CS 173, Spring 19 Examlet 12, white

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $f : \mathbb{Z}_{12} \to \mathbb{P}(\mathbb{Z}_{12})$ be defined by $f(x) = \{y \in \mathbb{Z}_{12} \mid y^2 = x\}$. Let $S = \{f(x) \mid x \in \mathbb{Z}_{12}\}$.

(3 points) $S =$

(Write elements of $\mathbb{Z}_{12}$ as plain integers, without brackets.)

(3 points) Is S a partition of $\mathbb{Z}_{12}$? Check the partition properties that are satisfied.

No Empty set ☐ No Partial Overlap ☐ Covers base set ☐

(7 points) Suppose that $A_1, A_2, \ldots, A_n$ are non-empty subsets of A, and let $P = \{A_1, A_2, \ldots, A_n\}$. Also suppose that $A_1 \cap A_2 \cap \ldots \cap A_n = \emptyset$ and $A_1 \cup A_2 \cup \ldots \cup A_n = A$. Is P a partition of A? Explain why or why not.

(2 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is

an integer ☐ a set of integers ☐ undefined ☐

a power set ☐ one or more integers ☐

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(7 points) Suppose that A is a set and P is a collection of subsets of A. Using precise language and/or notation, state the conditions P must satisfy to be a partition of A.

(2 points) $\{\{p,q\} \;:\; p \in \mathbb{Z}^+, q \in \mathbb{Z}^+, \text{ and } pq = 6\} \;=$

(6 points) Check the (single) box that best characterizes each item.

$\{\{a,b\},c\} = \{a,b,c\}$ true ☐ false ☐

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(3)$ is a rational ☐ a set of rationals ☐ undefined ☐ a power set ☐ one or more rationals ☐

$\binom{k}{k-1}$ 1 ☐ 2 ☐ k-1 ☐ k ☐ undefined ☐

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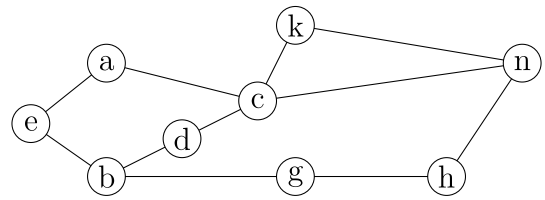
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is at right.
V is the set of nodes. E is the set of edges.



Let $M : (V, \mathbb{N}) \to \mathbb{P}(V)$ such that $M(x, n) = \{y \in V \mid \text{ there is a path of length } n \text{ from } x \text{ to } y\}$.
Let $P(x) = \{M(x, n) \mid n \in \mathbb{N}\}$.

(3 points) $M(c, 2) =$

(3 points) Is $P(c)$ a partition of V? Check the partition properties that are satisfied.

No Empty set ☐ No Partial Overlap ☐ Covers base set ☐

(7 points) Let $f : X \to Y$ be any function, and let A and B be subsets of X. For any subset S of X define its image $f(S)$ by $f(S) = \{f(s) \in Y \mid s \in S\}$. Is it the case that $f(A) \cap f(B) = f(A \cap B)$? Informally explain why this is true or give a concrete counter-example showing why it is not.

(2 points) Check the (single) box that best characterizes each item.

$\{4, 5, 6\} \cap \{6, 7\}$ 6 ☐ $\{6\}$ ☐ $\{\{6\}\}$ ☐

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(7 points) Suppose that $f : A \to B$ is a function. Let's define $T : B \to \mathbb{P}(A)$ by $T(m) = \{x \in A \mid f(x) = m\}$. Then let $P = \{T(m) \mid m \in B\}$. Under what conditions is P a partition of A? Briefly justify your answer.

