CS 173, Spring 19 Examlet 5, white

Name: _____

NetID: _____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 12-letter strings can be made be rearranging the letters in the word ``apalachicola''? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is the ____ of f.
   domain ☐ co-domain ☐ image ☐ none of these ☐

   $f : \mathbb{N}^2 \to \mathbb{N}$, $f(p, q) = pq$
   onto ☐ not onto ☐ not a function ☐

   $g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$, $g(x, y) = \gcd(x, y)$
   one-to-one ☐ not one-to-one ☐ not a function ☐

   We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on exactly two mailboxes.
   true ☐ false ☐

   $\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ y \leq x$
   true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Hermione Grainger has 7000 socks in her magically expanding drawer. The socks are colored purple, magenta, and shocking pink. How many socks must she pull out of the drawer before she is guaranteed to have two socks of the same color. Briefly justify your answer.

2. (10 points) Check the (single) box that best characterizes each item.

   A function is onto if and only if its image is the same as its co-domain. true ☐ false ☐

   $g : \mathbb{R} \to [-1, 1]$
   $g(x) = \sin(x)$ onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{R}^2 \to \mathbb{R}^2$
   $g(x, y) = (y, 3x)$ one-to-one ☐ not one-to-one ☐ not a function ☐

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least three elves have charm. true ☐ false ☐

   $\exists t \in \mathbb{N},\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = p$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) 8 presidential candidates (including Bernie and Hilary) need to line up for a photo. The new editor would like Bernie and Hilary to stand next to each other. How many different ways can we arrange the eight people?

2. (10 points) Check the (single) box that best characterizes each item.

Suppose $f : A \to B$. For all $x, y \in A$, if $x = y$, then $f(x) = f(y)$.

onto ☐ one-to-one ☐ neither ☐

$g : \mathbb{R} \to [0, 1]$
$g(x) = \sin(x)$

onto ☐ not onto ☐ not a function ☐

$f : \mathbb{R} \to \mathbb{Z}$
$f(x) = x$

one-to-one ☐ not one-to-one ☐ not a function ☐

Each dorm room is given an integer access code between 1 and 10 (inclusive). According to the pigeonhole principle, if there are 21 dorm rooms, then every access code must be shared by at least two rooms.

true ☐ false ☐

$\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ x - y < 100$

true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) To make exam grading anonymous and therefore hopefully more fair, each of the 200 students in CS 241 has been assigned a unique 3-character exam code. The character set is $\{\alpha, \beta, \gamma, \delta\}$. Use the Pigeonhole Principle to explain what's wrong with this plan.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the real numbers is the _____ of f.
   domain ☐ co-domain ☐ image ☐ none of these ☐

   $f : \mathbb{Z} \to \mathbb{Z}$
   $f(x) = x + 3$ (x even),
   $f(x) = x - 22$ (x odd)
   onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{N}^2 \to \mathbb{N}$
   $g(x, y) = \gcd(x, y)$
   one-to-one ☐ not one-to-one ☐ not a function ☐

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, there must be at least three elves with the same gift.
   true ☐ false ☐

   $\exists y \in \mathbb{R}^+,\ \forall x \in \mathbb{R}^+,\ xy = 1$
   ($\mathbb{R}^+$ is the positive real numbers.)
   true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that $|A| = p$ and $|B| = q$. How many different functions are there from A to B?

2. (10 points) Check the (single) box that best characterizes each item.

   A function is one-to-one if and only if each value in the co-domain has at most one pre-image. true ☐ false ☐

   $g : \mathbb{R}^2 \to \mathbb{R}$
   $g(x, y) = \lfloor x \rfloor + y$ onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{Z}^2 \to \mathbb{Z}^2$
   $g(x, y) = (y, 3x)$ one-to-one ☐ not one-to-one ☐ not a function ☐

   Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. At least three of these shirts must have the same slogan. true ☐ false ☐

   $\forall x \in \mathbb{Q},\ \exists m, n \in \mathbb{Z},\ x = \frac{m}{n}$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) 15 men and 15 women showed up to this week's meeting of the UIUC Swing Dance Society. How many different ways can we form all of them into pairs, each pair containing one man and one woman?

2. (10 points) Check the (single) box that best characterizes each item.

