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# Optical models of the human eye

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**David A Atchison*** DSc
**Larry N Thibos**† PhD

*School of Optometry & Vision Science and Institute of Health & Biomedical Innovation, Queensland University of Technology, Brisbane, Queensland, Australia
†School of Optometry, Indiana University, Indiana, USA
E-mail: d.atchison@qut.edu.au

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Optical models of the human eye have been used in visual science for purposes such as providing a framework for explaining optical phenomena in vision, for predicting how refraction and aberrations are affected by change in ocular biometry and as computational tools for exploring the limitations imposed on vision by the optical system of the eye. We address the issue of what is understood by optical model eyes, discussing the 'encyclopaedia' and 'toy train' approaches to modelling. An extensive list of purposes of models is provided. We discuss many of the theoretical types of optical models (also schematic eyes) of varying anatomical accuracy, including single, three and four refracting surface variants. We cover the models with lens structure in the form of nested shells and gradient index. Many optical eye models give accurate predictions only for small angles and small fields of view. If aberrations and image quality are important to consider, such 'paraxial' model eyes must be replaced by 'finite model' eyes incorporating features such as aspheric surfaces, tilts and decentrations, wavelength-dependent media and curved retinas. Many optical model eyes are population averages and must become adaptable to account for age, gender, ethnicity, refractive error and accommodation. They can also be customised for the individual when extensive ocular biometry and optical performance data are available. We consider which optical model should be used for a particular purpose, adhering to the principle that the best model is the simplest fit for the task. We provide a glimpse into the future of optical models of the human eye. This review is interwoven with historical developments, highlighting the important people who have contributed so richly to our understanding of visual optics.

Key words: finite models, optical models, paraxial models, schematic eyes, visual optics

During their investigations in vision science, both authors have relied heavily on optical models of the eye. Their reasons for developing and using these include establishing a framework for explaining optical phenomena in vision, for predicting how aberrations are affected by change in ocular biometry and as a computational tool for exploring the limitations imposed on vision by the optical system of the eye. It seems fitting to assemble our ideas on the subject here, as well as to acknowledge our forebears and colleagues. To develop the flow of ideas, we have omitted equations.

We begin by asking 'What is an optical model eye, anyway?' A short answer is that optical models summarise and organise our understanding of the eye as an optical system and provide a conceptual framework for thinking about how the retinal image is formed to launch the visual process. Eye models fall into two different categories. One is the 'encyclopaedia' type of model, which means that the model is a mechanistic summary of everything we know about the eye's optical system and how it works. The encyclopaedic model is a compact, working representation of knowledge about ocular mechanisms but its comprehensiveness can also be a disadvantage, if it is too complicated for solving practical problems. The other category is 'the toy train' type of model, which is meant to be a working tool that mimics the behaviour of real eyes but does not necessarily attempt to be anatomically or mechanistically accurate. This type of model can have a variety of embodiments: it can be a physical device used to test and calibrate instrumentation, or a purely mathematical entity that provides analytical descriptions of the eye's optical behaviour or it could be a collection of computer programs that provide numerical descriptions of the eye's aberrations. The 'toy train' or working optical eye model has the advantage that real-world problems get solved but has the possible disadvantage of oversimplifying (both structurally and mechanistically) important features of the eye.

Encyclopaedic model eyes have a long history dating back to the ancients, and still form the basic curriculum for teaching the theory of visual optics in optometry, so they are the main focus of this review. For practical calculations in everyday clinical optometry, nothing beats the simplified approach of Gaussian optics and the reduced 'toy train' model of the eye.

We can now ask 'to what purposes can optical model eyes be put'? A non-exhaustive list includes the following.