(2 points) $\{p + q^2 \ \mid \ p \in \mathbb{Z},\ q \in \mathbb{Z},\ 1 \leq p \leq 2 \text{ and } 1 \leq q \leq 3\} \ =$

(6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$ always ☐ sometimes ☐ never ☐

Set B is a partition of a finite set A. Then $|B| \leq 2^{|A|}$ ☐ $|B| \leq |A|$ ☐ $|B| = 2^{|A|}$ ☐ $|B| \leq |A + 1|$ ☐

Pascal's identity states that $\binom{n}{k}$ is equal to $\binom{n-1}{k} + \binom{n-1}{k-1}$ ☐ $\binom{n-1}{k} + \binom{n-1}{k+1}$ ☐ $\binom{n-1}{k} + \binom{n-2}{k}$ ☐

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Let $f : \mathbb{R} \to \mathbb{P}(\mathbb{R})$ such that $f(x) = \{p \in \mathbb{R} \mid \lfloor x \rfloor = \lfloor p \rfloor\}$. Let $T = \{f(x) \mid x \in \mathbb{R}\}$.

(3 points) Describe (at a high level) the elements of $f(7)$:

(3 points) Is T a partition of $\mathbb{R}$? Check the partition properties that are satisfied.

No Empty set ☐ No Partial Overlap ☐ Covers base set ☐

(7 points) Define $f : \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z})$ by $f(x, k) = \{y \in \mathbb{Z} : x = y + kn \text{ for some } n \in \mathbb{Z}\}$. Suppose that $k|p$. Compare $f(r, k)$ and $f(r, p)$. Justify your answer.

(2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(\emptyset)$	$\emptyset$ ☐	$\{\emptyset\}$ ☐	$\{\{\emptyset\}\}$ ☐	$\{\emptyset, \{\emptyset\}\}$ ☐

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(7 points) Let $f : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ be defined by $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$. Suppose that $f(a) = f(b) \cap f(c)$. Express a in terms of b and c. Briefly justify your answer.

(2 points) $\{\{p\} \mid p \in \{2,3,4\}\ \} =$

(6 points) Check the (single) box that best characterizes each item.

$|\{A \subseteq \mathbb{Z}_4\ :\ |A| \text{ is even}\}|$ 1 ☐ 6 ☐ 7 ☐ 8 ☐ infinite ☐

There is a set A such that $|\mathbb{P}(A)| \leq 2$. true ☐ false ☐

$\binom{n}{1}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

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Suppose that $A = \{2, 3, 5, 13, 17\}$. Define a function $F : A \to \mathbb{P}(A)$ and a set S by $F(x) = \{y \in A \mid y \text{ is a factor of } x\}$ $\quad S = \{F(x) \mid x \in A\}$

(3 points) $S =$

(3 points) Is S a partition of A? Check the partition properties that are satisfied.

No Empty set ☐ No Partial Overlap ☐ Covers base set ☐

(7 points) Let $f : X \to Y$ be any function, and let A and B be subsets of X. For any subset S of X define its image $f(S)$ by $f(S) = \{f(s) \in Y \mid s \in S\}$. Is it the case that $f(A) \cup f(B) = f(A \cup B)$? Informally explain why this is true or give a concrete counter-example showing why it is not.

(2 points) Check the (single) box that best characterizes each item.

A partition of a set A contains $\emptyset$ always ☐ sometimes ☐ never ☐

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(7 points) Give an example of a partition P of $\mathbb{N}$ where the set P is infinite. Be specific.

(2 points) $\{pq \mid p \in \mathbb{N},\ q \in \mathbb{N},\ p+q=6\} =$

(6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$ always ☐ sometimes ☐ never ☐

$|\{\emptyset\}|$ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ undefined ☐

$\{4,5\} \cap \{6,7\}$ $\emptyset$ ☐ $\{\emptyset\}$ ☐ nothing ☐ undefined ☐

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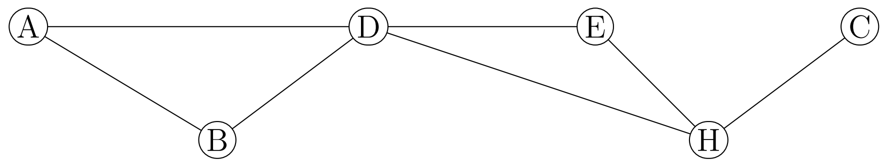
Discussion: Friday 11 12 1 2 3 4

Graph G is at right.
V is the set of nodes in G.
$M = \{0, 1, 2, 3, 4\}$



Define $f : M \to \mathbb{P}(V)$ by $f(n) = \{p \in V \ : \ d(p, E) = n\}$, where $d(a, b)$ is the (shortest-path) distance between a and b. Let $P = \{f(n) \mid n \in M\}$.