Suppose $f : A \to B$. For all $x \in A$, there is a $y \in B$, $f(x) = y$.	onto ☐	one-to-one ☐	neither ☐
$g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$ $g(x, y) = \gcd(x, y)$	onto ☐	not onto ☐	not a function ☐
$f : \mathbb{N} \to \mathbb{R}$ $f(x) = x^2 + 2$	one-to-one ☐	not one-to-one ☐	not a function ☐
Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. There is a slogan that appears on at least two shirts.	true ☐	false ☐	
$\forall x \in \mathbb{Z},\ \exists y \in \mathbb{N},\ x^2 = y$	true ☐	false ☐	

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that $|A| = 2$ and $|B| = 3$. How many onto functions are there from A to B? Briefly justify or show work.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the integers is the ____ of f.
   domain ☐ co-domain ☐ image ☐ none of these ☐

   $f : \mathbb{N}^2 \to \mathbb{Z}$
   $f(p, q) = 2^p 3^q$
   onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$
   $g(x) = x|x|$
   one-to-one ☐ not one-to-one ☐ not a function ☐

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least one elf has stamina.
   true ☐ false ☐

   $\forall x \in \mathbb{R}^+,\ \exists y \in \mathbb{R}^+,\ xy = 1$
   ($\mathbb{R}^+$ is the positive real numbers.)
   true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Prof. Snape is teaching potions to 52 girls and 73 boys. Quiz 1 has integer scores between zero and 100 (inclusive). Assuming no one missed the quiz, what is the probability that two students got the same score? Briefly justify your answer.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : A \to B$ is one-to-one, then $|A| \geq |B|$ ☐ $|A| \leq |B|$ ☐ $|A| = |B|$ ☐

$g : \mathbb{N}^2 \to \mathbb{N}$
$g(x, y) = \gcd(x, y)$ onto ☐ not onto ☐ not a function ☐

$g : \mathbb{R} \to \mathbb{R}^2$
$g(x) = (x, 3x^2 + 2)$ one-to-one ☐ not one-to-one ☐ not a function ☐

We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on at least two mailboxes. true ☐ false ☐

$\exists m, n \in \mathbb{Z},\ \forall x \in \mathbb{Q},\ x = \frac{m}{n}$ true ☐ false ☐

CS 173, Fall 18 Examlet 5, Part B

Name:_____

NetID:_____ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

1. (5 points) Suppose that $|A| = p$ and $|B| = q$, $p \leq q$. How many different one-to-one functions are there from A to B?

2. (10 points) Check the (single) box that best characterizes each item.

   If a function from $\mathbb{R}$ to $\mathbb{R}$ is increasing, it must be one-to-one. true ☐ false ☐

   $f : \mathbb{N} \to \mathbb{R}$, $f(x) = x^2 + 2$ onto ☐ not onto ☐ not a function ☐

   $f : \mathbb{N} \to \mathbb{N}$, $f(x) = 3 - x$ one-to-one ☐ not one-to-one ☐ not a function ☐

   We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, every color appears on at least two mailboxes. true ☐ false ☐

   $\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ x = xy$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

1. (5 points) Suppose that $|A| = p$, $|B| = q$, $|C| = n$. How many different functions are there from A to $B \times C$?

2. (10 points) Check the (single) box that best characterizes each item.

If a function from $\mathbb{R}$ to $\mathbb{R}$ is strictly increasing, it must be one-to-one. true ☐ false ☐

$g : \mathbb{N} \to \mathbb{Z}$
$g(x) = |x|$ one-to-one ☐ not one-to-one ☐ not a function ☐

$g : \mathbb{R} \to \mathbb{R}$
$g(x) = \sin(x)$ onto ☐ not onto ☐ not a function ☐

We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on at least two mailboxes. true ☐ false ☐

$\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ x \neq y$ and $x + y = 0$ true ☐ false ☐

CS 173, Fall 17 Examlet 5, Part B

Name: _____

NetID: _____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 7-letter strings can be made by selecting and rearranging letters from the word ‘‘`metalworking`’’? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is the ____ of f.

   domain ☐ co-domain ☐ image ☐ none of these ☐

   $f : \mathbb{Z} \to \mathbb{Z}$
   $f(x) = x + 4$ (x even),
   $f(x) = x - 22$ (x odd)

   onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$
   $g(x) = \lfloor x \rfloor$

   one-to-one ☐ not one-to-one ☐ not a function ☐

   We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on exactly two mailboxes.

   true ☐ false ☐

   $\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ y \leq x$

   true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) 10 men and 15 women showed up to this week's meeting of the UIUC Swing Dance Society. How many different ways can they line up (left to right) in front of the stage without any men being next to another man?