1. Physical models used for calibrating instruments. These are frequently used in instruments such as keratometers, autorefractors and partial coherence interferometry for measuring intraocular lengths
2. Retinal image size. This is of interest in considering differences between eyes that may affect binocular vision, such as

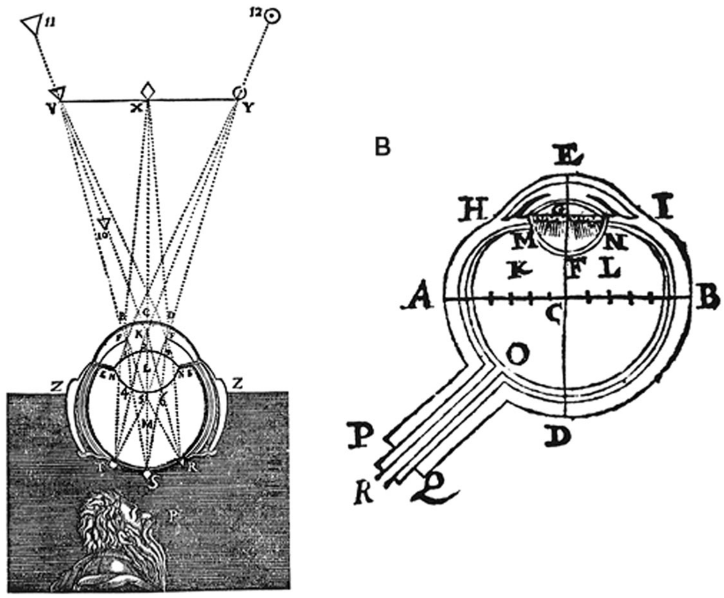


**Figure 1. A) Scheiner's 1619 drawing of the eye, B) Scheiner's method of viewing the retinal image of an excised animal eye. Reproduced from Wade,[3] pages 80 and 30.**

high levels of anisometropia that may be natural or surgically induced.

3. Retinal light levels. This is important for safety purposes, such as using ophthalmic lasers.
4. Refractive errors arising from variations or changes in ocular dimensions.
5. Power of intraocular lenses following cataract surgery.
6. Aberrations and retinal image quality with or without optical or surgical intervention.
7. Designing spectacles, contact lenses, intraocular lenses and corneal refractive surgery.
8. Customisation for individuals.
9. Incorporation into the design of imaging instruments to predict retinal spot sizes, magnification, field of view and irradiance levels.[1,2]
10. One-off types of problems. An example is provided later in the paper.

## SOME HISTORICAL DEVELOPMENT OF MODEL EYES

Much of the information in this section is derived from Wade,[3] including Figures 1 to 4.

The ancient Greeks had an incorrect understanding of the optics of image formation and their descriptions of the eye were often based on philosophy rather than on observation. For example, their four elements of earth, air, fire and water led to the suggestion that there must be four coats to the eye. There was conjecture about whether vision occurred in the lens, at the object or somewhere external to the eye before the object. Democritus described the eye as two coats containing a humour that passed along a hollow pipe called the optic nerve from the brain to the eye. Arabic scholars, such as Ibn al-Haytham (Alhazen), preserved Greek teaching after the fall of Rome until the Renaissance 1,000 years later.

Religious prohibition of dissection of dead bodies in Europe and the Islamic world delayed progress toward understanding the eye's anatomy. Vesalius launched the renaissance of anatomy in the 16th century, finally overcoming unquestioned adherence to the teachings of the Greek physician Aelius Galenus (Galen, second century A.D.).

In the early 17th century, Scheiner demonstrated the retinal image by direct observation in an animal eye. By removing the sclera from the back of the eye, he could directly observe the inverted retinal image

