CS 173, Spring 19 Examlet 3, white sheet

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A and B, $(A \cap B) \cup (A \cap \overline{B}) = A$.

   Solution: This claim is true. If x is an element of A, there are exactly two possibilities: either x is in B or x is not in B (i.e. x is in $\overline{B}$).

2. (4 points) Check the (single) box that best characterizes each item.

   If $x \in A \cap B$, then $x \in A$.

   true for all sets A and B [√] true for some sets A and B [ ]

   false for all sets A and B [ ]

   For all positive integers n, if $n! < -10$, then $n > 8$.

   true [√] false [ ] undefined [ ]

3. (7 points) In $\mathbb{Z}_7$, find the value of $[3]^{37}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 6$.

   Solution: $[3]^2 = [9] = [2]$

   $[3]^4 = [2]^2 = [4]$

   $[3]^8 = [4]^2 = [16] = [2]$

   $[3]^{16} = [2]^2 = [4]$

   $[3]^{32} = [4]^2 = [2]$

   $[3]^{37} = [3]^{32} \cdot [3]^4 \cdot [3] = [2][4][3] = [24] = [3]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{4, 5, 9\}$ $B = \{\text{arya}, \text{bran}\}$ $C = \{\ 2, 4, 10\ \}$

   $(A \cap C) \times B =$

   Solution: $\{4\} \times B = \{(4, \text{arya}), (4, \text{bran})\}$

   $|A \times B \times C| =$

   Solution: $3 \times 2 \times 3 = 18$

2. (4 points) Check the (single) box that best characterizes each item.

   $A \times A = A$ (Assume $A \neq \emptyset$)
   true for all sets A ☐ false for all sets A ☑
   true for some sets A ☐

   $\emptyset \subseteq A$
   true for all sets A ☑ true for some sets A ☐
   false for all sets A ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[6]^{42}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[6]^2 = [36] = [3]$

   $[6]^4 = [3]^2 = [9]$

   $[6]^8 = [9]^2 = [81] = [4]$

   $[6]^{16} = [4]^2 = [16] = [5]$

   $[6]^{32} = [5]^2 = [25] = [3]$

   $[6]^{42} = [6]^{32} \cdot [6]^8 \cdot [6]^2 = [3][4][3] = [36] = [3]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $M = \{\text{cereal}, \text{toast}\}$ $N = \{\text{milk}, \text{coffee}, \text{wine}\}$
   $P = \{\text{wine}, \text{beer}, (\text{coffee}, \text{ham}), (\text{milk}, \text{ham})\}$

   $M \times (N - P) =$

   Solution: $M \times (N - P) = M \times \{\text{milk}, \text{coffee}\}$
   $= \{(\text{cereal}, \text{milk}), (\text{cereal}, \text{coffee}), (\text{toast}, \text{milk}), (\text{toast}, \text{coffee})\}$

   $|M \times N \times P| =$

   Solution: $|M \times N \times P| = 2 \cdot 3 \cdot 4 = 24$

2. (4 points) Check the (single) box that best characterizes each item.

   $\overline{A \cup B} = \overline{A} \cap \overline{B}$ true for all sets A and B [√] true for some sets A and B [ ] false for all sets A and B [ ]

   $\{\emptyset\} \times \{\emptyset\} =$ $\emptyset$ [ ] $\{\emptyset\}$ [ ] $\{\emptyset, \emptyset\}$ [ ] $\{(\emptyset, \emptyset)\}$ [√]

3. (7 points) In $\mathbb{Z}_{17}$, find the value of $[5]^{37}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 16$.

   Solution:

   $[5]^2 = [25] = [8]$

   $[5]^4 = [8]^2 = [64] = [13] = [-4]$

   $[5]^8 = [-4]^2 = [16] = [-1]$

   $[5]^{16} = [-1]^2 = [1]$

   $[5]^{32} = [1]^2 = [1]$

   So

   $[5]^{37} = [5]^{32} \cdot [5]^4 \cdot [5] = [1][-4][5] = [-20] = [14]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{trump}, \text{rubio}\}$ $B = \{\text{clinton}, \text{sanders}\}$
   $C = \{$ (trump, clinton), (sanders, rubio) $\}$
   $(B \times A) - C =$
   Solution: $\{(\text{clinton}, \text{trump}), (\text{clinton}, \text{rubio}), (\text{sanders}, \text{trump})\}$
   $(A \cap C) \times B =$
   Solution: $\emptyset \times B = \emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $A \cap B = A \cup B$ true for all sets A and B ☐ true for some sets A and B ☑
   false for all sets A and B ☐