2. (10 points) Check the (single) box that best characterizes each item.

A function is onto if and only if its image is the same as its co-domain. true ☐ false ☐

$f : \mathbb{Z} \to \mathbb{Z}$
$f(x) = x + 3$ (x even),
$f(x) = x - 21$ (x odd)
one-to-one ☐ not one-to-one ☐ not a function ☐

$g : \mathbb{Z} \to \mathbb{R}$
$g(x) = x + 2.137$
onto ☐ not onto ☐ not a function ☐

Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least three elves have charm. true ☐ false ☐

$\exists y \in \mathbb{R}^+,\ \forall x \in \mathbb{R}^+,\ xy = 1$
($\mathbb{R}^+$ is the positive real numbers.) true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that $|A| = p$, $|B| = q$, $|C| = n$. How many different functions are there from A to $B \times C$?

2. (10 points) Check the (single) box that best characterizes each item.

   A function from $\mathbb{R}$ to $\mathbb{R}$ is strictly increasing if and only if it is one-to-one. true ☐ false ☐

   $f : \mathbb{Z} \to \mathbb{Z}$
   $f(x) = x + 3$ (x even),
   $f(x) = x - 22$ (x odd) onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{R} \to \mathbb{Z}$
   $g(x) = |x|$ one-to-one ☐ not one-to-one ☐ not a function ☐

   We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there are two mailboxes with the same color. true ☐ false ☐

   $\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ y \leq x$ true ☐ false ☐

CS 173, Fall 17 Examlet 5, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that $|A| = 3$ and $|B| = 3$. How many onto functions are there from A to B? Briefly justify or show work.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the integers is the _____ of f.
   domain ☐ co-domain ☐ image ☐ none of these ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = |x|$
   one-to-one ☐ not one-to-one ☐ not a function ☐

   $g : \mathbb{R} \to [0, 1]$, $g(x) = \sin(x)$
   onto ☐ not onto ☐ not a function ☐

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, there must be at least three elves with the same gift.
   true ☐ false ☐

   $\exists y \in \mathbb{N},\ \forall x \in \mathbb{Z},\ x^2 = y$
   true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Let n and k be integers. Consider the integer powers of n from n^0 to n^k. Use the Pigeonhole Principle to show that there are two distinct (i.e. not equal) integers i and j, both between 0 and k (inclusive), such that $n^i \equiv n^j \pmod{k}$. (Your solution should be clear but does not need to be very formal.)

2. (10 points) Check the (single) box that best characterizes each item.

   If a function is onto, then each value in the co-domain has exactly one pre-image. true ☐ false ☐

   $g : \mathbb{R} \to \mathbb{R}^2$
   $g(x) = (x, 3x^2 + 2)$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $f : \mathbb{N} \to \mathbb{R}$
   $f(x) = x^2 + 2$ onto ☐ not onto ☐ not a function ☐

   If $f : A \to B$ is one-to-one, then $|A| \geq |B|$ ☐ $|A| \leq |B|$ ☐ $|A| = |B|$ ☐

   $\exists t \in \mathbb{Z}^+,\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = 1$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 14-letter strings can be made be rearranging the letters in the word ‘‘`classification`’’? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{Z}$ is a function such that $f(x) = -|x|$ then $\mathbb{N}$ is the ____ of f. ☐ domain ☐ co-domain ☐ image ☐ none of these

$f : \mathbb{N}^2 \to \mathbb{N}$
$f(p, q) = pq$ ☐ one-to-one ☐ not one-to-one ☐ not a function

$g : \mathbb{Z} \to \mathbb{Z}$
$g(x) = |x|$ ☐ onto ☐ not onto ☐ not a function

We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on at least two mailboxes. ☐ true ☐ false

$\exists t \in \mathbb{N},\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = p$ ☐ true ☐ false

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that $|A| = p$ and $|B| = q$, $p \leq q$. How many different one-to-one functions are there from A to B?

