

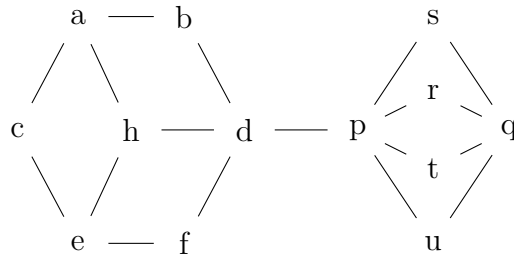
Name: \_\_\_\_\_

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Lecture:    A    B

Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (10 points) How many isomorphisms are there from  $G$  (below) to itself? Justify your answer and/or show your work clearly .



2. (5 points) The wheel graph  $W_{73}$  has 73 nodes on the rim. How many edges does it have?

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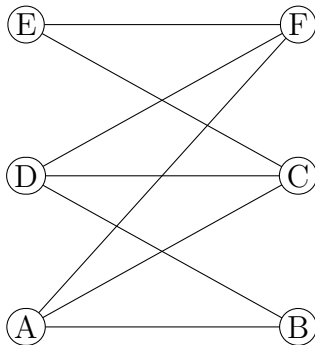
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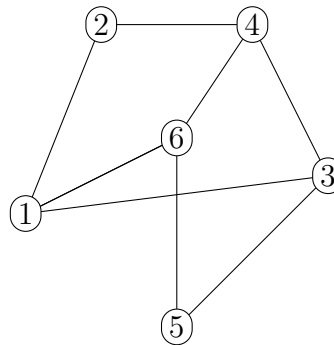
Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) Show four distinct (i.e. not isomorphic) graphs, each of which is connected and has six nodes and no cycles.

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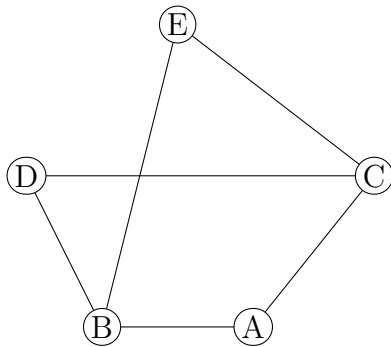
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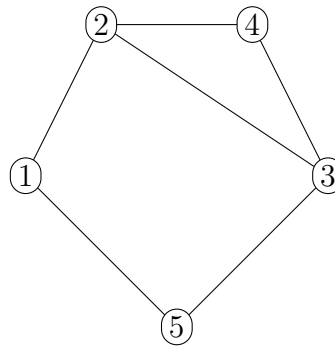
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1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) Is the cycle graph  $C_4$  a subgraph of graph  $K_{3,3}$ ? Briefly justify your answer.

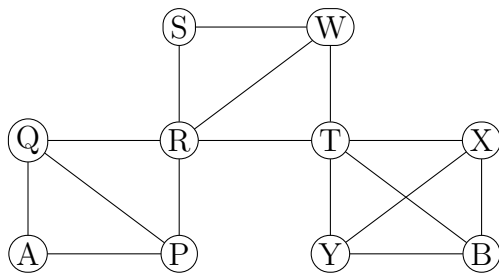
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1. (10 points) How many isomorphisms are there from  $G$  (below) to itself? Justify your answer and/or show your work clearly .



2. (5 points) What is the difference between a path and an open walk?

Name: \_\_\_\_\_

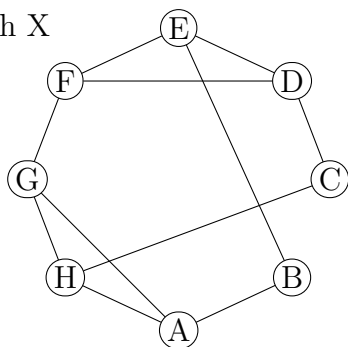
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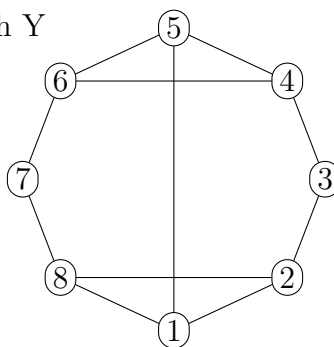
Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Suppose that graph G has degree sequence 1, 1, 1, 1, 1, 1. How many connected components does G have?