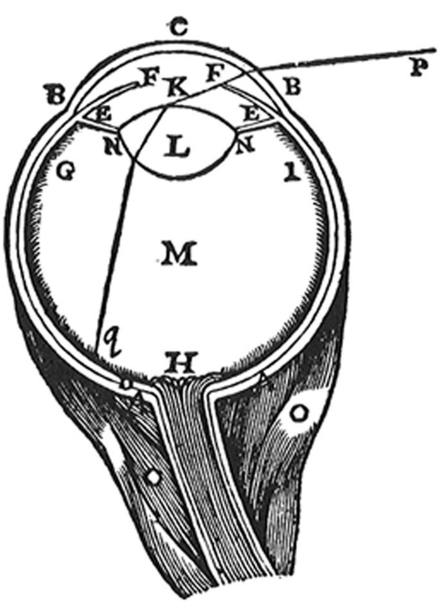

**Figure 2. Descartes' ray-tracing diagram for peripheral vision. Reproduced from Wade,[3] page 82.**

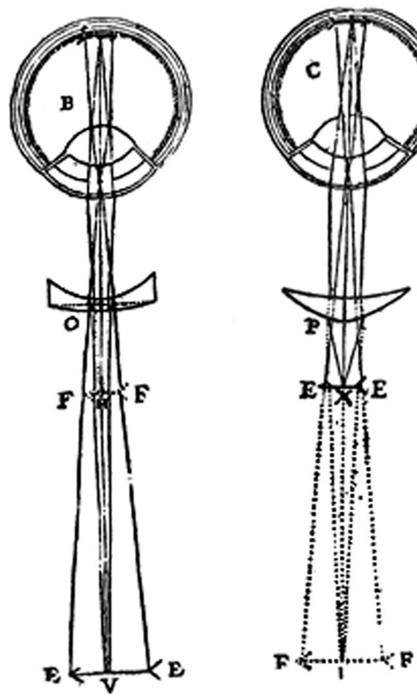

**Figure 3. Descartes' diagram showing correction (left) for myopia and (right) presbyopia. Reproduced from Wade,[3] page 56.**

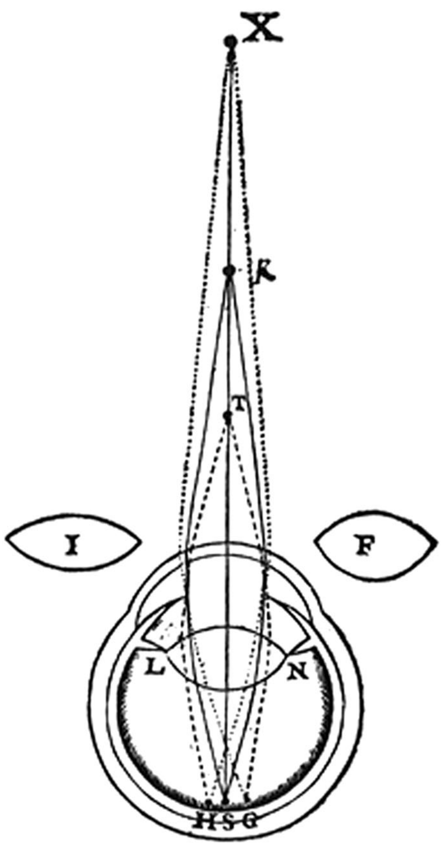


**Figure 4. Descartes' description of accommodation. He wrote 'In order to represent point X distinctly, it is necessary that the whole shape of the humor LN be changed and that it become slightly flatter, like that marked I; and to represent point T it is necessary that it become slightly more arched, like that marked F'. Reproduced from Wade.[3] page 41, quotation on page 39.**

being formed on the retina (Figure 1A). Figure 1B shows the first known schematic eye that was sufficient to show how light rays from objects are refracted by the eye's optical components to produce the retinal image.

René Descartes exploited the optical model eye concept to understand the mapping of the visual hemisphere onto the retinal surface (Figure 2). Note that the optical power of the lens is much greater than that of the cornea and that the bending of light is excessive but apart from this, it is a modern-looking optical model of the eye.

One of the earliest applications of optical model eyes was to explain how and why spectacles work. Lenses had been used as spectacles since the 13th century to correct myopia and presbyopia but no-one understood how and why they worked prior to the 17th century. Johannes Kepler (1604) was the first to realise the existence of the inverted retinal image and to understand how to correct refractive errors. He wrote: 'Those who see remote objects distinctly and near objects confusedly (that is, presbyopes) require glasses that are in relief (convex, positive power); however, those who see remote objects confusedly and near objects distinctly (that is, myopes) are helped by depressed lenses (concave, negative power).' Scheiner used his schematic eye to think about the cause of refractive error and to invent the first optometer (a device for measuring refractive error). Descartes (1637) used his powers of analysis to create a schematic representation of the eye plus a correcting lens system that explains how spectacles work (Figure 3).

In addition to the role played by schematic eyes in understanding the cause and cure of refractive error, schematic eyes have been used since the 17th century to think about the mechanism of accommodation. Descartes clearly understood that to change the eye's focus the lens needs to change shape (Figure 4).

Christian Huygens (1629–1695) made a physical model of the eye, consisting of two hemispheres representing the cornea and retina, with the retinal hemisphere having a radius of curvature that was three times that of the corneal hemisphere. The hemispheres were filled with water and there was a diaphragm between them.

