Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) How many different 12-letter strings can be made be rearranging the letters in the word ``apalachicola''? Show your work.

   Solution: There are 12 letters total to rearrange, with 4 copies of a, 2 copies of l, and 2 copies of c. So the total number of possibilities is

   $$\frac{12!}{4!2!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is the _____ of f.

   domain ☐ co-domain ☐
   image ☑ none of these ☐

   $f : \mathbb{N}^2 \to \mathbb{N}$
   $f(p, q) = pq$

   onto ☑ not onto ☐ not a function ☐

   $g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$
   $g(x, y) = \gcd(x, y)$

   one-to-one ☐ not one-to-one ☑ not a function ☐

   We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on exactly two mailboxes.

   true ☐ false ☑

   $\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ y \leq x$

   true ☐ false ☑

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Hermione Grainger has 7000 socks in her magically expanding drawer. The socks are colored purple, magenta, and shocking pink. How many socks must she pull out of the drawer before she is guaranteed to have two socks of the same color. Briefly justify your answer.

   Solution: She needs to pull out four socks. By the pigeonhole principle, four socks and only three colors means that two must have the same color.

2. (10 points) Check the (single) box that best characterizes each item.

A function is onto if and only if its image is the same as its co-domain. true ☑ false ☐

$g : \mathbb{R} \to [-1, 1]$
$g(x) = \sin(x)$ onto ☑ not onto ☐ not a function ☐

$g : \mathbb{R}^2 \to \mathbb{R}^2$
$g(x, y) = (y, 3x)$ one-to-one ☑ not one-to-one ☐ not a function ☐

Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least three elves have charm. true ☐ false ☑

$\exists t \in \mathbb{N},\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = p$ true ☑ false ☐

Name:______________________________________

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) 8 presidential candidates (including Bernie and Hilary) need to line up for a photo. The new editor would like Bernie and Hilary to stand next to each other. How many different ways can we arrange the eight people?

   Solution: Bernie and Hilary can be moved as a unit. So we have 7 objects to permute, so 7! permutations. However, Bernie might stand either to the left or to the right of Hilary. So the total number of possibilities is $2 \cdot 7!$.

2. (10 points) Check the (single) box that best characterizes each item.

Suppose $f : A \to B$. For all $x, y \in A$, if $x = y$, then $f(x) = f(y)$.	onto ☐	one-to-one ☐	neither $\boxed{\surd}$
$g : \mathbb{R} \to [0, 1]$ $g(x) = \sin(x)$	onto ☐	not onto ☐	not a function $\boxed{\surd}$
$f : \mathbb{R} \to \mathbb{Z}$ $f(x) = x$	one-to-one ☐	not one-to-one ☐	not a function $\boxed{\surd}$
Each dorm room is given an integer access code between 1 and 10 (inclusive). According to the pigeonhole principle, if there are 21 dorm rooms, then every access code must be shared by at least two rooms.	true ☐	false $\boxed{\surd}$	
$\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ x - y < 100$	true ☐	false $\boxed{\surd}$	

Name: ______________________________

NetID: ____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) To make exam grading anonymous and therefore hopefully more fair, each of the 200 students in CS 241 has been assigned a unique 3-character exam code. The character set is $\{\alpha, \beta, \gamma, \delta\}$. Use the Pigeonhole Principle to explain what's wrong with this plan.

   Solution: Since there are four distinct characters, there are $4^3 = 64$ different 3-character codes. Since there are more students than codes, the pigeonhole principle implies that there is at least one pair of students with the same code.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the real numbers is the _____ of f.

   domain ☐ co-domain ☑ image ☐ none of these ☐

   $f : \mathbb{Z} \to \mathbb{Z}$
   $f(x) = x + 3$ (x even),
   $f(x) = x - 22$ (x odd)

   onto ☐ not onto ☑ not a function ☐

   $g : \mathbb{N}^2 \to \mathbb{N}$
   $g(x, y) = \gcd(x, y)$

   one-to-one ☐ not one-to-one ☐ not a function ☑

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, there must be at least three elves with the same gift.

   true ☑ false ☐

   $\exists y \in \mathbb{R}^+,\ \forall x \in \mathbb{R}^+,\ xy = 1$
   ($\mathbb{R}^+$ is the positive real numbers.)

   true ☐ false ☑

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Suppose that $|A| = p$ and $|B| = q$. How many different functions are there from A to B?

