

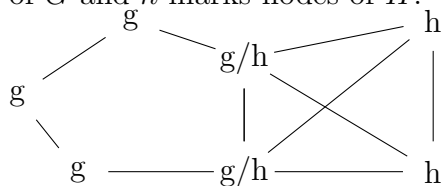
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture:    A    B

Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (11 points) If  $G$  is a graph, recall that  $\chi(G)$  is its chromatic number. Suppose that  $G$  is a graph with at least one edge and  $H$  is another graph with at least one edge, not connected to  $G$ . Now, pick a specific edge  $e$  from  $G$  and an edge  $f$  from  $H$  and merge the two edges, creating a combined graph  $T$ . For example, suppose that  $G$  is  $C_5$  and  $H$  is  $K_4$ . Then  $T$  might look as follows, where  $g$  marks nodes of  $G$  and  $h$  marks nodes of  $H$ .



Describe how  $\chi(T)$  is related to  $\chi(G)$  and  $\chi(H)$ , justifying your answer. Your answer should handle any choice for  $G$  and  $H$ .

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{i=1}^{p-1} i \quad \frac{p(p-1)}{2} \quad \frac{(p-1)^2}{2} \quad \frac{p(p+1)}{2} \quad \frac{(p-1)(p+1)}{2}$$

Leal team's bridge held 100 pounds without collapsing. 100 pounds is \_\_\_\_\_ on how much the bridge can hold.

an upper bound on  
a lower bound on

☐  
☐

exactly  
not a bound on

☐  
☐

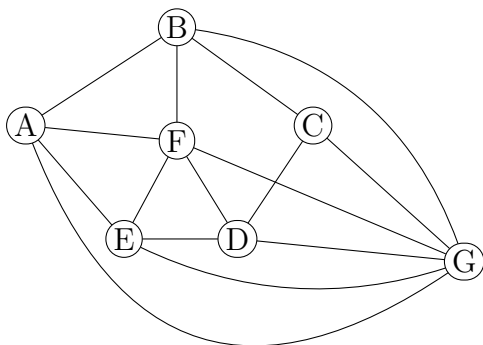
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Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph  
containing a  $C_7$ .

$\geq 2$  ☐     $\geq 3$  ☐     $\leq 3$  ☐    can't tell ☐

$$\sum_{i=1}^{p-1} i$$

$$\frac{(p-1)^2}{2}$$

☐

$$\frac{(p-1)(p+1)}{2}$$

☐

$$\frac{p(p+1)}{2}$$

☐

$$\frac{p(p-1)}{2}$$

☐

$$\tau \leq 1.3$$

an upper bound on  $\tau$

☐

exactly  $\tau$

☐

a lower bound on  $\tau$

☐

not a bound on  $\tau$

☐

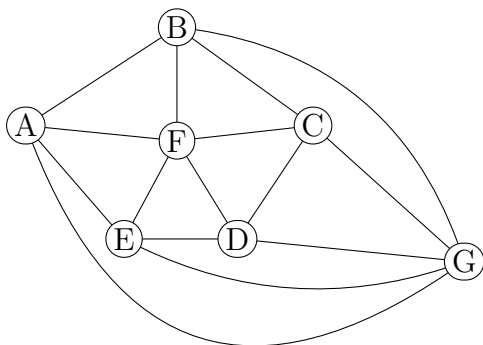
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1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph  
containing a  $W_n$ .

$\leq 3$  ☐     $\geq 3$  ☐     $\geq n$  ☐    can't tell ☐

$$\sum_{i=1}^{p-1} \frac{i}{p}$$

$$\frac{p(p-1)}{2}$$

☐

$$\frac{p(p+1)}{2}$$

☐

$$\frac{(p+1)}{2}$$

☐

$$\frac{(p-1)}{2}$$

☐

Putting 10 people in the canoe  
caused it to sink. 10 is \_\_\_\_\_ how  
many people the canoe can carry.

an upper bound on

☐  
☐

exactly

☐  
☐

a lower bound on

not a bound on

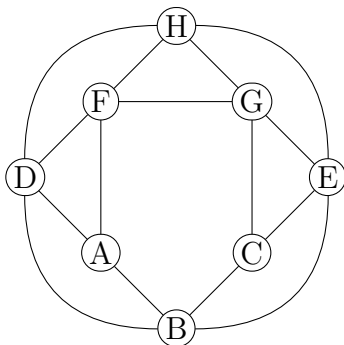
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1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of an  
connected acyclic graph  
with 5 nodes.

