# CS 173, Spring 19 Examlet 11, white sheet

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

Algorithm A takes $\log_2 n$ time. On one input, A takes x time. How long will it take if I double the input size?

$x+1$ [√] $2x$ [ ] $2^x$ [ ] $x^2$ [ ]

$T(1) = c$
$T(n) = 4T(n/2) + n$

$\Theta(n)$ [ ] $\Theta(n \log n)$ [ ] $\Theta(n^2)$ [√] $\Theta(n^3)$ [ ]
$\Theta(n^{\log_3 2})$ [ ] $\Theta(n^{\log_2 3})$ [ ] $\Theta(2^n)$ [ ] $\Theta(3^n)$ [ ]

The running time of binary search is $O(n \log n)$.

true [√] false [ ]

For a problem to satisfy the definition of NP, a "yes" answer must have a succinct justification.

true [√] false [ ]

Deciding whether an input logic expression be made true by appropriate choice of input values.

polynomial [ ] exponential [ ] in NP [√]

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**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

Karatsuba's integer multiplication algorithm

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☐ | $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ |
| $\Theta(n^3)$ | ☐ | $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☑ | $\Theta(2^n)$ | ☐ |

$T(1) = d$
$T(n) = 2T(n/4) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☐ | $\Theta(\sqrt{n})$ | ☐ | $\Theta(n)$ | ☑ | $\Theta(n \log n)$ | ☐ |
| $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

The running time of merge is recursively defined by $T(1) = d$ and $T(n) =$

| | | | |
|---|---|---|---|
| $T(n-1) + c$ | ☑ | $T(n-1) + cn$ | ☐ |
| $2T(n-1) + c$ | ☐ | $2T(n-1) + cn$ | ☐ |

Circuit satisfiability can be solved in polynomial time. true ☐ false ☐ not known ☑

For a problem to satisfy the definition of co-NP, a "no" answer must have a succinct justification. true ☑ false ☐

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**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

The running time of merge

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☐ | $\Theta(n)$ | ☑ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ |
| $\Theta(n^3)$ | ☐ | $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ |

$T(1) = d$
$T(n) = T(n-1) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☑ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

$T(1) = d$
$T(n) = 2T(n/3) + d$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☑ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

The solution to the Tower of Hanoi puzzle with $n$ disks requires $\Theta(2^n)$ steps

true ☑ false ☐ not known ☐

The chromatic number of a graph with $n$ nodes can be found in polynomial time.

true ☐ false ☐ not known ☑

**Name:**_____

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**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(6 points) Fill in the missing bits of a recursive implementation of Merge, which merges two lists of integers sorted in increasing order. Use the functions first (first element), rest (everything after first element), and and cons (adds number to list).

Merge($L_1$,$L_2$: sorted lists of real numbers)

if ($L_1$ is empty and $L_2$ is empty) return emptylist

else if ($L_2$ is empty or first($L_1$) <= first($L_2$))

**Solution:** return cons(first($L_1$),merge(rest($L_1$),$L_2$))

else

**Solution:** return cons(first($L_2$),merge($L_1$,rest($L_2$)))

(9 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n/3) + c$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | √ | $\Theta(n \log n)$ | | $\Theta(n^2)$ | | $\Theta(n^3)$ | |
| $\Theta(n^{\log_3 2})$ | | $\Theta(n^{\log_2 3})$ | | $\Theta(2^n)$ | | $\Theta(3^n)$ | |

The Towers of Hanoi puzzle requires exponential time. true √ false ☐ not known ☐

Finding the chromatic number of a graph with $n$ nodes requires $\Theta(2^n)$ time. true ☐ false ☐ not known √

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

The running time of Karatsuba's algorithm is recursively defined by $T(1) = d$ and $T(n) =$

| | | | |
|---|---|---|---|
| $4T(n/2) + cn$ | ☐ | $4T(n/2) + c$ | ☐ |
| $2T(n/2) + cn$ | ☐ | $3T(n/2) + cn$ | ☑ |

$T(1) = d$
$T(n) = 2T(n-1) + c$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☑ | $\Theta(3^n)$ | ☐ |

