

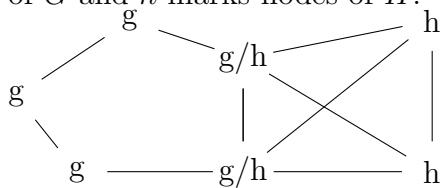
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (11 points) If  $G$  is a graph, recall that  $\chi(G)$  is its chromatic number. Suppose that  $G$  is a graph with at least one edge and  $H$  is another graph with at least one edge, not connected to  $G$ . Now, pick a specific edge  $e$  from  $G$  and an edge  $f$  from  $H$  and merge the two edges, creating a combined graph  $T$ . For example, suppose that  $G$  is  $C_5$  and  $H$  is  $K_4$ . Then  $T$  might look as follows, where  $g$  marks nodes of  $G$  and  $h$  marks nodes of  $H$ .



Describe how  $\chi(T)$  is related to  $\chi(G)$  and  $\chi(H)$ , justifying your answer. Your answer should handle any choice for  $G$  and  $H$ .

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{i=1}^{p-1} i$$

$$\frac{p(p-1)}{2}$$

$$\frac{(p-1)^2}{2}$$

$$\frac{p(p+1)}{2}$$

$$\frac{(p-1)(p+1)}{2}$$

Leal team's bridge held 100 pounds without collapsing. 100 pounds is \_\_\_\_\_ on how much the bridge can hold.

an upper bound on  
a lower bound on

exactly  
not a bound on

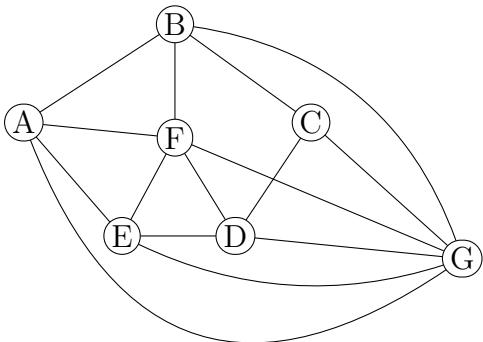
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a  $C_7$ .       $\geq 2$         $\geq 3$         $\leq 3$        can't tell

$$\sum_{i=1}^{p-1} i \quad \frac{(p-1)^2}{2} \quad \boxed{\phantom{00}} \quad \frac{(p-1)(p+1)}{2} \quad \boxed{\phantom{00}} \quad \frac{p(p+1)}{2} \quad \boxed{\phantom{00}} \quad \frac{p(p-1)}{2} \quad \boxed{\phantom{00}}$$

$\tau \leq 1.3$       an upper bound on  $\tau$        exactly  $\tau$    
                           a lower bound on  $\tau$        not a bound on  $\tau$

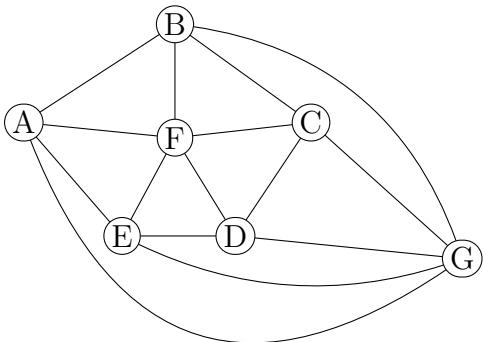
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a  $W_n$ .       $\leq 3$         $\geq 3$         $\geq n$        can't tell

$$\sum_{i=1}^{p-1} \frac{i}{p}$$

$\frac{p(p-1)}{2}$         $\frac{p(p+1)}{2}$         $\frac{(p+1)}{2}$         $\frac{(p-1)}{2}$

Putting 10 people in the canoe caused it to sink. 10 is \_\_\_\_\_ how many people the canoe can carry.

