Name:_____

NetID:_____　　　　**Lecture:**　　A　　B

Discussion:　　**Thursday**　　**Friday**　　9　　10　　11　　12　　1　　2　　3　　4　　5　　6

1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is onto. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}$ by $g(x, y) = f(x - 7)f(y)$. Prove that g is onto.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : C \to M$ to be "onto." You must use explicit quantifiers. Do not assume the reader knows what the image of the function is.

Name:_____

NetID:_____   **Lecture:**   **A**   **B**

Discussion: **Thursday**   **Friday**   **9**   **10**   **11**   **12**   **1**   **2**   **3**   **4**   **5**   **6**

1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $g(x, y) = (2f(x) + f(y), f(x) - f(y))$. Prove that g is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : C \to M$ to be "one-to-one." You must use explicit quantifiers; do not use words like "unique".

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Discussion: **Thursday**   **Friday**   **9   10   11   12   1   2   3   4   5   6**

1. (10 points) Suppose that A and B are sets. Suppose that $f : B \to A$ and $g : A \to B$ are functions such that $f(g(x)) = x$ for every $x \in A$. Prove that f is onto.

2. (5 points) Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is one-to-one but not onto. Your answer must include a specific formula.

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Discussion: **Thursday**   **Friday**   9   10   11   12   1   2   3   4   5   6

1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is onto. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}$ by $g(x, y) = f(x) + 2f(y) - 6$.
   Prove that g is onto.

2. (5 points) $A = \{0, 2, 4, 6, 8, 10, 12, \ldots\}$, i.e. the even integers starting with 0.

   $B = \{1, 4, 9, 16, 25, 36, 49, \ldots\}$, i.e. perfect squares starting with 1.

   Give a specific formula for a bijection $f : A \to B$. (You do not need to prove that it is a bijection.)

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Discussion: **Thursday**   **Friday**   9   10   11   12   1   2   3   4   5   6

1. (10 points) Suppose that $f : A \rightarrow B$ and $g : B \rightarrow C$ are one-to-one. Prove that $g \circ f$ is one-to-one.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : \mathbb{R} \rightarrow \mathbb{R}$ to be "increasing." You must use explicit quantifiers.

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Discussion: **Thursday**   **Friday**   **9**   **10**   **11**   **12**   **1**   **2**   **3**   **4**   **5**   **6**

1. (10 points) Let P be the set of pairs of positive integers. Suppose that $f : P \to \mathbb{R}^2$ is defined by $f(x, y) = (\frac{x}{y}, x + y)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is onto but not one-to-one. Your answer must include a specific formula.

Name:_____

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Discussion: **Thursday**   **Friday**   **9**   **10**   **11**   **12**   **1**   **2**   **3**   **4**   **5**   **6**

1. (10 points) Suppose that $f : \mathbb{Z}^2 \to \mathbb{Z}$ is defined by $f(x, y) = 3x + 5y$. Prove that f is onto.

2. (5 points) $A = \{0, 1, 4, 9, 16, 25, 36, \ldots\}$, i.e. perfect squares starting with 0.

   $B = \{2, 4, 6, 8, 10, 12, 14, \ldots\}$, i.e. the even integers starting with 2.

   Give a specific formula for a bijection $f : A \to B$. (You do not need to prove that it is a bijection.)

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Discussion: **Thursday**   **Friday**   **9**   **10**   **11**   **12**   **1**   **2**   **3**   **4**   **5**   **6**

1. (10 points) Let's define the function $f : \mathbb{R}^+ \to \mathbb{R}$ by $f(x) = \dfrac{4x - 1}{2x + 5}$. ($\mathbb{R}^+$ is the positive reals.)
   Prove that f is one to one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : \mathbb{R} \to \mathbb{R}$ to be "strictly increasing." You must use explicit quantifiers.
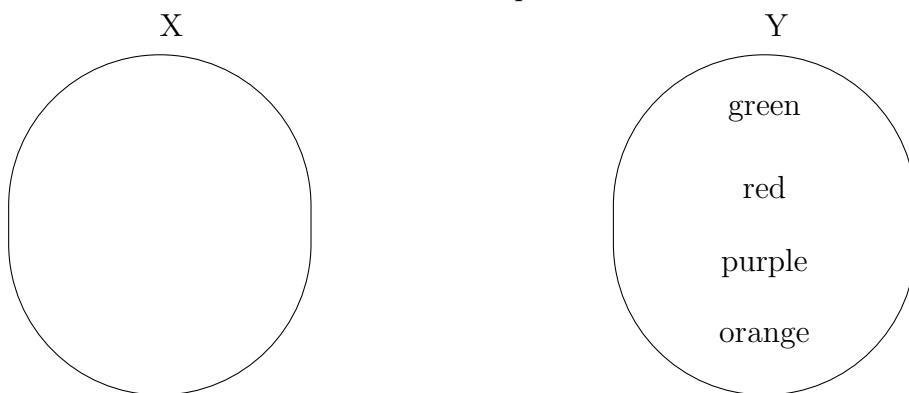
**Name:**_____

