

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every dragon d , if d is green, then d is not large or d is fat.

Solution: For all dragons d , if d is large and d is not fat, then d is not green.

2. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For any student s , if s rides a bicycle, then s wears a helmet or s has no fear of death.

Solution: There is a student s who rides a bicycle but doesn’t wear a helmet and fears death.

3. (5 points) Solve $\frac{3}{x} + m = \frac{3}{p}$ for x , expressing your answer as a single fraction. Simplify your answer and show your work.

Solution: Multiplying by xp gives you $3p + mxp = 3x$.

So $3x - mxp = 3p$.

So $x(3 - mp) = 3p$.

So $x = \frac{3p}{3 - mp}$.

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$$(p \wedge q) \vee q \equiv q$$

p	q	$p \wedge q$	$(p \wedge q) \vee q$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	F

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every cat c , if c is not fierce or c wears a collar, then c is a pet.

Solution: For every cat c , if c is not a pet, then c is fierce and c does not wear a collar.

3. (5 points) Solve $16p^2 - 81 = 0$ for p . Simplify your answer and show your work.

Solution: $16p^2 - 81 = (4p - 9)(4p + 9)$

$(4p - 9)(4p + 9) = 0$ when either $4p = 9$ or $4p = -9$. That is $p = \pm \frac{9}{4}$

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every dragon d , if d is not large, then d is green or d not hungry.

Solution: There is a dragon d such that d is not large but d is not green and d is hungry.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every tree t , if t is in Illinois and t is not hardy, then t is indoors.

Solution: For every tree t , if t is not indoors, then t is not in Illinois or t is hardy.

3. (5 points) Suppose that k is a positive integer, x is a positive real number, and $\frac{1}{k} = x + \frac{1}{6}$. What are the possible values for k ? (Hint: k is an INTEGER.) Briefly explain or show work.

Solution: Observe that we can rearrange the equation as follows:

Since x is positive, $\frac{1}{k} = x + \frac{1}{6}$ implies that $\frac{1}{k} > \frac{1}{6}$. So k must be smaller than 6. But we were told that k was a positive integer. The only positive integers smaller than 6 are 1, 2, 3, 4, and 5.

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Show that the following two expressions are not logically equivalent, by giving specific values of p, q for which they produce different values.

$$p \rightarrow (q \rightarrow p)$$

$$(p \rightarrow q) \rightarrow p$$

Solution: Set p and q to be false.

Then $p \rightarrow (q \rightarrow p)$ is true because its hypothesis is false.

$p \rightarrow q$ is also true. So $(p \rightarrow q) \rightarrow p$ is false because its hypothesis is true but its conclusion (p) is false.

A similar argument works if you set p to be false and q to be true.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every car c , if c is a Tesla, then c is new or c is not fast.

Solution: For every car c , if c is not new and c is fast, then c is not a Tesla.

3. (5 points) Suppose that k is a positive integer, x is a positive real number, and $\frac{1}{k} + x = \frac{1}{6}$. What are the possible values for k ? (Hint: k is an INTEGER.) Briefly explain or show work.

Solution: Observe that we can rearrange the equation as follows:

Since x is positive, $\frac{1}{k} + x = \frac{1}{6}$ implies that $\frac{1}{k} < \frac{1}{6}$. So k must be an integer greater than 6.

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every tree t , if t grows in Canada, then t is not tall or t is a conifer.

Solution: There is a tree t , such that g is tall and t is not a conifer, but t grows in Canada.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every garbage can c , if c was supplied by the city, then c is small or c has wheels.

Solution: For every garbage can c , if c is large and c does not have wheels, then c was not supplied by the city.

3. (5 points) Solve $3x + 2m = \frac{w}{y}$ for x , expressing your answer as a single fraction. Simplify your answer and show your work.

Solution:

$$\begin{aligned} 3x + 2m &= \frac{w}{y} \\ 3x &= \frac{w}{y} - 2m \\ 3x &= \frac{w - 2ym}{y} \\ x &= \frac{w - 2ym}{3y} \end{aligned}$$

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$$(p \rightarrow q) \wedge (p \rightarrow \neg q) \equiv \neg p$$

p	q	$p \rightarrow q$	$p \rightarrow \neg q$	$(p \rightarrow q) \wedge (p \rightarrow \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	T	T	T
F	F	T	T	T

2. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every book b , if b is blue or b is not heavy, then b is not a math book.

Solution: There is a book b , such that b is blue or b is not heavy, but b is a math book.

3. (5 points) Solve $\frac{2m^2 - m - 6}{m - 2} = 9$ for m . (Assume $m \neq 2$.)

Solution: Notice that $2m^2 - m - 6 = (m - 2)(2m + 3)$. So $\frac{2m^2 - m - 6}{m - 2} = 2m + 3$.

So our problem reduces to solving $2m + 3 = 9$. That is, $2m = 6$. So $m = 3$.

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

There is a soup s such that s is tasty and s does not contain meat.

Solution: For every soup s , s is not tasty or s contains meat.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For any bear b , if b is blue and b talks, then b is fuzzy.

Solution: For any bear b , if b is not fuzzy, then b is not blue or b doesn’t talk.

3. (5 points) Suppose that G and H are functions whose inputs and outputs are real numbers, defined by $G(x) = x - 2$ and $H(x) = \sqrt{2x + 1}$, where the square root function returns only the positive root. Compute the value of $H(G(G(8)))$, showing your work.

Solution: $G(8) = 6$

So $G(G(8)) = 4$

So $H(G(G(8))) = \sqrt{2 \cdot 4 + 1} = \sqrt{9} = 3$

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Show that the following two expressions are not logically equivalent, by giving specific values of p , q , and r for which they produce different values.

$$(p \rightarrow q) \wedge r$$

$$p \rightarrow (q \wedge r)$$

Solution: Set p and r to be false and q to be true. Then $(p \rightarrow q)$ is true (because its hypothesis is false) and $(p \rightarrow q) \wedge r$. But $p \rightarrow (q \wedge r)$ is true because its hypothesis is false.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every jedi j , if j has a light saber and j is not sick, then j can defeat the Dark Side.

Solution: For every jedi j , if j cannot defeat the Dark Side. then j does not have a light saber or j is sick.

3. (5 points) Suppose that x is an integer and $x^2 + 3x - 18 < 0$. What are the possible values of x ? Show your work.

Solution: $x^2 + 3x - 18 = (x + 6)(x - 3)$. So we have $(x + 6)(x - 3) < 0$. So one of $(x + 6)$ and $(x - 3)$ is negative and the other positive. Because $(x + 6)$ is larger, $(x + 6)$ must be the positive one.

So we have $x + 6 > 0$ and $x - 3 < 0$. So $x > -6$ and $x < 3$. Since x is an integer, it must be one of the following values:

$$-5, -4, -3, -2, -1, 0, 1, 2$$

Name: _____

NetID: _____ Lecture: B

Discussion: Friday 11 12 1 2 3 4

1. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every tree t , if t grows in Canada, then t is not tall or t is a conifer.

Solution: For every tree t , if g is tall and t is not a conifer, then t doesn’t grow in Canada.

2. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every computer game g , if g has trendy music or g has an interesting plotline, then g is not cheap.

Solution: There is a computer game g such that g has trendy music or an interesting plotline but g is cheap.

3. (5 points) Suppose that G and H are functions whose inputs and outputs are real numbers, defined by $G(x) = x - 5$ and $H(x) = \sqrt{x + 1}$. Compute the value of $H(H(G(13)))$, showing your work.

Solution: $G(13) = 8$. So $H(G(13)) = \sqrt{9} = 3$. So $H(H(G(13))) = \sqrt{4} = 2$.