an upper bound on   
a lower bound on  exactly   
not a bound on

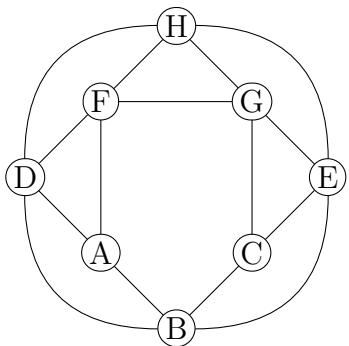
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of an  
connected acyclic graph       $\leq 2$         $= 2$         $\leq 5$        can't tell

with 5 nodes.

$$\sum_{i=0}^{k-1} (k \cdot i + 2)$$

$\frac{k^2(k+1)}{2} + 2k$	<input type="checkbox"/>	$\frac{k(k+1)}{2} + 2(k - 1)$	<input type="checkbox"/>
$\frac{k^2(k-1)}{2} + 2k$	<input type="checkbox"/>	$\frac{k(k-1)}{2} + 2(k - 1)$	<input type="checkbox"/>

$$\pi \geq 1.3$$

an upper bound on $\pi$ a lower bound on $\pi$	<input type="checkbox"/> <input checked="" type="checkbox"/>	exactly $\pi$ not a bound on $\pi$	<input type="checkbox"/> <input checked="" type="checkbox"/>
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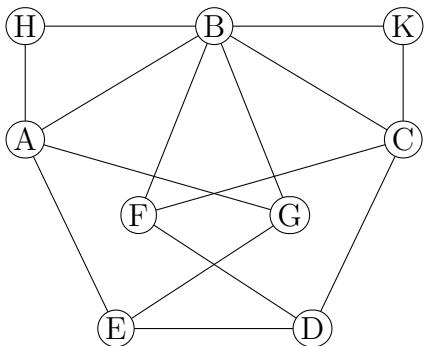
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a  
connected graph with 10 nodes.       $\leq 2$         $= 2$         $\geq 2$        can't tell

$$\sum_{k=-2}^n k^2 \quad \sum_{p=0}^{n+2} (p+2)^2 \quad \sum_{p=0}^{n-2} (p-2)^2 \quad \sum_{p=0}^{n+2} (p-2)^2 \quad \sum_{p=0}^{n+2} p^2$$

We have 30 tablespoons of filling.  
Each bun requires exactly one  
tablespoon of filling. 30 is \_\_\_\_\_ on  
how many buns we can make.

an upper bound on   
exactly   
a lower bound on   
not a bound on

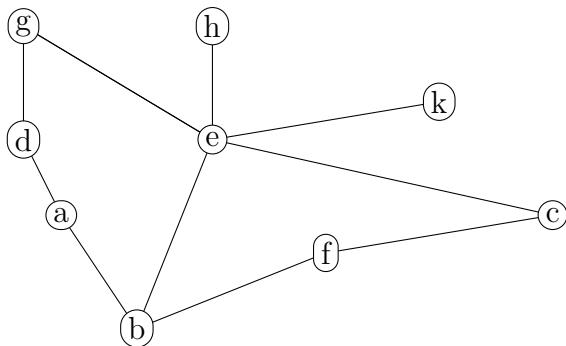
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a  $W_7$ .       $\geq 3$         $\geq 4$         $\geq 7$        can't tell

$$\sum_{k=1}^n k! \quad \sum_{p=0}^{n+1} (p+1)! \quad \sum_{k=0}^{n+1} (k-1)! \quad \sum_{k=0}^{n-1} (k+1)! \quad \sum_{p=0}^{n+1} k!$$

10 people rowed across Lake Tahoe in my canoe. 10 is \_\_\_\_\_ how many people the canoe can carry.