NetID:_____   **Lecture:**   B

Discussion: **Friday**   **11**   **12**   **1**   **2**   **3**   **4**

1. (10 points) If a is any real number, (a, ∞) is the set of all real numbers greater than a. Let's define the function $f : (0, \infty) \to \left(\frac{1}{3}, \infty\right)$ by $f(x) = \dfrac{x^2 + 2}{3x^2}$. Prove that f is onto.

2. (5 points) Complete this picture to make an example of a function that is onto but not one-to-one, by adding elements to the domain and arrows showing how input values map to output values. The elements of the domain must be letters of the alphabet.

   X   Y

   green

   red

   purple

   orange

Name:_____

NetID:_____   **Lecture:**   **B**

Discussion: **Friday**   **11**   **12**   **1**   **2**   **3**   **4**
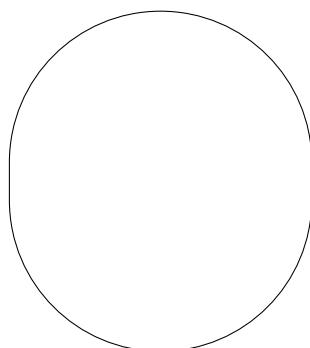
1. (10 points) Suppose that $h : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $f : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $f(x,y) = (h(x) - y, 3h(x) + 1)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Complete this picture to make an example of a function that is one-to-one but not onto, by adding elements to the co-domain and arrows showing how input values map to output values. The elements of the co-domain must be integers.

X   Y

coffee

horchata

tea

Name:_____

NetID:_____   Lecture: A   B

Discussion: Thursday   Friday   9   10   11   12   1   2   3   4   5   6

1. (10 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is onto. Recall that the max function returns the larger of its two inputs. Let's define $g : \mathbb{N}^2 \to \mathbb{N}$ by $g(x, y) = f(\max(x - 7, 0)) + f(y)$. Prove that g is onto.

2. (5 points) Recall that $\mathbb{Z}^+$ is the set of positive integers. Let's define the function $h : (\mathbb{Z}^+)^2 \to \mathbb{R}$ such that $h(d, e) = 2^d + \frac{1}{e}$. Is h one-to-one? Briefly justify your answer.

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Discussion: **Thursday**   **Friday**   9   10   11   12   1   2   3   4   5   6

1. (10 points) Let's define the function $f : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $f(x, y) = (x + y, 2x - 3y)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{Z}$ are functions. Let's define the function $f + g$ by $(f + g)(x) = f(x) + g(x)$. Adele claims that if f and g are onto, then $f + g$ is onto. Is this correct? Briefly explain why it is or give a counter-example.

Name:_____
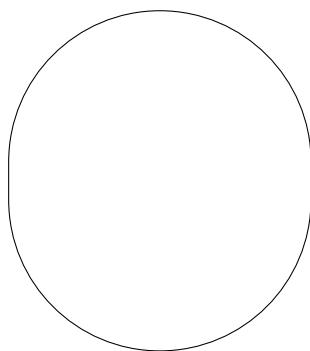
NetID:_____   **Lecture:**   **A**   **B**

Discussion: **Thursday**   **Friday**   **9**   **10**   **11**   **12**   **1**   **2**   **3**   **4**   **5**   **6**
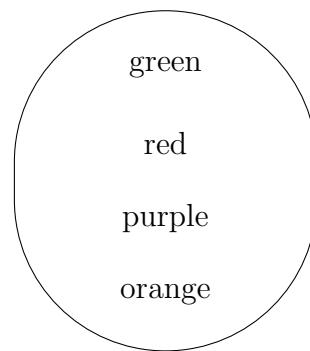
1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $g : \mathbb{Z} \to \mathbb{Z}^2$ by $g(n) = (|n|, f(n)|n|)$. Prove that g is one-to-one.

2. (5 points) Complete this picture to make an example of a function that is one-to-one but not onto, by adding elements to the domain and arrows showing how input values map to output values. The elements of the domain must be letters of the alphabet.