   For all reals n, if $n^2 = 101$, then $n > 11$. true ☐ false ☑ undefined ☐

3. (7 points) In $\mathbb{Z}_9$, find the value of $[4]^6 \times [5]^{20}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.
   Solution: $[5]^2 = [25] = [7]$
   $[5]^4 = [7]^2 = [49] = [4]$
   $[5]^8 = [4]^2 = [16] = [7]$
   $[5]^{16} = [7]^2 = [49] = [4]$
   $[4]^2 = [16] = [7]$
   $[4]^4 = [49] = [4]$
   $[4]^6 \times [5]^{20} = [4]^4 \cdot [4]^2 \cdot [5]^{16} \cdot [5]^4 = [4] \cdot [7] \cdot [4] \cdot [4] = [28] \cdot [16] = [1] \cdot [7] = [7]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{ginger, clove, nutmeg}\}$ $B = \{\text{ginger, vanilla, pepper}\}$ $C = \{\text{ (clove, nutmeg) }\}$

   $A \cap B =$

   Solution: {ginger}

   $A \cap C =$

   Solution: $\emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   For any sets A and B,
   if $x \in A - B$, then $x \in A$. true [√] false [ ]

   $\{\emptyset\} \subseteq A$ true for all sets A [ ] true for some sets A [√]
   false for all sets A [ ]

3. (7 points) In $\mathbb{Z}_{17}$, find the value of $[5]^{42}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 16$.

   Solution:

   $[5]^2 = [25] = [8]$

   $[5]^4 = [8]^2 = [64] = [-4]$

   $[5]^8 = [-4]^2 = [16] = [-1]$

   $[5]^{16} = [-1]^2 = [1]$

   $[5]^{32} = [1]^2 = [1]$

   So

   $[5]^{42} = [5]^{32} \cdot [5]^8 \cdot [5]^2 = [1][-1][8] = [-8] = [9]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A and B, $A \cup (B - A) = A \cup B$.

   Solution: This claim is true. If x is in A, x is clearly in both sets. So consider an x that isn't in A. If x is in $A \cup (B - A)$ then $x \in (B - A)$, so x is in B. Going the other way, if x is in $A \cup B$ but not in A, then x is in B but not in A, so x is in $B - A$.

2. (4 points) Check the (single) box that best characterizes each item.

   Let A and B be disjoint. $|A - B| = |A| - |B|$

   true for all sets A and B ☐ true for some sets A and B ☑

   false for all sets A and B ☐

   $\{1, 2\} \cap \emptyset =$

$\emptyset$	☑	$\{(1, \emptyset), (2, \emptyset)\}$	☐	$\{1, 2, \emptyset\}$	☐
$\{\emptyset\}$	☐	$\{1, 2\}$	☐	undefined	☐

3. (7 points) In $\mathbb{Z}_7$, find the value of $[3]^{41}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 6$.

   Solution: $[3]^2 = [9] = [2]$

   $[3]^4 = [2]^2 = [4]$

   $[3]^8 = [4]^2 = [16] = [2]$

   $[3]^{16} = [2]^2 = [4]$

   $[3]^{32} = [4]^2 = [2]$

   $[3]^{41} = [3]^{32} \cdot [3]^8 \cdot [3] = [2][2][3] = [12] = [5]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \cap B = \emptyset$ and $B \cap C = \emptyset$ then $A \cap C = \emptyset$.

   Solution: This claim is false. Consider $A = C = \{1\}$ and $B = \{2\}$. Then $A \cap B = \emptyset$ and $B \cap C = \emptyset$, but $A \cap C = \{1\} \neq \emptyset$.

2. (4 points) Check the (single) box that best characterizes each item.