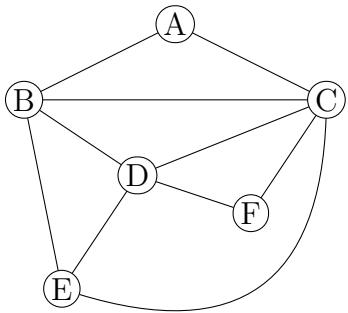
an upper bound on   
a lower bound on  exactly   
not a bound on

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a  $K_n$ .       $\leq n$         $= n$         $\geq n$        can't tell

$$\sum_{k=0}^n k! \quad \sum_{p=1}^{n+1} (p+1)! \quad \boxed{\phantom{00}} \quad \sum_{k=1}^{n+1} (k-1)! \quad \boxed{\phantom{00}} \quad \sum_{k=1}^{n-1} (k+1)! \quad \boxed{\phantom{00}} \quad \sum_{p=1}^{n+1} k! \quad \boxed{\phantom{00}}$$

I heated 2 liters of milk in my big pot. 2 liters is \_\_\_\_\_ how much the pot holds.

an upper bound on   
a lower bound on  exactly   
not a bound on

Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (11 points) Let's define two sets as follows:

$$A = \{(x, y) \in \mathbb{R}^2 : y = x^2 - 4x + 3\}$$

$$B = \{(t + 2, t^2 - 1) : t \in \mathbb{R}\}$$

Prove that  $A = B$  by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

Chromatic number of  $K_{m,n}$ .  
(Assume  $m \geq 1, n \geq 1$ .)      2       3       4       can't tell

$\pi \leq 10$

an upper bound on  $\pi$    
a lower bound on  $\pi$   exactly  $\pi$    
not a bound on  $\pi$

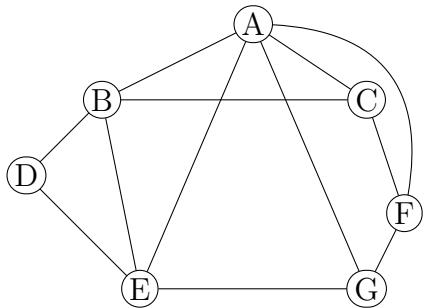
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: B

Discussion: Friday 11 12 1 2 3 4

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a bipartite  
graph with at least one edge

1  2  3  can't tell

Suppose I want to estimate  $\frac{103}{20}$ .  
3 is \_\_\_\_\_

an upper bound   
a lower bound  an exact answer   
not a bound on

$$\sum_{k=3}^n k^7$$

$$\sum_{p=1}^{n-2} p^9 \quad \boxed{\phantom{0}}$$

$$\sum_{p=1}^{n-2} k^7 \quad \boxed{\phantom{0}}$$

$$\sum_{p=1}^{n-2} k^9 \quad \boxed{\phantom{0}}$$

$$\sum_{p=1}^{n-2} (p+2)^7 \quad \boxed{\phantom{0}}$$

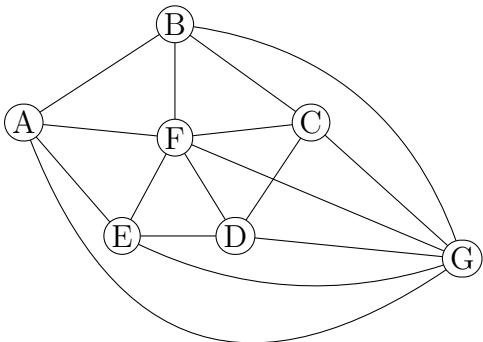
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: B

Discussion: Friday 11 12 1 2 3 4

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of  $W_n$ .      2       3        $\leq 3$         $\leq 4$

All elements of  $M$  are also elements of  $X$ .       $M = X$         $M \subseteq X$         $X \subseteq M$