Important tools for understanding the optical system became available in the 17th to 19th centuries. Willebrord Snel van Royen, around 1621, described the relationship between angles of incidence and refraction, upon which the subsequent technical advances in optical instrument manufacture were based. Snell's law was elaborated by Descartes 16 years later but was first given by Ibn Sahl in *On Burning Mirrors and Lenses* published in the year 984.

Newton (1642–1727) pioneered the use of eye models to understand how retinal images are affected by monochromatic aberrations (in addition to the refractive errors already mentioned) caused by irregularities in the eye's refracting surfaces and chromatic aberrations due to dispersion of the ocular media.[4] Aberrations make analysis of basic properties difficult — this was overcome by Gauss, who in 1841 published his paraxial theory of optics. Gauss' simplified method remains the standard method for optical calculation of image location and size that is the basis for routine optometric calculation of refractive error and magnification. Further development of optical models of the eye requires the determination of surface radii of curvature, intraocular distances and refractive indices, and methods of finding these appeared from the 17th century onward.

## SCHEMATIC EYES

From this brief review of the historical roots of eye models, we jump into the 19th century and describe the levels of complexity that occur in optical models of the eye that were developed from this time onward. These eyes are often referred to as schematic eyes, which is just another term for optical model eyes. This section describes the trend for increasing complexity, which has accelerated in recent years by refinement of measurement techniques and better technology.

Before introducing the models, we will mention some important reference or 'cardinal' points, associated with eye models. These cardinal points that arose from the work of Gauss, enable calculation of image position and size without concern for anatomical details, and good optical model eyes will have them in accurate positions. There are two principal points of the eye (Figure 5). For ray-tracing and in particular vergence equations, we can often relate everything on the object side of the system to the anterior principal point P and everything on the image side of the system of the posterior principal point P'. There are two focal points of the eye. The posterior focal point of the eye *F*' is found by ray-tracing into the eye from infinity. The anterior focal point *F* of the eye is found by ray-tracing out of the eye, as if from infinity. There are two nodal points of the eye. A ray directed to the anterior nodal point will pass through to the retina at the same angle but as if it came from the posterior nodal point. Although these cardinal points may be difficult to locate in any individual

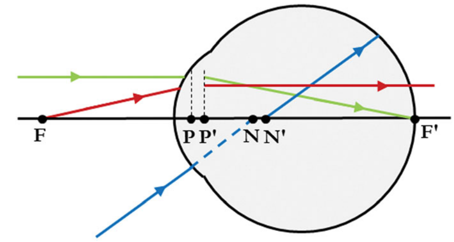


**Figure 5. The cardinal points of the eye**

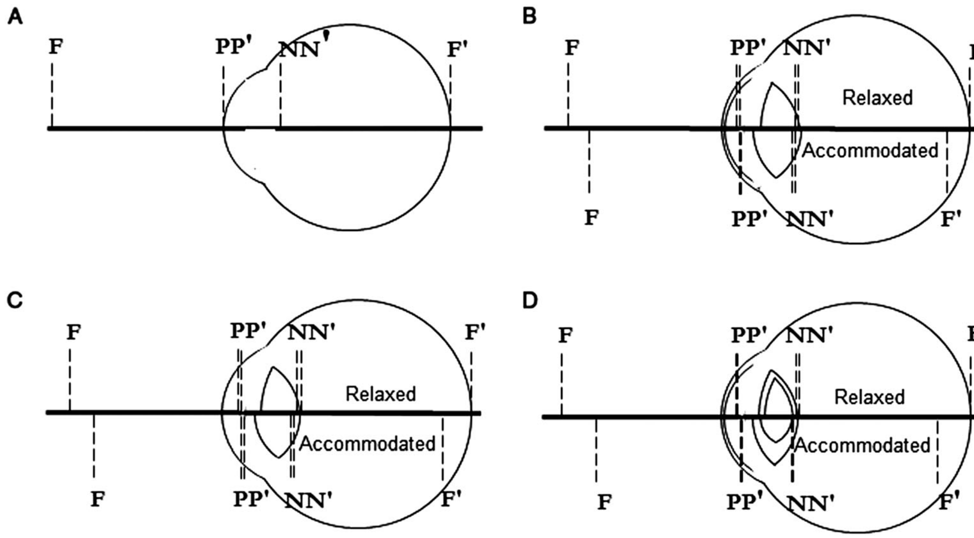


**Figure 6. Paraxial schematic eyes. A) Emsley reduced, B) Gullstrand–Emsley simplified eye, C) Le Grand full theoretical, D) Gullstrand number 1 'exact'.**

eye, Gaussian theory assures us that they exist even in eyes with astigmatic refracting surfaces that may or may not have collinear centres of curvature.[5]