The running time of the Towers of Hanoi solver is recursively defined by $T(1) = d$ and $T(n) =$

| | | | |
|---|---|---|---|
| $2T(n-1) + c$ | ☑ | $2T(n-1) + cn$ | ☐ |
| $2T(n/2) + c$ | ☐ | $2T(n/2) + cn$ | ☐ |

For a problem to satisfy the definition of co-NP, a "yes" answer must have a succinct justification. true ☐ false ☑

The Towers of Hanoi puzzle can be solved in polynomial time. true ☐ false ☑ not known ☐

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

Adding element to start of array (array gets longer)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(1)$ | ☐ | $\Theta(\log n)$ | ☐ | $\Theta(n)$ | ☑ | $\Theta(n \log n)$ | ☐ |
| $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

$T(1) = d$
$T(n) = 3T(n/2) + d$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☑ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

$T(1) = c$
$T(n) = 2T(n/2) + n^2$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☑ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

Problems in class NP require exponential time

true ☐ false ☐ not known ☑

The Marker Making problem can be solved in polynomial time.

true ☐ false ☐ not known ☑

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $T(1) = d$<br>$T(n) = T(n/3) + c$ | $\Theta(\log n)$ | √ | $\Theta(\sqrt{n})$ | | $\Theta(n)$ | | $\Theta(n \log n)$ | |
| | $\Theta(n^2)$ | | $\Theta(n^3)$ | | $\Theta(2^n)$ | | $\Theta(3^n)$ | |
| Dividing a linked list in half | $\Theta(1)$ | | $\Theta(\log n)$ | | $\Theta(n)$ | √ | $\Theta(n \log n)$ | |
| | $\Theta(n^2)$ | | $\Theta(n^3)$ | | $\Theta(2^n)$ | | $\Theta(3^n)$ | |
| The running time of the Towers of Hanoi solver is recursively defined by $T(1) = d$ and $T(n) =$ | $2T(n/2) + c$ | | | | $2T(n/2) + cn$ | | | |
| | $2T(n-1) + c$ | √ | | | $2T(n-1) + cn$ | | | |
| Producing all parses for a sentence. | polynomial | | exponential | √ | in NP | | | |
| The Travelling Salesman Problem | polynomial | | exponential | | in NP | √ | | |

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n/2) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☐ | $\Theta(\sqrt{n})$ | ☐ | $\Theta(n)$ | ☑ | $\Theta(n \log n)$ | ☐ |
| $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

Algorithm A takes $2^n$ time. On one input, A takes x time. How long will it take if I add one to the input size?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x + 2$ | ☐ | $2x$ | ☑ | $2^x$ | ☐ | $x^2$ | ☐ |

$T(1) = d$
$T(n) = 3T(n/2) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☑ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

Problems in class P (as in P vs. NP) require exponential time

| | | | | | |
|---|---|---|---|---|---|
| true | ☐ | false | ☑ | not known | ☐ |

The Travelling Salesman problem can be solved in polynomial time.

| | | | | | |
|---|---|---|---|---|---|
| true | ☐ | false | ☐ | not known | ☑ |

# CS 173, Fall 18 Examlet 11, Part B

**Name:**_____

**NetID:**_____ **Lecture: B**

**Discussion: Friday 11 12 1 2 3 4**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n/2) + d$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | √ | $\Theta(\sqrt{n})$ | | $\Theta(n)$ | | $\Theta(n \log n)$ | |
| $\Theta(n^2)$ | | $\Theta(n^3)$ | | $\Theta(2^n)$ | | $\Theta(3^n)$ | |

$T(1) = d$
$T(n) = 2T(n/4) + c$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | | $\Theta(\sqrt{n})$ | √ | $\Theta(n)$ | | $\Theta(n \log n)$ | |
| $\Theta(n^2)$ | | $\Theta(n^3)$ | | $\Theta(2^n)$ | | $\Theta(3^n)$ | |

Finding a value in a sorted array is $\Theta(2^n)$. true ☐ false √

Problems in class P (as in P vs. NP) require exponential time true ☐ false √ not known ☐