   $|A \cup B| \leq |A| + |B|$ true for all sets A and B [√] true for some sets A and B [ ] false for all sets A and B [ ]

   $\forall x \in \mathbb{Q}$, if $x^2 = 3$, then $x > 1000$. true [√] false [ ] undefined [ ]

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{19}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10]$

   $[7]^4 = [100] = [9]$

   $[7]^8 = [9]^2 = [81] = [3]$

   $[7]^{16} = [3]^2 = [9]$

   $[7]^{19} = [7]^{16} \cdot [7]^{[}2] \cdot [7] = [9] \cdot [10] \cdot [7]$

   $[9] \cdot [10] \cdot [7] = [90] \cdot [7] = [-1] \cdot [7] = [-7] = [6]$

   So $[7]^{19} = [6]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{oak}, \text{apple}, \text{maple}, \text{elm}\}$ $B = \{\text{tree}, \text{leaf}, \text{oak}\}$ $C = \{(\text{oak}, \text{tree})\}$

   $|A \times (B - C)| =$

   Solution: $(B - C) = B$. So $|A \times (B - C)| = 4 \times 3 = 12$

   $A \cap B =$

   Solution: $A \cap B = \{\text{oak}\}$

2. (4 points) Check the (single) box that best characterizes each item.

   Sets A and B are disjoint

$A - B = B - A$	☐	$A = \overline{B}$	☐
$A \cap B = \{\emptyset\}$	☐	$A \cap B = \emptyset$	☑

   $\{1, 2\} \times \emptyset =$

$\emptyset$	☑	$\{(1, \emptyset), (2, \emptyset)\}$	☐	$\{1, 2, \emptyset\}$	☐
$\{\emptyset\}$	☐	$\{1, 2\}$	☐	undefined	☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{21}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10] = [-3]$

   $[7]^4 = ([7]^2)^2 = [-3]^2 = [9]$

   $[7]^8 = ([7]^4)^2 = [9]^2 = [81] = [3]$

   $[7]^{16} = ([7]^8)^2 = [3]^2 = [9]$

   $[7]^{21} = [7]^{16} \cdot [7]^4 \cdot [7] = [9] \cdot [9] \cdot [7] = [81] \cdot [7] = [3] \cdot [7] = [21] = [8]$

CS 173, Fall 18 Examlet 3, Part B

Name:_____

NetID:_____ **Lecture: B**

Discussion: Thursday Friday 11 12 1 2 3 4

1. (4 points) $A = \{\text{fox}, \text{cat}\}$ $B = \{3, 4\}$ $C = \{3, 7\}$

   $A \times (B \cap C) =$

   Solution: $A \times (B \cap C) = A \times \{3\} = \{(\text{fox}, 3), (\text{cat}, 3)\}$

   $A \cap B =$

   Solution: $A \cap B = \emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $A \cap (B \cup C)$ $= (A \cap B) \cup (A \cap C)$ true for all sets A,B,C [√] true for some sets A,B,C [ ] false for all sets A,B,C [ ]

   $\forall x \in \mathbb{N}$, if $x < -10$, then $x = \pi$. (π is the familiar constant.) true [√] false [ ] undefined [ ]

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{12} + [9]^5$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = [5^2] = [25] = [3]$

   $[7]^8 = [3^2] = [9]$

   So $[7]^{12} = [7]^8 \cdot [7]^4 = [3] \cdot [9] = [27] = [5]$

   $[9]^2 = [81] = [4]$

   $[9]^4 = [4]^2 = [16] = 5$

   So $[9]^5 = [9] \cdot [5] = [45] = [1]$

   So $[7]^{12} + [9]^5 = [5] + [1] = [6]$.

Name:_____

NetID:_____ **Lecture: B**

Discussion: Thursday Friday 11 12 1 2 3 4

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \times C \subseteq B \times C$, then $A \subseteq B$.

   Solution: This is false. Suppose that $A = \{1, 2\}$, $B = \{10, 11\}$, and $C = \emptyset$. Then $A \times C = \emptyset = B \times C$, so $A \times C \subseteq B \times C$. But $A \not\subseteq B$.

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset$ is an element of $\mathbb{Z}$ ☐ a subset of $\mathbb{Z}$ ☑ both ☐ neither ☐

   $|A \cup B| = |A| + |B|$ true for all sets A and B ☐ true for some sets A and B ☑ false for all sets A and B ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{38}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = ([7]^2)^2 = [5]^2 = [25] = [3]$

   $[7]^8 = ([7]^4)^2 = [3]^2 = [9] = [-2]$

   $[7]^16 = ([7]^8)^2 = [-2]^2 = [4]$

   $[7]^32 = ([7]^{16})^2 = [4]^2 = [16] = [5]$

   $[7]^{38} = [7]^{32} \cdot [7]^4 \cdot [7]^2 = [5] \cdot [3] \cdot [5] = [15] \cdot [5] = [4] \cdot [5] = [20] = [9]$

CS 173, Spring 18 Examlet 3, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{fox}, \text{cat}\}$ $B = \{\text{rat}, \text{mouse}\}$

   $A \cap B =$

   Solution: $\emptyset$

   $\{p^2 + q \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ 1 \leq p \leq 2 \text{ and } 1 \leq q \leq 3\} =$

   Solution: $\{2, 3, 4, 5, 6, 7\}$

2. (4 points) Check the (single) box that best characterizes each item.

