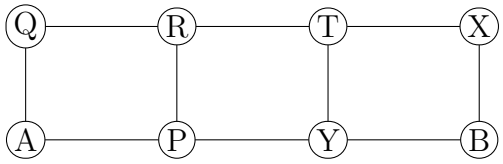
CS 173, Spring 19 Examlet 6, white

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many cycle subgraphs (i.e. subgraphs isomorphic to C_n for some n) does the graph below contain? Count two cycles as the same if they have the same set of nodes and the same set of edges. Don't worry about which node is the start/end node. Briefly justify and/or show work.



Solution: There is one cycle containing all 8 nodes. There are two cycles containing 6 nodes. And there are three cycles containing only 4 nodes. So there are a total of 6 cycles.

(2 points) Is the above graph acyclic?

Solution: No

(2 points) Does the above graph have an Euler circuit?

Solution: No

(2 points) What is the largest complete (K_n) subgraph of the above graph?

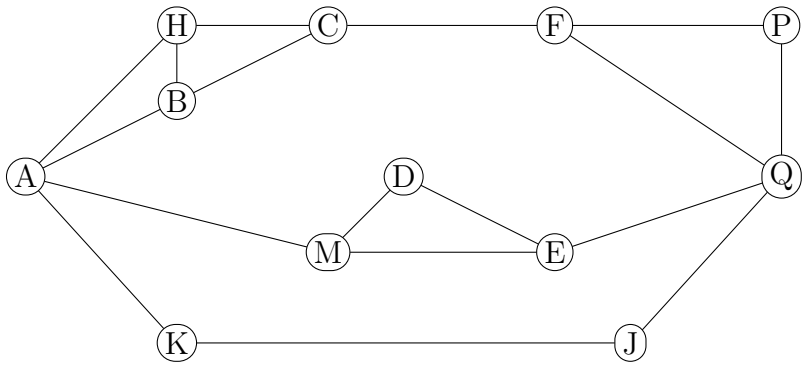
Solution: 2

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there from A to Q in the graph below? Explain or show work.



Solution: There are 8 paths across the top (4 ways to get from A to C, 2 ways to get from F to Q). Then there are two paths via the middle route and one path along the bottom. So a total of $8+2+1 = 11$ paths.

(2 points) Does the above graph contain a 4-node cycle?

Solution: Yes

(2 points) How many connected components does the above graph have?

Solution: One

(2 points) What is the largest complete (K_n) subgraph of the above graph?

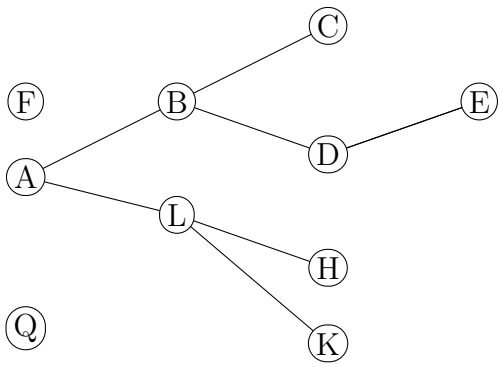
Solution: 3

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) In the graph below, how many paths are there from one node to a distinct (aka different) node? Consider all choices of start and end nodes. Explain or show work.



Solution: Nodes F and Q don't connect to any other distinct node.

In the large component, if you pick any two nodes, there is exactly one path between them. There are 8 choices for the start node and 7 choices for the (distinct) end node, so $8 \cdot 7 = 56$ paths.

(2 points) Is the above graph acyclic?

Solution: Yes

(2 points) How many connected components does the above graph have?

Solution: Three

(2 points) What is the largest complete (K_n) subgraph of the above graph?

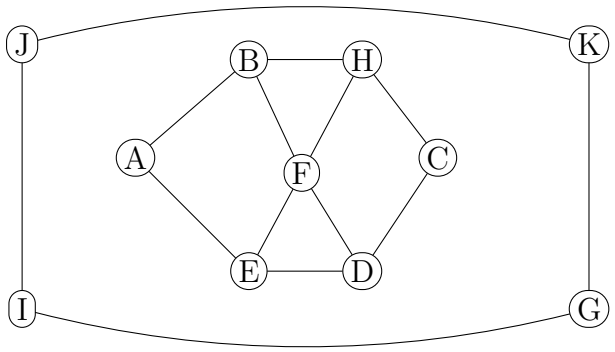
Solution: 2

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there from A to C in the graph below? Explain or show work.