 $\leq 2$  ☐ $= 2$  ☐ $\leq 5$  ☐can't tell ☐

$$\sum_{i=0}^{k-1} (k \cdot i + 2)$$

$$\frac{k^2(k+1)}{2} + 2k$$
 ☐

$$\frac{k(k+1)}{2} + 2(k-1)$$
 ☐

$$\frac{k^2(k-1)}{2} + 2k$$
 ☐

$$\frac{k(k-1)}{2} + 2(k-1)$$
 ☐

$$\pi \geq 1.3$$

an upper bound on  $\pi$ ☐exactly  $\pi$ ☐a lower bound on  $\pi$ ☐not a bound on  $\pi$ ☐

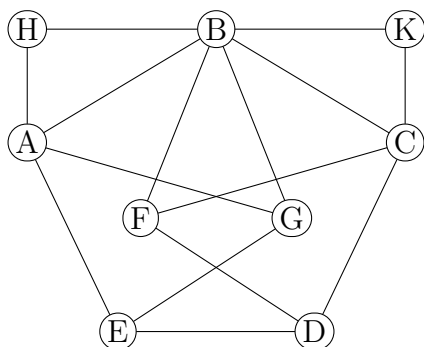
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Lecture:    A    B

Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a  
connected graph with 10 nodes.  $\leq 2$  ☐  $= 2$  ☐  $\geq 2$  ☐ can't tell ☐

$\sum_{k=-2}^n k^2$   $\sum_{p=0}^{n+2} (p+2)^2$  ☐  $\sum_{p=0}^{n-2} (p-2)^2$  ☐  $\sum_{p=0}^{n+2} (p-2)^2$  ☐  $\sum_{p=0}^{n+2} p^2$  ☐

We have 30 tablespoons of filling.  
Each bun requires exactly one  
tablespoon of filling. 30 is \_\_\_\_\_ on  
how many buns we can make.

an upper bound on

☐  
☐

a lower bound on

exactly

☐  
☐

not a bound on

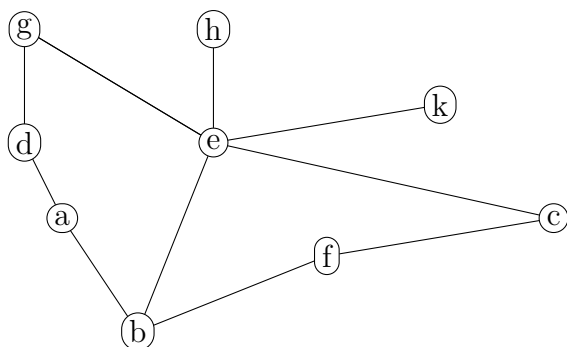
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1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph  
containing a  $W_7$ .

$\geq 3$  ☐     $\geq 4$  ☐     $\geq 7$  ☐    can't tell ☐

$\sum_{k=1}^n k!$   $\sum_{p=0}^{n+1} (p+1)!$  ☐     $\sum_{k=0}^{n+1} (k-1)!$  ☐     $\sum_{k=0}^{n-1} (k+1)!$  ☐     $\sum_{p=0}^{n+1} k!$  ☐

10 people rowed across Lake Tahoe  
in my canoe. 10 is \_\_\_\_\_ how many  
people the canoe can carry.

an upper bound on ☐  
a lower bound on ☐

exactly ☐  
not a bound on ☐

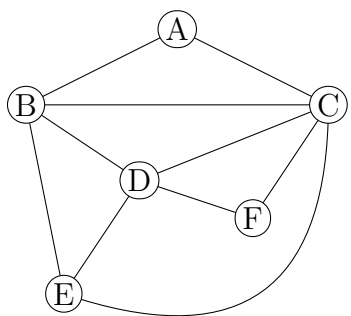
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Lecture:    A    B

Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph  
containing a  $K_n$ .