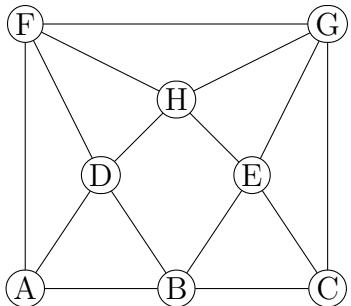
$$\sum_{k=0}^n \frac{1}{2^k} \quad 1 - (\frac{1}{2})^{n-1} \quad \boxed{\phantom{00}} \quad 2 - (\frac{1}{2})^n \quad \boxed{\phantom{00}} \quad 1 - (\frac{1}{2})^n \quad \boxed{\phantom{00}} \quad 2 - (\frac{1}{2})^{n-1} \quad \boxed{\phantom{00}}$$

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph with no cycles and at least one edge

1  2  3  can't tell

15 guests are invited to brunch. Each guest will eat at least two buns. 20 is \_\_\_\_ on how many buns we will need.

an upper bound on   
a lower bound on  exactly   
not a bound on

$$\sum_{k=0}^{n-1} \frac{1}{2^k}$$

$$1 - (\frac{1}{2})^{n-1} \quad \boxed{\phantom{0}}$$

$$2 - (\frac{1}{2})^n \quad \boxed{\phantom{0}}$$

$$1 - (\frac{1}{2})^n \quad \boxed{\phantom{0}}$$

$$2 - (\frac{1}{2})^{n-1} \quad \boxed{\phantom{0}}$$

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (11 points) Let's define two sets as follows:

$$A = \{x \in \mathbb{R} : |x + 1| \leq 2\}$$

$$B = \{w \in \mathbb{R} : w^2 + 2w - 3 \leq 0\}$$

Prove that  $A = B$  by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{i=1}^p i \quad \frac{p(p-1)}{2} \quad \boxed{\phantom{00}} \quad \frac{(p-1)^2}{2} \quad \boxed{\phantom{00}} \quad \frac{p(p+1)}{2} \quad \boxed{\phantom{00}} \quad \frac{(p-1)(p+1)}{2} \quad \boxed{\phantom{00}}$$

Chromatic number of  $C_n$ .      2          3           $\leq 3$            $\leq 4$

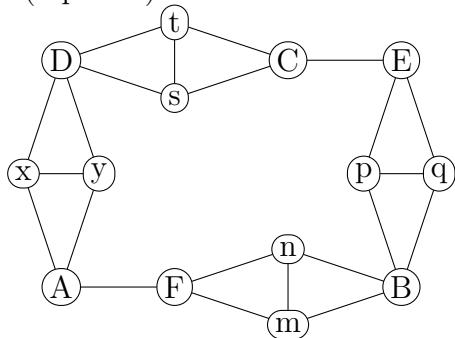
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^{n-1} \frac{1}{2^k}$$

$$1 - (\frac{1}{2})^n \quad \square$$

$$2 - (\frac{1}{2})^n \quad \square$$

$$1 - (\frac{1}{2})^{n-1} \quad \square$$

$$2 - (\frac{1}{2})^{n-1} \quad \square$$

10 guests are invited to brunch.  
Each guest will eat at least two  
buns. 30 is \_\_\_\_ on how many  
buns we will need.

an upper bound on   
a lower bound on

exactly   
not a bound on

Chromatic number of a graph  
with maximum vertex degree  $D$

$= D$    
 $\leq D + 1$

$= D + 1$    
 $\geq D + 1$

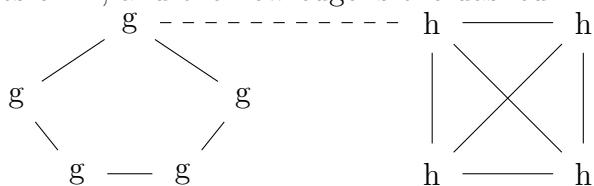
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (11 points) If  $G$  is a graph, recall that  $\chi(G)$  is its chromatic number. Suppose that  $G$  is a graph and  $H$  is another graph, not connected to  $G$ . Now, create a new graph  $T$  which consists of a copy of  $G$ , a copy of  $H$ , and a new edge that connects some node of  $G$  to some node of  $H$ . For example, suppose that  $G$  is  $C_5$  and  $H$  is  $K_4$ . Then  $T$  might look as follows, where  $g$  marks nodes of  $G$  and  $h$  marks nodes of  $H$ , and the new edge is the dashed line.