Solution: A graph from A to C must go via B or E. There are six ways to get from B to C: BHC, BHFDC, BHFEDC, BFHC, BFDC, BFEDC. Similarly, there are six ways to get from E to C. So there are 12 paths total from A to C.

(2 points) Does the above graph contain a 6-node cycle?

Solution: Yes

(2 points) How many connected components does the above graph have?

Solution: Two

(2 points) Is the above graph bipartite?

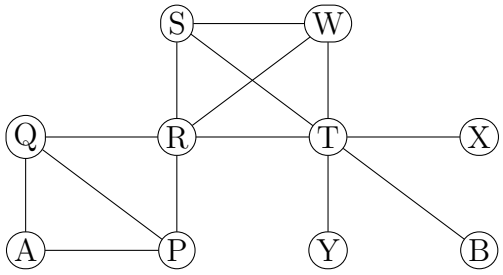
Solution: No

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many cycle subgraphs (i.e. subgraphs isomorphic to C_n for some n) does the graph below contain? Count two cycles as the same if they have the same set of nodes and the same set of edges. Don't worry about which node is the start/end node. Briefly justify and/or show work.



Solution: Since a cycle cannot re-use a node, it must live entirely within the set of nodes $\{A, P, Q, R\}$ or the set of nodes $\{R, S, T, W\}$.

In $\{A, P, Q, R\}$, there is one 4-cycle and two 3-cycles.

In $\{R, S, T, W\}$, there are three 4-cycles (SRTWS, SWRTS, STWRS) and four 3-cycles.

So there are a total of 10 cycles in this graph

(2 points) Does the above graph have a cut edge?

Solution: Yes

(2 points) How many connected components does the above graph have?

Solution: One

(2 points) What is the diameter of the above graph?

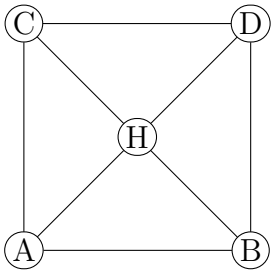
Solution: Four

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many cycle subgraphs (i.e. subgraphs isomorphic to C_n for some n) does the graph below contain? Count two cycles as the same if they have the same set of nodes and the same set of edges. Don't worry about which node is the start/end node. Briefly justify and/or show work.



Solution: There are four 3-cycles. There are four 4-cycles that are rotations of CDBHC and one four-cycle that doesn't include the hub H.

There are also four 5-cycles: CDBHAC and three rotated versions of it.

So there are either 13 cycles.

(2 points) What is the largest complete (K_n) subgraph of the above graph?

Solution: 3

(2 points) How many connected components does the above graph have?

Solution: One

(2 points) What is the diameter of the above graph?

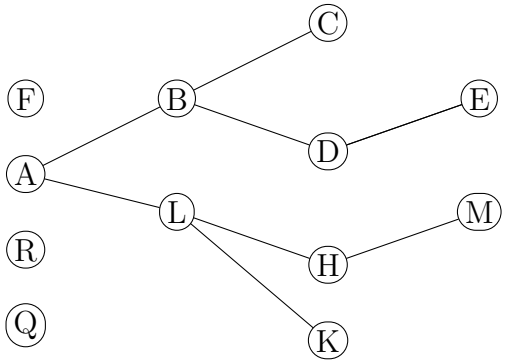
Solution: 2

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there in the graph below? Consider all choices of start and end nodes. Explain or show work.



Solution: There are three zero-length paths not in the large component: one using just the node R, on using just the node Q, and one using just the node F.

In the large component, if you pick any two nodes, there is exactly one path between them. Since there are 9 nodes, there are $k \cdot 9 = 81$ paths.

So there are 84 paths total.

(2 points) Is the above graph acyclic?