(6 points) Fill in the following values:

f(0) =

f(1) =

P =

(7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always ☐ sometimes ☐ never ☐

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Name:____

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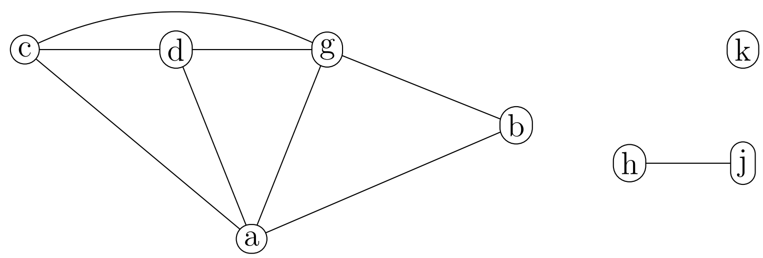
Discussion: Friday 11 12 1 2 3 4

Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(E)$ be defined by $f(n) = \{e \in E \mid n \text{ is an endpoint of } e\}$. And let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|V| =$

$f(d) =$

$f(h) =$

(7 points) Is T a partition of E? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

(2 points) State the definition of $\binom{n}{k}$, i.e. express $\binom{n}{k}$ in terms of more basic arithmetic operations.

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(7 points) Let $f : X \to Y$ be any function, and let A and B be subsets of X. For any subset S of X define its image $f(S)$ by $f(S) = \{f(s) \in Y \mid s \in S\}$. Is it the case that $f(A \cap B) \subseteq f(A) \cap f(B)$? Informally explain why this is true or give a concrete counter-example showing why it is not.

(8 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A \cap B) \subseteq \mathbb{P}(A \cup B)$ always ☐ sometimes ☐ never ☐

Pascal's identity states that $\binom{n}{k}$ is equal to $\binom{n-1}{k} + \binom{n-1}{k-1}$ ☐ $\binom{n-1}{k} + \binom{n-1}{k+1}$ ☐ $\binom{n-1}{k} + \binom{n-2}{k}$ ☐

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(1.73)$ is a rational ☐ a set of rationals ☐ undefined ☐ one or more rationals ☐ a power set ☐

Set B is a partition of a finite set A. Then $|A| = |B|$. always ☐ sometimes ☐ never ☐

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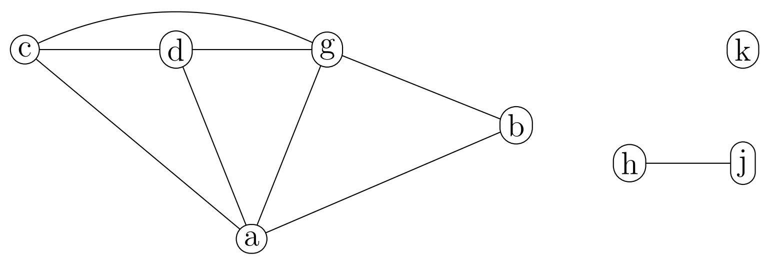
Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(E)$ be defined by $f(n) = \{e \in E \mid n \text{ is an endpoint of } e\}$. And let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|V| =$

$f(d) =$

$f(h) =$

(7 points) Is T a partition of E? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{P}(\mathbb{Q}) \to \mathbb{N}$ then $f(\{3\})$ is

an integer ☐	a set of integers ☐	undefined ☐
one or more integers ☐	a power set ☐	

Name:____

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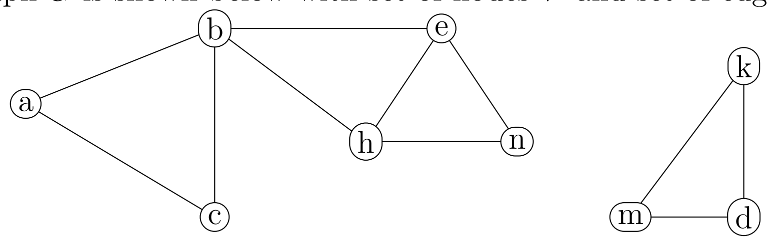
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

Graph G is shown below with set of nodes V and set of edges E.