   For all integers n, if $n^2 = 101$, then $n > 11$. true [√] false [ ] undefined [ ]

   If $x \in A \cup B$, then $x \in A$. true for all sets A and B [ ] true for some sets A and B [√] false for all sets A and B [ ]

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[6]^6 + [5]^3$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[6]^2 = [36] = [3]$

   $[6]^6 = [3]^3 = [27] = [5]$

   $[5]^3 = [125] = [4]$

   $[6]^6 + [5]^3 = [5] + [4] = [9]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \subseteq B$ then $A \times C \subseteq B \times C$.

   Solution: This is true. Suppose we pick (x, y) from $A \times C$. Then x is in A and y is in C. Since x is in A and $A \subseteq B$, x is in B. So (x, y) is in $B \times C$.

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset$ is an element of $\mathbb{Z}$ ☐ both ☐ a subset of $\mathbb{Z}$ ☑ neither ☐

   Sets A and B are disjoint $A \cap B = \{\emptyset\}$ ☐ $A \cap B = \emptyset$ ☑ $|A \cap B| = 1$ ☐ $A = \overline{B}$ ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[8]^{22}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[8]^2 = [64] = 9$

   $[8]^4 = [9]^2 = [81] = [4]$

   $[8]^8 = [4]^2 = [16] = [5]$

   $[8]^{16} = [5]^2 = [3]$

   $[8]^{22} = [8]^{16} \cdot [8]^4 \cdot [8]^2 = [3][4][9]$

   $[3][4][9] = [3][36] = [3][3] = [9]$

   So $[8]^{22} = [9]$

CS 173, Spring 18 Examlet 3, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $A \cup (B - C) \subseteq (A \cup B) - C$

   Solution: This is false. Suppose that $A = \{1, 2\}$, $B = \{3, 4\}$, and $C = \{2, 3\}$.
   Then $A \cup (B - C) = \{1, 2\} \cup \{4\} = \{1, 2, 4\}$.
   But $(A \cup B) - C = \{1, 2, 3, 4\} - \{2, 3\} = \{1, 4\}$.
   So $A \cup (B - C) \not\subseteq (A \cup B) - C$.

2. (4 points) Check the (single) box that best characterizes each item.

   $A \times B = A$

   true for all sets A and B ☐ false for all sets A and B ☐
   true for some sets A and B ☑

   $\emptyset \times \emptyset =$

   $\emptyset$ ☑ $\{\emptyset\}$ ☐ $\{\emptyset, \emptyset\}$ ☐ $\{(\emptyset, \emptyset)\}$ ☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[6]^8 + [5]^{20}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:
   $[6]^2 = [36] = [10]$
   $[6]^4 = [10]^2 = [100] = [9]$
   $[6]^8 = [9]^2 = [81] = [3]$
   $[5]^2 = [25] = [-1]$
   $[5]^4 = [1]$. So $[5]^{16} = [1]$. So $[5]^{20} = [5]^{16} \times [5]^4 = [1][1] = [1]$.
   $[6]^8 + [5]^{20} = [3] + [1] = [4]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \subseteq B$ then $A \cap C \subseteq B \cap C$.

   Solution: This is true. An element of $A \cap C$ must be in both A and C. I $A \subseteq B$, then it's also in B. But then it's in $B \cap C$.

2. (4 points) Check the (single) box that best characterizes each item.

   $A = \overline{A}$ (Assume the universe is not empty.)

   true for all sets A ☐ true for some sets A ☐

   false for all sets A ☑

   $\forall x \in \mathbb{Q}$, if $x^2 = 3$, then $x > 1000$.

   true ☑ false ☐ undefined ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{40}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = ([7]^2)^2 = [5]^2 = [25] = [3]$

   $[7]^8 = ([7]^4)^2 = [3]^2 = [9] = [-2]$

   $[7]^16 = ([7]^8)^2 = [-2]^2 = [4]$

   $[7]^32 = ([7]^{16})^2 = [4]^2 = [16] = [5]$

   $[7]^{40} = [7]^{32} \cdot [7]^8 = [5] \cdot [-2] = [-10] = [1]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{oak}, \text{apple}, \text{maple}, \text{elm}\}$ $B = \{\text{tree}, \text{oak}, \emptyset\}$

