# CS 173, Spring 19 Examlet 8, white

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function $f$ defined (for $n$ a power of 4) by

$$\begin{aligned} f(1) &= 0 \\ f(n) &= 2f(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$f(n) = 2^k f(n/4^k) + n \sum_{p=0}^{k-1} 1/2^p$$

Finish finding the closed form for $f(n)$ assuming that $n$ is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function $F$ defined (for $n$ a power of 2) by

$$\begin{aligned} F(2) &= c \\ F(n) &= F(n/2) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$F(n) = F(n/2^k) + \sum_{i=0}^{k-1} n \frac{1}{2^i}$$

Finish finding the closed form for $F$. Show your work and simplify your answer.

# CS 173, Spring 19 Examlet 8, white

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function $f$ defined by

$$
\begin{aligned}
f(0) &= f(1) = 3 \\
f(n) &= 5f(n-2) + d, \text{ for } n \geq 2
\end{aligned}
$$

where $d$ is a constant. Express $f(n)$ in terms of $f(n-6)$ (where $n \geq 6$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The chromatic number of the 4-dimensional hypercube $Q_4$ 2 ☐ 3 ☐ 4 ☐ 5 ☐

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function $f$ defined (for $n$ a power of 4) by

$$\begin{aligned} f(1) &= 0 \\ f(n) &= 2f(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Express $f(n)$ in terms of $f(n/4^{13})$ (assuming $n$ is large enough that this input hasn't reached the base case). Express your answer using a summation and show your work. Do **not** finish the process of finding the closed form for $f(n)$.

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function $g$ defined (for $n$ a power of 3) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 3g(n/3) + n \text{ for } n \geq 3 \end{aligned}$$

Express $g(n)$ in terms of $g(n/3^3)$ (where $n \geq 27$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The number of nodes in the 4-dimensional hypercube $Q_4$ 4 ☐ 16 ☐ 32 ☐ 64 ☐

**Name:**____________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function $g$ defined (for $n$ a power of 4) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 2g(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 2^k g(n/4^k) + n \sum_{p=0}^{k-1} \frac{1}{2^p}$$

Finish finding the closed form for $g(n)$ assuming that $n$ is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

**Name:**________________________________________

**NetID:**____________________ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Suppose we have a function $g$ defined (for $n$ a power of 2) by

$$\begin{aligned} g(1) &= 1 \\ g(n) &= 4g(n/2) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 4^k g(n/2^k) + kn^2$$

Finish finding the closed form for $g$. Show your work and simplify your answer.

2. (2 points) Check the (single) box that best characterizes each item.

The Fibonacci numbers can be defined recursively by $F(0) = 0$, $F(1) = 1$, and $F(n) = F(n-1) + F(n-2)$ for all integers ... $n \geq 0$ ☐ $n \geq 1$ ☐ $n \geq 2$ ☐

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function $g$ defined (for $n$ a power of 2) by

$$\begin{aligned} g(1) &= 3 \\ g(n) &= 4g(n/2) + n \text{ for } n \geq 2 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 4^k g(n/2^k) + \sum_{p=0}^{k-1} n2^p$$

Finish finding the closed form for $g(n)$ assuming that $n$ is a power of 2. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

# CS 173, Fall 18 Examlet 8, Part B 

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **B**

**Discussion:** **Friday** **11** **12** **1** **2** **3** **4**

1. (8 points) Suppose we have a function $g$ defined by

$$\begin{aligned} g(0) &= g(1) = c \\ g(n) &= kg(n-2) + n^2, \text{ for } n \geq 2 \end{aligned}$$

where $k$ and $c$ are constants. Express $g(n)$ in terms of $g(n-6)$ (where $n \geq 6$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is such that $f(n) = n!$. Give a recursive definition of $f$

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **B**

**Discussion:** **Friday** **11** **12** **1** **2** **3** **4**

1. (8 points) Suppose we have a function $f$ defined by

$$\begin{aligned} f(1) &= 5 \\ f(n) &= 3f(n-1) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Express $f(n)$ in terms of $f(n-3)$ (where $n \geq 4$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

2. (2 points) Suppose that $G_0$ is the graph consisting of a single vertex. Also suppose that the graph $G_n$ consists of a copy of $G_{n-1}$ plus an extra vertex $v$ and edges joining $v$ to each vertex in $G_{n-1}$. Give a clear picture or precise description of $G_4$.

