CS 173, Spring 19 Examlet 10, colored

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Use (strong) induction to prove the following claim:

Claim: $\sum_{p=1}^{n} \frac{p}{p+1} \leq \frac{n^2}{n+1}$ for all positive integers n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: $(2n)!^2 < (4n)!$ for all positive integers.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

Claim: For any positive integer n and any reals $a_1, \ldots, a_n$ between 0 and 1 (inclusive)

$$\prod_{p=1}^{n}(1 - a_p) \geq 1 - \sum_{p=1}^{n} a_p$$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

CS 173, Spring 19 Examlet 10, colored

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Use (strong) induction to prove the following claim.

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{(-1)^{p-1}}{p} > 0$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hint: divide into cases based on whether k is even or odd. Consider removing more than term from the summation in one case.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: $\frac{(2n)!}{n!n!} > 2^n$, for all integers $n \geq 2$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Use (strong) induction to prove the following claim:

Claim: For any natural number n and any real number x, where $0 < x < 1$, $(1-x)^n \geq 1-nx$.

Let x be a real number, where $0 < x < 1$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: $\displaystyle\sum_{p=1}^{n} \frac{1}{p} \leq \frac{n}{2} + 1$, for any positive integer n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hint: recall that if $x \leq y$, then $\frac{1}{y} \leq \frac{1}{x}$

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

Claim: For any positive integer n and any positive reals $a_1, \ldots, a_n$,

$$\prod_{p=1}^{n}(1+a_p) \geq 1 + \sum_{p=1}^{n} a_p$$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:________________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4**

(15 points) Use (strong) induction to prove the following claim:

Claim: For all integers $n \geq 2$, $(2n)! > 2^n n!$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:______________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4**

(15 points) Use (strong) induction to prove the following claim:

Claim: For any natural number n and any real number $x > -1$, $(1+x)^n \geq 1+nx$.

Let x be a real number with $x > -1$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: For any natural number n and any real number $x > -1$, $(1+x)^n \geq 1 + nx$.

Let x be a real number with $x > -1$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} \geq \sqrt{n}$

You may use the fact that $\sqrt{n+1} \geq \sqrt{n}$ for any natural number n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Let function $f : \mathbb{Z}^+ \to \mathbb{R}$ be defined by

$$f(1) = f(2) = 1$$

$$f(n) = \frac{1}{2}f(n-1) + \frac{1}{f(n-2)}$$

Use (strong) induction to prove that $1 \leq f(n) \leq 2$ for all positive integers n.

Hint: prove both inequalities together using one induction.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

CS 173, Spring 18 Examlet 10, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Recall the following fact about real numbers

Triangle Inequality: For any real numbers x and y, $|x+y| \leq |x| + |y|$.

Use this fact and (strong) induction to prove the following claim:

Claim: For any real numbers $x_1, x_2, \ldots, x_n$ $(n \geq 2)$, $|x_1 + x_2 + \ldots + x_n| \leq |x_1| + |x_2| + \ldots + |x_n|$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name: ______________________________

NetID: ____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

Claim: $\prod_{p=1}^{n} \frac{2p-1}{2p} < \frac{1}{\sqrt{2n+1}}$ for all integers $n \geq 1$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hints: Work backwards from the goal, then rewrite into logical order. Try squaring both sides. For positive numbers a and b, $a < b$ if and only if $\sqrt{a} < \sqrt{b}$.

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Suppose that $0 < q < \frac{1}{2}$. Use (strong) induction to prove the following claim:

Claim: $(1+q)^n \leq 1 + 2^n q$, for all positive integers n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Notice that $1 \leq 2^{n-1}$ for any positive integer n.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

Claim: For any positive integer n and any positive reals $a_1, \ldots, a_n$,

$$\prod_{p=1}^{n}(1+a_p) \geq 1 + \sum_{p=1}^{n} a_p$$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: For all integers $n \geq 2$, $(2n)! > 2^n n!$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: For any sets A_1, A_2, ..., A_n, $|A_1 \cup A_2 \cup \ldots \cup A_n| \leq |A_1| + |A_2| + \ldots + |A_n|$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hint: remember the "Inclusion-Exclusion" formula for computing $|A \cup B|$ in terms of $|A|$, $|B|$, $|A \cap B|$.

CS 173, Fall 17 Examlet 10, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim. You may use the fact that $\sqrt{2} \leq 1.5$.

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} \geq 2\sqrt{n+1} - 2$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hint: notice that $(\sqrt{x+1} - \sqrt{x+2}\ )^2 \geq 0$. What does this imply about $2\sqrt{x+1}\sqrt{x+2}$?

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Let function $f : \mathbb{Z}^+ \to \mathbb{N}$ be defined by

$f(1) = 0$

$f(n) = 1 + f(\lfloor n/2 \rfloor)$, for $n \geq 2$,

Use (strong) induction on n to prove that $f(n) \leq \log_2 n$ for any positive integer n. You cannot assume that n is a power of 2. However, you can assume that the log function is increasing (if $x \leq y$ then $\log x \leq \log y$) and that $\lfloor x \rfloor \leq x$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

CS 173, Fall 17 Examlet 10, Part A

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

Claim: For any positive integer n and any positive reals $a_1, \ldots, a_n$,

$$\prod_{p=1}^{n}(1 - a_p) \geq 1 - \sum_{p=1}^{n} a_p$$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: $\frac{(2n)!}{n!n!} > 2^n$, for all integers $n \geq 2$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: For any natural number n and any real number x, where $0 < x < 1$, $(1-x)^n \geq 1-nx$.

Let x be a real number, where $0 < x < 1$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(15 points) Use (strong) induction to prove the following claim:

Claim: $(2n)!^2 < (4n)!$ for all positive integers.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: $\sum_{p=1}^{n} \frac{1}{p^2} > \frac{3n}{2n+1}$ for all integers $n \geq 2$

First prove a lemma: if $k \geq 2$, then $\frac{3k}{2k+1} - \frac{3(k-1)}{2(k-1)+1} < \frac{1}{k^2}$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Let function $f : \mathbb{Z}^+ \to \mathbb{R}$ be defined by

$f(1) = f(2) = 1$

$f(n) = \frac{1}{2}f(n-1) + \frac{1}{f(n-2)}$

Use (strong) induction to prove that $1 \leq f(n) \leq 2$ for all positive integers n.

Hint: prove both inequalities together using one induction.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use (strong) induction to prove the following claim:

Claim: $\frac{(2n)!}{n!n!} < 4^n$, for all integers $n \geq 2$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: