CS 173, Spring 19 Examlet 9, white

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Consider the following grammar G, with start symbol S and terminals a and b.

$$S \rightarrow a\ S\ a \mid b\ S\ b \mid a\ S\ b \mid b\ S\ a \mid a \mid b$$

Amy claims that this generates all non-empty strings containing a's and/or b's. Is this correct? Justify your answer.

2. (4 points) Check the (single) box that best characterizes each item.

Total number of leaves in a full and complete 5-ary tree of height h	5^h ☐	$\leq 5^h$ ☐	$\geq 5^h$ ☐	$5^{h+1}-1$ ☐
The level of a leaf node in a full and complete binary tree of height h.	0 ☐	1 ☐	$h-1$ ☐	$\leq h$ ☐	h ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string `abba` that match this grammar.

$$S \rightarrow \quad S\ S \mid a\ S \mid S\ a \mid b$$

2. (4 points) Check the (single) box that best characterizes each item.

A full m-ary tree with i internal nodes has $mi + 1$ nodes total. always ☐ sometimes ☐ never ☐

A binary tree of height h has at least $2^{h+1} - 1$ nodes. true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Consider the following grammar G

   $S \to S\ b\ S \mid a \mid c\ d$

   S is the only start symbol. The terminal symbols are a, b, c, and d.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   `aaacd` `bbbbb`

2. (4 points) Check the (single) box that best characterizes each item.

   The mathematical symbol for an empty (zero-length) string ☐ $\emptyset$ ☐ e ☐ ϵ ☐ NULL

   Number of bit strings of length $\leq k$. ☐ 2^k ☐ $2^k - 1$ ☐ 2^{k-1} ☐ $2^{k+1} - 1$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Min's virus detection code needs to generate all strings of the form $a^n b^n$. That is, all strings that consist of a sequence of one or more a's followed by the same number of b's. Write a context-free grammar G that will do this.

2. (4 points) Check the (single) box that best characterizes each item.

The number of nodes in a binary tree of height h

$\geq 2^h$	☐	$2^{h+1} - 1$	☐
$\leq 2^{h+1} - 1$	☐	$\geq 2^{h+1} - 1$	☐

A tree node is a descendent of itself.

always ☐ sometimes ☐ never ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Consider the following grammar G

   $S \to a\ S\ b \mid b\ S\ b \mid c$

   S is the only start symbol. The terminal symbols are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   `ababb` `babcbbb`

2. (4 points) Check the (single) box that best characterizes each item.

   The level of the root node in a tree of height h. 0 ☐ 1 ☐ $h-1$ ☐ h ☐ $h+1$ ☐

   A tree node is a proper ancestor of itself. always ☐ sometimes ☐ never ☐

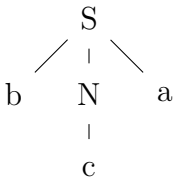
Name:_____

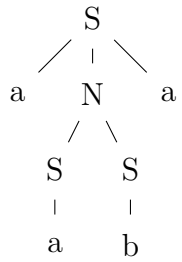
NetID:_____ **Lecture: A B**

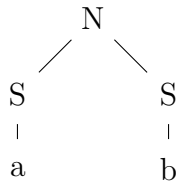
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, and c. Circle the trees that match the grammar.

$$\begin{aligned} S &\rightarrow a\ N\ a \mid b\ N\ b \mid a \mid b \\ N &\rightarrow S\ S \mid c \end{aligned}$$







2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at least $2^h - 1$ nodes.

true ☐ false ☐

A full m-ary tree with i internal nodes has ____ nodes total.

$mi - 1$ ☐ mi ☐

$mi + 1$ ☐ $\leq mi + 1$ ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Consider the following grammar G

   $S \to a\ S\ b \mid b\ S\ b \mid a \mid b$

   S is the only start symbol. The terminal symbols are a and b.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   `bababbb` `aaaab`

2. (4 points) Check the (single) box that best characterizes each item.