Let $F : V \to \mathbb{P}(V)$ such that $F(n) = \{v \in V \mid \text{ there is a cycle containing } n \text{ and } v\}$.
Let $T = \{F(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$F(k) =$

$F(b) =$

$|T| =$

(7 points) Is T a partition of V? For each of the three conditions required to be a partition, explain why T does or doesn't satisfy that condition.

(2 points) State Pascal's identity.

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(7 points) Suppose that R is a relation on $\mathbb{Z}$ which is reflexive and symmetric, but not transitive. Let's define $T(n) = \{a \in \mathbb{Z} \mid aRn\}$. Notice that $n \in T(n)$ for any integer n. The collection of all sets $T(n)$ does not form a partition of $\mathbb{Z}$. Explain (informally but clearly) why the fact that R is not transitive can cause one of the partition properties to fail.

(8 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{P}(\mathbb{Q}) \to \mathbb{N}$ then $f(3)$ is

an integer ☐ a set of integers ☐ undefined ☐
one or more integers ☐ a power set ☐

$\{\mathbb{N}\}$ is a partition of $\mathbb{N}$. true ☐ false ☐

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$ always ☐ sometimes ☐ never ☐

$\binom{n}{0}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(7 points) Suppose that $g : A \to B$ is an onto function. Let's define $F(y) = \{x \in A \mid g(x) = y\}$. Then define $P = \{F(y) \mid y \in B\}$. Is P a partition of A? Briefly justify your answer.

(2 points) State the binomial theorem.

(6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always ☐ sometimes ☐ never ☐

$\binom{0}{0}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

$\mid \mathbb{P}(\{4,5,6,7,8\} \times \emptyset) \mid$ $\emptyset$ ☐ $\{\emptyset\}$ ☐ 0 ☐ 1 ☐ 25 ☐ 2^5 ☐

Name:____

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Let $f : \mathbb{R}^2 \to \mathbb{P}(\mathbb{R}^2)$ be defined by $f(x, y) = \{(p, q) \in \mathbb{R}^2 \mid \exists \alpha \in \mathbb{R}, (p, q) = \alpha(x, y)\}$.
Let $T = \{f(x, y) \mid (x, y) \in \mathbb{R}^2\}$.

(6 points) Answer the following questions:

$f(0, 0) =$

Describe (at a high level) the elements of $f(0, 36)$:

Give an element of $\mathbb{P}(\mathbb{R}^2) - T$:

(7 points) Is T a partition of $\mathbb{R}^2$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Q} \to \mathbb{P}(\mathbb{Q})$ then $f(1.73)$ is

a rational ☐	a set of rationals ☐	undefined ☐
one or more rationals ☐	a power set ☐	

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(7 points) Let $f : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ be defined by $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$. Suppose that $m = ab$, where a and b are two different primes. Express $f(m)$ in terms of $f(a)$ and $f(b)$. Briefly justify your answer.

(8 points) Check the (single) box that best characterizes each item.

$|\{A \subseteq \mathbb{Z}_4 \ : \ |A| \text{ is even}\}|$ 1 ☐ 6 ☐ 7 ☐ 8 ☐ infinite ☐

$\binom{n}{1}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

There is a set A such that $|\mathbb{P}(A)| \leq 2$. true ☐ false ☐

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is an integer ☐ a set of integers ☐ undefined ☐ one or more integers ☐ a power set ☐

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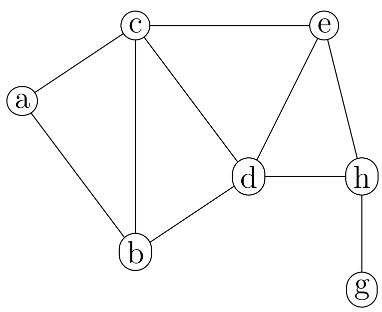
Graph G with set of nodes V is shown below. Recall that $deg(n)$ is the degree of node n. Let's define $f : \mathbb{N} \to \mathbb{P}(V)$ by $f(k) = \{n \in V : \deg(n) = k\}$. Also let $T = \{f(k) \mid k \in \mathbb{N}\}$.