## SINGLE REFRACTING SURFACE (REDUCED EYES)

The single refractive surface optical model eyes, also called reduced eyes, are the simplest of the schematic eyes. These are anatomically inaccurate because there is no crystalline lens and this is compensated by an extra-powerful cornea and having a short length. Apart from the fact that they cannot demonstrate accommodation, they can be functionally accurate, with the cardinal points near to correct positions. The simplicity of reduced eyes is responsible for their popularity among optometric students learning about refractive error, astigmatism, blur and their effects on the retinal image.

Reduced eyes have a genealogy stretching back 360 years to Huygens.[6–8] The best known example is Emsley's reduced eye[7] (Figure 6A), with a power of 60 D produced by a corneal radius of curvature of 50/9 mm (or $5.5\dot{5}$ mm) and a refractive index of 4/3 (or $1.3\dot{3}$). It has no intrinsic aperture stop but this can be placed at the cornea or slightly inside the eye. Allowing the refractive index of the model to vary with wavelength makes the reduced eye a useful introduction to ocular chromatic aberrations and their effects on vision.[9]

## THREE REFRACTING SURFACES (SIMPLIFIED EYES)

The next level of sophistication is to have three refracting surfaces, one for the cornea and two for the lens. In such models, the aperture stop is placed in an anatomically correct position at the front of the lens, accommodated forms can be provided and the cardinal points can be accurately placed.

The genealogy for these models stretches back 210 years.[6–8,10–14] Such models are preferred for refractive error and accommodation calculations, as often there is little to be gained by more complex models. A good example is the Gullstrand number 2 eye as modified by Emsley — the Gullstrand-Emsley Eye[7] (Figure 6B). This comes in relaxed and 10.9 D accommodated forms, with the lens moving forward and being more curved in the latter.

## FOUR REFRACTING SURFACES

These models have two corneal and two lens refracting surfaces. A good example is Le Grand's full theoretical eye[12], which comes in relaxed and 7.1 D accommodated forms (Figure 6C). To change from the relaxed to the accommodated form, the lens becomes more curved, the anterior lens surface moves forward 0.4 mm and the posterior surface moves backward by 0.1 mm.

From such models, 'adaptive' optical model eyes have been developed, with equations showing how parameters vary with accommodation and age.[15–17]

## MODELS WITH LENS STRUCTURE

The refractive index of the crystalline lens is not constant but is inhomogeneous in that the refractive index increases from the edge toward the centre, that is, there is a gradient index. The gradient index has its own power independent of the surface powers, which causes the total refracting power of the lens to be greater than would be expected from its surface powers. In the three and four refracting surface models, such as the ones mentioned above, the lack of a gradient index has been compensated by increasing surface power by having an 'equivalent' index

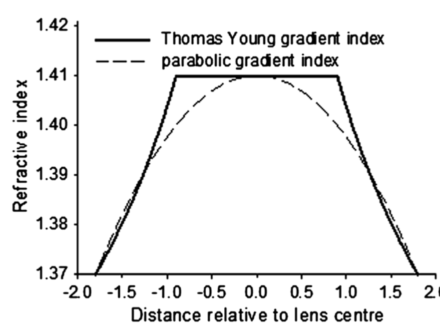


**Figure 7. Refractive index distribution according to the model developed by Thomas Young. A parabolic distribution is shown for comparison.**

that is higher than occurs anywhere in the real lens.

An early attempt to model the gradient index of the eye was made by the English polymath Thomas Young.[18,19] His function of the refractive index against position along the optical axis, is shown in Figure 7, together with a more realistic parabolic function.