**Name:**________________________________________

**NetID:**______________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function $F$ defined (for $n$ a power of 2) by

$$\begin{aligned} F(2) &= 17 \\ F(n) &= 3F(n/2), \text{ for } n \geq 4 \end{aligned}$$

Use unrolling to find the closed form for $F$. Show your work and simplify your answer.

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function $g$ defined (for $n$ a power of 2) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 4g(n/2) + d \text{ for } n \geq 2 \end{aligned}$$

Express $g(n)$ in terms of $g(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

$f(n) = n!$ can be defined recursively by $f(0) = 1$, and $f(n+1) = (n+1)f(n)$ for all integers ... $n \geq 0$ ☐ $n \geq 1$ ☐ $n \geq 2$ ☐

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function $g$ defined (for $n$ a power of 3) by

$$\begin{aligned} g(9) &= 5 \\ g(n) &= 3g(n/3) + n \text{ for } n \geq 27 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 3^k g(n/3^k) + kn$$

Finish finding the closed form for $g$. Show your work and simplify your answer.

2. (2 points) Suppose that $G_0$ is the graph consisting of a single vertex. Also suppose that the graph $G_n$ consists of a copy of $G_{n-1}$ plus an extra vertex $v$ and edges joining $v$ to each vertex in $G_{n-1}$. Give a clear picture or precise description of $G_4$.

# CS 173, Spring 18 Examlet 8, Part B

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function $F$ defined (for $n$ a power of 3) by

$$\begin{aligned} F(1) &= 5 \\ F(n) &= 3F(n/3) + 7 \text{ for } n \geq 3 \end{aligned}$$

Your partner has already figured out that

$$F(n) = 3^k F(n/3^k) + 7\sum_{p=0}^{k-1} 3^p$$

Finish finding the closed form for $F$. Show your work and simplify your answer. Recall the following useful closed form (for $r \neq 1$): $\sum_{k=0}^{n} r^k = \frac{r^{n+1}-1}{r-1}$

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function $g$ defined (for $n$ a power of 2) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 4g(n/2) + n \text{ for } n \geq 2 \end{aligned}$$

Express $g(n)$ in terms of $g(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The diameter of the 4-dimensional hypercube $Q_4$ 1 ☐ 2 ☐ 4 ☐ 16 ☐

**Name:**__________________________________________

**NetID:**________________________ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Suppose we have a function $f$ defined (for $n$ a power of 2) by

$$\begin{aligned} f(1) &= 5 \\ f(n) &= 3f(n/2) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Express $f(n)$ in terms of $f(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

| | | | |
|---|---|---|---|
| The n-dimensional hypercube $Q_n$ has an Euler circuit. | always ☐ | sometimes ☐ | never ☐ |

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function $f$ defined by

$$\begin{aligned} f(0) &= f(1) = 3 \\ f(n) &= 5f(n-2) + d, \text{ for } n \geq 2 \end{aligned}$$

where $d$ is a constant. Your partner has already figured out that

$$f(n) = 5^k f(n-2k) + \sum_{p=0}^{k-1} d5^p$$

Finish finding the closed form for $f(n)$ assuming that $n$ is even. Show your work and simplify your answer. Recall the following useful closed form (for $r \neq 1$): $\sum_{k=0}^{n} r^k = \frac{r^{n+1}-1}{r-1}$

# CS 173, Fall 17 Examlet 8, Part B

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function $f$ defined by

$$\begin{aligned} f(0) &= f(1) = 3 \\ f(n) &= 5f(n-2) + d, \text{ for } n \geq 2 \end{aligned}$$

where $d$ is a constant. Express $f(n)$ in terms of $f(n-6)$ (where $n \geq 6$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