   $A \cap B =$

   Solution: $\{\text{oak}\}$

   $\{(p, q) \ : \ p \in \mathbb{Z}^+, q \in \mathbb{Z}^+, \text{ and } pq = 6\} =$

   Solution: $\{(p, q) \ : \ p \in \mathbb{Z}^+, q \in \mathbb{Z}^+, \text{ and } pq = 6\} = \{(1, 6), (6, 1), (2, 3), (3, 2)\}$

2. (4 points) Check the (single) box that best characterizes each item.

$\emptyset \in A$	true for all sets A	☐	true for some sets A	☑
	false for all sets A	☐		
$\lvert A \cup B\rvert = \lvert A\rvert + \lvert B\rvert$	true for all sets A and B	☐	true for some sets A and B	☑
	false for all sets A and B	☐		

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[8]^{37}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[8]^2 = [64] = 9$

   $[8]^4 = [9]^2 = [81] = [4]$

   $[8]^8 = [4]^2 = [16] = [5]$

   $[8]^{16} = [5]^2 = [3]$

   $[8]^{32} = [3]^2 = [9]$

   $[8]^{37} = [8]^{32} \cdot [8]^4 \cdot [8] = [9] \cdot [4] \cdot [8] = [36] \cdot [8] = [3] \cdot [8] = [24] = [2]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{fox, tiger, wolf}\}$ $B = \{3, 4\}$ $C = \{6, 7, 8\}$

   $| A \times (B \cup C) | =$

   Solution: $|\text{fox, tiger, wolf} \times \{3, 4, 6, 7, 9\}| = 3 \times 5 = 15$

   $\{p + q \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ 1 \leq p \leq 3 \text{ and } 1 \leq q \leq 3\} =$

   Solution: $\{2, 3, 4, 5, 6\}$

2. (4 points) Check the (single) box that best characterizes each item.

   $\{1, 2\} \cup \emptyset =$

$\emptyset$		$\{\emptyset\}$		$\{1, 2\}$	√
$\{(1, \emptyset), (2, \emptyset)\}$		$\{1, 2, \emptyset\}$		undefined	

   $A \cup B = A$

true for all sets A and B		false for all sets A and B	
true for some sets A and B	√		

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[10]^{43} + [7]^{10}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[10] = [-1]$. So $[10]^{43} = [-1]^{43} = -1$.

   $[7]^2 = [49] = [5]$

   $[7]^4 = [5]^2 = [25] = [3]$

   $[7]^8 = [3]^2 = [9]$

   So $[7]^{10} = [7]^2 \times [7]^8 = [5][9] = [45] = [1]$

   So $[10]^{43} + [7]^{10} = [1] + [-1] = [0]$.

CS 173, Fall 17 Examlet 3, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{oak}, \text{apple}, \text{maple}, \text{elm}\}$ $B = \{\text{tree}, \text{oak}, \emptyset\}$

   $(A \times \emptyset) \cap B =$

   Solution: $A \times \emptyset = \emptyset$ So $(A \times \emptyset) \cap B = \emptyset \cap B = \emptyset$

   $\{\frac{p}{q} \ : \ p \in \mathbb{Z}^+, q \in \mathbb{Z}^+, \text{ and } pq = 6\} =$

   Solution: $\{\frac{p}{q} \ : \ p \in \mathbb{Z}^+, q \in \mathbb{Z}^+, \text{ and } pq = 6\} = \{\frac{1}{6}, \frac{2}{3}, \frac{3}{2}, 6\}$

2. (4 points) Check the (single) box that best characterizes each item.

   For all positive integers n,
   if $n! < -10$, then $n > 8$.

   true [√] false [ ] undefined [ ]

   Let A and B be disjoint.
   $|A - B| = |A| - |B|$

   true for all sets A and B [ ] true for some sets A and B [√]
   false for all sets A and B [ ]

3. (7 points) In $\mathbb{Z}_7$, find the value of $[3]^{41}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 6$.