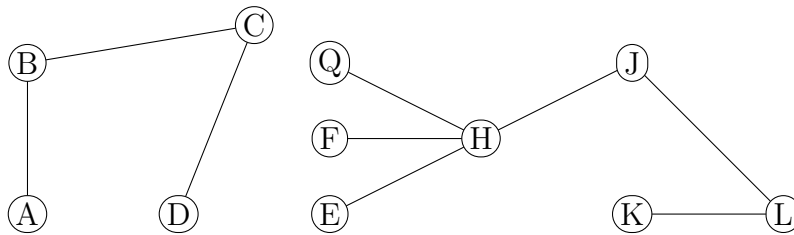
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1. (10 points) How many isomorphisms are there from  $G$  (below) to itself? Justify your answer and/or show your work clearly .



2. (5 points) Is the cycle graph  $C_{17}$  a subgraph of the wheel graph  $W_{23}$ ? Briefly justify your answer.

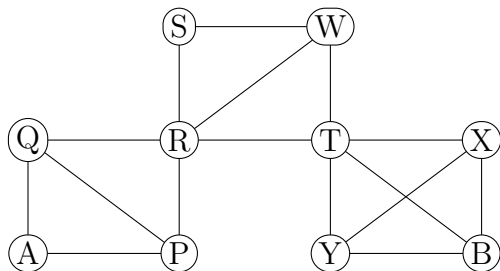
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1. (10 points) How many isomorphisms are there from  $G$  (below) to itself? Justify your answer and/or show your work clearly .



2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Suppose graph  $G$  has degree sequence 1, 1, 1, 1, 2. Is  $G$  connected? Briefly justify your answer.

Name: \_\_\_\_\_

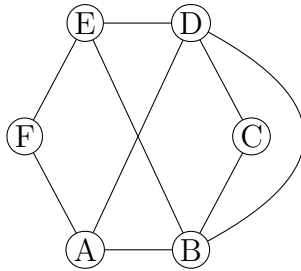
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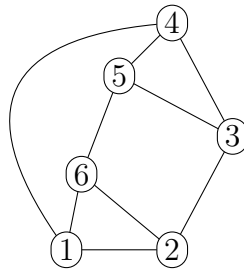
Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) Is the graph  $C_{10}$  bipartite? Briefly justify your answer.



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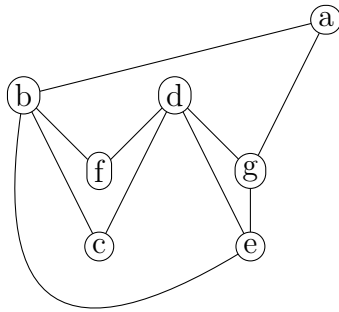
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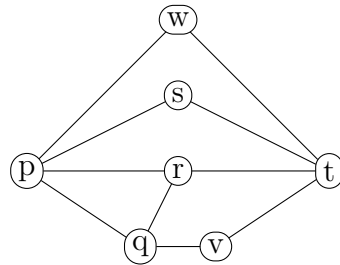
Discussion: Friday 11 12 1 2 3 4

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



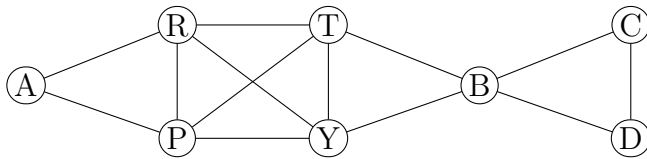
2. (5 points) The complete graph  $K_8$  contains 8 nodes. How many edges does it have?

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1. (10 points) How many isomorphisms are there from  $G$  (below) to itself? Justify your answer and/or show your work clearly .



2. (5 points) Complete this statement of the Handshaking Theorem.  
For any graph  $G$  with set of nodes  $V$  and set of edges  $E$ , ...

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(10 points) Suppose that  $g : \mathbb{N} \rightarrow \mathbb{N}$  is one-to-one. Let's define the function  $f : \mathbb{N}^2 \rightarrow \mathbb{N}^2$  by the equation  $f(x, y) = (x + g(y), g(x))$ . Prove that  $f$  is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

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(10 points) Suppose that  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  is onto. Let's define  $g : \mathbb{Z}^2 \rightarrow \mathbb{Z}^2$  by  $g(x, y) = (f(x) + y, y + 3)$ . Prove that  $g$  is onto.