Solution: Yes

(2 points) How many connected components does the above graph have?

Solution: Four

(2 points) Is the above graph bipartite?

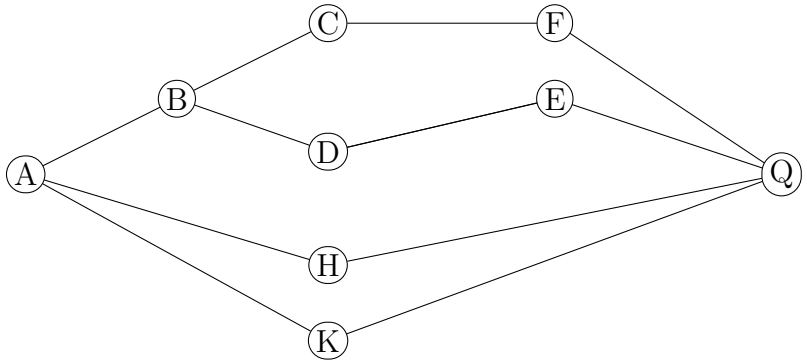
Solution: Yes

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many cycle subgraphs (i.e. subgraphs isomorphic to C_n for some n) does the graph below contain? Count two cycles as the same if they have the same set of nodes and the same set of edges. Don't worry about which node is the start/end node. Briefly justify and/or show work.



Solution: Six. One is BCFQED. A second is AHQK. Then there are four cycles that choose one of the upper paths from A to Q (AFDEQ or ABCFQ) followed by one of the lower paths from Q to A (QHA or QKA).

(2 points) What is the diameter of the above graph?

Solution: 3

(2 points) How many connected components does the above graph have?

Solution: One

(2 points) Is the above graph bipartite?

Solution: Yes

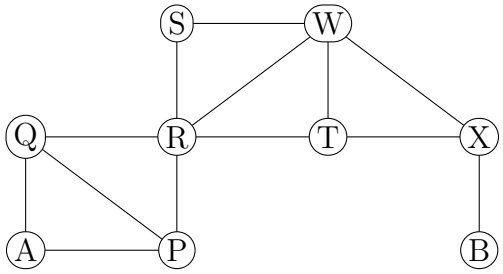
CS 173, Fall 18 Examlet 6, Part B

Name:_____

NetID:_____ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

(9 points) How many paths are there from A to B in the graph below? Explain or show work.



Solution: There are four ways to get from A to R. Then there are six ways to get from R to X: RSWX, RSWTX, RWX, RWTX, RTWX, RTX. And then there is only one way to get from X to B. So there are a total of $4 \cdot 6 = 24$ paths from A to B.

(2 points) How many connected components does the above graph have?

Solution: One

(2 points) What is the diameter of the above graph?

Solution: Five

(2 points) Does the above graph have a cut edge?

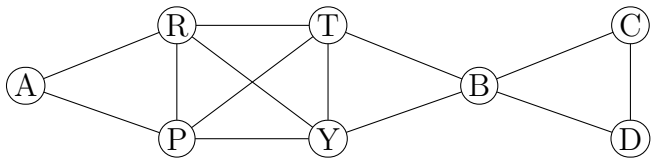
Solution: Yes

Name:_____

NetID:_____ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

(9 points) How many paths are there from A to C in the graph below? Explain or show work.



Solution: Suppose we go first to node R. We can continue to B via the following 10 sequences of nodes: T, Y, TY, YT, PT, PY, PTY, PYT, TPY, YPT. If, instead, we first go to P, there is a similar set of 10 ways to reach B. So there are a total of 20 ways to get from A to B. And then two ways to get from B to C. So there are a total of 40 paths.

(2 points) How many connected components does the above graph have?

Solution: One

(2 points) Does the above graph have a cut edge?

Solution: No

(2 points) Does the above graph have an Euler circuit?