The Travelling Salesman problem can be solved in polynomial time. true ☐ false ☐ not known √

**Name:**_____

**NetID:**_____ **Lecture: B**

**Discussion: Friday 11 12 1 2 3 4**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n/3) + c$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Theta(n)$ | √ | $\Theta(n \log n)$ | | $\Theta(n^2)$ | | $\Theta(n^3)$ | |
| $\Theta(n^{\log_3 2})$ | | $\Theta(n^{\log_2 3})$ | | $\Theta(2^n)$ | | $\Theta(3^n)$ | |

$T(1) = d$
$T(n) = T(n/2) + n$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | | $\Theta(\sqrt{n})$ | | $\Theta(n)$ | √ | $\Theta(n \log n)$ | |
| $\Theta(n^2)$ | | $\Theta(n^3)$ | | $\Theta(2^n)$ | | $\Theta(3^n)$ | |

Algorithm A takes $2^n$ time. On one input, A takes x time. How long will it take if I double the input size?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x + 2$ | | $2x$ | | $2^x$ | | $x^2$ | √ |

Problems in class NP require exponential time

| | | | | | |
|---|---|---|---|---|---|
| true | | false | | not known | √ |

Circuit satisfiability can be solved in polynomial time.

| | | | | | |
|---|---|---|---|---|---|
| true | | false | | not known | √ |

# CS 173, Spring 18 Examlet 11, Part B

**Name:** _____

**NetID:** _____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n-1) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | √ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

$T(1) = d$
$T(n) = 2T(n/4) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☐ | $\Theta(\sqrt{n})$ | ☐ | $\Theta(n)$ | √ | $\Theta(n \log n)$ | ☐ |
| $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

The running time of mergesort is $O(n^3)$. true √ false ☐

For a problem to satisfy the definition of NP, a "no" answer must have a succinct justification. true ☐ false √

The Travelling Salesman Problem polynomial ☐ exponential ☐ in NP √

# CS 173, Spring 18 Examlet 11, Part B

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 2T(n/2) + n$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☑ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

$T(1) = d$
$T(n) = 3T(n-1) + c$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☑ |

Algorithm A takes $n^5$ time. On one input, A takes x time. How long will it take if I double the input size?

$2x$ ☐ $5x$ ☐ $32x$ ☑ $x^5$ ☐

The Marker Making problem can be solved in polynomial time.

true ☐ false ☐ not known ☑

Problems in class P (as in P vs. NP) require exponential time

true ☐ false ☑ not known ☐

# CS 173, Spring 18 Examlet 11, Part B

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n/2) + d$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☑ | $\Theta(\sqrt{n})$ | ☐ | $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ |
| $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

$T(1) = d$
$T(n) = 3T(n/3) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☑ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

All ways to assign True/False values to $n$ input variables

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☐ | $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ |
| $\Theta(n^3)$ | ☐ | $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☑ |

Producing all parses for a sentence.

polynomial ☐ exponential ☑ in NP ☐

The chromatic number of a graph with $n$ nodes can be found in polynomial time.

true ☐ false ☐ not known ☑

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n/2) + n$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☑ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

$T(1) = d$
$T(n) = T(n-1) + c$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☐ | $\Theta(\sqrt{n})$ | ☐ | $\Theta(n)$ | ☑ | $\Theta(n \log n)$ | ☐ |
| $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

The running time of the Towers of Hanoi solver is $\Theta(n!)$ true ☐ false ☑

Circuit satisfiability can be solved in exponential time. true ☑ false ☐ not known ☐

Producing all parses for a sentence requires exponential time. true ☑ false ☐ not known ☐

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n/2) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☐ | $\Theta(\sqrt{n})$ | ☐ | $\Theta(n)$ | ☑ | $\Theta(n \log n)$ | ☐ |
| $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

$T(1) = d$
$T(n) = 2T(n/3) + d$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☑ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

Algorithm A takes $2^n$ time. On one input, A takes x time. How long will it take if I add one to the input size?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x + 2$ | ☐ | $2x$ | ☑ | $2^x$ | ☐ | $x^2$ | ☐ |

Problems in class P (as in P vs. NP) can be solved in exponential time

| | | | | | |
|---|---|---|---|---|---|
| true | ☑ | false | ☐ | not known | ☐ |

Deciding if a graph is 2-colorable

| | | | | | |
|---|---|---|---|---|---|
| polynomial | ☑ | exponential | ☐ | in NP | ☐ |

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(6 points) Your partner has implemented the function Merge(A,B), which merges two sorted linked lists of integers. Using Merge, fill in the missing parts of this implementation of Mergesort.