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(10 points) Suppose that  $f : (0, \infty) \rightarrow (\frac{5}{4}, \infty)$  is defined by  $f(x) = \frac{5x^2+3}{4x^2}$ . Proof that  $f$  is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

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(10 points) Suppose that  $f : \mathbb{Z}^2 \rightarrow \mathbb{Z}$  is defined by  $f(x, y) = xy + yx^2 - x^2$ . Prove that  $f$  is onto.

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(10 points) Suppose that  $A$  and  $B$  are sets. Suppose that  $f : B \rightarrow A$  and  $g : A \rightarrow B$  are functions such that  $f(g(x)) = x$  for every  $x \in A$ . Prove that  $g$  is one-to-one.

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(10 points) Suppose that  $f : [0, \frac{1}{2}] \rightarrow [1, \frac{5}{2}]$  is defined by  $f(x) = \frac{x^2+1}{1-2x^2}$ . Prove that  $f$  is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.



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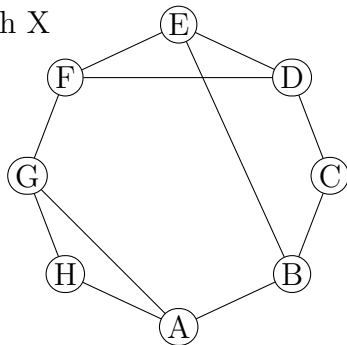
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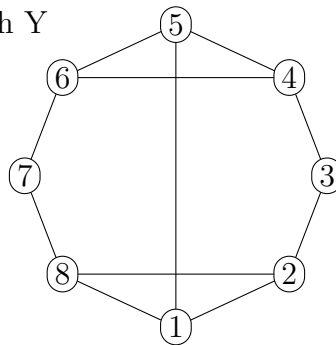
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1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) Draw a picture of the graph  $K_{2,3}$ .

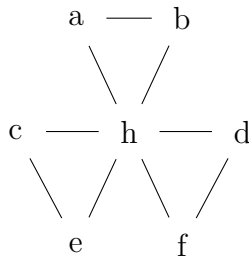
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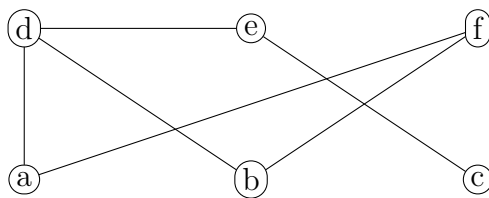
Lecture:    A    B

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1. (10 points) How many isomorphisms are there from  $G$  (below) to itself? Justify your answer and/or show your work clearly .



2. (5 points) Is this graph bipartite? Briefly justify your answer.



Name: \_\_\_\_\_

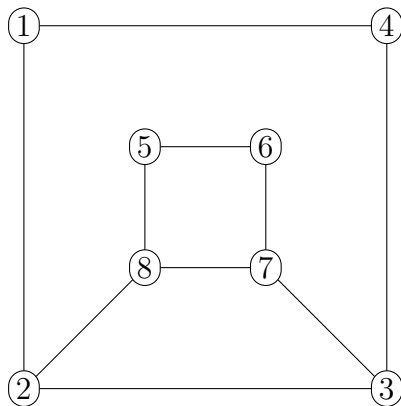
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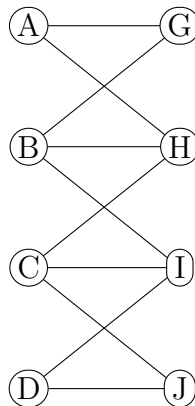
Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Is it possible to construct a (simple) graph with degree sequence: 4, 3, 3, 2, 0? Show how or briefly explain why this isn't possible.