Gradient index optics complicates analysis. Model eye builders in the 20th century responded by approximating the true, gradient index nature of the lens with nested, homogeneous shells with different refractive indices. The first of these was Gullstrand's No. 1 'exact' eye[10] in 1909, which had two corneal and four lens surfaces. It has both relaxed and accommodated (10.9 D) forms (Figure 6D). The outer cortex had a refractive index of 1.386 and the inner nucleus had a refractive index of 1.406. The lens power is greater than it would be if made of a homogeneous material with a refractive index of that of the nucleus. Other models have followed with greater numbers of shells.[20–24]

With the development of computers, ray-tracing through gradient lens media has become commonplace. There is no longer a need for the lens to be modelled as a series of shells. Models of gradient index have developed as more is understood about the internal optical structure of the lens.[6,25–34] Figure 8 shows iso-indical contours of some lens models — the step size is 0.005. From left to right they are the distribution of Gullstrand from which his shell lens model was developed,[10] the lens of the Liou and Brennan schematic eye[35] and a distribution pattern according to Navarro, Palos and Gonzáles.[31] For the latter, the point of highest refractive index has moved toward the back of the lens, as has been found experimentally[36] but the lens shape is not anatomically accurate at the equator. Bahrami and Goncharov[27] developed a 'geometric-invariant' refractive index structure, similar to that of Navarro, Palos and Gonzáles, except for smoothing so that the anterior and posterior surfaces and iso-indical contours smoothly meet.

## PARAXIAL VERSUS FINITE OPTICAL MODEL EYES

Many optical model eyes give accurate predictions of retinal image quality only for small pupils and if the object is close to the optical axis — these are referred to as 'paraxial' models. If these conditions are not fulfilled, their aberrations and retinal image quality are worse than usually occurs in real eyes. To improve predictions of optical imaging, 'finite' model eyes began to appear in the 1970s. Better known ones include those of Lotmar,[37] Drasdo and Fowler,[38] Kooijman,[39] Navarro Santamaría and Bescós[16] and Liou and Brennan.[35]

Several finite optical model eyes are adaptations of existing paraxial model eyes. Adaptations include: asphering one or more of the surfaces, for example, as conicoids, placing the fovea off-centre in models, as it is about five degrees away from the best fit optical axis in real eyes, and tilting and decentring surfaces and the aperture. If we are concerned about aberrations and retinal image quality in the periphery, we need to include a curved retina. While curved retinas are shown in Figure 6 for the paraxial eyes, these are not part of the model. If we want to determine the chromatic aberrations or determine image quality in polychromatic light, we need to vary the refractive indices of media as a function of wavelength.[16,40,41]

Some of the new finite model eyes are very sophisticated, encyclopaedic in scope, with all of the features mentioned in the previous paragraph and gradient index distributions.[42] Some use 'reverse engineering', in which measured on-axis and off-axis aberration and ocular biometry in a population are used with an optimisation routine in an optical design program to determine other parameters.[43–45] Although the derived parameters may not be anatomically accurate, the model may nevertheless be useful for describing the eye's functional capabilities. The most recently presented model appears to do an excellent job of matching the mean aberrations in the population from which it was derived.[46]

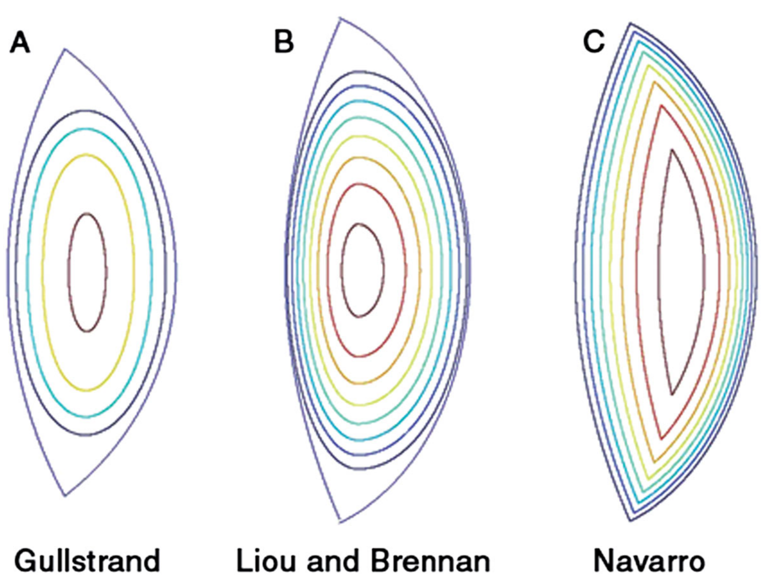


**Figure 8. Lens shapes and iso-incidal contours of A) the model on which the unaccommodated version of the Gullstrand number 1 eye is based,[10] B) the Liou and Brennan eye[35] and C) a distribution based on Navarro, Palos and Gonzáles[31] in 2007.**