   Solution: q^p

2. (10 points) Check the (single) box that best characterizes each item.

   A function is one-to-one if and only if each value in the co-domain has at most one pre-image. true ☑ false ☐

   $g : \mathbb{R}^2 \to \mathbb{R}$
   $g(x, y) = \lfloor x \rfloor + y$ onto ☑ not onto ☐ not a function ☐

   $g : \mathbb{Z}^2 \to \mathbb{Z}^2$
   $g(x, y) = (y, 3x)$ one-to-one ☑ not one-to-one ☐ not a function ☐

   Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. At least three of these shirts must have the same slogan. true ☑ false ☐

   $\forall x \in \mathbb{Q},\ \exists m, n \in \mathbb{Z},\ x = \frac{m}{n}$ true ☑ false ☐

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) 15 men and 15 women showed up to this week's meeting of the UIUC Swing Dance Society. How many different ways can we form all of them into pairs, each pair containing one man and one woman?

   Solution: We're constructing a bijection from the women to the men (or vice versa). Since there are 15 people in each set, there are 15! bijections.

2. (10 points) Check the (single) box that best characterizes each item.

Suppose $f : A \to B$. For all $x \in A$, there is a $y \in B$, $f(x) = y$.	onto ☐	one-to-one ☐	neither ☑
$g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$ $g(x, y) = \gcd(x, y)$	onto ☑	not onto ☐	not a function ☐
$f : \mathbb{N} \to \mathbb{R}$ $f(x) = x^2 + 2$	one-to-one ☑	not one-to-one ☐	not a function ☐
Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. There is a slogan that appears on at least two shirts.	true ☑	false ☐	
$\forall x \in \mathbb{Z},\ \exists y \in \mathbb{N},\ x^2 = y$	true ☑	false ☐	

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Suppose that $|A| = 2$ and $|B| = 3$. How many onto functions are there from A to B? Briefly justify or show work.

   Solution: There are no onto functions from A to B, because $|A|$ is smaller than $|B|$.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the integers is the _____ of f.	domain	☑	co-domain	☐
	image	☐	none of these	☐

$f : \mathbb{N}^2 \to \mathbb{Z}$ $f(p, q) = 2^p 3^q$	onto	☐	not onto	☑	not a function	☐
$g : \mathbb{Z} \to \mathbb{Z}$ $g(x) = x\|x\|$	one-to-one	☑	not one-to-one	☐	not a function	☐

Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least one elf has stamina.	true	☐	false	☑
$\forall x \in \mathbb{R}^+,\ \exists y \in \mathbb{R}^+,\ xy = 1$ ($\mathbb{R}^+$ is the positive real numbers.)	true	☑	false	☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Prof. Snape is teaching potions to 52 girls and 73 boys. Quiz 1 has integer scores between zero and 100 (inclusive). Assuming no one missed the quiz, what is the probability that two students got the same score? Briefly justify your answer.

   Solution: There are 101 distinct quiz scores and 125 students. By the pigeonhole principle, two students got the same score. So probability 1.0 (or 100% if you prefer that notation).

2. (10 points) Check the (single) box that best characterizes each item.