Describe how  $\chi(T)$  is related to  $\chi(G)$  and  $\chi(H)$ , justifying your answer. Your answer should handle any choice for  $G$  and  $H$ .

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=0}^{n-1} 2^k \quad 2^n - 2 \quad \boxed{\phantom{00}} \quad 2^n - 1 \quad \boxed{\phantom{00}} \quad 2^{n-1} - 1 \quad \boxed{\phantom{00}} \quad 2^{n+1} - 1 \quad \boxed{\phantom{00}}$$

All elements of  $X$  are also elements of  $M$ .

$$M = X \quad \boxed{\phantom{00}} \quad M \subseteq X \quad \boxed{\phantom{00}} \quad X \subseteq M \quad \boxed{\phantom{00}}$$

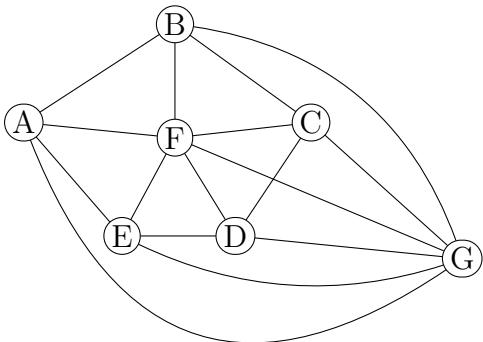
Name: \_\_\_\_\_

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Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^n \frac{1}{2^k}$$

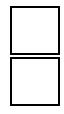
 $1 - (\frac{1}{2})^{n-1}$  
 $2 - (\frac{1}{2})^n$  
 $1 - (\frac{1}{2})^n$  
 $2 - (\frac{1}{2})^{n-1}$  

Graph  $H$  has 6 nodes. 7 is \_\_\_\_\_  
the chromatic number of  $H$ .

an upper bound on  
a lower bound on



exactly  
not a bound on

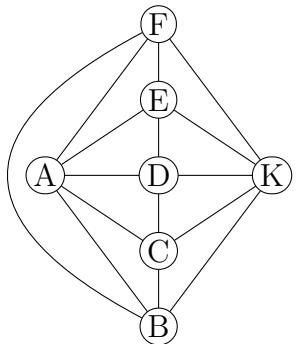
Chromatic number of  $G$  $\mathcal{C}(G)$   $\phi(G)$   $\chi(G)$   $\|G\|$

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^n k$$

$$\sum_{p=1}^n (n-p+1)$$

$$\sum_{p=1}^n (n-p)$$

$$\sum_{p=0}^n (n-p)$$

$$\sum_{p=1}^{n+1} (n-p)$$

10 students drove home in John's van. 10 is \_\_\_\_\_ how many students the van can carry.

an upper bound on  
a lower bound on

exactly  
not a bound on

Chromatic number of a graph (with at least one node) and no edges.

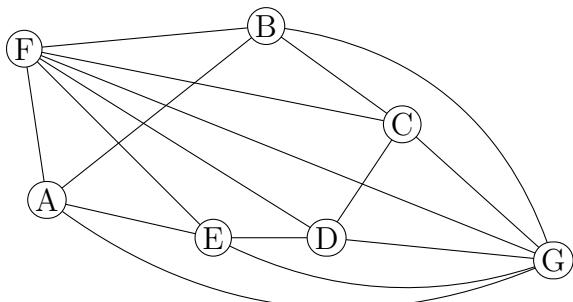
1  2  3  can't tell

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph with maximum vertex degree $D$	$= D$ <input type="checkbox"/> $= D + 1$ <input type="checkbox"/> $\geq D + 1$ <input type="checkbox"/> $\leq D + 1$ <input type="checkbox"/>
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We have 30 tablespoons of filling  
 and each bun will require one  
 tablespoon. 30 is \_\_\_\_\_ on how  
 many buns we can make.