Name: _____

NetID: _____ Lecture: B

Discussion: Friday 11 12 1 2 3 4

1. (5 points) Show that the following two expressions are not logically equivalent, by giving specific values of p , q , and r for which they produce different values.

$$(p \rightarrow q) \wedge r$$

$$p \rightarrow (q \wedge r)$$

Solution: Set p and r to be false and q to be true. Then $(p \rightarrow q)$ is true (because its hypothesis is false) and $(p \rightarrow q) \wedge r$. But $p \rightarrow (q \wedge r)$ is true because its hypothesis is false.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every dinosaur d , if d is small and d is not a juvenile, then d is not a sauropod.

Solution: For every dinosaur d , if d is a sauropod, then d is not small or d is a juvenile.

3. (5 points) Suppose that F and G are functions whose inputs and outputs are positive real numbers, defined by $F(x) = x^2 + 14x$ and $G(x) = \sqrt{x + 49}$. Compute the value of $G(F(p))$. Simplify your answer and show your work.

Solution: Notice that p is given to be positive, so $p + 7$ is also positive.

$$G(F(p)) = G(p^2 + 14p) = \sqrt{(p^2 + 14p) + 49} = \sqrt{(p + 7)^2} = p + 7$$

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every dinosaur k , if k is blue, then k is not vegetarian or k is friendly.

Solution: There is a dinosaur k such that k is blue but k is vegetarian and k is not friendly.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every book b , if b is blue or b is not heavy, then b is not a math book.

Solution: For every book b , if b is a math book, then b is not blue and b is heavy.

3. (5 points) List all solutions to the equation $abc = 2$, where a , b , and c are integers. Notice that a solution where $a = 8$ and $b = 3$ would be different from a solution with $a = 3$ and $b = 8$.

Solution: Solution:

Writing the values for a , b , and c , in order, the possibilities are:

2, 1, 1 -2, -1, 1 -2, 1, -1 2, -1, -1

1, 2, 1 -1, -2, 1 -1, 2, -1 1, -2, -1

1, 1, 2 -1, -1, 2 -1, 1, -2 1, -1, -2

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$$(p \rightarrow q) \wedge (p \rightarrow \neg q) \equiv \neg p$$

p	q	$p \rightarrow q$	$p \rightarrow \neg q$	$(p \rightarrow q) \wedge (p \rightarrow \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	T	T	T
F	F	T	T	T

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every tree t , if t grows in Canada, then t is not tall or t is a conifer.

Solution: For every tree t , if t is tall and t is not a conifer, then t doesn’t grow in Canada.

3. (5 points) Suppose that m and p are positive integers such that $2p^2 + mp < 6$. What are the possible values for m ? Briefly explain or show work.

Solution: Since $2p^2 + mp < 6$, $mp < 6 - 2p^2$. Since p is a positive integer $2p^2 \geq 2$. So $6 - 2p^2 \leq 4$. So $mp < 4$. Since m is a positive integer, this implies that m is 1, 2, or 3.

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Show that the following two expressions are not logically equivalent, by giving specific values of p , q , r for which they produce different values.

$$p \rightarrow (q \rightarrow r)$$

$$p \wedge (q \wedge r)$$

Solution: Set p true, and set q and r false.

Then $(q \rightarrow r)$ is true because its hypothesis is false. Therefore $p \rightarrow (q \rightarrow r)$ is true.

However, $(q \wedge r)$ is false because both inputs are false. So $p \wedge (q \wedge r)$ is also false.

(It's sufficient to give one set of values that work. The general pattern is that p and/or q must be false.)

2. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

There is a relish r such that r is orange but r is not spicy.

Solution: For every relish r , r is not orange or r is spicy.

3. (5 points) Suppose that G and H are functions whose inputs and outputs are real numbers, defined by $G(x) = x + 7$ and $H(x) = \sqrt{x - 1}$. Compute the value of $G(H(H(2)))$, showing your work.

Solution: $H(2) = \sqrt{1} = 1$

So $H(H(2)) = \sqrt{0} = 0$.

So $G(H(H(8))) = 0 + 7 = 7$

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Are the following two expressions logically equivalent? Briefly justify your answer.

$$(p \wedge q) \rightarrow r \qquad \qquad (p \wedge \neg r) \rightarrow \neg q$$

Solution: These two expressions are equivalent. The first is false exactly when $(p \wedge q)$ is true and r is false. That is, it's false exactly when p and q are true and r is false.

