Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(7 points) A triomino is a triangular tile with a number on each edge, visible on both front and back. In our set of triominos, the numbers range from 0 to 5. So possible tiles include 5-3-4, 0-4-4, and 3-3-3. A tile is the same if you turn it over or rotate it. So 5-3-4 is the same tile as 3-4-5, 4-5-3, and also 5-4-3. How many distinct tiles are in our set? Briefly justify your answer and/or show work.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every dinosaur k, if k is blue, then k is not vegetarian or k is friendly.

(2 points) Check the (single) box that best characterizes each item.

The number of bit strings of length 20 with exactly 8 1's.

$\binom{26}{7}$ ☐ $\binom{27}{7}$ ☐ $\binom{20}{8}$ ☐

$\binom{20}{13}$ ☐ $\binom{20}{14}$ ☐ 2^{20} ☐

Name:____

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) This evening, Ollie the Owl wants to say `hoot` 8 times, `krick` 7 times, and `yeet` 3 times. How many distinct sequences of the 18 noises could he produce? Briefly justify your answer and/or show work.

(8 points) Use proof by contradiction to show that, in a party of n people ($n \geq 2$), there are (at least) two people who danced with the same number of different partners. Assume people always dance in pairs. Don't assume everyone danced.

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Each time Nancy presses her 2FA login token, it generate a sequence of 6 decimal digits. What is the chance that this sequence will contain at least one duplicate digit? Give an exact formula; don't try to figure out the decimal equivalent. Briefly justify your answer and/or show work.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every relish r, if r is orange and r is not spicy, then r is pungent.

(2 points) Check the (single) box that best characterizes each item.

If you want to take 4 classes next semester, out of 35 classes being offered, how many different choices do you have?	$\frac{38!}{35!3!}$ ☐	$\frac{35!}{31!4!}$ ☐	35^4 ☐
	4^{35} ☐	$\frac{35!}{31!}$ ☐	4^4 ☐

Name:____

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Suppose a car dealer is planning to buy a collection of Civics, Accords, and Fits (three kinds of cars). The dealer will buy ten cars in total and can buy any number of each type. How many different choices does he have? The collection is unordered, so three Civics and seven Fits is the same as seven Fits and three Civics. Briefly justify your answer and/or show work.

(8 points) Use proof by contradiction to show that if x and y are positive integers, $x^2 - y^2 \neq 6$.

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(7 points) Pearl's start-up sells sets of circular bands for labelling wine glasses. Each band has four beads, each of which can be orange, blue, or silver. Two bands are the same if they can be made to look the same by rotation and/or turning over. E.g. all bands with three blue beads and one silver bead are the same. How many distinct bands can she put in each set? Briefly justify your answer and/or show work.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every mountain m, if m is tall or m is not in the north, then m has a snow cap.

(2 points) Check the (single) box that best characterizes each item.

Suppose you want to take 5 classes next semester, out of 25 classes being offered. You must take STATS 100. How many different choices do you have?

5^{24} ☐	$\frac{(24+3)!}{24!3!}$ ☐	$\frac{24!}{20!}$ ☐
24^5 ☐	$\frac{24!}{20!4!}$ ☐	$\frac{25!}{20!5!}$ ☐

Name:____

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Suppose that A is a set containing p elements and B is a set containing n elements. How many functions are there from A to $\mathbb{P}(B)$? How many of these functions are one-to-one? Briefly justify your answer and/or show work.

(8 points) Researchers recorded phone calls between pairs of two (different) people, never repeating the same pair of people. The experiment used n people ($n \geq 2$), but it's possible some of these n people were not in any conversation. Use proof by contradiction to show that two people were in the same number of conversations.

Name:____

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) CMU's new robotic hummingbird Merrill travels in 3D. Each command makes it move one foot in a specified cardinal direction e.g. up/down, north/south, or east/west, but not diagonally. How many different sequences of 30 commands will get Merrill from position $(1, 10, 3)$ to position $(10, 6, 20)$? Briefly justify your answer and/or show work.