## POPULATION AND CUSTOMISED MODEL EYES

Most optical model eyes have been generic, representing population averages. These can be developed for clinically normal and abnormal situations and can be stratified by age, gender, ethnicity, refractive error and accommodation.

As an example of a population study, Table 1 summarises a study conducted on optical models for emmetropic and myopic eyes in a young adult population.[47] The models had four refracting surfaces and a lens gradient index. The table shows the refractive indices, radii of curvature, asphericities and internal distances of the models. The following parameters changed with refraction: anterior radius of curvature of the cornea, the vitreous chamber depth, and both the radius of curvature and asphericity of the retina. Note also that beyond about 2.00 D of myopia that the

| Medium | Refractive index | Radius of curvature (mm) | Asphericity | Distance to next surface (mm) |
|---|---|---|---|---|
| Air | 1.0 | +7.77 + 0.022 *SR* | | |
| Cornea | 1.376 | +6.40 | | 0.55 |
| Aqueous | 1.3374 | +11.48 | | 3.15 |
| Anterior lens | $1.371 + 0.037r^2$ | Infinity | | 1.44 |
| Posterior lens | $1.416 - 0.037r^2$ | –5.90 | | 2.16 |
| Vitreous | 1.336 | *x*: –12.91 – 0.094 *SR* | *x*: +0.27 + 0.026 *SR* | 16.28 – 0.299*SR* |
| | | *y*: –12.72 + 0.004 *SR* | *y*: +0.25 + 0.017 *SR* | |
| Retina | | | | |

**Table 1. Parameters of optical model eyes, as a function of spectacle refraction *SR* in dioptres. Based on Table 1 of Atchison.[47] '*r*' is the relative distance from the centre of the lens to the edge in any direction.**

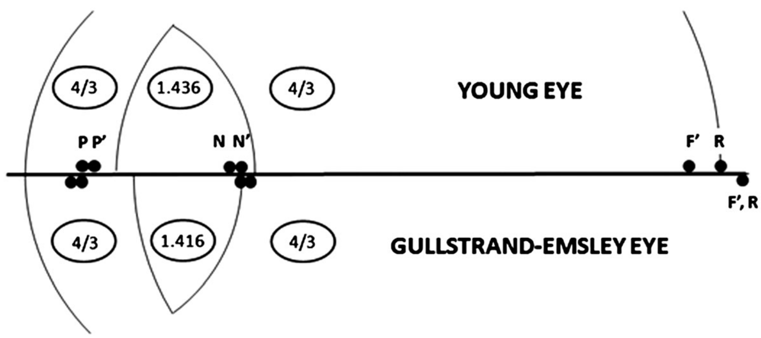


**Figure 9. Model eye based on Thomas Young's parameters of his left eye.[14] The Gullstrand–Emsley eye is shown for comparison. Refractive indices of media are shown in ellipses. Cardinal points are P, P', F, F', N and N' and the retina is given by R. We thank the *Journal of Vision* for permission to reproduce this figure from Atchison and Charman.[52]**

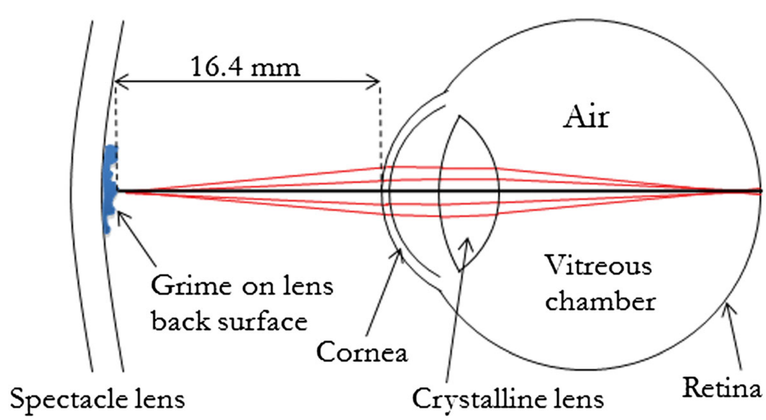


**Figure 10. Ray-trace from the back surface of a lens into the Le Grand full theoretical eye when the vitreous has been replaced by air. Reproduced from Efron[50]. I thank *mivision* for permission to republish this figure.**

retina has smaller radius of curvature, that is, it is steeper, along the horizontal meridian than along the vertical meridian.