(6 points) Fill in the following values:

$f(4) =$

$f(1) =$

$|T| =$

(7 points) Is T a partition of V? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always ☐ sometimes ☐ never ☐

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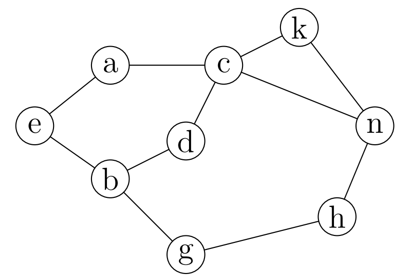
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is shown at right with set of nodes V and set of edges E. Let $M : (V, \mathbb{N}) \to \mathbb{P}(V)$ be defined by $M(x, n) = \{y \in V \mid \text{ there is a path of length } n \text{ from } x \text{ to } y\}$. Let $P(x) = \{M(x, n) \mid n \in \mathbb{N}\}$.



(6 points) Give the value of $M(c, n)$, for all values of n from 0 to 3.

(7 points) Is $P(c)$ a partition of V? For each of the three conditions required to be a partition, explain why $P(c)$ does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$ always ☐ sometimes ☐ never ☐

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Let $f : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ be defined by $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$. For which natural numbers a and b is $f(a)$ a subset of $f(b)$? Briefly justify your answer.

(8 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(3)$ is

a rational ☐ a set of rationals ☐ undefined ☐
one or more rationals ☐ a power set ☐

$\{\{a, b\}, c\} = \{a, b, c\}$

true ☐ false ☐

Set B is a partition of a finite set A. Then

$|B| \leq 2^{|A|}$ ☐ $|B| \leq |A|$ ☐
$|B| = 2^{|A|}$ ☐ $|B| \leq |A + 1|$ ☐

$\binom{n}{1}$

-1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

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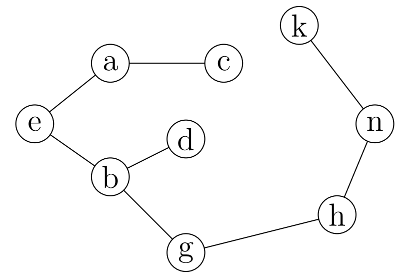
NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is shown at right with set of nodes V and set of edges E. Let $M : (V, \mathbb{N}) \to \mathbb{P}(V)$ be defined by
$M(x, n) = \{y \in V \mid$ there is a path of length n from x to $y\}$.
Let $P(x) = \{M(x, n) \mid n \in \mathbb{N}\}$.



(6 points) Give the value of $M(g, n)$, for all values of n from 0 to 3.

(7 points) Is $P(g)$ a partition of V? For each of the three conditions required to be a partition, explain why $P(g)$ does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

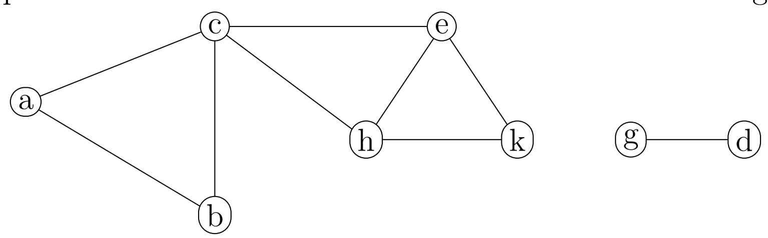
$\binom{n}{n}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is shown below with set of nodes V and set of edges E.



Let $F : V \to \mathbb{P}(V)$ such that $F(n) = \{v \in V \mid \text{ there is a cycle containing } n \text{ and } v\}$. Let $T = \{F(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$F(g) =$

$F(b) =$

$F(k) =$

(7 points) Is T a partition of V? For each of the three conditions required to be a partition, explain why T does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

$\binom{n}{0}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

Name:____

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Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Can we create a set C such that C is a partition of $\mathbb{R}$ but $|C|$ is finite? Give a specific set C that works or briefly explain why it's impossible.