(6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

There is a mushroom f such that f is not poisonous or f is blue.

(2 points) Check the (single) box that best characterizes each item.

E is the edge set of a tree with n nodes. $\lvert\mathbb{P}(E)\rvert =$	2^{n-1} ☐	2^n ☐	can't tell ☐
	2^{n+1} ☐	n ☐	$n-1$ ☐

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Prof. Howard has 17 tuba players, ordered by ability, who must be distributed among three bands. Stellar will get the best players, Dismal the worst ones, and Normal the group in the middle of the range. Each band must be given at least one tuba player. How many options does he have? Briefly justify your answer and/or show work.

(8 points) Suppose we know that $\sqrt{6}$ is irrational. Use proof by contradiction to show that $\sqrt{2}+\sqrt{3}$ is irrational. (You must use the definition of "rational." You may not use facts about adding/subtracting rational numbers.)

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Discussion: **Friday** **11** **12** **1** **2** **3** **4**

(8 points) Suppose that I have a set of p nodes, labelled 1 through p. How many different graphs can I make with this fixed set of nodes? (Isomorphic graphs with differently labelled nodes count as different for this problem.) Briefly justify your answer.

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

> For every computer game g, if g has trendy music or g has an interesting plotline, then g is not cheap.

(2 points) Check the (single) box that best characterizes each item.

The number of ways to select a set of 17 flowers chosen from 4 possible varieties (zero or more of each variety).	$\binom{17}{5}$ ☐	$\binom{20}{4}$ ☐	$\binom{20}{3}$ ☐
	$\binom{17}{4}$ ☐	$\binom{21}{4}$ ☐	$\frac{17!}{4!}$ ☐

Name:____

NetID:____________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4**

(9 points) Use proof by contradiction to show that $\log_5 2$ is irrational.

(6 points) Margaret's home is defended from zombies by wallnuts, peashooters, and starfruit. At each timestep, she can make one move, which adds or deletes one plant from her arsenal. If she starts with 3 wallnuts, 2 peashooters, and 19 starfruit, how many different sequences of 25 moves will get her to a configuration with 7 wallnuts, 13 peashooters, and 9 starfruit?

Name:____

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(8 points) The Emerald City bakery allows customers to special-order fruit pies. Each pie can contain one fruit or a mixture of 2 or 3 (distinct) fruits. The available fruits are raspberry, blueberry, pear, apple, cherry, apricot, and peach. How many choices do you have?

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

There is a dorm room d, such that d has green walls and d has no window.

(2 points) Check the (single) box that best characterizes each item.

The number of ways to select an ordered sequence of 17 flowers chosen from 4 possible varieties.	$\binom{16}{3}$ ☐	$\binom{16}{4}$ ☐	$\binom{20}{3}$ ☐
	$\binom{20}{4}$ ☐	$\binom{21}{3}$ ☐	4^{17} ☐

Name:____

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) Use proof by contradiction to show that there are no positive integer solutions to the equation $4x^2 - y^2 = 1$.

(6 points) In the game Tic-tac-toe is played on a 3x3 grid and a move consists of the first player putting an X into one of the squares, or the second player putting an O into one of the squares. The board cannot be rotated, e.g. an X in the upper right corner is different from an X in the lower left corner. How many different board configurations are possible after four moves (i.e. two moves by each player)?

Name:____

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(8 points) Anna has to climb 6 stairs to get onto the podium. She can climb a single step or two steps at a time. E.g. one possible (ordered) sequence of actions is: one step, a double step, three single steps. How many ways could she climb the stairs?

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

> There is a bug b, such that for every plant p, if b pollinates p and p is showy, then p is poisonous.

(2 points) Check the (single) box that best characterizes each item.

The number of ways to select a set of 17 flowers chosen from 4 possible varieties (zero or more of each variety).

