

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

$$A = \{(x, y) \in \mathbb{R}^2 : y = x^2 - 4\}$$

$$B = \{(p, q) \in \mathbb{Z}^2 : q < 0\}$$

$$C = \{(a, b) \in \mathbb{R}^2 : |a| \leq 1\}$$

Prove that $A \cap B \subseteq C$.

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$$A = \{(x, y, z) \in \mathbb{R}^3 \mid 0 < x < y - 1\}$$

$$B = \{(a, b, c) \in \mathbb{R}^3 \mid b^2 + 2 < c^2\}$$

$$C = \{(p, q, r) \in \mathbb{R}^3 \mid p^2 < r^2\}$$

Prove that $A \cap B \subseteq C$.

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$$A = \{\alpha(2, -4) + (1 - \alpha)(-3, 6) \mid \alpha \in \mathbb{R}\}$$

$$B = \{(a, b) \in \mathbb{R}^2 \mid a \geq 1\}$$

$$C = \{(p, q) \in \mathbb{R}^2 \mid q \leq 0\}$$

Prove that $A \cap B \subseteq C$.

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$$A = \{(x, y) \in \mathbb{R}^2 : y = x^2 - 2x - 1\}$$

$$B = \{(p, q) \in \mathbb{R}^2 : |p| \geq 3\}$$

$$C = \{(m, n) \in \mathbb{R}^2 : n \geq 0\}$$

Prove that $A \cap B \subseteq C$.

Name: _____

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

$$A = \{(x, y, z) \in \mathbb{R}^3 : y = x^2 - 2x + 11\}$$

$$B = \{(a, b, c) \in \mathbb{R}^3 : b \leq c\}$$

$$C = \{(p, q, r) \in \mathbb{R}^3 : r \geq 5\}$$

Prove that $A \cap B \subseteq C$.

Name: _____

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

$$A = \{(x, y, z) \in \mathbb{R}^3 : |x + y + z| = 20\}$$

$$B = \{(a, b, c) \in \mathbb{N}^3 : a + b < 5\}$$

$$C = \{(p, q, r) \in \mathbb{R}^3 : r > 10\}$$

Prove that $A \cap B \subseteq C$.

Name: _____

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

$$A = \{(x, y) \in \mathbb{R}^2 : xy \leq -7\}$$

$$B = \{(p^3, p^2) : p \in \mathbb{R}\}$$

$$C = \{(a, b) \in \mathbb{R}^2 : a < 0\}$$

Prove that $A \cap B \subseteq C$.

Name: _____

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

$$A = \{a(1, 0) + b(3, 1) + c(2, 4) : a, b, c \text{ are positive reals and } a + b + c = 1\}$$

$$B = \{(x, y) \in \mathbb{R}^2 : x \leq 3 \text{ and } y \geq 0\}$$

Prove that $A \subseteq B$.

Name: _____

NetID: _____ Lecture: B

Discussion: Thursday Friday 11 12 1 2 3 4

$$A = \{(a, b) \in \mathbb{R}^2 : a = 3 - b^2\}$$

$$B = \{(x, y) \in \mathbb{R}^2 : |x| \geq 1 \text{ or } |y| \geq 1\}$$

Prove that $A \subseteq B$. Hint: you may find proof by cases helpful.

Name: _____

NetID: _____ Lecture: B

Discussion: Thursday Friday 11 12 1 2 3 4

$$A = \{(x, y) \in \mathbb{Z}^2 \mid 2xy + 6y - 5x - 15 \geq 0\}$$

$$B = \{(a, b) \in \mathbb{Z}^2 \mid a \geq 0\}$$

$$C = \{(p, q) \in \mathbb{Z}^2 \mid q \geq 0\}$$

Prove that $(A \cap B) \subseteq C$.

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For any integers s and t define $L(s, t)$ as follows:

$$L(s, t) = \{sx + ty \mid x, y \in \mathbb{Z}\}$$

Thus, $L(s, t)$ consists of all integers that can be expressed as the sum of multiples of s and t . Prove the following claim using your best mathematical style and the following definition of congruence mod k : $p \equiv q \pmod{k}$ if and only if $p = q + kn$ for some integer n .

Claim: For any integers a, b, r , where r is positive, if $a \equiv b \pmod{r}$, then $L(a, b) \subseteq L(r, b)$.

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$$A = \{(p, q) \in \mathbb{R}^2 \mid p = 0\}$$

$$B = \{(x, y) \in \mathbb{R}^2 \mid (x - 1)^2 + y^2 = 4\}$$

$$C = \{(s, t) \in \mathbb{R}^2 \mid (s + 1)^2 + t^2 = 4\}$$

Prove that $B \cap C \subseteq A$.

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$$A = \{(x, y, z) \in \mathbb{Z}^3 : 2x + y = z - 1\}$$

$$B = \{(a, b, c) \in \mathbb{Z}^3 : 2a - b = c - 3\}$$

$$C = \{(p, q, r) \in \mathbb{Z}^3 : r \text{ is even}\}$$

Prove that $A \cap B \subseteq C$. (Work directly from the definition of “even.”)

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$$B = \{(x, y) \in \mathbb{R}^2 : |x| \geq 1 \text{ or } |y| \geq 1\}$$

Prove that $A \subseteq B$. Hint: you may find proof by cases helpful.

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$$A = \{\lambda(3, 2) + (1 - \lambda)(5, 0) \mid \lambda \in [0, 1]\}$$

$$B = \{(x, y) \in \mathbb{R}^2 \mid x \geq y\}$$

Prove that $A \subseteq B$.

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$$A = \{(x, y, z) \in \mathbb{Z}^3 : 0 < x \leq y \leq z\}$$

$$B = \{(a, b, c) \in \mathbb{R}^3 : c^2 \leq a\}$$

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$$A = \{(x, y) \in \mathbb{R}^2 \mid x = \lfloor 3y + 5 \rfloor\}$$

$$B = \{(p, q) \in \mathbb{Z}^2 \mid 2p + q \equiv 3 \pmod{7}\}$$

Prove that $A \cap \mathbb{Z}^2 \subseteq B$.

Use the following definition of congruence mod k : if s, t, k are integers, k positive, then $s \equiv t \pmod{k}$ if and only if $s = t + nk$ for some integer n .

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$$B = \{(p, q) \in \mathbb{R}^2 \mid p \geq 0\}$$

$$C = \{(m, n) \in \mathbb{R}^2 \mid n \geq 1\}$$

Prove that $A \subseteq B \cup C$.

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$$B = \{(p, q) \in \mathbb{R}^2 : |p - q| < 3\}$$

$$C = \{(a, b) \in \mathbb{R}^2 : |ab| < 10\}$$

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$$B = \{(a, b) \in \mathbb{R}^2 \mid b \leq -1\}$$

$$C = \{(p, q) \in \mathbb{R}^2 \mid p \geq 0\}$$

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