Solution: Yes (ARTBCDBYTPYRPA)

CS 173, Spring 18 Examlet 6, Part B

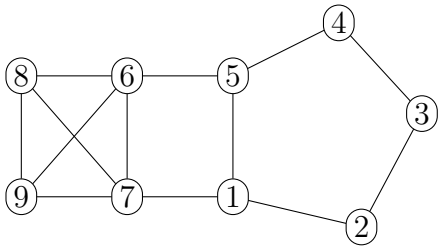
Name:_____

NetID:_____ **Lecture: A B**

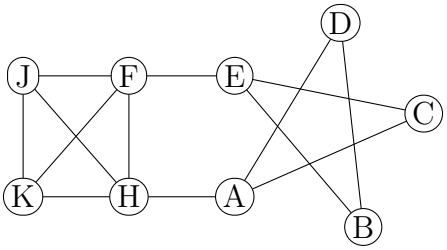
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



Solution: No, they are not isomorphic. Both graphs have three degree-2 nodes. In Graph X one degree-2 node (3) has neighbors that are both degree 2. In Graph Y, one of the degree-2 nodes (C) has only degree-3 neighbors.

Alternatively, consider the two degree-3 nodes that are adjacent to a degree-2 node. In Graph X, these nodes (1 and 5) are adjacent. In Graph Y, these nodes (A and E) are not neighbors.

Alternatively, each graph contains a K_4 and a C_5 that don't overlap. Look at where the two connect. In Graph X, they connect at adjacent nodes. In graph Y, they connect at nodes that aren't neighbors.

2. (5 points) Use the pigeonhole principle to briefly explain why a graph with n nodes ($n \geq 2$) must have two nodes with the same degree. Hint: if one node has degree 0, what is the maximum degree of any other node? How many possible degree values are there?

Solution: Node degrees range from 0 to $n-1$. However, it's impossible to have both a node with degree 0 and a node with degree $n-1$. So the degrees in any specific graph must either be between 0 and $n-2$, or else between 1 and $n-1$. In both cases, there are only $n-1$ distinct degrees. But there are n nodes. So two nodes must have the same degree.

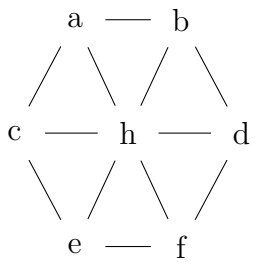
CS 173, Spring 18 Examlet 6, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: 12. There are 6 choices for how to map node a. Then node b can map to either of the two adjacent nodes. After that, the rest of the mapping is forced.

2. (5 points) Complete this statement of the Handshaking Theorem.

   For any graph G with set of nodes V and set of edges E, ...

   Solution: The sum of the degrees of all the nodes is equal to twice the number of edges.

CS 173, Spring 18 Examlet 6, Part B

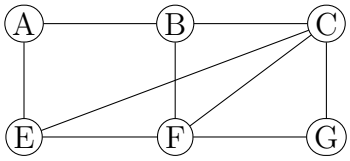
Name:_____

NetID:_____ **Lecture: A B**

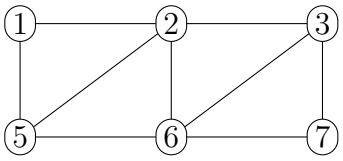
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



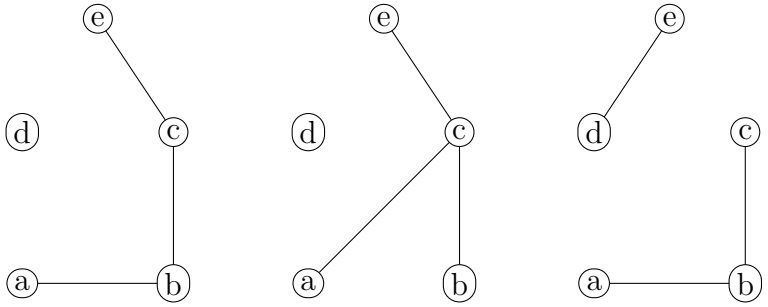
Graph Y



Solution: No, these are not isomorphic. In graph Y, both of the degree-2 nodes are in 3-cycles. In graph X, one of them (A) is not in a 3-cycle.

2. (5 points) Show three distinct (i.e. not isomorphic) graphs, each of which has five nodes, three edges, and no cycles.

Solution:

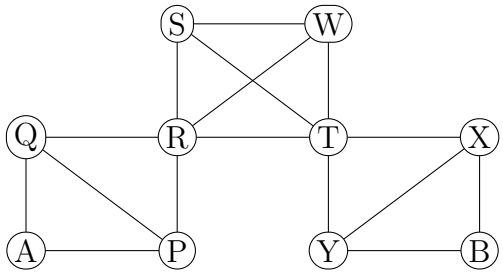


Name:_____

NetID:_____ **Lecture: A B**

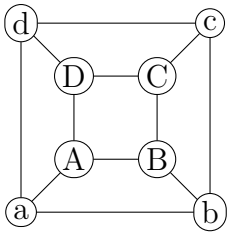
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: There are two choices for mapping R: to itself or to T. After that, Q and P can be interchanged (or not). And, X and Y can be interchanged (or not). Less obviously, S and W can also be interchanged. So there are $2 \cdot 2 \cdot 2 \cdot 2 = 2^4$ isomorphisms.

2. (5 points) Is this graph bipartite? Briefly justify your answer.



Solution: Yes, this is bipartite. Put nodes a, c, B, and D into one set and nodes A, C, b, and d into the other set.

CS 173, Spring 18 Examlet 6, Part B

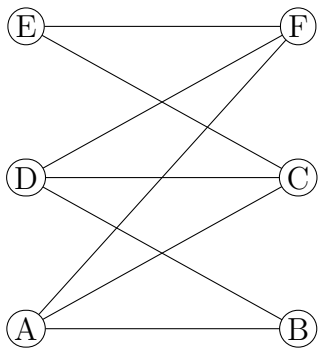
Name:_____

NetID:_____ **Lecture: A B**

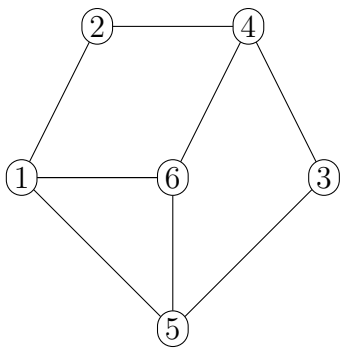
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

   Graph X

   Graph Y

   Solution: No, they aren't isomorphic. Graph Y has a 5-cycle but graph X has only even cycles. Also, one of the degree-3 nodes in graph Y is connected to three degree-3 nodes, but in graph X each degree-3 node is connected to only two other degree-3 nodes.

2. (5 points) Does the complete graph K_8 have an Euler circuit? Briefly justify your answer.

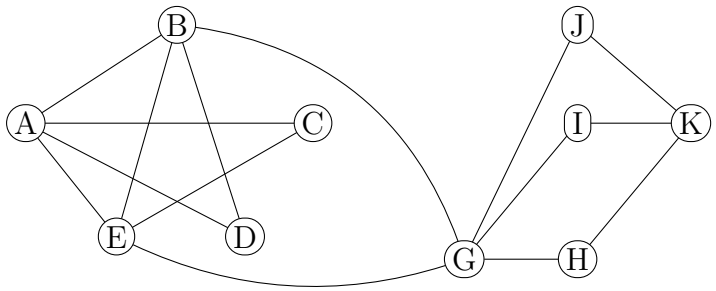
   Solution: No. Each node has degree 7, which is odd. You can't find an Euler circuit if there are any nodes with odd degree.

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: Nodes A, G, and K must map onto themselves.

B and E can swap (2 choices). This fixes C and D. I, J, and H can be permuted (6 choices). So there are a total of $2 \cdot 6 = 12$ choices.

2. (5 points) If G is a graph, its complement G' has the same nodes as G but G' has an edge between nodes x and y if and only G does not have an edge between x and y. Give a succinct high-level description of the complement of W_5 (5-cycle joined to a hub node). Briefly justify or show work.

Solution: Suppose we label the hub node h and the rim nodes as a, b, c, d, and e. In the complement, we have edges ac, ad, bd, be, and ce. Rearranging these edges gives us: ac, ce, eb, bd, da. So we have a five-cycle, plus an isolated node (h).

