# CS 173, Spring 19 Examlet 7, colored

**Name:**________________________________________

**NetID:**________________________ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Working directly from the definition of divides, use (strong) induction to prove the following claim:

Claim: $n^3 + 5n$ is divisible by 6, for all positive integers $n$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim.

Claim: For any positive integer $n$, $\sum_{p=1}^{n} \log(p^2) = 2\log(n!)$

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

# CS 173, Spring 19 Examlet 7, colored

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Let $A$ be a constant integer. Use (strong) induction to prove the following claim. Remember that $0! = 1$.

Claim: For any integer $n \geq A$, $\sum_{p=A}^{n} \frac{p!}{A!(p-A)!} = \frac{(n+1)!}{(A+1)!(n-A)!}$

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

# CS 173, Spring 19 Examlet 7, colored

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

$\prod_{p=2}^{n}(1 - \frac{1}{p^2}) = \frac{n+1}{2n}$ for any integer $n \geq 2$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**CS 173, Spring 19** **Examlet 7, colored**

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: for all natural numbers $n$, $\sum_{j=0}^{n} 2(-7)^j = \frac{1-(-7)^{n+1}}{4}$

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim.

Claim: For any positive integer $n$, $\displaystyle\sum_{p=1}^{n} \frac{1}{\sqrt{p-1}+\sqrt{p}} = \sqrt{n}$

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**: (Recall that $\frac{1}{a+b} = \frac{a-b}{(a-b)(a+b)} = \frac{a-b}{a^2-b^2}$.)

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: $\displaystyle\sum_{j=1}^{n} \frac{1}{j(j+1)} = \frac{n}{n+1}$ for all positive integers $n$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**________________________________________

**NetID:**____________________ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

(15 points) Use (strong) induction and the fact that $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$ to prove the following claim:

For all natural numbers $n$, $\left(\sum_{i=0}^{n} i\right)^2 = \sum_{i=0}^{n} i^3$

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**: (Start by removing the top term from the sum on the lefthand side.)

**Name:**________________________________________

**NetID:**______________________ **Lecture:** **B**

**Discussion:** **Friday** **11** **12** **1** **2** **3** **4**

Use (strong) induction to prove the following claim:

Claim: For all integers $a$, $b$, $n$, $n \geq 1$, if $a \equiv b \pmod 7$ then $a^n \equiv b^n \pmod 7$.

Use this definition in your proof: $x \equiv y \pmod p$ if and only if $x = y + kp$ for some integer $k$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

# CS 173, Fall 18 Examlet 7, Part A

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **B**

**Discussion:** **Friday** **11** **12** **1** **2** **3** **4**

Use (strong) induction to prove the following claim

Claim: $\sum_{k=0}^{n} p^k = \frac{p^{n+1}-1}{p-1}$, for all natural numbers n and all real numbers $p \neq 1$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

# CS 173, Spring 18 Examlet 7, Part A

**Name:** ____________________________________________

**NetID:** ______________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Use (strong) induction to prove the following claim:

Claim: $\sum_{p=0}^{n}(p \cdot p!) = (n+1)! - 1$, for all natural numbers $n$.

Recall that 0! is defined to be 1.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:** ______________________________

**NetID:** ____________________ **Lecture:** A B

**Discussion:** Thursday Friday 9 10 11 12 1 2 3 4 5 6

Use (strong) induction to prove the following claim:

Claim: $\sum_{j=2}^{n} \frac{1}{j(j-1)} = \frac{n-1}{n}$ for all integers $n \geq 2$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

# CS 173, Spring 18 Examlet 7, Part A

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Working directly from the definition of divides, use (strong) induction to prove the following claim:

Claim: $(4n)!$ is divisible by $8^n$, for all positive integers $n$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

# CS 173, Spring 18 Examlet 7, Part A 

**Name:**________________________________________

**NetID:**______________________ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

Use (strong) induction to prove the following claim:

Claim: $\sum_{j=1}^{n} j(j+1) = \frac{n(n+1)(n+2)}{3}$, for all positive integers $n$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

# CS 173, Spring 18 Examlet 7, Part A

**Name:**________________________________________

**NetID:**________________________ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

Use (strong) induction to prove the following claim:

For all positive integers $n$, $\sum_{p=1}^{n} p2^p = (n-1)2^{n+1} + 2$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

# CS 173, Spring 18 Examlet 7, Part A

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Use (strong) induction to prove the following claim.