   Solution: $[3]^2 = [9] = [2]$

   $[3]^4 = [2]^2 = [4]$

   $[3]^8 = [4]^2 = [16] = [2]$

   $[3]^{16} = [2]^2 = [4]$

   $[3]^{32} = [4]^2 = [2]$

   $[3]^{41} = [3]^{32} \cdot [3]^8 \cdot [3] = [2][2][3] = [12] = [5]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A and B, $(A - B) \cup (B - A) \subseteq (A \cup B) - (A \cap B)$

   Solution: This is true. An element of $(A - B) \cup (B - A)$ must be in exactly one of the two sets. So it must be in $(A \cup B)$ but not in $(A \cap B)$.

2. (4 points) Check the (single) box that best characterizes each item.

   $A \times A = A$ (Assume $A \neq \emptyset$)
   - true for all sets A ☐
   - false for all sets A ☑
   - true for some sets A ☐

   $\{1, 2\} \times \emptyset =$
   - $\emptyset$ ☑
   - $\{(1, \emptyset), (2, \emptyset)\}$ ☐
   - $\{1, 2, \emptyset\}$ ☐
   - $\{\emptyset\}$ ☐
   - $\{1, 2\}$ ☐
   - undefined ☐

3. (7 points) In $\mathbb{Z}_9$, find the value of $[5]^{38}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution: $[5]^2 = [25] = [7]$

   $[5]^4 = [7]^2 = [49] = [4]$

   $[5]^8 = [4]^2 = [16] = [7]$

   $[5]^{16} = [7]^2 = [49] = [4]$

   $[5]^{32} = [4]^2 = [16] = [7]$

   $[5]^{38} = [5]^{32} \cdot [5]^4 \cdot [5]^2 = [7] \cdot [4] \cdot [7] = [28] \cdot [7] = [1] \cdot [7] = [7]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $(A - B) - C \subseteq A - C$

   Solution: This is true. Suppose that x is in $(A - B) - C$. Then x must be in A, but not in B or C. Since x is in A but not in C, x is in $A - C$.

2. (4 points) Check the (single) box that best characterizes each item.

   $|A - B| = |A| - |B|$ true for all sets A and B ☐ true for some sets A and B ☑
   false for all sets A and B ☐

   For all reals n, if $n^2 = 101$, then $n > 11$. true ☐ false ☑ undefined ☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{21}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10] = [-3]$

   $[7]^4 = ([7]^2)^2 = [-3]^2 = [9]$

   $[7]^8 = ([7]^4)^2 = [9]^2 = [81] = [3]$

   $[7]^{16} = ([7]^8)^2 = [3]^2 = [9]$

   $[7]^{21} = [7]^{16} \cdot [7]^4 \cdot [7] = [9] \cdot [9] \cdot [7] = [81] \cdot [7] = [3] \cdot [7] = [21] = [8]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{earth}, \text{air}, \text{fire}\}$ $B = \{\ (\text{fire}, 3),\ (\text{water}, 2)\ \}$ $C = \{\ 1, 2, 3\ \}$

   $(A \times C) \cap B =$

   Solution: $\{\ (\text{fire}, 3)\}$

   $\{p + q \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ pq = 6\} =$

   Solution: $\{7, -7, 5, -5\}$

2. (4 points) Check the (single) box that best characterizes each item.

   $A = \overline{A}$ (Assume the universe is not empty.)

   true for all sets A ☐ true for some sets A ☐

   false for all sets A ☑

   $\{1, 2\} \times \{\emptyset\} =$

   $\emptyset$ ☐ $\{(1, \emptyset), (2, \emptyset)\}$ ☑ $\{1, 2, \emptyset\}$ ☐

   $\{\emptyset\}$ ☐ $\{1, 2\}$ ☐ undefined ☐

3. (7 points) In $\mathbb{Z}_9$, find the value of $[5]^{41}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution: $[5]^2 = [25] = [7]$

   $[5]^4 = [7]^2 = [49] = [4]$

   $[5]^8 = [4]^2 = [16] = [7]$

   $[5]^{16} = [7]^2 = [49] = [4]$

   $[5]^{32} = [4]^2 = [16] = [7]$

   $[5]^{41} = [5]^{32} \cdot [5]^8 \cdot [5] = [7] \cdot [7] \cdot [5] = [49] \cdot [5] = [4] \cdot [5] = [20] = [2]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \times C \subseteq B \times C$, then $A \subseteq B$.

   Solution: This is false. Suppose that $A = \{1, 2\}$, $B = \{10, 11\}$, and $C = \emptyset$. Then $A \times C = \emptyset = B \times C$, so $A \times C \subseteq B \times C$. But $A \not\subseteq B$.