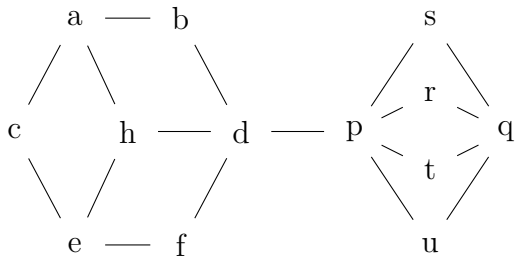
CS 173, Fall 17 Examlet 6, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there from c to q in the graph below? Explain or show work.



Solution: Every path from c to q must go via the nodes d and p. There are 6 ways to get from c to d: cabd, cahd, cahefd, cefd, cehd, cehabd And then there are 4 ways to get from p to q. So there are a total of 24 paths from c to q.

(2 points) Is the above graph acyclic?

Solution: No

(2 points) How many connected components does the above graph have?

Solution: One

(2 points) Does the above graph contain a 5-node cycle?

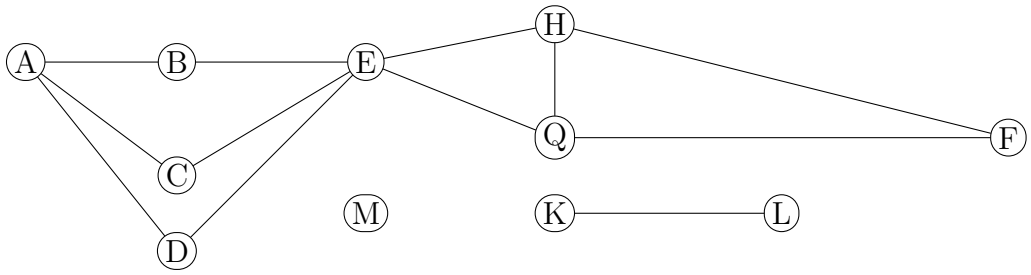
Solution: No

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there from A to F in the graph below? Explain or show work.



Solution: There are three ways to get from A to E. There are four ways to get from E to F. So there are $3 \cdot 4 = 12$ paths total.

(2 points) Does the above graph contain a 5-node cycle?

Solution: No

(2 points) How many connected components does the above graph have?

Solution: Three

(2 points) Does the above graph have an Euler circuit?

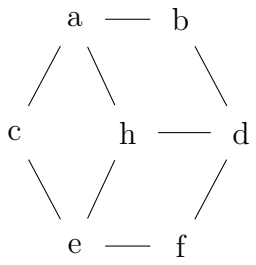
Solution: No

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many cycle subgraphs (i.e. subgraphs isomorphic to C_n for some n) does the graph below contain? Count two cycles as the same if they have the same set of nodes and the same set of edges. Don't worry about which node is the start/end node. Briefly justify and/or show work.



Solution: There are three cycles with 4 nodes: cahec, bdhab, and fdhef. Then there are 4 cycles with 6 nodes: abdfeca, abdfeha, dfecahd, ehdbace.

(2 points) Does the above graph have an Euler circuit?

Solution: No

(2 points) Is the above graph bipartite?

Solution: Yes

(2 points) Does the above graph contain a 4-node cycle?

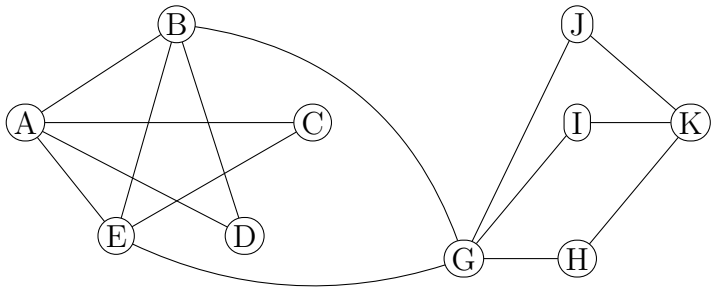
Solution: Yes

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there from A to K in the graph below? Explain or show work.



Solution: There are 4 ways to get from A to G via B (ABG, ABEG, ADBG, ACEBG) and similarly 4 ways to get from A to G via E. There are then 3 paths from G to K. So a total of $8 \cdot 3 = 24$ paths.