If $f : A \to B$ is one-to-one, then	$\lvert A\rvert \geq \lvert B\rvert$ ☐	$\lvert A\rvert \leq \lvert B\rvert$ ☑ $\surd$	$\lvert A\rvert = \lvert B\rvert$ ☐
$g : \mathbb{N}^2 \to \mathbb{N}$ $g(x, y) = \gcd(x, y)$	onto ☐	not onto ☐	not a function ☑ $\surd$
$g : \mathbb{R} \to \mathbb{R}^2$ $g(x) = (x, 3x^2 + 2)$	one-to-one ☑ $\surd$	not one-to-one ☐	not a function ☐
We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on at least two mailboxes.	true ☑ $\surd$	false ☐	
$\exists m, n \in \mathbb{Z},\ \forall x \in \mathbb{Q},\ x = \frac{m}{n}$	true ☐	false ☑ $\surd$	

Name:____

NetID:____________________ **Lecture:** **B**

Discussion: **Friday 11 12 1 2 3 4**

1. (5 points) Suppose that $|A| = p$ and $|B| = q$, $p \le q$. How many different one-to-one functions are there from A to B?

   Solution: $\frac{q!}{(q-p)!}$

2. (10 points) Check the (single) box that best characterizes each item.

If a function from $\mathbb{R}$ to $\mathbb{R}$ is increasing, it must be one-to-one.	true	$\square$	false	$\surd$		
$f : \mathbb{N} \to \mathbb{R}$ $f(x) = x^2 + 2$	onto	$\square$	not onto	$\surd$	not a function	$\square$
$f : \mathbb{N} \to \mathbb{N}$ $f(x) = 3 - x$	one-to-one	$\square$	not one-to-one	$\square$	not a function	$\surd$
We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, every color appears on at least two mailboxes.	true	$\square$	false	$\surd$		
$\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ x = xy$	true	$\surd$	false	$\square$		

CS 173, Fall 18 Examlet 5, Part B

Name:____

NetID:____________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4**

1. (5 points) Suppose that $|A| = p$, $|B| = q$, $|C| = n$. How many different functions are there from A to $B \times C$?

   Solution: There are qn elements in $B \times C$. So there are $(qn)^p$ ways to build a function from A to $B \times C$.

2. (10 points) Check the (single) box that best characterizes each item.

If a function from $\mathbb{R}$ to $\mathbb{R}$ is strictly increasing, it must be one-to-one.	true	$\surd$	false			
$g : \mathbb{N} \to \mathbb{Z}$ $g(x) = \|x\|$	one-to-one	$\surd$	not one-to-one		not a function	
$g : \mathbb{R} \to \mathbb{R}$ $g(x) = \sin(x)$	onto		not onto	$\surd$	not a function	
We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on at least two mailboxes.	true	$\surd$	false			
$\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ x \neq y$ and $x + y = 0$	true		false	$\surd$		

CS 173, Fall 17 Examlet 5, Part B

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) How many different 7-letter strings can be made by selecting and rearranging letters from the word ``metalworking''? Show your work.

   Solution: The word ``metalworking'' contains 12 letters with no duplicates. We are selecting an ordered list of 7 from this set. By the permutation formula, this can be done in $\frac{12!}{5!}$ ways.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is the _____ of f.	domain ☐	co-domain ☐	image ☑	none of these ☐

$f : \mathbb{Z} \to \mathbb{Z}$ $f(x) = x + 4$ (x even), $f(x) = x - 22$ (x odd)	onto ☑	not onto ☐	not a function ☐
$g : \mathbb{Z} \to \mathbb{Z}$ $g(x) = \lfloor x \rfloor$	one-to-one ☑	not one-to-one ☐	not a function ☐

We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on exactly two mailboxes.	true ☐	false ☑
$\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ y \leq x$	true ☐	false ☑

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) 10 men and 15 women showed up to this week's meeting of the UIUC Swing Dance Society. How many different ways can they line up (left to right) in front of the stage without any men being next to another man?

   Solution: There are 15! ways to arrange the women.

   Then there are 16 places to put a man: between two women or at one of the ends. We have $\frac{16!}{6!}$ ways to arrange the men in these spaces.

   So the total number of different lines is is $15! \cdot \frac{16!}{6!}$.

2. (10 points) Check the (single) box that best characterizes each item.