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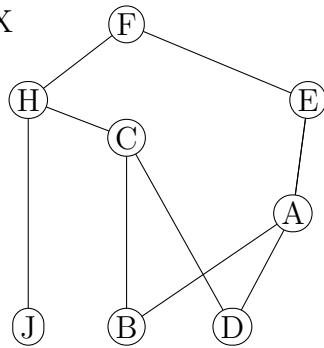
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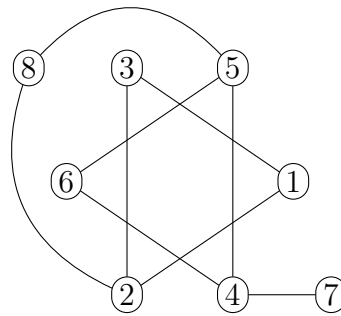
Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) Suppose that  $d(u, v)$  is the distance between nodes  $u$  and  $v$  (i.e. along the shortest path). Agent K claims that  $d(u, v) + d(v, w) = d(u, w)$  for any nodes  $u$ ,  $v$ , and  $w$ . Is he correct? Briefly explain why or give a counter-example.

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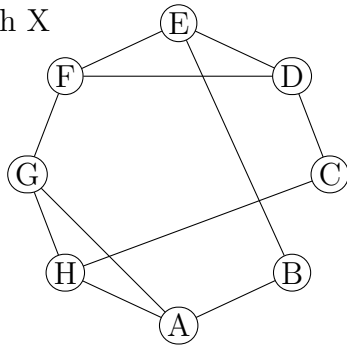
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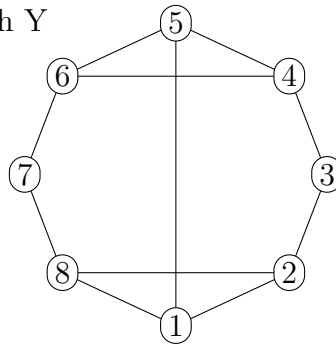
Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Is it possible to construct a (simple) graph with degree sequence: 4, 3, 3, 2, 2, 1? Show how or briefly explain why this isn't possible.

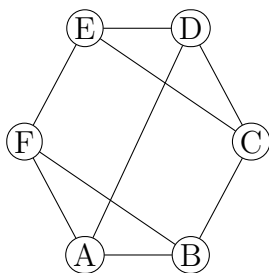
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Lecture:    A    B

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1. (10 points) How many isomorphisms are there from  $G$  (below) to itself? Justify your answer and/or show your work clearly .



2. (5 points) How many edges are in the complete bipartite graph  $K_{10,5}$ ?

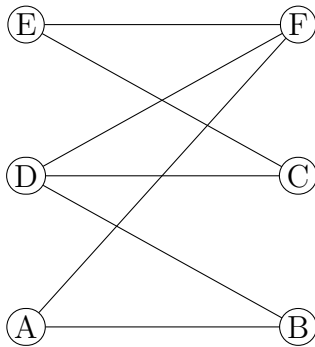
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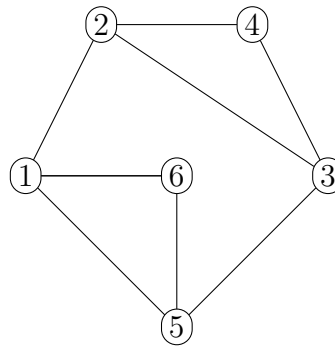
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1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) Show three graphs, each with exactly four nodes and three edges, none of which are isomorphic.

Name: \_\_\_\_\_

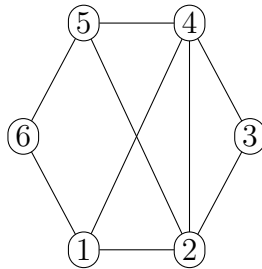
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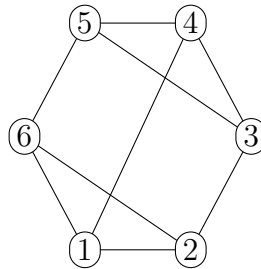
Discussion:    Thursday   Friday   10   11   12   1   2   3   4   5   6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Is it possible to construct a graph with degree sequence: 5, 3, 2, 2, 2, 0? Show how or briefly explain why this isn't possible.



