Wiley StatsRef:
Statistics Reference Online

Sampling With and Without Replacement

By *Jason Hsia*

Keywords: *sampling, finite population*

Abstract: Sampling with replacement is a class of sampling procedures in which the selected elements are replaced in the selection pool following each "draw" and may be reselected on subsequent draws. In contrast, sampling without replacement is a class of procedures in which the already selected elements are not replaced in the pool and cannot be selected again.

Sampling with replacement is a class of sampling procedures in which the selected elements are replaced in the selection pool following each "draw" and may be reselected on subsequent draws. In contrast, *sampling without replacement* is a class of procedures in which the already selected elements are not replaced in the pool and cannot be selected again [1, p. 37].

Under **simple random sampling** without replacement of a sample of size n from a population of size N, the first element of the sample is selected with a probability of $1/N$; the second element of the sample is then selected from the remaining $N - 1$ elements with a probability of $1/(N - 1)$; and so on. Finally, the nth element of the sample is selected with a probability of $1/(N - n + 1)$. It can also be shown from elementary combinatorial theory that the probability of the kth element of the population being included in the sample, under simple random sampling without replacement is n/N.

Under simple random sampling with replacement of a sample of size n from a population of size N, n independent draws are made such that in each draw, each element of the population has an equal probability, $1/N$, of being selected. Since, after each draw, the selected element is replaced into the population, some elements in the sample may be drawn more than once. Thus, the probability that the kth element of the population is *not* included in the sample is $(1 - 1/N)^n$ and its inclusion probability is $1 - (1 - 1/N)^n$ [2, p. 49].

Some further points concerning the relationship between sampling with and without replacement are as follows:

1. The variance of conventional estimators (e.g. means, total, or ratios) under simple random sampling without replacement is $(N - n)/(N - 1)$ times its value under simple random sampling with replacement (*see* **Finite Population Correction**), when conventional estimators are applied [3, pp.

Centers for Disease Control & Prevention, Atlanta, GA, USA

This article was originally published online in 2005 in Encyclopedia of Biostatistics, © John Wiley & Sons, Ltd and republished in Wiley StatsRef: Statistics Reference Online, 2014.

29–30]. That is, the variance in sampling without replacement is smaller than that in sampling with replacement. However, the variances under the two sampling schemes are nearly equal when the population size, N, is large and the sample size, n, is small.

2. In practice, almost all sampling is conducted without replacement, and the "classical" methods of **finite population sampling** theory were developed primarily for this class of sampling designs.
3. Under certain conditions, results derived under the assumptions of sampling with replacement are approximately equivalent to those that would have been obtained under sampling without replacement. One example is **sampling with probability proportional to size**.

References

[1] Kish, L. (**1965**). *Survey Sampling*. Wiley, New York.
[2] Sarndal, C. E., Swensson, B. & Wretman, J. (**1992**). *Model Assisted Survey Sampling*. Springer-Verlag, New York.
[3] Cochran, W. G. (**1977**). *Sampling Techniques*. Wiley, New York.