Thomas Young[14] introduced the idea of making a schematic eye for the individual, which he did for himself using measurements on his own eye: 'I have endeavoured to express the form of every part of my eye, as nearly as I have been able to ascertain it.' Thomas Young's experiments were rather heroic. These were done by himself and required the use of mirrors for corneal measurements. The length of the eye was measured with a modified pair of dividers with small rings at both points. Young inverted his eye as much as possible and the rings were placed outside the cornea and the macula. The pressure at the back of the eye produced an entoptic ring phosphene, which he kept in the centre of the visual field. He subtracted 0.8 mm to allow for the coats of the thicknesses of the eyes to get an internal axial length of 23.1 mm. Figure 9 shows a schematic diagram of his eye, with the Gullstrand–Emsley model eye shown for comparison. More recent work on customised models includes that of Navarro, Gonzáles and Hernandez-Matamoros.[48]

## WHICH OPTICAL MODEL EYE TO USE?

Someone who wants to use optical model eyes has to decide which one to use. He or she could decide to use the most anatomically correct model that is available; however, it is possible that this is too complex and unwieldy to be useful for other applications and the increasing complexity of models may make it harder to use them as useful thinking tools.

A good guide to aid the choice is the law of parsimony (Occam's razor) that 'entities should not be multiplied needlessly and the simplest of two competing theories is to be preferred.'[49]

Applying this to optical model eyes means using the simplest model that is adequate for an application. This may be a model that is functionally accurate but anatomically inaccurate.

A case study demonstrating the law of parsimony follows. An academic colleague came to see one of us about a problem he was noticing with his vision following a vitrectomy of one of his eyes. A vitrectomy means that air had, temporarily, replaced the vitreous of the eye. When wearing his spectacles, the colleague noticed blobs of gelatinous-like matter that moved with slight movements of the spectacles and which he suspected was grime on the spectacle lens. Modelling was performed with the classical four-surface Le Grand schematic eye (Figure 6C) but replacing the 1.336 vitreous index by 1.0 corresponding to air (Figure 10). The colleague had been turned into a 61 D myope, confirming his suspicion that he was indeed focusing at the back surface of his spectacle lens. A simple model was used here. It was not necessary to aspherise surfaces, tilt surfaces, include a gradient index et cetera to demonstrate the phenomenon being experienced. A more colourful account of this study is available.[50]

## WHERE TO IN THE FUTURE?

In this review, we have covered what are model eyes, their purposes, some history, the different levels of complexity and which optical model eye should be used in an application.

As more studies are done of ocular biometry in populations, optical eye models will increase in number. As we learn more about the optical structure of the eye and in particular the lens gradient index and retinal shape, most likely models will increase in complexity.

In addition, there may be a role for models which become simpler and more abstract. Features of such models might include few refracting surfaces, which are free-form or phase-plate in nature.

## ACKNOWLEDGEMENTS

One of the greatest joys of a research career in academia is being inspired by and collaborating with, other researchers. While acknowledging the help of many people in his career, the first author would like to single out a few individuals. First, he is grateful for the research culture developed by Barry Cole and Barry Collin, while he was a student in the Department of Optometry at The University of Melbourne, and he thanks Ken Bowman and Leo Carney for replicating this culture at the School of Optometry at the Queensland University of Technology. A physicist called George Smith took a lecturing position in the department in Melbourne and was trying to learn about vision at the same time that Atchison was trying to learn about optics. This started a fruitful collaboration that was to last for 30 years. This paper is based in part on a paper written by George Smith 20 years ago.[51] The first author also thanks the second author for many stimulating discussions over the last 30 years and finally thanks his mentor and colleague Neil Charman, who taught him that research can be a lot of fun.

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