The second is false exactly when $(p \wedge \neg r)$ is true and $\neg q$ is false. That is, it's false exactly when p is true, r is false, and q is true.

So the two expressions are false under exactly the same conditions and therefore they must be logically equivalent.

2. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every jedi j , if j has a light saber and j is not sick, then j can defeat the Dark Side.

Solution: There is a jedi j , such that j has a light saber and j is not sick, but j cannot defeat the Dark Side.

3. (5 points) Suppose that G and H are functions whose inputs and outputs are real numbers, defined by $G(x) = x^2$ and $H(x) = 2x - 10$. Compute the value of $G(H(G(3)))$, showing your work.

Solution: $G(3) = 9$

So $H(G(3)) = 2 \cdot 9 - 10 = 8$

So $G(H(G(3))) = 8^2 = 64$.

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

There is a mushroom f such that f is not poisonous or f is blue.

Solution: For every mushroom f , f is poisonous and f is not blue.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every alien A , if A has three fingers or A is not tall, then A is friendly.

Solution: For every alien A , if A is not friendly, then A does not have three fingers and A is tall.

3. (5 points) Find all integer solutions to $x^2 - 2x - 3 < 0$. Show your work.

Solution: Factoring the lefthand side, we get $(x + 1)(x - 3) < 0$. Since $x + 1$ is larger than $x - 3$, this means that $x + 1 > 0$ and $x - 3 < 0$. So $x > -1$ and $x < 3$. The only integers in this range are 0, 1, and 2.

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every dog d , if d is a terrier, then d is not large and d is noisy.

Solution: There is a dog d , such that d is a terrier, but d is large or d is not noisy.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every dragon d , if d is green, then d is not large or d is fat.

Solution: For all dragons d , if d is large and d is not fat, then d is not green.

3. (5 points) Solve $5x + m = \frac{n}{5}$ for x , expressing your answer as a single fraction. Simplify your answer and show your work.

Solution:

$$\begin{aligned} 5x + m &= \frac{n}{5} \\ 5x &= \frac{n}{5} - m \\ x &= \frac{n}{25} - \frac{m}{5} = \frac{n - 5m}{25} \end{aligned}$$

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

There is a violin v , such that v is not old but the maker of v is not known;

Solution: For every violin v , v is old or the maker of v is known;

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every tiger k , if k is orange, then k is large and k is not friendly.

Solution: For every tiger k , if k is not large or k is friendly, then k is not orange.

3. (5 points) Suppose that F and G are functions whose inputs and outputs are positive real numbers, defined by $F(x) = x$ and $G(x) = x^2$. Compute the value of $G(F(G(x)))$. Simplify your answer and show your work.

Solution: Solution:

$$G(x) = x^2$$

$$\text{So } F(G(X)) = F(x^2) = x^2.$$

$$\text{So } G(F(G(X))) = G(x^2) = x^4.$$

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every computer game g , if g has trendy music or g has an interesting plotline, then g is not cheap.

Solution: There is a computer game g such that g has trendy music or an interesting plotline but g is cheap.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every tree t , if t is in Illinois and t is not hardy, then t is indoors.

Solution: For every tree t , if t is not indoors, then t is not in Illinois or t is hardy.

3. (5 points) List all solutions to the equation $abc = 6$, where a , b , and c are natural numbers.

Solution: **Solution:**

Writing the values for a , b , and c , in order, the possibilities are:

6, 1, 1 1, 6, 1 1, 1, 6

1, 3, 2 3, 2, 1 2, 1, 3

1, 2, 3 2, 3, 1 3, 1, 2

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$$(p \rightarrow p) \rightarrow p \equiv p$$

p	$p \rightarrow p$	$(p \rightarrow p) \rightarrow p$
T	T	T
F	T	F

2. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every dragon d , if d is green, then d is not large or d is fat.

Solution: There is a dragon d such that d is green but/and d is large and d is not fat.