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = |x|$ then $\mathbb{N}$ is the ____ of f. domain ☐ co-domain ☐ image ☐

   $g : \mathbb{R}^2 \to \mathbb{R}$
   $g(x, y) = \lfloor x \rfloor + y$ onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{R}^2 \to \mathbb{R}^2$
   $g(x, y) = (y, 3x)$ one-to-one ☐ not one-to-one ☐ not a function ☐

   Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, every color appears on at least two vertices. true ☐ false ☐

   $\forall x \in \mathbb{Q},\ \exists m, n \in \mathbb{Z},\ x = \frac{m}{n}$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Xin plans to randomly draw a hand of cards from a standard deck of 52 cards (evenly divided among 4 suits). He'd like to be sure the hand includes 3 cards with the same suit. How large must the hand be? Briefly justify your answer.

2. (10 points) Check the (single) box that best characterizes each item.

   A function is one-to-one if and only if each value in the domain has exactly one image. true ☐ false ☐

   $g : \mathbb{R}^2 \to \mathbb{R}$
   $g(x, y) = \lfloor x \rfloor + y$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $g : \mathbb{R} \to [-1, 1]$
   $g(x) = \sin(x)$ onto ☐ not onto ☐ not a function ☐

   $f : \mathbb{N}^2 \to \mathbb{Z}$
   $f(p, q) = 2^p 3^q$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $\forall x \in \mathbb{Z}^+,\ \exists y \in \mathbb{Z}^+,\ xy = 1$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that $|A| = 50$ and $B = \{5, 6\}$. How many onto functions are there from A to B? Briefly justify or show work. (Hint: how many non-onto functions are there?)

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{N} \to \mathbb{Z}$ is a function such that $f(x) = -|x|$ then $\mathbb{N}$ is the ____ of f. domain ☐ co-domain ☐ image ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = x|x|$ onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = 7 - \lfloor \frac{x}{3} \rfloor$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = x|x|$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ x - y < 100$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 10-letter strings can be made be rearranging the characters in the word ``minimalist''? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

If a function is onto, then each value in the co-domain has at least one pre-image. true ☐ false ☐

$g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$
$g(x, y) = \gcd(x, y)$ one-to-one ☐ not one-to-one ☐ not a function ☐

$g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$
$g(x, y) = \gcd(x, y)$ onto ☐ not onto ☐ not a function ☐

$f : \mathbb{R} \to \mathbb{Z}$
$f(x) = x$ one-to-one ☐ not one-to-one ☐ not a function ☐

$\exists m, n \in \mathbb{Z},\ \forall x \in \mathbb{Q},\ x = \frac{m}{n}$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that A is a set containing $k+1$ (distinct) integers. Use the Pigeonhole Principle to show that there are x and y in A ($x \neq y$) such that $x - y$ is a multiple of k.

2. (10 points) Check the (single) box that best characterizes each item.

A function is one-to-one if and only if each value in the co-domain has at most one pre-image. true ☐ false ☐

$g : \mathbb{Z}^2 \to \mathbb{Z}^2$
$g(x, y) = (y, 3x)$ one-to-one ☐ not one-to-one ☐ not a function ☐

$g : \mathbb{Z} \to \mathbb{N}$
$g(x) = x$ one-to-one ☐ not one-to-one ☐ not a function ☐

$g : \mathbb{N}^2 \to \mathbb{N}$
$g(x, y) = \gcd(x, y)$ onto ☐ not onto ☐ not a function ☐

$\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ x - y < 100$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 10-letter strings can be made be rearranging the characters in the word ``tattletale''? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the real numbers is the ____ of f. domain ☐ co-domain ☐ image ☐

   $g : \mathbb{N}^2 \to \mathbb{N}$
   $g(x, y) = \gcd(x, y)$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $f : \mathbb{N}^2 \to \mathbb{N}$
   $f(p, q) = pq$ onto ☐ not onto ☐ not a function ☐

   Each dorm room is given an access code between 1 and 10 (inclusive). According to the pigeonhole principle, if there are 21 dorm rooms, then every access code must be shared by at least two rooms. true ☐ false ☐

   $\forall m, n \in \mathbb{Z},\ \exists x \in \mathbb{Q},\ x = \frac{m}{n}$ true ☐ false ☐