2. (4 points) Check the (single) box that best characterizes each item.

   $\forall x \in \mathbb{N}$, if $x^2 < -3$, then $x > 1000$. true [√] false [ ] undefined [ ]

   $A \cap B \subseteq A$ true for all sets A and B [√] true for some sets A and B [ ] false for all sets A and B [ ]

3. (7 points) In $\mathbb{Z}_{17}$, find the value of $[5]^{42}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 16$.

   Solution:

   $[5]^2 = [25] = [8]$

   $[5]^4 = [8]^2 = [64] = [-4]$

   $[5]^8 = [-4]^2 = [16] = [-1]$

   $[5]^{16} = [-1]^2 = [1]$

   $[5]^{32} = [1]^2 = [1]$

   So

   $[5]^{42} = [5]^{32} \cdot [5]^8 \cdot [5]^2 = [1][-1][8] = [-8] = [9]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) State the Inclusion Exclusion Principle/Formula for two sets.

   Solution: For any sets A and B, $|A \cup B| = |A| + |B| - |A \cap B|$

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset \times A = A \times \emptyset$ — true for all sets A [√] | true for some sets A [ ] | false for all sets A [ ]

   $A \cap B = A \cup B$ — true for all sets A and B [ ] | true for some sets A and B [√] | false for all sets A and B [ ]

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{19}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10]$

   $[7]^4 = [100] = [9]$

   $[7]^8 = [9]^2 = [81] = [3]$

   $[7]^{16} = [3]^2 = [9]$

   $[7]^{19} = [7]^{16} \cdot [7]^{[}2] \cdot [7] = [9] \cdot [10] \cdot [7]$

   $[9] \cdot [10] \cdot [7] = [90] \cdot [7] = [-1] \cdot [7] = [-7] = [6]$

   So $[7]^{19} = [6]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{water}, \text{beer}, \text{wine}\}$ $B = \{\ \text{cup}, \text{mug}\ \}$ $C = \{\text{wine},\ (\text{water}, \text{beer})\ \}$

   $(A - C) \times B =$

   Solution: $\{\ (\text{water}, \text{cup}), (\text{beer}, \text{cup}), (\text{water}, \text{mug}), (\text{beer}, \text{mug}),\ \}$

   $A \cap B =$

   Solution: $\emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $\forall x \in \mathbb{N}$, if $x < -10$, then $x = \pi$.
   (π is the familiar constant.) true [√] false [ ] undefined [ ]

   $|A \times B| = |A| \times |B|$ true for all sets A [√] true for some sets A [ ] false for all sets A [ ]

3. (7 points) In $\mathbb{Z}_7$, find the value of $[3]^{37}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 6$.

   Solution: $[3]^2 = [9] = [2]$

   $[3]^4 = [2]^2 = [4]$

   $[3]^8 = [4]^2 = [16] = [2]$

   $[3]^{16} = [2]^2 = [4]$

   $[3]^{32} = [4]^2 = [2]$

   $[3]^{37} = [3]^{32} \cdot [3]^4 \cdot [3] = [2][4][3] = [24] = [3]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $(A - B) \cup (B - C) = (A \cup B) - (A \cap B \cap C)$

   Solution: This is not true. Suppose that A is the empty set, and $B = C = \{1\}$. Then 1 is in $(A \cup B)$ but not in $(A \cap B \cap C)$, So 1 is in $(A \cup B) - (A \cap B \cap C)$. However, 1 is neither in $A - B$ nor in $B - C$. So it's not in $(A - B) \cup (B - C)$

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset \subseteq A$ true for all sets A [√] true for some sets A [ ] false for all sets A [ ]

   For any sets A and B, if $x \in A - B$, then $x \in A$. true [√] false [ ]

3. (7 points) In $\mathbb{Z}_9$, find the value of $[5]^{21}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution: $[5]^2 = [25] = [7]$

   $[5]^4 = [7]^2 = [49] = [4]$

   $[5]^8 = [4]^2 = [16] = [7]$

   $[5]^{16} = [7]^2 = [49] = [4]$

   $[5]^{21} = [5]^{16} \cdot [5]^4 \cdot [5] = [4][4][5] = [80] = [8]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $(A - B) - C = A - C$

   Solution: This is not true. Suppose that C is the empty set, and $A = B = \{1\}$. Then 1 is not $(A - B)$, therefore not in $(A - B) - C$. However, 1 is in $A - C$, because it's in A but not in C. So the two sets aren't equal.