Mergesort($L = (a_1, a_2, \ldots, a_n)$) \\ input is a linked list L containing n integers

**Solution:** if (n=1) return L

p = floor(n/2)

**Solution:**

$L_a = (a_1, \ldots, a_p)$

$L_b = (a_{p+1}, \ldots, a_n)$

return Merge(Mergesort($L_a$), Mergesort($L_b$))

(9 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n/2) + d$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☑ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

Merging two sorted lists

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(1)$ | ☐ | $\Theta(\log n)$ | ☐ | $\Theta(n)$ | ☑ | $\Theta(n \log n)$ | ☐ |
| $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

Circuit satisfiability can be solved in polynomial time.

true ☐ false ☐ not known ☑

# CS 173, Fall 17 Examlet 11, Part B

**Name:** _____

**NetID:** _____ **Lecture:** A B

**Discussion:** Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n/2) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☑ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

$T(1) = d$
$T(n) = 2T(n/2) + c$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☑ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

Adding element to head of linked list

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(1)$ | ☑ | $\Theta(\log n)$ | ☐ | $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ |
| $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

Problems in class NP (as in P vs. NP) can be solved in polynomial time

true ☐ false ☐ not known ☑

The solution to the Tower of Hanoi puzzle with $n$ disks requires $\Theta(2^n)$ steps

true ☑ false ☐ not known ☐

# CS 173, Fall 17 Examlet 11, Part B

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n/2) + d$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☑ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

The running time of binary search is recursively defined by $T(1) = d$ and $T(n) =$

| | | | |
|---|---|---|---|
| $T(n/2) + c$ | ☑ | $T(n/2) + cn$ | ☐ |
| $2T(n/2) + c$ | ☐ | $2T(n/2) + cn$ | ☐ |

Algorithm A takes $\log_2 n$ time. On one input, A takes x time. How long will it take if I double the input size?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x + 1$ | ☑ | $2x$ | ☐ | $2^x$ | ☐ | $x^2$ | ☐ |

The Towers of Hanoi puzzle can be solved in polynomial time.

| | | | | | |
|---|---|---|---|---|---|
| true | ☐ | false | ☑ | not known | ☐ |

Problems in class NP (as in P vs. NP) can be solved in exponential time

| | | | | | |
|---|---|---|---|---|---|
| true | ☑ | false | ☐ | not known | ☐ |

# CS 173, Fall 17 Examlet 11, Part B

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 4T(n/2) + n$

| | | | | |
|---|---|---|---|---|
| $\Theta(n)$ ☐ | $\Theta(n \log n)$ ☐ | $\Theta(n^2)$ ☑ | $\Theta(n^3)$ ☐ |
| $\Theta(n^{\log_3 2})$ ☐ | $\Theta(n^{\log_2 3})$ ☐ | $\Theta(2^n)$ ☐ | $\Theta(3^n)$ ☐ |

Adding element to start of array (array gets longer)

| | | | |
|---|---|---|---|
| $\Theta(1)$ ☐ | $\Theta(\log n)$ ☐ | $\Theta(n)$ ☑ | $\Theta(n \log n)$ ☐ |
| $\Theta(n^2)$ ☐ | $\Theta(n^3)$ ☐ | $\Theta(2^n)$ ☐ | $\Theta(3^n)$ ☐ |

Algorithm A takes $2^n$ time. On one input, A takes x time. How long will it take if I double the input size?