(2 points) Does the above graph contain a 5-node cycle?

Solution: Yes

(2 points) What is the diameter of the above graph?

Solution: Four

(2 points) Is the above graph bipartite?

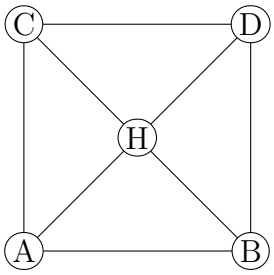
Solution: No

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there from B to C in the graph below? Explain or show work.



Solution: There are 9 paths. Starting with BH: BHC, BHDC, BHAC. Starting with BA: BAC, BAHC, BAHDC. Starting with BD: BDC, BDHC, BDHAC.

(2 points) Is the above graph acyclic?

Solution: No

(2 points) Does the above graph have a cut edge?

Solution: No

(2 points) What is the diameter of the above graph?

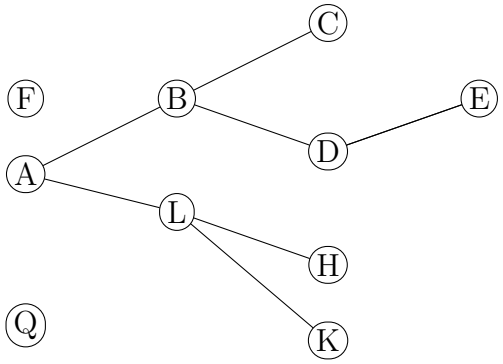
Solution: 2

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there in the graph below? Consider all choices of start and end nodes. Explain or show work.



Solution: There are two zero-length paths not in the large component: one using just the node F and the other using just the node Q.

In the large component, if you pick any two nodes, there is exactly one path between them. Since there are 8 nodes, there are $8 \cdot 8 = 64$ paths.

So there are 66 paths total.

(2 points) How many connected components does the above graph have?

Solution: Three

(2 points) Is the above graph bipartite?

Solution: Yes

(2 points) Does the above graph contain a 4-node cycle?

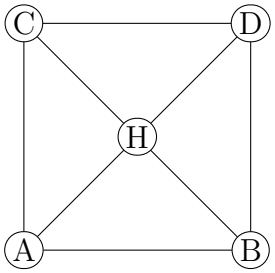
Solution: No

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(9 points) How many cycle subgraphs (i.e. subgraphs isomorphic to C_n for some n) does the graph below contain? Count two cycles as the same if they have the same set of nodes; don't worry about (for example) which node is the start/end node. Briefly justify and/or show work.



Solution: There are four 3-cycles. There are four 4-cycles that are rotations of CDBHC and one four-cycle that doesn't include the hub H.

There are also either one or four 5-cycles, depending on how you interpreted the criterion about "have the same set of nodes." The issue is that all four rotated verions of CDBHAC have the same set of nodes, but the difference in ordering means that they would normally be considered different subgraphs.

So there are either 10 or 13 cycles.

(2 points) What is the diameter of the above graph?

Solution: 2

(2 points) Does the above graph have a cut edge?

Solution: No

(2 points) How many connected components does the above graph have?

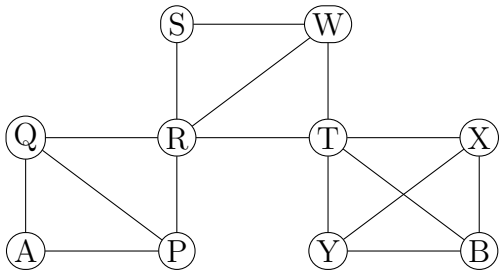
Solution: One

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there from A to B in the graph below? Explain or show work.



Solution: There are 4 paths from A to R. Then there are 3 paths from R to T. And 5 paths from T to B. So there are $4 \cdot 3 \cdot 5 = 60$ paths total.

(2 points) Does the above graph have an Euler circuit?

Solution: No

(2 points) Is the above graph bipartite?

Solution: No

(2 points) How many connected components does the above graph have?