$\binom{17}{5}$ ☐	$\binom{20}{4}$ ☐	$\binom{20}{3}$ ☐
$\binom{17}{4}$ ☐	$\binom{21}{4}$ ☐	$\frac{17!}{4!}$ ☐

Name:____

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(8 points) Ignatius Eggbert flips a coin 10 times. The coin is fair, i.e. equal chance of getting a head vs. a tail. What is the chance that he gets exactly 7 heads? Give an exact formula; don't try to figure out the decimal equivalent. Briefly explain your answer and/or show work.

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every Meerkat m, if m is in New York, then m is not in the wild or m is lost.

(2 points) Check the (single) box that best characterizes each item.

The number of bit strings of length 20 with exactly 7 1's.	$\binom{26}{7}$ ☐	$\binom{27}{7}$ ☐	$\binom{26}{6}$ ☐
	$\binom{20}{13}$ ☐	$\binom{20}{14}$ ☐	2^{20} ☐

CS 173, Spring 18 Examlet 12, Part A

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(9 points) Use proof by contradiction to show that $\sqrt{2} + \sqrt{3} \leq 4$.

(6 points) Use the binomial theorem to find a closed form for the summation $\sum_{k=0}^{n}(-1)^k\binom{n}{k}$. Make sure it's clear how you used the theorem.

CS 173, Spring 18 Examlet 12, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(8 points) If w is a string of characters, a *substring* of w is a contiguous string that forms part of w. For example, "rtho" is a substring of "warthog" but "ahog" is not. Suppose that w is a string without any repeated characters, of length k. How many non-empty substrings does it have?

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

There is a soup s such that s is tasty and s does not contain meat.

(2 points) Check the (single) box that best characterizes each item.

How many ways can I choose 6 bagels from among 8 varieties, if I can have any number of bagels from any type?	$\frac{8!}{6!2!}$ ☐	$\frac{13!}{6!7!}$ ☐	$\frac{14!}{9!5!}$ ☐
	$\frac{14!}{6!7!}$ ☐	8^6 ☐	6^8 ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(8 points) Marsha has 25 cups, identical except that 10 are red, 8 are blue, and 7 are green. How many ways can she make an (ordered) sequence of 25 cups?

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

There is a dorm room d, such that d has green walls and d has no window.

(2 points) Check the (single) box that best characterizes each item.

Suppose you want to take 5 classes next semester, out of 25 classes being offered. You must take STATS 100. How many different choices do you have?

$\frac{24!}{20!}$ ☐ $\frac{24!}{20!4!}$ ☐

$\frac{(24+3)!}{24!3!}$ ☐ $\frac{25!}{20!5!}$ ☐

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(9 points) Use proof by contradiction to show that, for any graph G and any two nodes a and b, the shortest walk from a to b does not contain any repeated nodes.

(6 points) Suppose a set S has 11 elements. How many subsets of S have an even number of elements? Express your answer as a summation. Briefly justify or show work.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(8 points) Suppose we know (e.g. from the binomial theorem) that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$. Use this to show that $\sum_{k=1}^{n} k \cdot \binom{n}{k} = n \cdot 2^{n-1}$. Show your work.

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every tree t, if t grows in Canada, then t is not tall or t is a conifer.

(2 points) Check the (single) box that best characterizes each item.

How many ways can I choose a set of 5 bagels, if there are 10 types of bagels and all 5 bagels must be different types?

$\frac{10!}{5!5!}$ ☐	$\frac{14!}{10!4!}$ ☐	$\frac{14!}{9!5!}$ ☐
$\frac{15!}{10!5!}$ ☐	10^5 ☐	5^{10} ☐

CS 173, Fall 17 **Examlet 12, Part A**

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(9 points) Researchers recorded phone calls between pairs of two (different) people, never repeating the same pair of people. The calls used n people ($n \geq 2$), but it's possible some people were not in any conversation. Use proof by contradiction to show that two people were in the same number of conversations.

(6 points) Use the binomial theorem to find a closed form for the summation $\sum_{k=0}^{n} \binom{n}{k}$. Make sure it's clear how you used the theorem.