an upper bound on <input type="checkbox"/> a lower bound on <input type="checkbox"/>	exactly <input type="checkbox"/> not a bound on <input type="checkbox"/>
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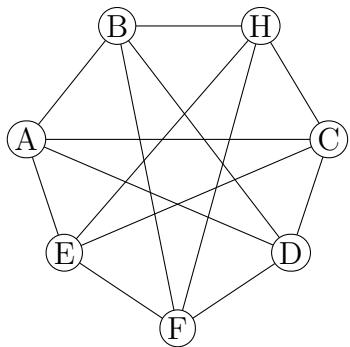
$$\sum_{k=0}^{n+1} 2^k \quad 2^{n+1} + 1 \quad \boxed{\phantom{00}} \quad 2^{n+2} - 1 \quad \boxed{\phantom{00}} \quad 2^{n+2} - 2 \quad \boxed{\phantom{00}} \quad 2^{n+1} - 1 \quad \boxed{\phantom{00}}$$

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{i=1}^{p-1} \frac{i}{p}$$





10 people rowed across Lake Tahoe in my canoe. 10 is \_\_\_\_\_ how many people the canoe can carry.

an upper bound on


exactly

a lower bound on


not a bound on


Chromatic number of a graph containing a  $W_7$ .

$\geq 3$

$\geq 4$

$\geq 7$

can't tell

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Tomas wants to plant his tomatoes so that plants are more than 1 foot apart. His garden bed is an equilateral triangle with each side 2 feet long. Prove that four is the maximum number of tomatoes he can plant.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{i=1}^{p-1} i \quad \frac{(p-1)^2}{2} \quad \boxed{\phantom{00}} \quad \frac{(p-1)(p+1)}{2} \quad \boxed{\phantom{00}} \quad \frac{p(p+1)}{2} \quad \boxed{\phantom{00}} \quad \frac{p(p-1)}{2} \quad \boxed{\phantom{00}}$$

Putting 10 people in the canoe caused it to sink. 10 is \_\_\_\_\_ how many people the canoe can carry.

an upper bound on   
 a lower bound on   
 exactly   
 not a bound on

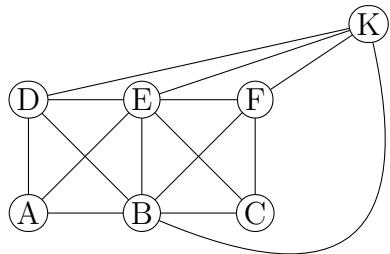
Chromatic number of a connected graph with 10 nodes  $\leq 2$    $= 2$    $\geq 2$   can't tell

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=0}^{n-1} 2^k$$

 $2^n - 2$  
 $2^n - 1$  
 $2^{n-1} - 1$  
 $2^{n+1} - 1$  

$C_5$  is a subgraph of graph  $H$ . 3 is  
\_\_\_\_\_ the chromatic number of  $H$ .

an upper bound on  
a lower bound on


exactly  
not a bound on


Exactly 40 books fit in my suitcase by volume, but I haven't checked their total weight. 40 is \_\_\_\_\_ how many books the suitcase can hold.