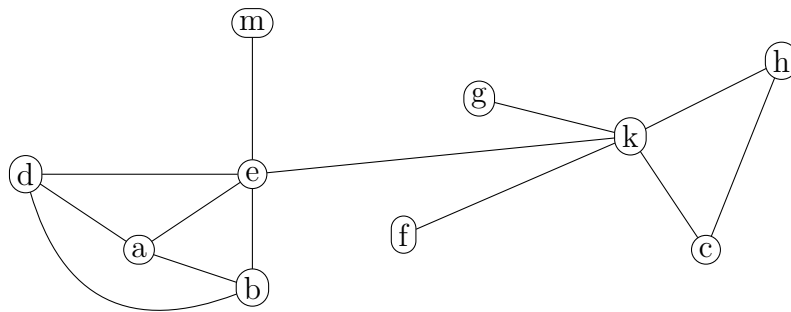
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1. (10 points) How many isomorphisms are there from  $G$  (below) to itself? Justify your answer and/or show your work clearly .



2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Is it possible to construct a graph with degree sequence: 5, 3, 3, 2, 2, 1? Show how or briefly explain why this isn't possible.

Name: \_\_\_\_\_

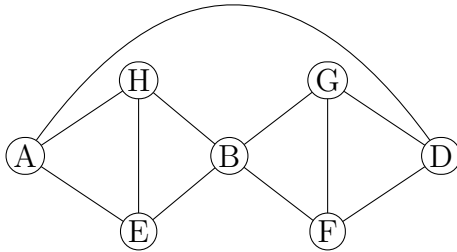
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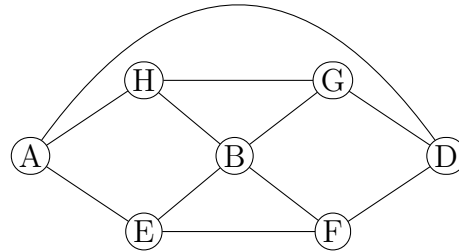
Discussion:    Thursday    Friday    10    11    12    1    2    3    4    5    6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) What is the difference between a cycle and a closed walk?

Name: \_\_\_\_\_

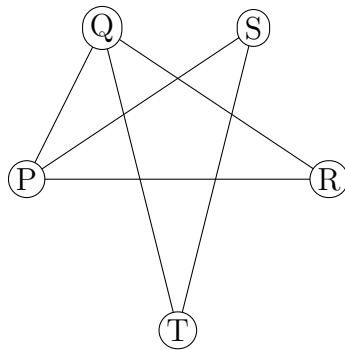
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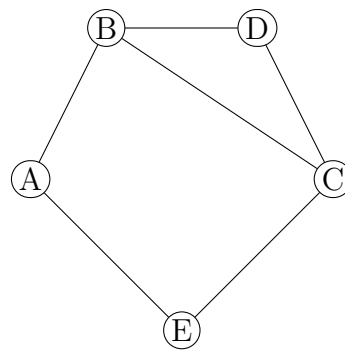
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1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



2. (5 points) If  $G$  is a graph, its complement  $G'$  has the same nodes as  $G$  but  $G'$  has an edge between nodes  $x$  and  $y$  if and only if  $G$  does not have an edge between  $x$  and  $y$ . Give a succinct high-level description of the complement of  $K_{2,3}$ . Briefly justify or show work.

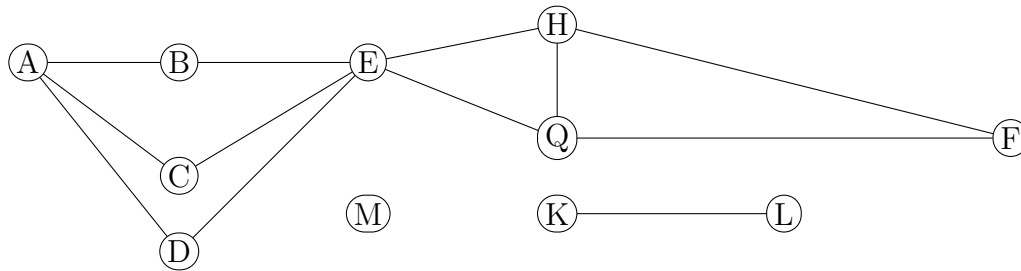
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1. (10 points) How many isomorphisms are there from  $G$  (below) to itself? Justify your answer and/or show your work clearly .



2. (5 points) What is the diameter of  $C_n$ ?