

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A

Discussion: Monday & Wednesday 1:30 2:30

(18 points) A Borg tree is a full binary tree whose nodes contain integers such that

- Every leaf contains the value 0.
- The value  $v(X)$  in a node X is (strictly) larger than the values in X's children.

Use (strong) induction to prove that the value in the root of a Borg tree is larger than the value in any other node of the tree.

The induction variable is named \_\_\_\_\_ and it is the \_\_\_\_\_ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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1. (8 points) Here is a grammar with start symbol  $S$  and terminal symbols  $a$  and  $b$ . Draw three parse trees for the string  $aab$  that match this grammar.

$$\begin{aligned} S &\rightarrow a N \mid N \\ N &\rightarrow a S \mid S \mid b \end{aligned}$$

2. (4 points) Check the (single) box that best characterizes each item.

The number of paths between two nodes in an  $n$ -node tree.

$$n \quad \boxed{\phantom{0}} \quad 2n \quad \boxed{\phantom{0}} \quad \frac{n(n-1)}{2} \quad \boxed{\phantom{0}}$$

Paths in opposite directions count as different.

$$n(n-1) \quad \boxed{\phantom{0}} \quad n^2 \quad \boxed{\phantom{0}} \quad \frac{n(n+1)}{2} \quad \boxed{\phantom{0}}$$

The level of the root node in a tree of height  $h$ .

$$-1 \quad \boxed{\phantom{0}} \quad 0 \quad \boxed{\phantom{0}} \quad 1 \quad \boxed{\phantom{0}} \quad h-1 \quad \boxed{\phantom{0}} \quad h \quad \boxed{\phantom{0}}$$