A function is onto if and only if its image is the same as its co-domain.	true	$\boxed{\surd}$	false	$\square$		
$f : \mathbb{Z} \to \mathbb{Z}$ $f(x) = x + 3$ (x even), $f(x) = x - 21$ (x odd)	one-to-one	$\boxed{\surd}$	not one-to-one	$\square$	not a function	$\square$
$g : \mathbb{Z} \to \mathbb{R}$ $g(x) = x + 2.137$	onto	$\square$	not onto	$\boxed{\surd}$	not a function	$\square$
Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least three elves have charm.	true	$\square$	false	$\boxed{\surd}$		
$\exists y \in \mathbb{R}^+,\ \forall x \in \mathbb{R}^+,\ xy = 1$ ($\mathbb{R}^+$ is the positive real numbers.)	true	$\square$	false	$\boxed{\surd}$		

CS 173, Fall 17 Examlet 5, Part B

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Suppose that $|A| = p$, $|B| = q$, $|C| = n$. How many different functions are there from A to $B \times C$?

   Solution: There are qn elements in $B \times C$. So there are $(qn)^p$ ways to build a function from A to $B \times C$.

2. (10 points) Check the (single) box that best characterizes each item.

A function from $\mathbb{R}$ to $\mathbb{R}$ is strictly increasing if and only if it is one-to-one. true ☐ false ☑

$f : \mathbb{Z} \to \mathbb{Z}$
$f(x) = x + 3$ (x even),
$f(x) = x - 22$ (x odd)
onto ☐ not onto ☑ not a function ☐

$g : \mathbb{R} \to \mathbb{Z}$
$g(x) = |x|$
one-to-one ☐ not one-to-one ☐ not a function ☑

We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there are two mailboxes with the same color. true ☑ false ☐

$\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ y \leq x$ true ☑ false ☐

CS 173, Fall 17 Examlet 5, Part B

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Suppose that $|A| = 3$ and $|B| = 3$. How many onto functions are there from A to B? Briefly justify or show work.

   Solution: Since the two sets have the same number of elements, an onto function must also be one-to-one. If two inputs mapped to the same output value, it would be impossible for the image to cover all of B. So we can use the formula for the number of permutations of 3 values: there are $3! = 6$ onto functions from A to B.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the integers is the ____ of f.
   domain ☑ co-domain ☐ image ☐ none of these ☐

   $g : \mathbb{Z} \to \mathbb{Z}$
   $g(x) = |x|$
   one-to-one ☐ not one-to-one ☑ not a function ☐

   $g : \mathbb{R} \to [0, 1]$
   $g(x) = \sin(x)$
   onto ☐ not onto ☐ not a function ☑

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, there must be at least three elves with the same gift.
   true ☑ false ☐

   $\exists y \in \mathbb{N},\ \forall x \in \mathbb{Z},\ x^2 = y$
   true ☐ false ☑

CS 173, Fall 17 Examlet 5, Part B

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Let n and k be integers. Consider the integer powers of n from n^0 to n^k. Use the Pigeonhole Principle to show that there are two distinct (i.e. not equal) integers i and j, both between 0 and k (inclusive), such that $n^i \equiv n^j \pmod{k}$. (Your solution should be clear but does not need to be very formal.)

   Solution: For each power n^p, look at the remainder when n^p is divided by k. There are only k possible remainders, but there are $k+1$ powers in our list. So there are two powers with the same remainder, call them n^i and n^j. Since they have the same remainder, $n^i \equiv n^j \pmod{k}$.

2. (10 points) Check the (single) box that best characterizes each item.