3. (5 points) Suppose that G and H are functions whose inputs and outputs are real numbers, defined by $G(x) = x - 5$ and $H(x) = \sqrt{x + 1}$. Compute the value of $H(H(G(13)))$, showing your work.

Solution: $G(13) = 8$. So $H(G(13)) = \sqrt{9} = 3$. So $H(H(G(13))) = \sqrt{4} = 2$.

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every mountain m , if m is tall or m is not in the north, then m has a snow cap.

Solution: There is a mountain m , such that m is tall or m is not in the north, but m does not have a snow cap.

2. (5 points) Solve $3x + 2m = \frac{w}{y}$ for x , expressing your answer as a single fraction. Simplify your answer and show your work.

Solution:

$$\begin{aligned} 3x + 2m &= \frac{w}{y} \\ 3x &= \frac{w}{y} - 2m \\ 3x &= \frac{w - 2ym}{y} \\ x &= \frac{w - 2ym}{3y} \end{aligned}$$

3. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$$(p \rightarrow q) \wedge (p \rightarrow \neg q) \equiv \neg p$$

p	q	$p \rightarrow q$	$p \rightarrow \neg q$	$(p \rightarrow q) \wedge (p \rightarrow \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	T	T	T
F	F	T	T	T

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

If the date is before 1800, then every monster m is either smelly or large.

Solution: The date is before 1800, but there is a monster that is not smelly and not large.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every egg E , if E floats, then E is not good or the water has been salted.

Solution: For every egg E , if E is good and the water has not been salted, then E will not float.

3. (5 points) Suppose that F and G are functions whose inputs and outputs are real numbers, defined by $F(x) = x^2 - 4x$ and $G(x) = x + 4$. Compute the value of $F(G(z))$, showing your work.

Solution: $G(z) = z + 4$.

$$\text{So } F(G(z)) = F(z + 4) = (z + 4)^2 - 4(z + 4) = z^2 + 8z + 16 - 4z - 16 = z^2 + 4z$$

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every dragon d , if d is not large, then d is green or d is not hungry.

Solution: There is a dragon d such that d is not large but d is green and d is hungry.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every car c , if c is a Tesla, then c is new or c is not fast.

Solution: For every car c , if c is not new and c is fast, then c is not a Tesla.

3. (5 points) Solve $\frac{3}{7x} + a = \frac{b}{7}$ for x , expressing your answer as a single fraction. Simplify your answer and show your work.

Solution: Multiplying everything by $7x$, we get $3 + 7ax = bx$.

So then $bx - 7ax = 3$. So $x(b - 7a) = 3$. So $x = \frac{3}{b-7a}$.

Notice that there is no solution if $b = 7a$.

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

John has a camera and there is a Meerkat m , such that m lives in New York and John has not photographed m

Solution: John does not have a camera or for every Meerkat m , m does not live in New York or John has photographed m

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every violin v , if v is old or the maker of v is not known, then v is not valuable.

Solution: For every violin v , if v is valuable, then v is not old and the maker of v is known.

3. (5 points) Suppose that x is an integer and $x^2 + 3x - 18 < 0$. What are the possible values of x ? Show your work.

Solution: $x^2 + 3x - 18 = (x + 6)(x - 3)$. So we have $(x + 6)(x - 3) < 0$. So one of $(x + 6)$ and $(x - 3)$ is negative and the other positive. Because $(x + 6)$ is larger, $(x + 6)$ must be the positive one.

So we have $x + 6 > 0$ and $x - 3 < 0$. So $x > -6$ and $x < 3$. Since x is an integer, it must be one of the following values:

$$-5, -4, -3, -2, -1, 0, 1, 2$$

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every Meerkat m , if m is in New York, then m is not in the wild or m is lost.

Solution: There is a Meerkat m , such that m is in New York, but m is in the wild and m is not lost.

2. (5 points) Solve $\frac{3}{x} + m = \frac{3}{p}$ for x , expressing your answer as a single fraction. Simplify your answer and show your work.