(8 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always ☐ sometimes ☐ never ☐

If $n \geq k \geq 0$, then $\binom{n}{k} = \binom{n}{n-k}$ true ☐ true for some n and k ☐ false ☐

$\binom{n}{0}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$ always ☐ sometimes ☐ never ☐

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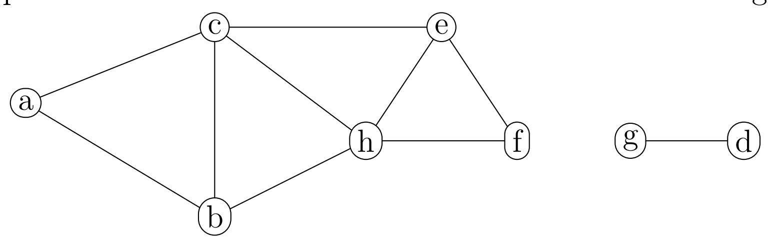
Graph G is shown below with set of nodes V and set of edges E.



Let $f : V \to \mathbb{P}(V)$ such that $f(n) = \{v \in V \mid \text{ there is a cycle containing } n \text{ and } v\}$.
Let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|E| =$

$f(b) =$

$f(h) =$

(7 points) Is T a partition of V? For each of the three conditions required to be a partition, explain why T does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$ always ☐ sometimes ☐ never ☐

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Let $f : \mathbb{R}^2 \to \mathbb{P}(\mathbb{R}^2)$ be defined by $f(x, y) = \{(p, q) \in \mathbb{R}^2 \mid x^2 + y^2 = p^2 + q^2\}$
Let $T = \{f(x, y) \mid (x, y) \in \mathbb{R}^2\}$.

(6 points) Answer the following questions:

$f(0, 0) =$

Describe (at a high level) the elements of $f(0, 36)$:

The cardinality of (aka the number of elements in) T is:

(7 points) Is T a partition of $\mathbb{R}^2$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

Let A be a non-empty set, $\{A\}$ is a partition of A.	always ☐	sometimes ☐	never ☐

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(7 points) Suppose that A and B are disjoint sets, C_A is a partition of A and C_B is a partition of B. Is $C_A \cup C_B$ a partition of $A \cup B$? Briefly justify your answer.

(8 points) Check the (single) box that best characterizes each item.

$|\mathbb{P}(\mathbb{P}(\emptyset))|$ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ undefined ☐

If $f : \mathbb{P}(\mathbb{Q}) \to \mathbb{N}$ then $f(3)$ is a rational ☐ a power set of rationals ☐ a set of rationals ☐ undefined ☐

$|\{\emptyset\}|$ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ undefined ☐

$\mathbb{P}(A \cup B) = \mathbb{P}(A) \cup \mathbb{P}(B)$ always ☐ sometimes ☐ never ☐

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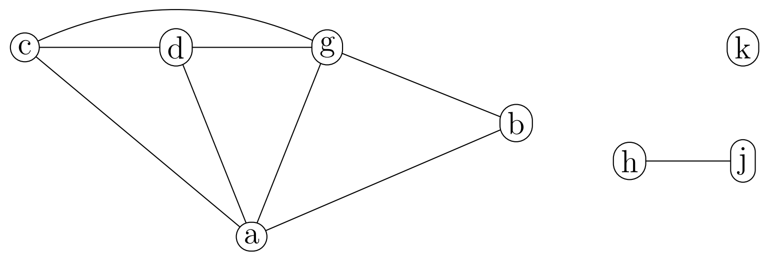
Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(V)$ be defined by $f(n) = \{v \in V \mid \text{there is a path from n to v}\}$. And let $T = \{f(n) \mid n \in V\}$.

6 points) Fill in the following values:

$f(k) =$

$f(d) =$

$T =$

(7 points) Is T a partition of V? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

$\binom{n}{1}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

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(7 points) Can a set A be a partition of the empty set? Briefly justify your answer.

(8 points) Check the (single) box that best characterizes each item.

Pascal's identity states that $\binom{n+1}{k}$ is equal to $\binom{n}{k} + \binom{n}{k+1}$ ☐ $\binom{n}{k} + \binom{n-1}{k}$ ☐ $\binom{n}{k} + \binom{n}{k-1}$ ☐

$\mathbb{P}(A \cap B) \subseteq \mathbb{P}(A \cup B)$ always ☐ sometimes ☐ never ☐

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is an integer ☐ a set of integers ☐ one or more integers ☐ a power set ☐

A partition of a set A contains A always ☐ sometimes ☐ never ☐