an upper bound on  
a lower bound on

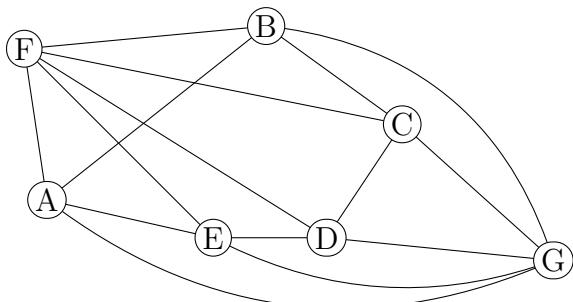

exactly  
not a bound on

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^n 2^k$$

$$2^{n+1} - 1$$

$$2^{n+1} - 2$$

$$2^{n+1} - 3$$

$$2^n - 1$$

$C_5$  is a subgraph of graph  $H$ . 5 is  
\_\_\_\_\_ the chromatic number of  $H$ .

an upper bound on  
a lower bound on


exactly  
not a bound on


Chromatic number of a bipartite  
graph with at least one edge

1

2

3

can't tell

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (11 points) Let's define two sets as follows:

$$A = \{(4 - t^2, t + 1) : t \in \mathbb{R}\}$$

$$B = \{(x, y) \in \mathbb{R}^2 : x = 3 + 2y - y^2\}$$

Prove that  $A = B$  by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=3}^n k^7$$

$$\sum_{p=1}^{n-2} p^9 \quad \square$$

$$\sum_{p=1}^{n-2} k^7 \quad \square$$

$$\sum_{p=1}^{n-2} k^9 \quad \square$$

$$\sum_{p=1}^{n-2} (p+2)^7 \quad \square$$

Chromatic number of  $K_{m,n}$ .

2

3

4

can't tell

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (11 points) Let's define two sets as follows:

$$A = \{(x, y) \in \mathbb{R}^2 : y = 3x + 7\}$$

$$B = \{\lambda(-2, 1) + (1 - \lambda)(1, 10) : \lambda \in \mathbb{R}\}$$

Prove that  $A = B$  by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=0}^n \frac{1}{2^k} \quad 1 - (\frac{1}{2})^{n-1} \quad \boxed{\phantom{00}} \quad 2 - (\frac{1}{2})^n \quad \boxed{\phantom{00}} \quad 1 - (\frac{1}{2})^n \quad \boxed{\phantom{00}} \quad 2 - (\frac{1}{2})^{n-1} \quad \boxed{\phantom{00}}$$

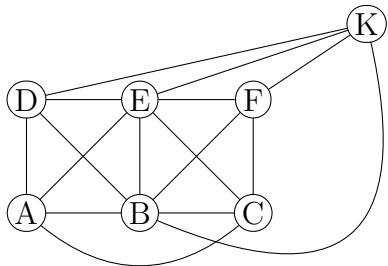
Chromatic number of  $C_n$ .      2          3           $\leq 3$            $\leq 4$

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^n k!$$

$$\sum_{p=0}^{n+1} (p+1)! \quad \square$$

$$\sum_{k=0}^{n+1} (k-1)! \quad \square$$

$$\sum_{k=0}^{n-1} (k+1)! \quad \square$$

$$\sum_{p=0}^{n+1} k! \quad \square$$

All elements of  $M$  are also elements of  $X$ .

$$M = X \quad \square \quad M \subseteq X \quad \square \quad X \subseteq M \quad \square$$

Chromatic number of  $G$

$$\mathcal{C}(G) \quad \square$$

$$\phi(G) \quad \square$$

$$\chi(G) \quad \square$$

$$\|G\| \quad \square$$

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1. (11 points) Recall that if  $G$  is a graph, then  $\chi(G)$  is its chromatic number. Suppose that  $G$  is a graph and  $H$  is another graph not connected to  $G$ . Suppose  $G$  and  $H$  each have at least two nodes and at least one edge. Dr. Evil picks two adjacent nodes  $a$  and  $b$  from  $G$ , and also two adjacent nodes  $c$  and  $d$  from  $H$ . He merges  $G$  and  $H$  into a single graph  $T$  by merging  $b$  and  $d$  into a single node, and adding an edge connecting  $a$  and  $c$ . So, if  $G$  and  $H$  are as shown on the left, then  $T$  might look as shown on the right.