If a function is onto, then each value in the co-domain has exactly one pre-image. true ☐ false ☑ √

$g : \mathbb{R} \to \mathbb{R}^2$
$g(x) = (x, 3x^2 + 2)$ one-to-one √ not one-to-one ☐ not a function ☐

$f : \mathbb{N} \to \mathbb{R}$
$f(x) = x^2 + 2$ onto ☐ not onto √ not a function ☐

If $f : A \to B$ is one-to-one, then $|A| \geq |B|$ ☐ $|A| \leq |B|$ √ $|A| = |B|$ ☐

$\exists t \in \mathbb{Z}^+, \ \forall p \in \mathbb{Z}^+, \ \gcd(p, t) = 1$ true √ false ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) How many different 14-letter strings can be made be rearranging the letters in the word ``‘‘classification’’``? Show your work.

   Solution: There are 14 letters total, with 2 copies of c, 3 copies of i, 2 copies of a, and 2 copies of s. So the number of possibilities is

   $$\frac{14!}{3!2!2!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{Z}$ is a function such that $f(x) = -\lvert x\rvert$ then $\mathbb{N}$ is the _____ of f.	domain ☐	co-domain ☐	image ☐	none of these ☑
$f : \mathbb{N}^2 \to \mathbb{N}$, $f(p, q) = pq$	one-to-one ☐	not one-to-one ☑	not a function ☐	
$g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = \lvert x\rvert$	onto ☐	not onto ☑	not a function ☐	
We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on at least two mailboxes.	true ☑	false ☐		
$\exists t \in \mathbb{N},\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = p$	true ☑	false ☐		

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Suppose that $|A| = p$ and $|B| = q$, $p \leq q$. How many different one-to-one functions are there from A to B?

   Solution: $\frac{q!}{(q-p)!}$

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = |x|$ then $\mathbb{N}$ is the _____ of f. domain ☐ co-domain ☐ image ☑ $\surd$

   $g : \mathbb{R}^2 \to \mathbb{R}$, $g(x, y) = \lfloor x \rfloor + y$ onto ☑ $\surd$ not onto ☐ not a function ☐

   $g : \mathbb{R}^2 \to \mathbb{R}^2$, $g(x, y) = (y, 3x)$ one-to-one ☑ $\surd$ not one-to-one ☐ not a function ☐

   Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, every color appears on at least two vertices. true ☐ false ☑ $\surd$

   $\forall x \in \mathbb{Q},\ \exists m, n \in \mathbb{Z},\ x = \frac{m}{n}$ true ☑ $\surd$ false ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Xin plans to randomly draw a hand of cards from a standard deck of 52 cards (evenly divided among 4 suits). He'd like to be sure the hand includes 3 cards with the same suit. How large must the hand be? Briefly justify your answer.

   Solution: He needs to draw 9 cards.

   Suppose the hand contained ≤ 2 cards from each suit. Since there are four suits, that means the total number of cards must be ≤ 8. Therefore, 9 cards guarantees 3 with the same suit.

2. (10 points) Check the (single) box that best characterizes each item.

A function is one-to-one if and only if each value in the domain has exactly one image.	true	☐	false	☑ $\surd$		
$g : \mathbb{R}^2 \to \mathbb{R}$ $g(x, y) = \lfloor x \rfloor + y$	one-to-one	☐	not one-to-one	☑ $\surd$	not a function	☐
$g : \mathbb{R} \to [-1, 1]$ $g(x) = \sin(x)$	onto	☑ $\surd$	not onto	☐	not a function	☐
$f : \mathbb{N}^2 \to \mathbb{Z}$ $f(p, q) = 2^p 3^q$	one-to-one	☑ $\surd$	not one-to-one	☐	not a function	☐
$\forall x \in \mathbb{Z}^+,\ \exists y \in \mathbb{Z}^+,\ xy = 1$	true	☐	false	☑ $\surd$		

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Suppose that $|A| = 50$ and $B = \{5, 6\}$. How many onto functions are there from A to B? Briefly justify or show work. (Hint: how many non-onto functions are there?)