 $\leq n$  ☐ $= n$  ☐ $\geq n$  ☐can't tell ☐

$$\sum_{k=0}^n k!$$

$$\sum_{p=1}^{n+1} (p+1)! \quad \text{☐$$

$$\sum_{k=1}^{n+1} (k-1)! \quad \text{☐$$

$$\sum_{k=1}^{n-1} (k+1)! \quad \text{☐$$

$$\sum_{p=1}^{n+1} k! \quad \text{☐$$

I heated 2 liters of milk in my big  
pot. 2 liters is \_\_\_\_\_ how much the  
pot holds.

an upper bound on

☐  
☐

a lower bound on

exactly

☐  
☐

not a bound on

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture:    A    B

Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (11 points) Let's define two sets as follows:

$$A = \{(x, y) \in \mathbb{R}^2 : y = x^2 - 4x + 3\}$$

$$B = \{(t + 2, t^2 - 1) : t \in \mathbb{R}\}$$

Prove that  $A = B$  by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

Chromatic number of  $K_{m,n}$ .  
(Assume  $m \geq 1, n \geq 1$ .)

2 ☐3 ☐4 ☐can't tell ☐

$\pi \leq 10$

an upper bound on  $\pi$

☐  
☐

a lower bound on  $\pi$

exactly  $\pi$

not a bound on  $\pi$

☐  
☐



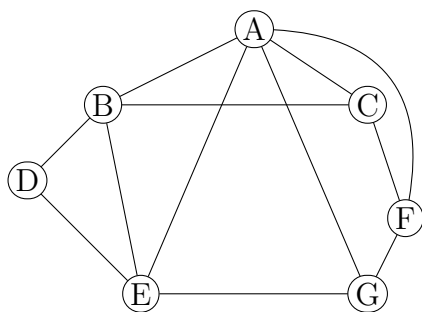
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Lecture: B

Discussion: Friday 11 12 1 2 3 4

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a bipartite graph with at least one edge

1 ☐ 2 ☐ 3 ☐ can't tell ☐

Suppose I want to estimate  $\frac{103}{20}$ .  
3 is \_\_\_\_\_

an upper bound

☐

an exact answer

☐

a lower bound

☐

not a bound on

☐

$$\sum_{k=3}^n k^7$$

$$\sum_{p=1}^{n-2} p^9$$

☐

$$\sum_{p=1}^{n-2} k^7$$

☐

$$\sum_{p=1}^{n-2} k^9$$

☐

$$\sum_{p=1}^{n-2} (p+2)^7$$

☐

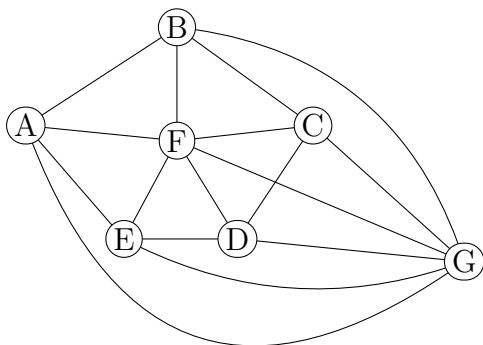
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Lecture: B

Discussion: Friday 11 12 1 2 3 4

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of  $W_n$ .2 ☐3 ☐ $\leq 3$  ☐ $\leq 4$  ☐All elements of  $M$  are also elements of  $X$ . $M = X$  ☐ $M \subseteq X$  ☐ $X \subseteq M$  ☐

$$\sum_{k=0}^n \frac{1}{2^k}$$

$1 - \left(\frac{1}{2}\right)^{n-1}$  ☐

$2 - \left(\frac{1}{2}\right)^n$  ☐

$1 - \left(\frac{1}{2}\right)^n$  ☐

$2 - \left(\frac{1}{2}\right)^{n-1}$  ☐

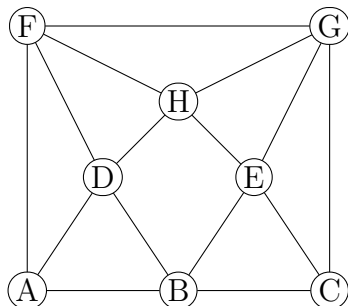
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Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph with  
no cycles and at least one edge

1 ☐    2 ☐    3 ☐    can't tell ☐

15 guests are invited to brunch.  
Each guest will eat at least two  
buns. 20 is \_\_\_\_\_ on how many  
buns we will need.

