ten or twenty times as many cells for a time $\frac{1}{10}a$ or $\frac{1}{20}a$. If I were to guess what might be called "the retardation," which in the observations between Greenwich and Brussels was found to be about $\frac{1}{10}$th of a second, I should say it corresponded to four or five times a, but this must depend on the kind of instrument used, and the mode of making and breaking contacts with the battery which was followed.

Equation of principal curve (I).

$$y=10a-20a(e-e^4+e^9-e^{16}+\&c.),\ \text{where}\ e=\left(\frac{3}{4}\right)^{\frac{x}{a}};$$

a being half the side of one of the squares.

If $y=f(x)$ denote the equation of the principal wave, and if $f(x)$ be supposed to vanish for all negative values of x, the series of derived curves are represented by the equations

(1) . . . . . $y=f(x)-f(x-a)$

(2) . . . . . $y=f(x)-f(x-2a)$

$$(3) \quad . \quad . \quad . \quad . \quad y=f(x)-f(x-3a)$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$(7) \quad . \quad . \quad . \quad . \quad y=f(x)-f(x-7a)$$

$$(II) \quad . \quad . \quad . \quad . \quad y=a\frac{df(x)}{da}.$$

I think clearly the right way of making observations on telegraph retardations would be to use either Weber's electro-dynamometer, or any instrument of suitable sensibility constructed on the same principle, that is, adapted to show deflections experienced by a moveable part of a circuit, in virtue of the mutual electro-dynamic force between it and the fixed part of the same circuit due to a current flowing for a very short time through the circuit. Such an instrument, and an ordinary galvanometer, (showing impulsive deflections of a steel needle,) both kept in the circuit at the remote end of the telegraph-wire from that at which the signal is made, would give the values of $\int_0^\infty y^2 dx$, and $\int_0^\infty y dx$ (or the area), for any of the curves; and the ratio of the time a of the diagrams to the time during which the battery was held in communication with the wire, might be deduced. The method will lose sensibility if the battery be held too long in communication, but will be quite sufficiently precise if this be not more than ten or twenty times a. I believe there will be no difficulty in applying the method to telegraph-wires of only twenty or thirty miles long, where no retardation would be noticed by ordinary observation. Before, however, planning any observations of this kind with a view to having them executed, I wished to form some estimate of the probable value of a certain element,—the number of electro-statical units in the electro-magnetic unit of electrical quantity,—which I hoped to be able to do from the observation of $\frac{1}{10}$th of a second as the apparent retardation of signals between Greenwich and Brussels. I therefore applied to the Astronomer Royal for some data regarding the mode of observation on the indications of the needle, and the dimensions and circumstances of insulation of the wire; and he was so good as to send me immediately all the information that was available for my purpose. This has enabled me to make the estimate, and so has convinced me that a kind of experiment which I proposed in a paper on Transient Electric Currents in the Philosophical Magazine for June

1853, and which I hope to be able before long to put in practice, will be successful in giving a tolerably accurate comparison of the electro-statical and electro-dynamic units; and, with a further investigation of the specific inductive capacity of gutta percha which will present no difficulty, will enable me to give all the data required for estimating telegraph retardations, without any data from telegraphic operations. This experiment is simply to put two plane-conducting discs in communication with the two poles of a Daniell's battery (or any other battery of which the electromotive force is known in electro-magnetic units), and to *weigh* the attraction between them. I now find that 100 cells of Daniell's so applied would give a force of not less than four grains between two discs each a square foot in area, and placed $\frac{1}{100}$th of a foot apart. As the force varies inversely as the square of the distance between the discs, the weighing will be rather troublesome in consequence of instability, but I think with a good balance it will be quite practicable.

In making this estimate, I suppose the retardation observed between Greenwich and Brussels to be chiefly due to the subterranean part of the wire, and I have taken it as if it were actually observed in 180 miles of coated copper wire. Not having worked out the theoretical problem in the case of a number of insulated wires under the same sheathing, I have considered the cases of a single wire excentrically placed in the iron sheathing, and insulated from it by gutta percha, and a single wire in its own gutta percha tube with the others removed, and itself symmetrically sheathed with tow and iron wire in the usual manner. In the former case the electro-statical capacity of the wire would be $\dfrac{I}{2\log_{\varepsilon}\dfrac{R^2-f^2}{RR'}}$ approximately*, if R, the inner radius of the conducting sheath, be a considerable multiple of R′ the radius of the copper wire, f denotirg he distance between their axes. In the latter case it is $\dfrac{I}{2\log\dfrac{R_{\prime}}{R'}}$, where $R_{\prime}$ is the inner radius of the

* The rigorous expression, which is very easily found by the method of "electrical images," need not be given here.

sheath. These become $\frac{1}{2{\cdot}45}$ and $\frac{1}{1{\cdot}35}$, if we take $I=2$ (as it probably is for gutta percha, nearly enough), and $R={\cdot}5$, $R_{\prime}=\frac{1}{8}$, $R'={\cdot}0325$, as the information given me by the Astronomer Royal indicates. Whatever the theory may show for the influence of the other wires, the result as regards retardation must be intermediate between what it would be if the other wires were removed, and if the one used were separated from them by a sheathing of its own. We may therefore apply the theoretical result by taking c something between $\frac{1}{2{\cdot}45}$ and $\frac{1}{1{\cdot}35}$. Hence if "the retardation" agree with the time corresponding to a in the diagrams, k must be intermediate between

$$\frac{\pi^2\times\frac{1}{10}}{\frac{1}{2{\cdot}45}\times(180\times5280)^2\times\log_\varepsilon\left(\frac{4}{3}\right)} \text{ and } \frac{\pi^2\times\frac{1}{10}}{\frac{1}{1{\cdot}35}\times(180\times5280)^2\times\log_\varepsilon\left(\frac{4}{3}\right)};$$

or again, if "the retardation" correspond to 9 a, k must be intermediate between

$$\frac{\pi^2\times\frac{1}{90}}{\frac{1}{2{\cdot}45}\times(180\times5280)^2\times\log_\varepsilon\left(\frac{4}{3}\right)} \text{ and } \frac{\pi^2\times\frac{1}{90}}{\frac{1}{1{\cdot}35}\times(180\times5280)^2\times\log_\varepsilon\left(\frac{4}{3}\right)}.$$