Claim: For any positive integer $n$, $2^{4n-1}$ ends in the digit 8. (I.e. when written out in base-10, the one's digit is 8.)

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

# CS 173, Fall 17 Examlet 7, Part A

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Working directly from the definition of divides, use (strong) induction to prove the following claim:

Claim: $2^{n+2} + 3^{2n+1}$ is divisible by 7, for all natural numbers $n$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

If $f$ is a function, recall that $f'$ is its derivative. Recall the product rule: if $f(x) = g(x)h(x)$, then $f'(x) = g'(x)h(x) + g(x)h'(x)$. Assume we know that the derivative of $f(x) = x$ is $f'(x) = 1$.

Use (strong) induction to prove the following claim:

For any positive integer $n$, if $f(x) = x^n$ then $f'(x) = nx^{n-1}$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Use (strong) induction to prove the following claim:

For any natural number $n$, $\sum_{p=0}^{n} 3(-1/2)^p = 2 + (-1/2)^n$

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

# CS 173, Fall 17 Examlet 7, Part A

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Use (strong) induction to prove the following claim:

For all natural numbers $n$, $\sum_{p=0}^{n}(2p+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let's say that a set of polygonal regions in the plane is "properly colored" if regions sharing an edge never have the same color.

Suppose that we draw $n$ lines in the plane, in general position (no lines are parallel, no point belongs to more than two lines). The lines divide up the plane into a set of regions. Use (strong) induction to prove that, for any positive integer $n$, this set of regions can be properly colored with two colors.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**________________________________________

**NetID:**____________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Use (strong) induction to prove the following claim:

Claim: For all integers $a$, $b$, $n$, $n \geq 1$, if $a \equiv b \pmod{7}$ then $a^n \equiv b^n \pmod{7}$.

Use this definition in your proof: $x \equiv y \pmod{p}$ if and only if $x = y + kp$ for some integer $k$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**________________________________________

**NetID:**________________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Use (strong) induction to prove the following claim:

Claim: $\sum_{p=1}^{n} \frac{2}{p^2+2p} = \frac{3}{2} - \frac{1}{(n+1)} - \frac{1}{(n+2)}$

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**__________________________________________

**NetID:**______________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

If $f$ is a function, recall that $f'$ is its derivative. Recall the product rule: if $f(x) = g(x)h(x)$, then $f'(x) = g'(x)h(x) + g(x)h'(x)$. Assume we know that the derivative of $f(x) = x$ is $f'(x) = 1$.

Use (strong) induction to prove the following claim:

For any positive integer $n$, if $f(x) = x^n$ then $f'(x) = nx^{n-1}$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**________________________________________

**NetID:**______________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $A$ be a constant integer. Use (strong) induction to prove the following claim. Remember that $0! = 1$.

Claim: For any integer $n \geq A$, $\sum_{p=A}^{n} \frac{p!}{A!(p-A)!} = \frac{(n+1)!}{(A+1)!(n-A)!}$

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**________________________________________

**NetID:**______________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Use (strong) induction to prove the following claim:

Claim: $\sum_{p=1}^{n} 2(-1)^p p^2 = (-1)^n n(n+1)$, for all positive integers $n$

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**:

**Name:**________________________________________

**NetID:**________________________ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

Use (strong) induction and the fact that $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$ to prove the following claim:

For all natural numbers $n$, $\left(\sum_{i=0}^{n} i\right)^2 = \sum_{i=0}^{n} i^3$

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**: (Start by removing the top term from the sum on the lefthand side.)

**Name:**________________________________________

**NetID:**______________________ **Lecture:** **A** **B**

**Discussion:** **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

$\prod_{p=2}^{n}(1 - \frac{1}{p^2}) = \frac{n+1}{2n}$ for any integer $n \geq 2$.

Proof by induction on $n$.

**Base case(s)**:

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step**: