CS 173, Spring 19 Examlet 10, white

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is even.

   $T(0) = 5$ $T(n) = 3T(n-2) + n^2$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   n $n\log(17n)$ $\sqrt{n}+18$ $8n^2$ $2^n+n!$ $2^{\log_4 n}+5^n$ $0.001n^3+3^n$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) Recall that f is $O(g)$ if and only if there are positive reals c and k such that $0 \leq f(x) \leq cg(x)$ for every $x \geq k$. Prof. Snape claims that there is a function f (from the reals to the reals) that can never be involved in a big-O relationship. Is he correct?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$ $T(n) = 2T(n/2) + n$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = T(n/2) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$n^{1.5}$ is $\Theta(n^{1.614})$ ☐ $O(n^{1.614})$ ☐ neither of these ☐

$n^{log_3 5}$ grows faster than n^2 ☐ slower than n^2 ☐ at the same rate as n^2 ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is even.

   $T(8) = 5$ $T(n) = 3T(n-2) + c$

   (a) The height:

   (b) The number of nodes at level k:

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $3n^2$ $\frac{n \log n}{7}$ $(10^{10^{10}})n$ $0.001n^3$ $30 \log(n^{17})$ $8n! + 18$ $3^n + 11^n$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for g to be $\Theta(f)$. Your definition can be in terms of other primitives such as $\ll$ and big-O.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n-1) + c$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

$T(1) = d$
$T(n) = 2T(n/2) + c$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

Suppose f and g produce only positive outputs and $f(n) \ll g(n)$.
Will $g(n)$ be $O(f(n))$?

no ☐ sometimes ☐ yes ☐

$n^{log_2 4}$ grows

faster than n^2 ☐ slower than n^2 ☐
at the same rate as n^2 ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a multiple of 3.

   $T(3) = 7$ $T(n) = 2T(n-3) + c$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Total work (sum of the nodes) at level k (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   n $n\log(17n)$ $\sqrt{n}+2^n+18$ $8n^2$ $2^n+n!$ $2^{\log_4 n}$ $0.001n^3+3^n$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$. Must $f(x)g(x)$ be $O(h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 3T(n/3) + n$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

$T(1) = c$
$T(n) = 2T(n/2) + n^2$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$ ☐ $k = m$ ☐
$k > m$ ☐ $km = 1$ ☐

$n^{log_2 5}$ grows

faster than n^2 ☐ slower than n^2 ☐
at the same rate as n^2 ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(8) = 7$ $T(n) = 4T\left(\frac{n}{2}\right) + n$

   (a) The height:

   (b) Total work (sum of the nodes) at level k (please simplify):

   (c) The number of leaves (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   3^n $4^{\log_2 n}$ 2^{3n} $3^{\log_2 4}$ $0.1n$ $(5n)!$ $\sqrt{n}$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely when $f \ll g$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$ $T(n) = 3T(n-1) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = T(n/2) + n$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
3^n is	$\Theta(5^n)$ ☐	$O(5^n)$ ☐	neither of these ☐	
3^n is	$\Theta(2^n)$ ☐	$O(2^n)$ ☐	neither of these ☐	

CS 173, Fall 18 Examlet 10, Part B

Name:_____

NetID:_____ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 4.

   $T(4) = 7$ $T(n) = 2T\left(\frac{n}{4}\right) + d$

   (a) The height:

   (b) Number of nodes at level k:

   (c) Sum of the work in all the leaves (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $2^n + 3^n$ n^3 $100 \log n$ 3^{31} $3n \log(n^3)$ $7n! + 2$ $173n - 173$

Name:_____

NetID:_____ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

1. (7 points) You found the following claim on a hallway whiteboard. Suppose that f and g are increasing functions from the reals to the reals, for which all output values are > 1. If $f(x)$ is $O(g(x))$, then $\log(f(x))$ is $O(\log(g(x)))$. Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n-1) + n$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

$T(1) = d$
$T(n) = 3T(n/3) + c$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

$\log_5 n$ is

$\Theta(\log_3 n)$ ☐ $O(\log_3 n)$ ☐ neither of these ☐

Dividing a problem of size n into m subproblems, each of size n/k, has the best big-Θ running time when

$k < m$ ☐ $k = m$ ☐
$k > m$ ☐ $km = 1$ ☐

CS 173, Spring 18 Examlet 10, Part B

Name: _____

NetID: _____ **Lecture:** A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 4.

   $T(1) = 7$ $T(n) = 2T\left(\frac{n}{4}\right) + n$

   (a) The height:

   (b) Number of leaves:

   (c) Total work (sum of the nodes) at level k (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $42n!$ 7^n $100 \log n$ $n \log(n^7)$ 2^{3n} $\log(2^n)$ $(n^3)^7$

CS 173, Spring 18 Examlet 10, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) In class, Prof. Snape made the following claim about all functions g and h from the reals to the reals whose output values are always > 1. If $g(x) \ll h(x)$, then $\log(g(x)) \ll \log(h(x))$. Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$ $T(n) = T(n/2) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = 2T(n-1) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

2^n is $\Theta(3^n)$ ☐ $O(3^n)$ ☐ neither of these ☐

Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $f(n)$ be $\Theta(g(n))$? no ☐ perhaps ☐ yes ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that f is $O(g)$ and g is $O(h)$. Must f be $O(h)$? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n/2) + n$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$T(1) = d$
$T(n) = 3T(n-1) + c$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$n^{log_2 5}$ grows

faster than n^2 ☐ slower than n^2 ☐
at the same rate as n^2 ☐

Suppose $f(n)$ is $O(g(n))$.
Will $g(n)$ be $O(f(n))$?

no ☐ perhaps ☐ yes ☐

CS 173, Spring 18 Examlet 10, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$. Must $f(x)g(x)$ be $O(h(x)h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

Suppose $f(n)$ is $O(g(n))$.
Will $f(n)$ be $\Theta(g(n))$? no ☐ perhaps ☐ yes ☐

$17n^3$ $\Theta(n^3)$ ☐ $O(n^3)$ ☐ neither of these ☐

$T(1) = c$
$T(n) = 2T(n/2) + n^2$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

$T(1) = d$
$T(n) = T(n/3) + c$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(1) = 1$ $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

   (a) Value in each node at level k:

   (b) Total work (sum of the nodes) at level k (please simplify):

   (c) Sum of the work in all internal (non-leaf) nodes (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $n \log n$ $\log(n^{17})$ $\sqrt{n} + n! + 18$ 2^n $8n^2$ $8^{\log_8 n}$ $0.001n^3$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for f to be $O(g)$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$ $T(n) = 3T(n/3) + n$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = 2T(n/2) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
2^n	$\Theta(n!)$ ☐	$O(n!)$ ☐	neither of these ☐	
Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $f(n)$ be $O(g(n))$?	no ☐	perhaps ☐	yes ☐	

CS 173, Fall 17 Examlet 10, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 3.

   $T(3) = 7$ $T(n) = 4T\left(\frac{n}{3}\right) + 5n$

   (a) The height:

   (b) Value in each node at level k:

   (c) Sum of the work in all the leaves (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $(3^n)^2$ 10 $0.001n^3$ $30 \log n$ $n \log(n^7)$ $8n! + 18$ $3n^2$

CS 173, Fall 17 Examlet 10, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x) \ll f(x))$. Must $f(x) + g(x)$ be $O(h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$ $T(n) = 3T(n/3) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = c$ $T(n) = 3T(n/3) + n$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$n!$ $O(2^n)$ ☐ $\Theta(2^n)$ ☐ neither of these ☐

$n^{log_2 4}$ grows faster than n^2 ☐ slower than n^2 ☐ at the same rate as n^2 ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $\Theta(h(x))$, $g(x)$ is $\Theta(h(x))$, and $f(x) > g(x)$ for any input x. Must $f(x) - g(x)$ be $\Theta(h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 2T(n/2) + n$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