   Solution: There's only two ways to create a function from A to B that is not onto: all input values map to 5 or all input values map to 6. The total number of functions from A to B is 2^{50}. So the total number of onto functions is $2^{50} - 2$.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{N} \to \mathbb{Z}$ is a function such that $f(x) = -|x|$ then $\mathbb{N}$ is the _____ of f. domain ☑ co-domain ☐ image ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = x|x|$ — onto ☐ not onto ☑ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = 7 - \left\lfloor \frac{x}{3} \right\rfloor$ — one-to-one ☐ not one-to-one ☑ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = x|x|$ — one-to-one ☑ not one-to-one ☐ not a function ☐

   $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ x - y < 100$ — true ☑ false ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) How many different 10-letter strings can be made be rearranging the characters in the word ``minimalist''? Show your work.

   Solution: There are 10 characters total, with two copies of m and three copies of i. So the total number of permutations is

$$\frac{10!}{3!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

If a function is onto, then each value in the co-domain has at least one pre-image.	true	$\boxed{\surd}$	false	$\boxed{}$		
$g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$ $g(x, y) = \gcd(x, y)$	one-to-one	$\boxed{}$	not one-to-one	$\boxed{\surd}$	not a function	$\boxed{}$
$g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$ $g(x, y) = \gcd(x, y)$	onto	$\boxed{\surd}$	not onto	$\boxed{}$	not a function	$\boxed{}$
$f : \mathbb{R} \to \mathbb{Z}$ $f(x) = x$	one-to-one	$\boxed{}$	not one-to-one	$\boxed{}$	not a function	$\boxed{\surd}$
$\exists m, n \in \mathbb{Z},\ \forall x \in \mathbb{Q},\ x = \frac{m}{n}$	true	$\boxed{}$	false	$\boxed{\surd}$		

Name: ____

NetID: ______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Suppose that A is a set containing $k+1$ (distinct) integers. Use the Pigeonhole Principle to show that there are x and y in A ($x \neq y$) such that $x - y$ is a multiple of k.

   Solution: (You don't need to be this formal for full credit.) Let $A = \{x_1, x_2, \ldots, x_{k+1}\}$. We can represent each integer x_i in terms of its quotient and remainder mod k, i.e. $x_i = kq_i + r_i$, where $0 \leq r_i < k$. There are k possible remainders, but $k+1$ numbers in A. So two numbers must share the same remainder, which implies that they differ by a multiple of k.

2. (10 points) Check the (single) box that best characterizes each item.

A function is one-to-one if and only if each value in the co-domain has at most one pre-image.	true ☑	false ☐	
$g: \mathbb{Z}^2 \to \mathbb{Z}^2$ $g(x, y) = (y, 3x)$	one-to-one ☑	not one-to-one ☐	not a function ☐
$g: \mathbb{Z} \to \mathbb{N}$ $g(x) = x$	one-to-one ☐	not one-to-one ☐	not a function ☑
$g: \mathbb{N}^2 \to \mathbb{N}$ $g(x, y) = \gcd(x, y)$	onto ☐	not onto ☐	not a function ☑
$\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ x - y < 100$	true ☐	false ☑	

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) How many different 10-letter strings can be made be rearranging the characters in the word ``tattletale''? Show your work.

   Solution: There are 10 letters total, with 4 copies of t, two a's, two e's, and two l's. So the total number of possibilities is

$$\frac{10!}{4!2!2!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the real numbers is the _____ of f. domain ☐ co-domain ☑ image ☐

$g : \mathbb{N}^2 \to \mathbb{N}$, $g(x, y) = \gcd(x, y)$ one-to-one ☐ not one-to-one ☐ not a function ☑

$f : \mathbb{N}^2 \to \mathbb{N}$, $f(p, q) = pq$ onto ☑ not onto ☐ not a function ☐

Each dorm room is given an access code between 1 and 10 (inclusive). According to the pigeonhole principle, if there are 21 dorm rooms, then every access code must be shared by at least two rooms. true ☐ false ☑

$\forall m, n \in \mathbb{Z},\ \exists x \in \mathbb{Q},\ x = \frac{m}{n}$ true ☐ false ☑