$x + 2$ ☐ $2x$ ☐ $2^x$ ☐ $x^2$ ☑

Deciding whether an input logic expression be made true by appropriate choice of input values.

polynomial ☐ exponential ☐ in NP ☑

For a problem to satisfy the definition of co-NP, a "no" answer must have a succinct justification.

true ☑ false ☐

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 2T(n/4) + c$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☐ | $\Theta(\sqrt{n})$ | ☑ | $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ |
| $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

The running time of the Towers of Hanoi solver is recursively defined by $T(1) = d$ and $T(n) =$

| | | | |
|---|---|---|---|
| $2T(n-1) + c$ | ☑ | $2T(n-1) + cn$ | ☐ |
| $2T(n/2) + c$ | ☐ | $2T(n/2) + cn$ | ☐ |

Algorithm A takes $n^2$ time. On one input, A takes x time. How long will it take if I double the input size?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x + 1$ | ☐ | $2x$ | ☐ | $4x$ | ☑ | $x^3$ | ☐ |

Determining whether a graph with $n$ edges is connected.

| | | | | | |
|---|---|---|---|---|---|
| polynomial | ☑ | exponential | ☐ | in NP | ☐ |

Problems in class NP require exponential time

| | | | | | |
|---|---|---|---|---|---|
| true | ☐ | false | ☐ | not known | ☑ |

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 4T(n/2) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☑ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

$T(1) = c$
$T(n) = 3T(n/3) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☑ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

Changing last value in array

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(1)$ | ☑ | $\Theta(\log n)$ | ☐ | $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ |
| $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

In a set of $n$ 2D points, which pair of points is closest? polynomial ☑ exponential ☐ in NP ☐

For a problem to satisfy the definition of NP, a "yes" answer must have a succinct justification. true ☑ false ☐

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(6 points) Fill in the missing bits of this recursive algorithm for returning the location of a number $k$ in a sorted list of numbers $a_p, a_2, ..., a_q$.

search(p,q,k) \\ assume $p \leq q$

$m := \lfloor (p+q)/2 \rfloor$

if $k = a_m$ then return m

else if $(k < a_m)$ and $p < m$ then

**Solution:** return search(p,m-1,k)

else if $(k > a_m)$ and $q > m$ then

**Solution:** return search(m+1,q,k)

else return -1 \\ i.e. error, not found

(9 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 4T(n/2) + n$

| | | | | |
|---|---|---|---|---|
| $\Theta(n)$ ☐ | $\Theta(n \log n)$ ☐ | $\Theta(n^2)$ ☑ | $\Theta(n^3)$ ☐ |
| $\Theta(n^{\log_3 2})$ ☐ | $\Theta(n^{\log_2 3})$ ☐ | $\Theta(2^n)$ ☐ | $\Theta(3^n)$ ☐ |

Changing last value in linked list

| | | | |
|---|---|---|---|
| $\Theta(1)$ ☐ | $\Theta(\log n)$ ☐ | $\Theta(n)$ ☑ | $\Theta(n \log n)$ ☐ |
| $\Theta(n^2)$ ☐ | $\Theta(n^3)$ ☐ | $\Theta(2^n)$ ☐ | $\Theta(3^n)$ ☐ |

For a problem to satisfy the definition of co-NP, a "yes" answer must have a succinct justification. true ☐ false ☑

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 2T(n/4) + n$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☑ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

The running time of mergesort

$\Theta(\log n)$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☑ $\Theta(n^2)$ ☐
$\Theta(n^3)$ ☐ $\Theta(n^{\log_3 2})$ ☐ $\Theta(n^{\log_2 3})$ ☐ $\Theta(2^n)$ ☐

All ways to assign True/False values to $n$ input variables

$\Theta(\log n)$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐
$\Theta(n^3)$ ☐ $\Theta(n^{\log_3 2})$ ☐ $\Theta(n^{\log_2 3})$ ☐ $\Theta(2^n)$ ☑

The Travelling Salesman Problem

polynomial ☐ exponential ☐ in NP ☑

NP Complete problems require exponential time.

true ☐ false ☐ not known ☑

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n-1) + d$

$\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐
$\Theta(n^{\log_3 2})$ ☐ $\Theta(n^{\log_2 3})$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☑

Dividing a list in half

$\Theta(\log n)$ ☐ $\Theta(n)$ ☑ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐
$\Theta(n^3)$ ☐ $\Theta(n^{\log_3 2})$ ☐ $\Theta(n^{\log_2 3})$ ☐ $\Theta(2^n)$ ☐

Algorithm A takes $n^5$ time. On one input, A takes x time. How long will it take if I double the input size?