an upper bound on ☐    exactly ☐  
a lower bound on ☐    not a bound on ☐

$$\sum_{k=0}^{n-1} \frac{1}{2^k}$$

$$1 - \left(\frac{1}{2}\right)^{n-1} \quad \boxed{\phantom{00}}$$

$$2 - \left(\frac{1}{2}\right)^n \quad \boxed{\phantom{00}}$$

$$1 - \left(\frac{1}{2}\right)^n \quad \boxed{\phantom{00}}$$

$$2 - \left(\frac{1}{2}\right)^{n-1} \quad \boxed{\phantom{00}}$$

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NetID: \_\_\_\_\_ Lecture:    A    B

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1. (11 points) Let's define two sets as follows:

$$A = \{x \in \mathbb{R} : |x + 1| \leq 2\}$$

$$B = \{w \in \mathbb{R} : w^2 + 2w - 3 \leq 0\}$$

Prove that  $A = B$  by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{i=1}^p i \quad \frac{p(p-1)}{2} \quad \frac{(p-1)^2}{2} \quad \frac{p(p+1)}{2} \quad \frac{(p-1)(p+1)}{2}$$

Chromatic number of  $C_n$ .    2    3     $\leq 3$      $\leq 4$

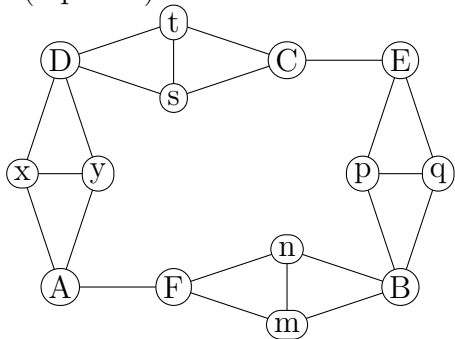
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NetID: \_\_\_\_\_

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Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^{n-1} \frac{1}{2^k} \quad 1 - \left(\frac{1}{2}\right)^n \quad \boxed{\phantom{0}} \quad 2 - \left(\frac{1}{2}\right)^n \quad \boxed{\phantom{0}} \quad 1 - \left(\frac{1}{2}\right)^{n-1} \quad \boxed{\phantom{0}} \quad 2 - \left(\frac{1}{2}\right)^{n-1} \quad \boxed{\phantom{0}}$$

10 guests are invited to brunch.  
Each guest will eat at least two  
buns. 30 is \_\_\_\_\_ on how many  
buns we will need.

an upper bound on  $\boxed{\phantom{0}}$     exactly  $\boxed{\phantom{0}}$   
a lower bound on  $\boxed{\phantom{0}}$     not a bound on  $\boxed{\phantom{0}}$

Chromatic number of a graph  
with maximum vertex degree  $D$

$$\begin{array}{ll} = D & \boxed{\phantom{0}} \\ \leq D + 1 & \boxed{\phantom{0}} \end{array} \quad \begin{array}{ll} = D + 1 & \boxed{\phantom{0}} \\ \geq D + 1 & \boxed{\phantom{0}} \end{array}$$

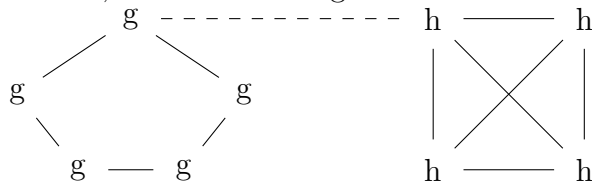
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1. (11 points) If  $G$  is a graph, recall that  $\chi(G)$  is its chromatic number. Suppose that  $G$  is a graph and  $H$  is another graph, not connected to  $G$ . Now, create a new graph  $T$  which consists of a copy of  $G$ , a copy of  $H$ , and a new edge that connects some node of  $G$  to some node of  $H$ . For example, suppose that  $G$  is  $C_5$  and  $H$  is  $K_4$ . Then  $T$  might look as follows, where  $g$  marks nodes of  $G$  and  $h$  marks nodes of  $H$ , and the new edge is the dashed line.



Describe how  $\chi(T)$  is related to  $\chi(G)$  and  $\chi(H)$ , justifying your answer. Your answer should handle any choice for  $G$  and  $H$ .