I think it quite certain that what was observed as the retardation must be in reality intermediate between a and $9a$ of the diagrams. Hence the true value of k for 1 foot of the wire must be between the greatest and least of the preceding estimates, that is, between

$$\frac{1}{108\times10^9} \text{ and } \frac{1}{176\times10^{10}}.$$

But the value of K (the "resistance" in British absolute electro-magnetic measure of 1 foot of the wire) must, according to Weber's observations on copper, be about 99810, or nearly enough 100,000*. Hence σ (the number of electro-statical units in the electro-magnetic unit) being equal to $\sqrt{\frac{K}{k}}$, must be between 104,000,000 and 419,000,000.

* See a paper on the application of the general principle of mechanical effect to the theory of electromotive forces, &c., published in the Philosophical Magazine, Dec. 1851.

According to the observations of Weber, Joule, and others, the quantity of water decomposed by a current of unit strength during the unit of time, that is, by the electro-magnetic unit of electricity, is very exactly $\frac{1}{50}$th of a grain. Hence from 2,000,000 to 8,200,000 electro-statical units are required to decompose a grain of water. A positive and a negative electro-statical unit at a foot distance attract one another with a force of $\frac{1}{32 \cdot 2}$ of the weight of a grain. Hence if the electricities separated in the decomposition of a grain be concentrated in two points a foot asunder, they will attract with a force of more than 10 tons, and less than 42 tons! Faraday long ago conjectured that less electricity passes in the greatest flash of lightning than in the decomposition of a drop of water, which is now I think rendered very probable.

The expression for the force, in British dynamic units, between two plates, each of area S, at a small distance, a, asunder, when connected with the two poles of a battery of which the electromotive force in electro-magnetic units is F, is $\frac{S}{8\pi}\left(\frac{F}{\sigma a}\right)^{2}$*, or in terms of the weight of a grain $\frac{1}{32 \cdot 2} \cdot \frac{S}{8\pi}\left(\frac{F}{\sigma a}\right)^{2}$. If F be the electromotive force of 100 cells of Daniell's, which, as I have found from Joule's observations, must be about 250,000,000†, and if a be $\frac{1}{10}$th of a foot, and S a square foot, I conclude from the preceding estimates for σ, that the force of attraction between the plates cannot be less than 4·4 grains, nor more than 72 grains.

It would be easy at any time to make a plan for observing telegraph indications by means of either Weber's electro-dynamometer, or an instrument constructed on the same principle, or by measuring thermal effects of intermittent currents, which could be put in practice by any one somewhat accustomed to make observations, and which would give a tolerably accurate determination of the element of time, even in cases where the observable retardation is considerably less than $\frac{1}{10}$th of a second. A single wire in a submarine cable would, as far as regards the physical deductions to be made from this determination, be to be preferred to one of a number of different wires insulated from one another under the same sheath-

* As was shown at the conclusion of a paper "On Transient Electric Currents," published in June 1853, in the Philosophical Magazine.

† See the paper referred to above, as published in the Phil. Mag., Dec. 1851.

ing. I have little doubt but the Varna and Balaklava wire will be the best yet made for the purpose.

Without knowing exactly what the "retardation" may be in terms of the element of time "a" of the diagrams, we may judge what the retardation, if similarly estimated, would be found to be in other cables of stated dimensions. Thus, if the retardation in 200 miles of submarine wire between Greenwich and Brussels be $\frac{1}{10}$th of a second, the retardation in a cable of equal and similar transverse section, extending half round the world (14,000 miles), would be

$$\left(\frac{14000}{200}\right)^2 \times \frac{1}{10} = 490 \text{ seconds, or } 8\tfrac{1}{6} \text{ minutes :}$$

and in the telegraphic cable (400 miles) between Varna and Balaklava, of which the electro-statical capacity per unit of length may be about one-half greater than in the other, while the conducting power of the wire is probably the same, the retardation may be expected to be

$$\left(\frac{400}{200}\right)^2 \times \frac{3}{2} \times \frac{1}{10} = \frac{3}{5} \text{ of a second.}$$

The rate at which distinct signals could be propagated to the remote end would perhaps be one signal in about a quarter of an hour in the former case, and nearly two signals in a second in the latter.

IV. "Observations on the Human Voice." By Manuel Garcia, Esq. Communicated by Dr. Sharpey, Sec. R.S. Received March 22, 1855.

The pages which follow are intended to describe some observations made on the interior of the larynx during the act of singing. The method which I have adopted is very simple. It consists in placing a little mirror, fixed on a long handle suitably bent, in the throat of the person experimented on against the soft palate and uvula. The party ought to turn himself towards the sun, so that the luminous rays falling on the little mirror, may be reflected on the larynx. If the observer experiment on himself, he ought, by means of a second mirror, to receive the rays of the sun, and direct them on the mirror, which is placed against the uvula. We shall