2. (4 points) Check the (single) box that best characterizes each item.

   $A \times B = B \times A$ — true for all sets A and B ☐ — false for all sets A and B ☐ — true for some sets A and B ☑

   $\{\emptyset\} \times \{\emptyset\} =$ — $\emptyset$ ☐ — $\{\emptyset\}$ ☐ — $\{\emptyset, \emptyset\}$ ☐ — $\{(\emptyset, \emptyset)\}$ ☑

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[6]^{42}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[6]^2 = [36] = [3]$

   $[6]^4 = [3]^2 = [9]$

   $[6]^8 = [9]^2 = [81] = [4]$

   $[6]^{16} = [4]^2 = [16] = [5]$

   $[6]^{32} = [5]^2 = [25] = [3]$

   $[6]^{42} = [6]^{32} \cdot [6]^8 \cdot [6]^2 = [3][4][3] = [36] = [3]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{water}, \text{beer}, \text{wine}\}$ $B = \{\text{ cup}, \text{mug }\}$ $C = \{\text{wine}, \ (\text{water}, \text{beer})\ \}$

   $A \times (B \cap C) =$

   Solution: $A \times \emptyset = \emptyset$

   $|A \times B \times C| =$

   Solution: $3 \times 2 \times 2 = 12$

2. (4 points) Check the (single) box that best characterizes each item.

   If $x \in A \cap B$, then $x \in A$.

   true for all sets A and B ☑ false for all sets A and B ☐ true for some sets A and B ☐

   $\{13, 14, 15\} \times \emptyset =$

   $\emptyset$ ☑ $\{\emptyset\}$ ☐ $\{13, 14, 15\}$ ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{12} + [9]^5$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = [5^2] = [25] = [3]$

   $[7]^8 = [3^2] = [9]$

   So $[7]^{12} = [7]^8 \cdot [7]^4 = [3] \cdot [9] = [27] = [5]$

   $[9]^2 = [81] = [4]$

   $[9]^4 = [4]^2 = [16] = 5$

   So $[9]^5 = [9] \cdot [5] = [45] = [1]$

   So $[7]^{12} + [9]^5 = [5] + [1] = [6]$.

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $A \times (B - C) = (A \times B) - (A \times C)$

   Solution: This is true. Elements in $A \times (B - C)$ need to have a first component from A and a second component that's in B but not in C. But these are the same conditions required for a element to be in $A \times B$ but not in $A \times C$.

2. (4 points) Check the (single) box that best characterizes each item.

   $\overline{A \cup B} = \overline{A} \cap \overline{B}$ true for all sets A [√] true for some sets A [ ] false for all sets A [ ]

   $\forall x \in \mathbb{R}$, if $\pi = 3$, then $x < 20$.
   (π is the familiar constant.) true [√] false [ ] undefined [ ]

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{18} + [7]^4$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10] = [-3]$

   $[7]^4 = [-3]^2 = [9]$

   $[7]^6 = ([7]^2)^3 = [-3]^3 = [-27] = [-1]$

   $[7]^{18} = ([7]^6)^3 = [-1]^3 = [-1] = [12]$

   So $[7]^{18} + [7]^4 = [12] + [9] = [21] = [8]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{ginger}, \text{clove}, \text{nutmeg}\}$ $B = \{\text{ginger}, \text{vanilla}, \text{pepper}\}$ $C = \{\ (\text{clove}, \text{nutmeg})\ \}$

   $A \cap B =$

   Solution: $\{\text{ginger}\}$

   $A \cap C =$

   Solution: $\emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $|A \cup B| \leq |A| + |B|$ true for all sets A [√] true for some sets A [ ] false for all sets A [ ]

   $\emptyset \times \emptyset =$ $\{\emptyset, \emptyset\}$ [ ] $\{\emptyset\}$ [ ] $\emptyset$ [√] undefined [ ]

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{38}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = ([7]^2)^2 = [5]^2 = [25] = [3]$

   $[7]^8 = ([7]^4)^2 = [3]^2 = [9] = [-2]$

   $[7]^16 = ([7]^8)^2 = [-2]^2 = [4]$

   $[7]^32 = ([7]^{16})^2 = [4]^2 = [16] = [5]$

   $[7]^{38} = [7]^{32} \cdot [7]^4 \cdot [7]^2 = [5] \cdot [3] \cdot [5] = [15] \cdot [5] = [4] \cdot [5] = [20] = [9]$