Describe how  $\chi(T)$  is related to  $\chi(G)$  and  $\chi(H)$ , justifying your answer.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=-2}^n k^2$$

$$\sum_{p=0}^{n+2} (p+2)^2 \quad \square$$

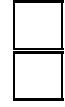
$$\sum_{p=0}^{n-2} (p-2)^2 \quad \square$$

$$\sum_{p=0}^{n+2} (p-2)^2 \quad \square$$

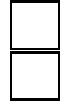
$$\sum_{p=0}^{n+2} p^2 \quad \square$$

$W_7$  is a subgraph of graph  $H$ . 4 is  
\_\_\_\_\_ the chromatic number of  $H$ .

an upper bound on  
a lower bound on



exactly  
not a bound on

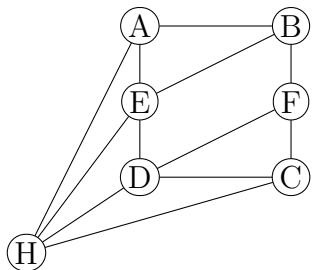


Name: \_\_\_\_\_

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1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{i=0}^{k-1} (k \cdot i + 2)$$

$$\frac{k^2(k+1)}{2} + 2k$$

$$\frac{k(k+1)}{2} + 2(k - 1)$$

$$\frac{k^2(k-1)}{2} + 2k$$

$$\frac{k(k-1)}{2} + 2(k - 1)$$

When I poured 5 gallons of water into the bucket, some spilled over the top. 5 gallons is \_\_\_\_\_ how much the bucket holds.

an upper bound on  
a lower bound on


exactly  
not a bound on


Chromatic number of a bipartite graph with at least two vertices.

1

2

3

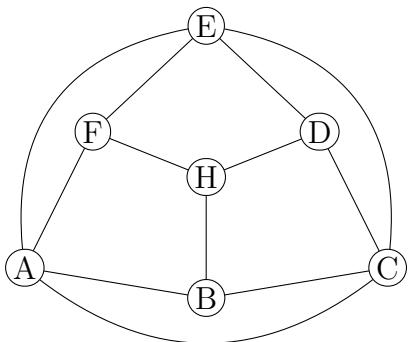
can't tell

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{i=1}^{p-1} i$$

$$\frac{p(p-1)}{2}$$

$$\frac{(p-1)^2}{2}$$

$$\frac{p(p+1)}{2}$$

$$\frac{(p-1)(p+1)}{2}$$

I heated 2 liters of milk in my big pot. 2 liters is \_\_\_\_\_ how much the pot holds.

an upper bound on  
a lower bound on


exactly  
not a bound on


Chromatic number of a graph containing a  $W_n$ .

$$\geq 2$$

$$\leq 3$$

$$\geq n$$

can't tell

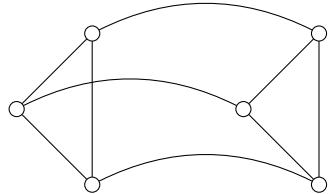
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (11 points) Recall that if  $G$  is a graph, then  $\chi(G)$  is its chromatic number. Let's define the "doubled" version of a graph  $G$  as follows: make two copies of  $G$  and add an edge joining each pair of corresponding nodes. For example, the doubled version of  $C_3$  looks like:



Suppose that  $T$  is the doubled version of a graph  $G$ . Describe how  $\chi(T)$  is related to  $\chi(G)$ , justifying your answer. Your answer should handle any choice for  $G$ , not just  $C_3$ .

2. (4 points) Check the (single) box that best characterizes each item.

Chromatic number of  $W_n$ .2  3   $\leq 3$    $\leq 4$  

10 people can row the canoe but  
 11 people caused it to sink. 10 is  
 \_\_\_\_ how many people the canoe  
 can carry.

an upper bound on  
 a lower bound on



exactly  
 not a bound on