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=0}^{n-1} 2^k \quad 2^n - 2 \quad \boxed{\phantom{0}} \quad 2^n - 1 \quad \boxed{\phantom{0}} \quad 2^{n-1} - 1 \quad \boxed{\phantom{0}} \quad 2^{n+1} - 1 \quad \boxed{\phantom{0}}$$

All elements of  $X$  are also elements of  $M$ .

$$M = X \quad \boxed{\phantom{0}}$$

$$M \subseteq X \quad \boxed{\phantom{0}}$$

$$X \subseteq M \quad \boxed{\phantom{0}}$$

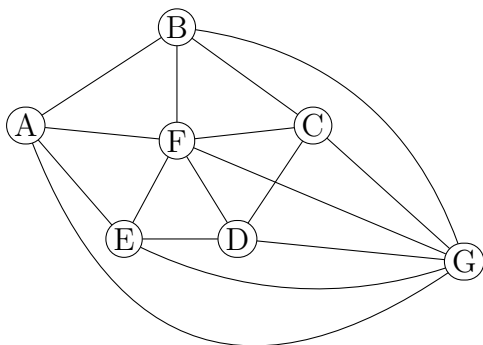
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1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^n \frac{1}{2^k} \quad 1 - \left(\frac{1}{2}\right)^{n-1} \quad \square \quad 2 - \left(\frac{1}{2}\right)^n \quad \square \quad 1 - \left(\frac{1}{2}\right)^n \quad \square \quad 2 - \left(\frac{1}{2}\right)^{n-1} \quad \square$$

Graph  $H$  has 6 nodes. 7 is \_\_\_\_\_  
the chromatic number of  $H$ .

an upper bound on  
a lower bound on

☐  
☐

exactly  
not a bound on

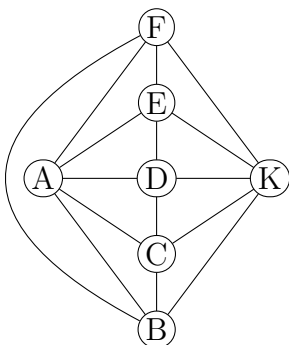
☐  
☐
Chromatic number of  $G$  $\mathcal{C}(G)$  ☐ $\phi(G)$  ☐ $\chi(G)$  ☐ $\|G\|$  ☐

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Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^n k \quad \sum_{p=1}^n (n-p+1) \quad \sum_{p=1}^n (n-p) \quad \sum_{p=0}^n (n-p) \quad \sum_{p=1}^{n+1} (n-p)$$

10 students drove home in John's van. 10 is \_\_\_\_\_ how many students the van can carry.

an upper bound on ☐ exactly ☐  
a lower bound on ☐ not a bound on ☐

Chromatic number of a graph (with at least one node) and no edges.

1 ☐ 2 ☐ 3 ☐ can't tell ☐





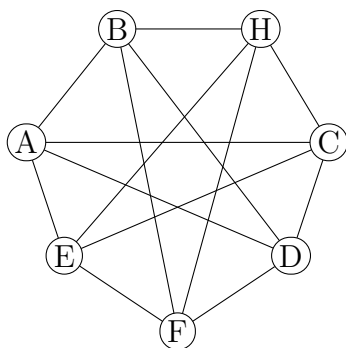
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NetID: \_\_\_\_\_

Lecture:    A    B

Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{i=1}^{p-1} \frac{i}{p} \quad \frac{p(p-1)}{2} \quad \frac{p(p+1)}{2} \quad \frac{(p+1)}{2} \quad \frac{(p-1)}{2}$$

10 people rowed across Lake Tahoe in canoe. 10 is \_\_\_\_\_ how many people the canoe can carry.

an upper bound on

☐  
☐

exactly

☐  
☐

a lower bound on

not a bound on

Chromatic number of a graph containing a  $W_7$ .

$\geq 3$  ☐

$\geq 4$  ☐

$\geq 7$  ☐

can't tell ☐

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1. (9 points) Tomas wants to plant his tomatoes so that plants are more than 1 foot apart. His garden bed is an an equilateral triangle with each side 2 feet long. Prove that four is the maximum number of tomatoes he can plant.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{i=1}^{p-1} i \quad \frac{(p-1)^2}{2} \quad \boxed{\phantom{0}} \quad \frac{(p-1)(p+1)}{2} \quad \boxed{\phantom{0}} \quad \frac{p(p+1)}{2} \quad \boxed{\phantom{0}} \quad \frac{p(p-1)}{2} \quad \boxed{\phantom{0}}$$

Putting 10 people in the canoe caused it to sink. 10 is \_\_\_\_\_ how many people the canoe can carry.

an upper bound on

☐

exactly

☐

a lower bound on

☐

not a bound on

☐

Chromatic number of a connected graph with 10 nodes.