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(9 points) Use proof by contradiction to show that, for any integer n, at least one of the three integers n, $2n+1$, $4n+3$ is not divisible by 7.

(6 points) Suppose a set S has 11 elements. How many subsets of S have five or fewer elements? Express your answer as a summation. Briefly justify or show work.

Name:______________________________________

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(8 points) Recall that nodes in a full binary tree have either zero or two children. Suppose we are building a full binary tree with unlabelled nodes whose leaves are all at levels k or $k+1$, with p leaves at level $k+1$. How many different ways can we construct such a tree? Briefly justify your answer.

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

If it is raining, then there is a cyclist c such that c is getting wet.

(2 points) Check the (single) box that best characterizes each item.

The number of ways to select a set of 17 flowers chosen from 4 possible varieties (one or more of each variety).	$\binom{16}{3}$ ☐	$\binom{16}{4}$ ☐	$\binom{20}{3}$ ☐
	$\binom{20}{4}$ ☐	$\binom{21}{3}$ ☐	4^{17} ☐

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(8 points) The Google interviewer suggests that $\binom{n}{k}$ can be computed very efficiently using the equation $\binom{n}{k} = \frac{n+1-k}{k}\binom{n}{k-1}$. Is this formula correct? Assume $k > 0$. Briefly justify your answer.

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every dog d, if d is a terrier, then d is not large and d is noisy.

(2 points) Check the (single) box that best characterizes each item.

V is the vertex set of a tree with n edges. $\lvert\mathbb{P}(V)\rvert =$	2^{n-1} ☐	2^n ☐	not determined ☐
	2^{n+1} ☐	n ☐	

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(9 points) Every hacker is a black hat or a white hat (and not both). White hats always tell the truth. Black hats always lie. Alfred says to you "I am a black hat." Use proof by contradiction to show that Alfred is not a hacker.

(6 points) If $x, y, z \in \mathbb{N}$, how many solutions are there to the equation $x + y + z = 25$?

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(8 points) A triomino is a triangular tile with a number on each edge. In our set of triominos, the numbers range from 0 to 5. So possible tiles include 5-3-4, 0-4-4, and 3-3-3. Tiles can be turned over: Also notice that a tile is the same if you rotate it. So 5-3-4 is the same tile as 3-4-5, 4-5-3, and also 5-4-3. How many distinct tiles are in our set?

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every tiger k, if k is orange, then k is large and k is not friendly.

(2 points) Check the (single) box that best characterizes each item.

The number of ways to select a set of 4 flowers chosen from 17 possible varieties (zero or more of each variety).	$\binom{17}{5}$ ☐	$\binom{20}{4}$ ☐	$\binom{20}{3}$ ☐
	$\binom{17}{4}$ ☐	$\binom{21}{4}$ ☐	$\frac{17!}{4!}$ ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(9 points) Use proof by contradiction to show that, in any group of 7 people, there is at least one person who knows an even number of people. (Assume that "knowing someone" is symmetric.)

(6 points) Margaret's home is defended from zombies by wallnuts, peashooters, and starfruit. She has a row of 20 pedestals on which they can stand, and she needs to use at least one starfruit. How many options does she have for the placing defenders on the pedestals?

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(9 points) Use proof by contradiction to show that $\sqrt{\sqrt{2}}$ is not rational. (You may use the fact that $\sqrt{2}$ is not rational.)

(6 points) In the town of West Fork, the streets are laid out in a uniform square grid. Alvin's school lies 6 blocks east and 9 blocks north of his house. So (since there are no diagonal roads) he travels 15 blocks to school. How many different 15-block paths can he choose from? Show your work or justify your answer.

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(9 points) Use proof by contradiction to show that there are no positive integer solutions to the equation $x^2 - y^2 = 10$.

(6 points) Margaret's home is defended from zombies by wallnuts, peashooters, and starfruit. She has room to place 20 of these on her lawn. How many options does she have for her set of defenders?