X   Y

green

red

purple

orange

Name:_____

NetID:_____   **Lecture:**   A   B

Discussion: **Thursday**   **Friday**   9   10   11   12   1   2   3   4   5   6

1. (10 points) If a is any real number, (a, ∞) is the set of all real numbers greater than a. Let's define
   the function $f : (0, \infty) \to (\frac{5}{4}, \infty)$ by $f(x) = \frac{5x^2+3}{4x^2}$. Prove that f is onto.

2. (5 points) What's wrong with this attempt to define $f \circ g$?

   If $f : A \to B$ and $g : B \to C$ are functions, then $f \circ g$ is the function from A to C defined
   by $(f \circ g)(x) = f(g(x))$.

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Discussion: **Thursday**   **Friday**   **9**   **10**   **11**   **12**   **1**   **2**   **3**   **4**   **5**   **6**

1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is onto. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}$ by $g(x, y) = f(x) + 2f(y) - 6$. Prove that g is onto.

2. (5 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is increasing (but perhaps not strictly increasing). Dumbledore claims that f must be one-to-one. Is he correct? Briefly explain why he is or give a concrete counter-example.

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NetID:—————————————————   **Lecture:**   **A**   **B**

Discussion: **Thursday**   **Friday**   **9**   **10**   **11**   **12**   **1**   **2**   **3**   **4**   **5**   **6**

1. (10 points) Suppose that $h : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $f : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $f(x, y) = (h(x) - y, 3h(x) + 1)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Recall that $\mathbb{Z}^+$ is the set of positive integers. Let's define the function $h : (\mathbb{Z}^+)^2 \to \mathbb{R}$ such that $h(d, e) = 2^d + \frac{1}{e}$. Is h onto? Briefly justify your answer.

Name:_____

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Discussion: Thursday   Friday   10   11   12   1   2   3   4   5   6

1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is onto. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}$ by $g(x, y) = (1 - f(x))f(y)$. Prove that g is onto.

2. (5 points) Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ which is one-to-one but not onto. Be specific.

Name:_____

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Discussion: **Thursday**   **Friday**   **10   11   12   1   2   3   4   5   6**

1. (10 points) Let's define the function $f : \mathbb{R}^+ \to \mathbb{R}$ by $f(x) = \dfrac{x^2 + 2}{3x^2}$. ($\mathbb{R}^+$ is the positive reals.) Prove that f is one to one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : M \to C$ to be "onto." You must use explicit quantifiers. Do not assume the reader knows what the image of the function is.

Name:_____

NetID:_____   **Lecture:**   A   B

Discussion: **Thursday   Friday   10   11   12   1   2   3   4   5   6**

1. (10 points) Let P be the set of pairs of positive integers. Suppose that $f : P \to \mathbb{R}^2$ is defined by $f(x, y) = (\frac{x}{y}, x + y)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) the behavior of increasing functions.

2. (5 points) $A = \{2, 3, 4, 5, 6, 7, 8 \ldots\}$, i.e. the integers ≥ 2

   $B = \{1, 2, 4, 8, 16, 32, 64, \ldots\}$, i.e. powers of 2 starting with 1

   Give a specific formula for a bijection $f : A \to B$. (You do not need to prove that it is a bijection.)

Name:_____

NetID:_____   **Lecture:**   A   B

Discussion: **Thursday**   **Friday**   **10**   **11**   **12**   **1**   **2**   **3**   **4**   **5**   **6**

1. (10 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is onto. Let's define $g : \mathbb{N}^2 \to \mathbb{Z}$ by $g(m, n) = (2 - n)f(m)$. Prove that g is onto.

2. (5 points) Suppose that $g : A \to B$ and $f : B \to C$. Prof. Snape claims that if g is onto, then $f \circ g$ is onto. Disprove this claim using a concrete counter-example in which A, B, and C are all small finite sets.

Name:⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

NetID:⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯   **Lecture:**   A   B

Discussion: Thursday   Friday   10   11   12   1   2   3   4   5   6

1. (10 points) Let $g : \mathbb{N} \to \mathbb{N}$ be onto, and let $f : \mathbb{N}^2 \to \mathbb{Z}$ be defined by $f(n,m) = (m-1)g(n)$. Prove that f is onto.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : M \to C$ to be "one-to-one." You must use explicit quantifiers; do not use words like "unique".

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NetID:_____   **Lecture:**   A   B

Discussion: **Thursday**   **Friday**   **10**   **11**   **12**   **1**   **2**   **3**   **4**   **5**   **6**

1. (10 points) Let's define the function $f : \mathbb{R}^+ \to \mathbb{R}$ by $f(x) = \dfrac{4x - 1}{2x + 5}$. ($\mathbb{R}^+$ is the positive reals.)
   Prove that f is one to one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ which is onto but not one-to-one. Be specific.