   The number of leaves in a binary tree of height h 2^h ☐ $2^{h+1}-1$ ☐ $\geq 2^h$ ☐ $\leq 2^h$ ☐

   The number of paths between two distinct nodes in an n-node tree. Paths in opposite directions count as the same. n ☐ $2n$ ☐ $\frac{n(n-1)}{2}$ ☐ $n(n-1)$ ☐ n^2 ☐ $\frac{n(n+1)}{2}$ ☐

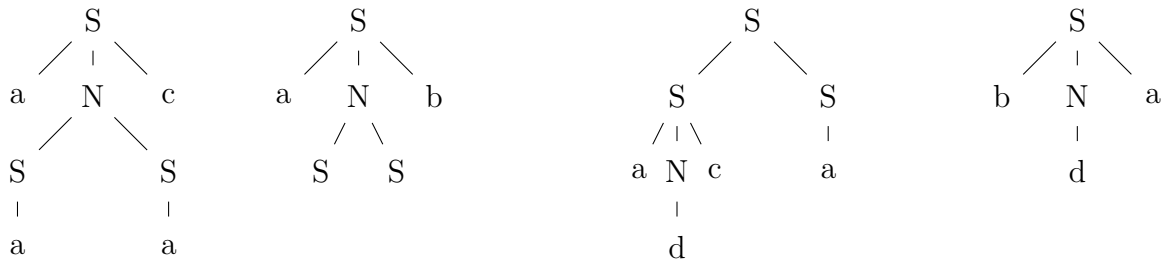
Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$\begin{aligned} S &\rightarrow a\ N\ b \mid a\ N\ c \mid a \\ N &\rightarrow S\ S \mid d \end{aligned}$$



2. (4 points) Check the (single) box that best characterizes each item.

The diameter of a tree of height h.

$\leq h$ ☐ h ☐ $h+1$ ☐

$2h$ ☐ $\leq 2h$ ☐

The number of nodes in a full complete binary tree of height h

$\geq 2^h$ ☐ $2^{h+1}-1$ ☐

$\leq 2^{h+1}-1$ ☐ $\geq 2^{h+1}-1$ ☐

CS 173, Fall 18 Examlet 9, Part B

Name:_____

NetID:_____ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

1. (8 points) Here is a grammar with start symbol S and terminal symbol b. Draw three parse trees for the string `bbb` that match this grammar.

$$S \rightarrow S\ S \mid S \mid b$$

2. (4 points) Check the (single) box that best characterizes each item.

Number of bit strings of length k.

2^k ☐ $2^k - 1$ ☐ 2^{k-1} ☐ k ☐

The chromatic number of a full 3-ary tree

1 ☐ 2 ☐ ≤ 2 ☐

3 ☐ ≤ 3 ☐ can't tell ☐

Name:_____

NetID:_____ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

1. (8 points) Here is a grammar with start symbol S and terminal symbol a. Draw three parse trees for the string `aaaaaa` that match this grammar.

$$S \rightarrow S\ S \mid a\ S\ a \mid a\ a$$

2. (4 points) Check the (single) box that best characterizes each item.

The chromatic number of a full 3-ary tree

1	☐	2	☐	≤ 2	☐		
3	☐	≤ 3	☐	can't tell	☐		

Number of bit strings of length k.

2^k	☐	$2^k - 1$	☐	2^{k-1}	☐	k	☐

CS 173, Spring 18 Examlet 9, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string `ababab` that match this grammar.

$$\begin{aligned} S &\rightarrow a\ N \mid a\ N\ S \\ N &\rightarrow b\ a\ N \mid b \end{aligned}$$

2. (4 points) Check the (single) box that best characterizes each item.