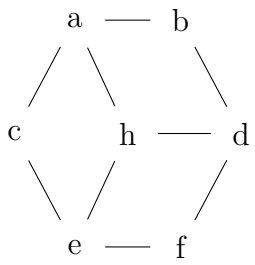
Solution: one

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there from a to f in the graph below? Explain or show work.



Solution: Paths starting with ab: abdf, abdhef

Paths starting with ac: acef, acehdf

Paths starting with ah: ahdf, ahef

So there are a total of 6 paths from a to f.

(2 points) How many connected components does the above graph have?

Solution: One

(2 points) Is the above graph bipartite?

Solution: Yes

(2 points) Does the above graph contain a 6-node cycle?

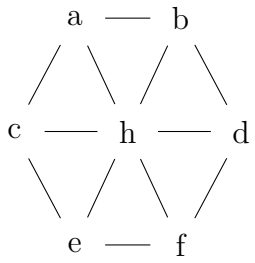
Solution: Yes

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there from h to d in the graph below? Explain or show work.



Solution: A path from h to d must go from h to one of the rim nodes, then (since it can't return to h) walk along the rim to d. There are 11 possible paths: HD, HBD, HBACEFD, HABD, HACEFD, HCABD, HCEFD, HEFD, HECABD, HFD, HFECAB.

(2 points) Does the above graph have an Euler circuit?

Solution: No

(2 points) Is the above graph bipartite?

Solution: No

(2 points) Does the above graph contain a 6-node cycle?

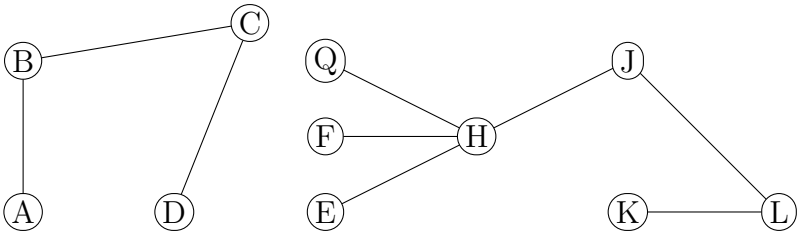
Solution: Yes

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there (with any starting/ending points) in the graph below? Explain or show work.



Solution: Within each component, every pair of nodes is connected by a path. There are 4 nodes in the lefthand component, so 16 paths. There are 7 nodes in the righthnad component, so 49 paths.

So there are $16 + 49 = 65$ paths total.

(2 points) Is the above graph acyclic?

Solution: Yes

(2 points) How many connected components does the above graph have?

Solution: Two

(2 points) Does the above graph have a cut edge?

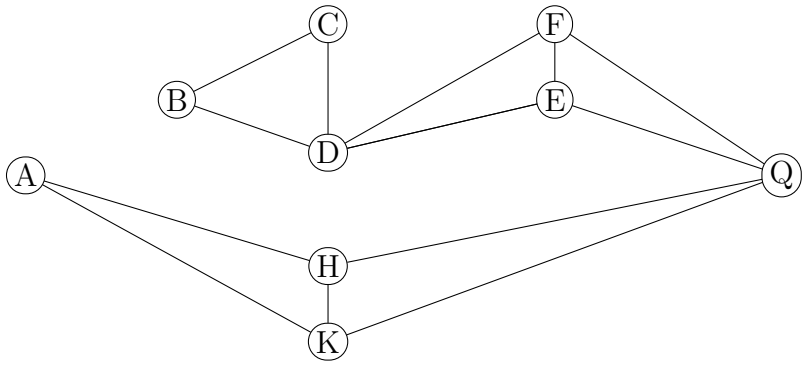
Solution: Yes

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(9 points) How many paths are there from A to B in the graph below? Explain or show work.



Solution: There are four ways to get from A to Q. Then there are four ways to get from Q to D. And two ways to get from D to B. So a total of $4 \cdot 4 \cdot 2 = 32$ paths.

(2 points) How many connected components does the above graph have?

Solution: One

(2 points) Is the above graph acyclic?

Solution: No

(2 points) Does the above graph have an Euler circuit?

Solution: No