$2x$ ☐ $5x$ ☐ $32x$ ☑ $x^5$ ☐

Problems in class P (as in P vs. NP) require exponential time

true ☐ false ☑ not known ☐

Producing all parses for a sentence requires exponential time.

true ☑ false ☐ not known ☐

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n/2) + d$

$\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐
$\Theta(n^{\log_3 2})$ ☐ $\Theta(n^{\log_2 3})$ ☑ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

Merging two sorted lists

$\Theta(\log n)$ ☐ $\Theta(n)$ ☑ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐
$\Theta(n^3)$ ☐ $\Theta(n^{\log_3 2})$ ☐ $\Theta(n^{\log_2 3})$ ☐ $\Theta(2^n)$ ☐

Algorithm A takes $2^n$ time. On one input, A takes x time. How long will it take if I add one to the input size?

$x + 2$ ☐ $2x$ ☑ $2^x$ ☐ $x^2$ ☐

Problems in class NP need exponential time

true ☐ false ☐ not known ☑

The chromatic number of a graph with $n$ nodes can be found in polynomial time.

true ☐ false ☐ not known ☑

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 2T(n/3) + d$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☑ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

The Towers of Hanoi solver

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☐ | $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ |
| $\Theta(n^3)$ | ☐ | $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☑ |

Algorithm A takes $\log_2 n$ time. On one input, A takes x time. How long will it take if I double the input size?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x + 1$ | ☑ | $2x$ | ☐ | $2^x$ | ☐ | $x^2$ | ☐ |

Deciding if a graph is 2-colorable

| | | | | | |
|---|---|---|---|---|---|
| polynomial | ☑ | exponential | ☐ | in NP | ☐ |

The Towers of Hanoi puzzle requires exponential time.

| | | | | | |
|---|---|---|---|---|---|
| true | ☑ | false | ☐ | not known | ☐ |

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

(15 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 3T(n/3) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☑ | $\Theta(n^2)$ | ☐ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

Karatsuba's integer multiplication algorithm

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☐ | $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ |
| $\Theta(n^3)$ | ☐ | $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☑ | $\Theta(2^n)$ | ☐ |

Finding a value in a sorted array is $\Theta(2^n)$. true ☐ false ☑

Circuit satisfiability is NP complete. true ☑ false ☐ not known ☐

The Marker Making problem can be solved in polynomial time. true ☐ false ☐ not known ☑

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

(6 points) Fill in the missing bits of the recursive algorithm for solving the Towers of Hanoi puzzle.

hanoi($A$,$B$,$C$: pegs, $d_1, d_2 \ldots d_n$: disks) \\ move n disks from peg A to peg B

if $(n = 1)$ move $d_1$ from $A$ to $B$

else

**Solution:**
hanoi($A$,$C$,$B$: pegs, $d_1, d_2 \ldots d_{n-1}$: disks) \\ move smaller disks to C

move $d_n$ from $A$ to $B$

**Solution:**
hanoi($C$,$B$,$A$: pegs, $d_1, d_2 \ldots d_{n-1}$: disks) \\ move smaller disks to B

(9 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 4T(n/2) + n$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☑ | $\Theta(n^3)$ | ☐ |
| $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ | $\Theta(3^n)$ | ☐ |

The running time of binary search

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Theta(\log n)$ | ☑ | $\Theta(n)$ | ☐ | $\Theta(n \log n)$ | ☐ | $\Theta(n^2)$ | ☐ |
| $\Theta(n^3)$ | ☐ | $\Theta(n^{\log_3 2})$ | ☐ | $\Theta(n^{\log_2 3})$ | ☐ | $\Theta(2^n)$ | ☐ |

Marker Making

| | | | | | |
|---|---|---|---|---|---|
| polynomial | ☐ | exponential | ☐ | in NP | ☑ |