$\leq 2$

☐

$= 2$

☐

$\geq 2$

☐

can't tell

☐

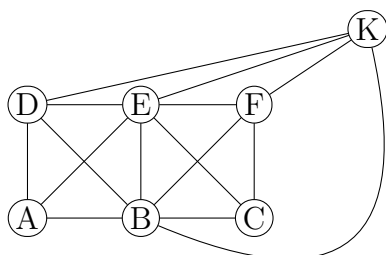
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1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=0}^{n-1} 2^k$$

$2^n - 2$  ☐

$2^n - 1$  ☐

$2^{n-1} - 1$  ☐

$2^{n+1} - 1$  ☐

$C_5$  is a subgraph of graph  $H$ . 3 is \_\_\_\_\_ the chromatic number of  $H$ .

an upper bound on

☐

exactly

☐

a lower bound on

☐

not a bound on

☐

Exactly 40 books fit in my suitcase by volume, but I haven't checked their total weight. 40 is \_\_\_\_\_ how many books the suitcase can hold.

an upper bound on

☐

exactly

☐

a lower bound on

☐

not a bound on

☐



Name: \_\_\_\_\_

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1. (11 points) Let's define two sets as follows:

$$A = \{(4 - t^2, t + 1) : t \in \mathbb{R}\}$$

$$B = \{(x, y) \in \mathbb{R}^2 : x = 3 + 2y - y^2\}$$

Prove that  $A = B$  by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=3}^n k^7$$

$$\sum_{p=1}^{n-2} p^9 \quad \square$$

$$\sum_{p=1}^{n-2} k^7 \quad \square$$

$$\sum_{p=1}^{n-2} k^9 \quad \square$$

$$\sum_{p=1}^{n-2} (p+2)^7 \quad \square$$

Chromatic number of  $K_{m,n}$ .

2      ☐

3      ☐

4      ☐

can't tell      ☐

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture:    A    B

Discussion:    Thursday    Friday    10    11    12    1    2    3    4    5    6

1. (11 points) Let's define two sets as follows:

$$A = \{(x, y) \in \mathbb{R}^2 : y = 3x + 7\}$$

$$B = \{\lambda(-2, 1) + (1 - \lambda)(1, 10) : \lambda \in \mathbb{R}\}$$

Prove that  $A = B$  by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=0}^n \frac{1}{2^k} \quad 1 - \left(\frac{1}{2}\right)^{n-1} \quad \square \quad 2 - \left(\frac{1}{2}\right)^n \quad \square \quad 1 - \left(\frac{1}{2}\right)^n \quad \square \quad 2 - \left(\frac{1}{2}\right)^{n-1} \quad \square$$

$$\text{Chromatic number of } C_n. \quad 2 \quad \square \quad 3 \quad \square \quad \leq 3 \quad \square \quad \leq 4 \quad \square$$

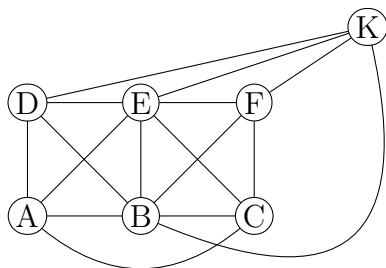
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture:      A      B

Discussion:      Thursday      Friday      10      11      12      1      2      3      4      5      6

1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^n k! \quad \sum_{p=0}^{n+1} (p+1)! \quad \sum_{k=0}^{n+1} (k-1)! \quad \sum_{k=0}^{n-1} (k+1)! \quad \sum_{p=0}^{n+1} k! \quad \square$$

All elements of  $M$  are also elements of  $X$ .

$$M = X \quad \square \quad M \subseteq X \quad \square \quad X \subseteq M \quad \square$$

Chromatic number of  $G$

$$\mathcal{C}(G) \quad \square \quad \phi(G) \quad \square \quad \chi(G) \quad \square \quad \|G\| \quad \square$$



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1. (11 points) Recall that if  $G$  is a graph, then  $\chi(G)$  is its chromatic number. Suppose that  $G$  is a graph and  $H$  is another graph not connected to  $G$ . Suppose  $G$  and  $H$  each have at least two nodes and at least one edge. Dr. Evil picks two adjacent nodes  $a$  and  $b$  from  $G$ , and also two adjacent nodes  $c$  and  $d$  from  $H$ . He merges  $G$  and  $H$  into a single graph  $T$  by merging  $b$  and  $d$  into a single node, and adding an edge connecting  $a$  and  $c$ . So, if  $G$  and  $H$  are as shown on the left, then  $T$  might look as shown on the right.