A tree node is a descendent of itself.

always ☐ sometimes ☐ never ☐

The number of nodes in a full complete binary tree of height h

$\geq 2^h$ ☐ $2^{h+1} - 1$ ☐

$\leq 2^{h+1} - 1$ ☐ $\geq 2^{h+1} - 1$ ☐

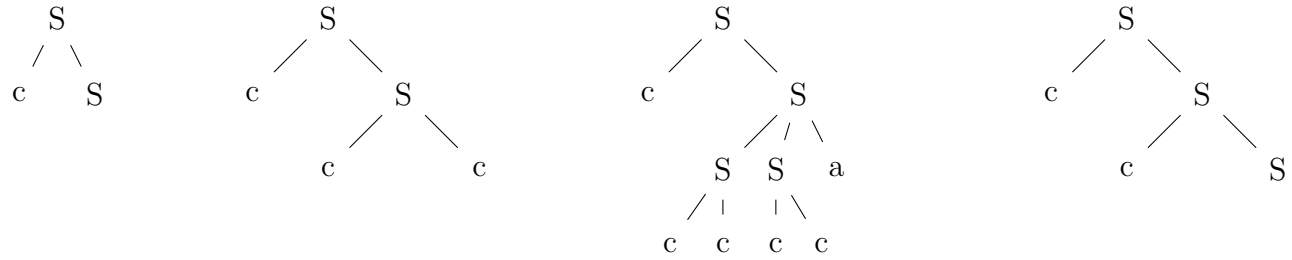
Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar, with start variable S and terminals a and c. Circle the trees that match the grammar.

$$S \rightarrow S\,S\,a \mid c\,S \mid c\,c$$



2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at most $2^{h+1} - 1$ nodes. true ☐ false ☐

A tree with n edges has ____ nodes. $n-1$ ☐ n ☐ $n+1$ ☐ $n/2$ ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Consider the following grammar G

   $S \to a\ S\ b \mid b\ S\ b \mid a \mid b$

   S is the only start symbol. The terminal symbols are a and b.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   `aabaaba` `aababaa`

2. (4 points) Check the (single) box that best characterizes each item.

   The root node of a tree is an internal node — always ☐ sometimes ☐ never ☐

   The level of the root node in a tree of height h. — -1 ☐ 0 ☐ 1 ☐ $h-1$ ☐ h ☐

CS 173, Spring 18 Examlet 9, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Give a context-free grammar that generates all strings of the form a^+b^+. That is, all strings that consist of a sequence of one or more a's followed by a sequence of one or more b's.

2. (4 points) Check the (single) box that best characterizes each item.

Number of non-empty bit strings of length k.

2^k ☐ $2^k - 1$ ☐ 2^{k-1} ☐ k ☐

The number of paths between two distinct nodes in an n-node tree. Paths in opposite directions count as the same.

n ☐ $2n$ ☐ $\frac{n(n-1)}{2}$ ☐

$n(n-1)$ ☐ n^2 ☐ $\frac{n(n+1)}{2}$ ☐

CS 173, Spring 18 Examlet 9, Part B

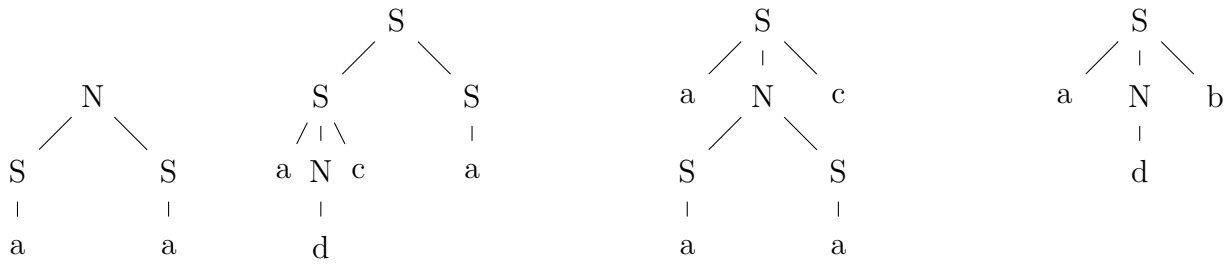
Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$\begin{aligned} S &\rightarrow a\ N\ b \mid a\ N\ c \mid a \\ N &\rightarrow S\ S \mid d \end{aligned}$$



2. (4 points) Check the (single) box that best characterizes each item.