Solution: Multiplying by xp gives you $3p + mp = 3x$.

So $3x - mp = 3p$.

So $x(3 - mp) = 3p$.

So $x = \frac{3p}{3 - mp}$.

3. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every dinosaur d , if d is small and d is not a juvenile, then d is not a sauropod.

Solution: For every dinosaur d , if d is a sauropod, then d is not small or d is a juvenile.

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Show that the following two expressions are not logically equivalent, by giving specific values of p , q for which they produce different values.

$$p \rightarrow (q \rightarrow p)$$

$$(p \rightarrow q) \rightarrow p$$

Solution: Set p and q to be false.

Then $p \rightarrow (q \rightarrow p)$ is true because its hypothesis is false.

$p \rightarrow q$ is also true. So $(p \rightarrow q) \rightarrow p$ is false because its hypothesis is true but its conclusion (p) is false.

A similar argument works if you set p to be false and q to be true.

2. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every cat c , if c is not fierce or c wears a collar, then c is a pet.

Solution: There exists a cat c that is either not fierce or wears a collar and is not a pet.

3. (5 points) Suppose that k is a positive integer, x is a positive real number, and $\frac{1}{k} = x + \frac{1}{6}$. What are the possible values for k ? (Hint: k is an INTEGER.) Briefly explain or show work.

Solution: Observe that we can rearrange the equation as follows:

Since x is positive, $\frac{1}{k} = x + \frac{1}{6}$ implies that $\frac{1}{k} > \frac{1}{6}$. So k must be smaller than 6. But we were told that k was a positive integer. The only positive integers smaller than 6 are 1, 2, 3, 4, and 5.

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

If it is raining, then there is a cyclist c such that c is getting wet.

Solution: It is raining and for every cyclist c , c is not getting wet.

2. (5 points) Describe all (real) solutions to the equation $2p^2 + p - 6 < 0$. Show your work.

Solution: $2p^2 + p - 6 = (2p - 3)(p + 2)$

So we have $(2p - 3)(p + 2) < 0$. Dividing by 2 gives us $(p - 1.5)(p + 2) < 0$.

$(p - 1.5)(p + 2)$ is negative when exactly one of the factors is positive. The positive factor must be $p + 2$ because it's larger. So we have $p + 2 > 0$, i.e. $p > -2$. And then also $p - 1.5 < 0$, i.e. $p < 1.5$.

So p is in the interval $(-2, 1.5)$.

3. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$$(r \rightarrow q) \rightarrow r = r$$

q	r	$r \rightarrow q$	$(r \rightarrow q) \rightarrow r$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	T	F

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$$(p \wedge q) \vee q = q$$

p	q	$p \wedge q$	$(p \wedge q) \vee q$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	F

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every elephant e , if e likes to dance and e has good taste, then e likes Juluka.

Solution: For every elephant e , if e does not like Juluka, then e doesn’t like to dance or e has bad taste.

3. (5 points) Solve $\frac{x}{2} - 1 < 3x + 9$ for x . (Assume x is real.) Show your work.

Solution: Multiplying both sides by 2 gives us $x - 2 < 6x + 18$. So $-20 < 5x$, and thus $x > -4$.

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) State the negation of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every violin v , if v is old or the maker of v is not known, then v is not valuable.

Solution: There is a violin v , such that v is old or the maker of v is not known, but v is valuable.

2. (5 points) Suppose that f and g are functions whose inputs and outputs are real numbers, defined by $f(x) = x^2 - 1$ and $g(x) = \frac{x}{2}$. Compute the value of $g(f(y + 1))$, showing your work.

Solution: $f(y + 1) = (y + 1)^2 - 1 = y^2 + 2y$

So $g(f(y + 1)) = \frac{y^2 + 2y}{2}$

3. (5 points) State the contrapositive of the following claim, moving all negations (e.g. “not”) so that they are on individual predicates.

For every garbage can c , if c was supplied by the city, then c is small or c has wheels.

Solution: For every garbage can c , if c is large and c does not have wheels, then c was not supplied by the city.