Describe how  $\chi(T)$  is related to  $\chi(G)$  and  $\chi(H)$ , justifying your answer.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=-2}^n k^2 \quad \sum_{p=0}^{n+2} (p+2)^2 \quad \sum_{p=0}^{n-2} (p-2)^2 \quad \sum_{p=0}^{n+2} (p-2)^2 \quad \sum_{p=0}^{n+2} p^2$$

$W_7$  is a subgraph of graph  $H$ . 4 is \_\_\_\_\_ the chromatic number of  $H$ .

an upper bound on \_\_\_\_\_  
a lower bound on \_\_\_\_\_

exactly \_\_\_\_\_

not a bound on \_\_\_\_\_

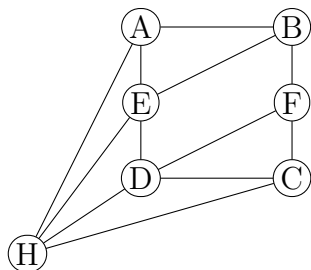
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1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{i=0}^{k-1} (k \cdot i + 2)$$

$$\frac{k^2(k+1)}{2} + 2k$$

☐

$$\frac{k(k+1)}{2} + 2(k-1)$$

☐

$$\frac{k^2(k-1)}{2} + 2k$$

☐

$$\frac{k(k-1)}{2} + 2(k-1)$$

☐

When I poured 5 gallons of water into the bucket, some spilled over the top. 5 gallons is \_\_\_\_\_ how much the bucket holds.

an upper bound on

☐

exactly

☐

a lower bound on

☐

not a bound on

☐

Chromatic number of a bipartite graph with at least two vertices.

1

☐

2

☐

3

☐

can't tell

☐

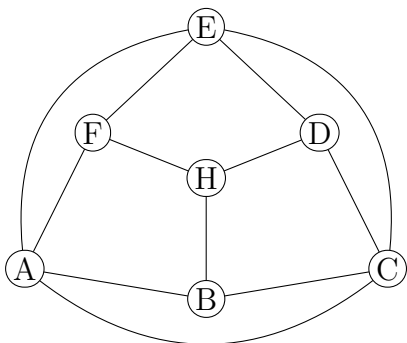
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Lecture:    A    B

Discussion:    Thursday    Friday    10    11    12    1    2    3    4    5    6

1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{i=1}^{p-1} i \quad \frac{p(p-1)}{2} \quad \frac{(p-1)^2}{2} \quad \frac{p(p+1)}{2} \quad \frac{(p-1)(p+1)}{2}$$

I heated 2 liters of milk in my big pot. 2 liters is \_\_\_\_\_ how much the pot holds.

an upper bound on ☐ exactly ☐  
a lower bound on ☐ not a bound on ☐

Chromatic number of a graph containing a  $W_n$ .

$\geq 2$  ☐  $\leq 3$  ☐  $\geq n$  ☐ can't tell ☐

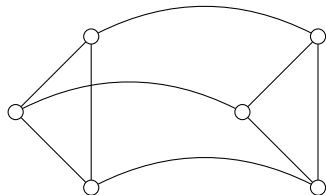
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Lecture:      A      B

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1. (11 points) Recall that if  $G$  is a graph, then  $\chi(G)$  is its chromatic number. Let's define the "doubled" version of a graph  $G$  as follows: make two copies of  $G$  and add an edge joining each pair of corresponding nodes. For example, the doubled version of  $C_3$  looks like:



Suppose that  $T$  is the doubled version of a graph  $G$ . Describe how  $\chi(T)$  is related to  $\chi(G)$ , justifying your answer. Your answer should handle any choice for  $G$ , not just  $C_3$ .

2. (4 points) Check the (single) box that best characterizes each item.

Chromatic number of  $W_n$ .2 ☐3 ☐ $\leq 3$  ☐ $\leq 4$  ☐

10 people can row the canoe but  
11 people caused it to sink. 10 is  
\_\_\_\_\_ how many people the canoe  
can carry.

an upper bound on

☐

exactly

☐

a lower bound on

☐

not a bound on

☐