A full m-ary tree with i internal nodes has _____ nodes total.

- $mi - 1$ ☐
- mi ☐
- $mi + 1$ ☐
- $\leq mi + 1$ ☐

Height of a binary tree with 2^n nodes.

- $\leq n - 1$ ☐
- $\leq n$ ☐
- $\leq 2^n$ ☐
- $\leq 2^n - 1$ ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbol a. Draw three parse trees for the string `aa` that match this grammar.

$$\begin{aligned} S &\rightarrow S\ S \mid N \mid a \\ N &\rightarrow a \end{aligned}$$

2. (4 points) Check the (single) box that best characterizes each item.

A tree node is a proper ancestor of itself.	always ☐	sometimes ☐	never ☐
Removing an edge from a tree (with at least one edge) produces two trees.	always ☐	sometimes ☐	never ☐

CS 173, Fall 17 Examlet 9, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string `abba` that match this grammar.

$$S \rightarrow S\ S \mid a\ S \mid S\ a \mid b$$

2. (4 points) Check the (single) box that best characterizes each item.

The mathematical symbol for an empty (zero-length) string

$\emptyset$ ☐ e ☐ ϵ ☐ NULL ☐

Number of nodes at level k in a full complete binary tree.

2^k ☐ $2^k - 1$ ☐ $2^{k+1} - 1$ ☐ 2^{k-1} ☐

CS 173, Fall 17 Examlet 9, Part B

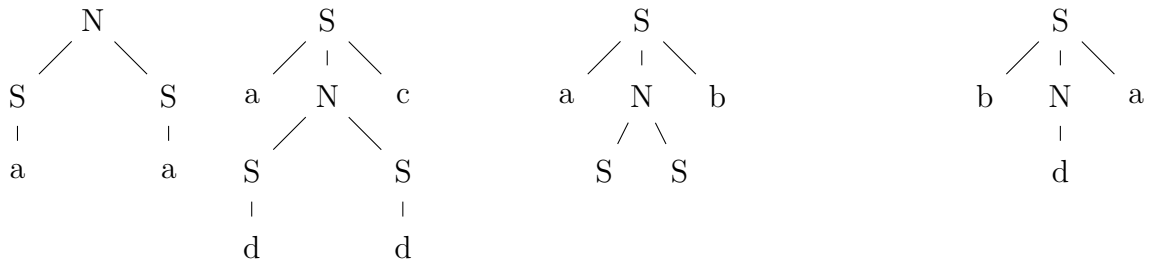
Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$\begin{aligned} S &\rightarrow b\ N\ a \mid a\ N\ c \mid a \\ N &\rightarrow S\ S \mid d \end{aligned}$$



2. (4 points) Check the (single) box that best characterizes each item.

Number of bit strings of length k. 2^k ☐ $2^k - 1$ ☐ 2^{k-1} ☐ k ☐

A full m-ary tree with i internal nodes has $mi + 1$ nodes total. always ☐ sometimes ☐ never ☐

CS 173, Fall 17 Examlet 9, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Consider the following grammar G

$$\begin{aligned} S &\rightarrow S\ S \mid a\ N \mid a\ a \\ N &\rightarrow S\ a \mid a\ b \end{aligned}$$

S is the only start symbol. The terminal symbols are a and b

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

`aabaa` `ab`

2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at least $2^{h+1} - 1$ nodes. true ☐ false ☐