$f(n) = n!$ can be defined recursively by $f(0) = 1$, and $f(n) = nf(n-1)$ for all integers ... $n \geq 0$ ☐ $n \geq 1$ ☐ $n \geq 2$ ☐

# CS 173, Fall 17 Examlet 8, Part B

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function $g$ defined (for $n$ a power of 3) by

$$
\begin{aligned}
g(1) &= c \\
g(n) &= 3g(n/3) + n \text{ for } n \geq 3
\end{aligned}
$$

Express $g(n)$ in terms of $g(n/3^3)$ (where $n \geq 27$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is such that $f(n) = n^2$. Give a recursive definition of $f$

# CS 173, Fall 17 Examlet 8, Part B

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function $g$ defined (for $n$ a power of 4) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 2g(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 2^k g(n/4^k) + n \sum_{p=0}^{k-1} \frac{1}{2^p}$$

Finish finding the closed form for $f(n)$ assuming that $n$ is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function $g$ defined (for $n$ a power of 2) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 4g(n/2) + n \text{ for } n \geq 2 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 4^k g(n/2^k) + n \sum_{p=0}^{k-1} 2^p$$

Finish finding the closed form for $g(n)$ assuming that $n$ is a power of 2. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function $g$ defined by

$$\begin{aligned} g(0) &= g(1) = c \\ g(n) &= kg(n-2) + n^2, \text{ for } n \geq 2 \end{aligned}$$

where $k$ and $c$ are constants. Express $g(n)$ in terms of $g(n-6)$ (where $n \geq 6$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The number of nodes in the 4-dimensional hypercube $Q_4$ 4 ☐ 16 ☐ 32 ☐ 64 ☐

## Name:________________________________

## NetID:____________________ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

(10 points) Suppose we have a function $f$ defined (for $n$ a power of 4) by

$$\begin{aligned} f(1) &= 0 \\ f(n) &= 2f(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Express $f(n)$ in terms of $f(n/4^{13})$ (assuming $n$ is large enough that this input hasn't reached the base case). Express your answer using a summation and show your work. Do **not** finish the process of finding the closed form for $f(n)$.

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function $g$ defined (for $n$ a power of 2) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 4g(n/2) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Express $g(n)$ in terms of $g(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The Fibonacci numbers can be defined recursively by $F(0) = 0$, $F(1) = 1$, and $F(n+1) = F(n) + F(n-1)$ for all integers ... $n \geq 0$ ☐ $n \geq 1$ ☐ $n \geq 2$ ☐

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function $f$ defined by

$$\begin{aligned} f(1) &= 5 \\ f(n) &= 3f(n-1) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Express $f(n)$ in terms of $f(n-3)$ (where $n \geq 4$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The Fibonacci numbers can be defined recursively by $F(0) = 0$, $F(1) = 1$, and $F(n+2) = F(n) + F(n+1)$ for all integers ... $n \geq 0$ ☐ $n \geq 1$ ☐ $n \geq 2$ ☐

## Name:______________________________________

## NetID:______________________ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

(10 points) Suppose we have a function $g$ defined (for $n$ a power of 2) by

$$\begin{aligned} g(1) &= 3 \\ g(n) &= 4g(n/2) + n \text{ for } n \geq 2 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 4^k g(n/2^k) + \sum_{p=0}^{k-1} n2^p$$

Finish finding the closed form for $g(n)$ assuming that $n$ is a power of 2. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(10 points) Suppose we have a function $f$ defined (for $n$ a power of 4) by

$$\begin{aligned} f(1) &= 0 \\ f(n) &= 2f(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$f(n) = 2^k f(n/4^k) + n\sum_{p=0}^{k-1} 1/2^p$$

Finish finding the closed form for $f(n)$ assuming that $n$ is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (8 points) Suppose we have a function $g$ defined (for $n$ a power of 4) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 2g(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Express $g(n)$ in terms of $g(n/4^3)$ (where $n \geq 64$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The chromatic number of the 4-dimensional hypercube $Q_4$ 2 ☐ 3 ☐ 4 ☐ 5 ☐