$T(1) = d$
$T(n) = 2T(n-1) + c$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

$n^{log_2 3}$ grows faster than n ☐ slower than n ☐ at the same rate as n ☐

Suppose $f(n)$ is $\Theta(g(n))$.
Will $g(n)$ be $\Theta(f(n))$? no ☐ perhaps ☐ yes ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) You found the following claim on a hallway whiteboard. Suppose that f and g are increasing functions from the reals to the reals, for which all output values are > 1. If $f(x)$ is $O(g(x))$, then $\log(f(x))$ is $O(\log(g(x)))$. Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

3^n is $\Theta(2^n)$ ☐ $O(2^n)$ ☐ neither of these ☐

Dividing a problem of size n into m subproblems, each of size n/k, has the best big-Θ running time when $k < m$ ☐ $k = m$ ☐ $k > m$ ☐ $km = 1$ ☐

$T(1) = d$
$T(n) = T(n/2) + n$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

$T(1) = d$
$T(n) = T(n-1) + n$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

CS 173, Fall 17 Examlet 10, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(4) = 7$ $T(n) = 5T\left(\frac{n}{2}\right) + n$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $30\log(n^{17})$ $\sqrt{n} + n! + 18$ $\frac{n \log n}{7}$ $(10^{10^{10}})n$ $0.001n^3$ 2^n $8n^2$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f and g are functions from the reals to the reals, such that f is $\Theta(g)$. Must g be $O(f)$?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$ $T(n) = 2T(n/2) + n^2$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = T(n-1) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$n^{log_4 2}$ grows faster than n^2 ☐ slower than n^2 ☐ at the same rate as n^2 ☐

$\log_5 n$ is $\Theta(\log_3 n)$ ☐ $O(\log_3 n)$ ☐ neither of these ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a multiple of 3.

   $T(3) = 7$ $T(n) = 2T(n-3) + c$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Total work (sum of the nodes) at level k (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   3^n $4^{\log_2 n}$ 2^{3n} $3^{\log_2 4}$ $0.1n$ $(5n)!$ $\sqrt{n}$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (7 points) Prof. Flitwick claims that for any functions f and g from the reals to the reals whose output values are always > 1, if $f(x) \ll g(x)$ then $\log(f(x)) \ll \log(g(x))$ Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$ $T(n) = 4T(n/2) + n$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = T(n/2) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

Suppose $f(n) \ll g(n)$. Is $g(n) \ll f(n)$?	no ☐	perhaps ☐	yes ☐
Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $g(n)$ be $O(f(n))$?	no ☐	perhaps ☐	yes ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $\Theta(h(x))$ and $g(x)$ is $\Theta(h(x))$. Must $f(x) - g(x)$ be $\Theta(h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n-1) + c$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$T(1) = d$
$T(n) = 2T(n/2) + c$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$ ☐ $k = m$ ☐

$k > m$ ☐ $km = 1$ ☐

$n^{log_3 2}$ grows

faster than n ☐ slower than n ☐

at the same rate as n ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 3.

   $T(9) = 7$ $T(n) = T\left(\frac{n}{3}\right) + n^2$

   (a) The height:

   (b) Number of nodes at level k:

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $(\sqrt{n})^4$ $200 \log_5 n$ $\log(2^n)$ $2^n + n!$ 7^n 3^{57} $55n \log n$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(8) = 7$ $T(n) = 4T\left(\frac{n}{2}\right) + n$

   (a) The height:

   (b) Total work (sum of the nodes) at level k (please simplify):

   (c) The number of leaves (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $15n!$ $\log(n^5)$ $127(2^n)$ $n \log_2 4$ 7^n $47n^3$ $20n$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for g to be $O(f)$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$ $T(n) = T(n-1) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = T(n/3) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

Suppose $f(n) \ll g(n)$.
Is $g(n) \ll f(n)$? no ☐ perhaps ☐ yes ☐

Suppose $f(n)$ is $\Theta(g(n))$.
Will $g(n)$ be $O(f(n))$? no ☐ perhaps ☐ yes ☐