The level of the root node in a tree of height h. 0 ☐ 1 ☐ $h-1$ ☐ h ☐ $h+1$ ☐

CS 173, Fall 17 Examlet 9, Part B

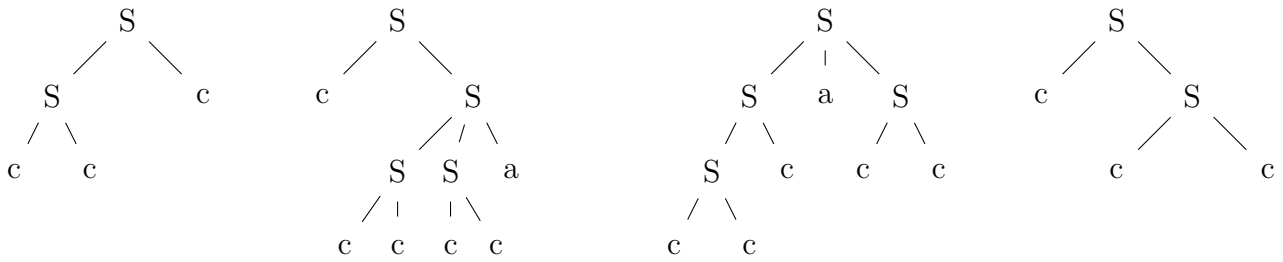
Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar, with start variable S and terminals a and c. Circle the trees that match the grammar.

$$S \rightarrow S\ a\ S \mid S\ c \mid c\ c$$



2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at least $2^h - 1$ nodes. true ☐ false ☐

Number of bit strings of length $\leq k$. 2^k ☐ $2^k - 1$ ☐ 2^{k-1} ☐ $2^{k+1} - 1$ ☐

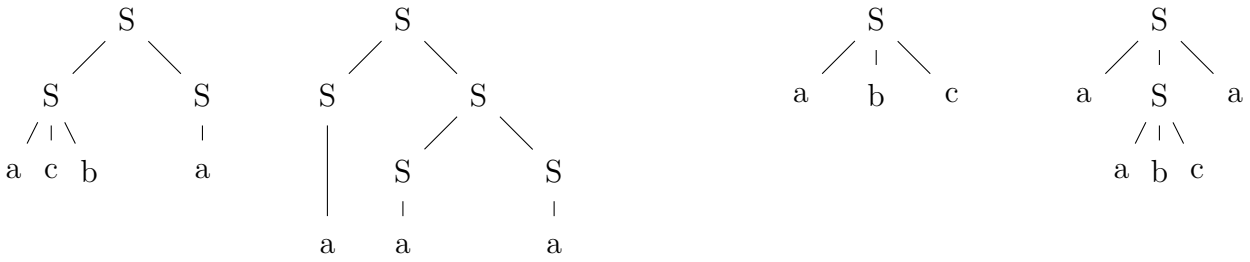
Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminals symbols a,b, and c. Circle the trees that match the grammar.

$$S \rightarrow S\ S \mid a\ b\ c \mid a$$



2. (4 points) Check the (single) box that best characterizes each item.

The number of nodes in a binary tree of height h

- $\geq 2^h$ ☐
- $2^{h+1} - 1$ ☐
- $\leq 2^{h+1} - 1$ ☐
- $\geq 2^{h+1} - 1$ ☐

The diameter of a tree of height h.

- $\leq h$ ☐
- h ☐
- $h + 1$ ☐
- $2h$ ☐
- $\leq 2h$ ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Consider the following grammar G

   $S \to b\ a\ S \mid S\ S \mid c \mid c\ a$

   S is the only start symbol. The terminal symbol are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   `baba` `bacca`

2. (4 points) Check the (single) box that best characterizes each item.

   The number of leaves in a binary tree of height h

   2^h ☐ $2^{h+1} - 1$ ☐ $\geq 2^h$ ☐ $\leq 2^h$ ☐

   The diameter of a full, complete 7-ary tree of height h.

   $\leq h$ ☐ h ☐ $h+1$ ☐

   $2h$ ☐ $7h$ ☐ $7h+1$ ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbol a. Draw three parse trees for the string $a\ a\ a\ a\ a\ a$ that match this grammar.

$$S \ \rightarrow \ \ S\ S \ \mid \ a\ S\ a \ \mid \ a\ a$$

2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at most $2^{h+1} - 1$ nodes. true ☐ false ☐

The root node of a tree is a leaf. always ☐ sometimes ☐ never ☐

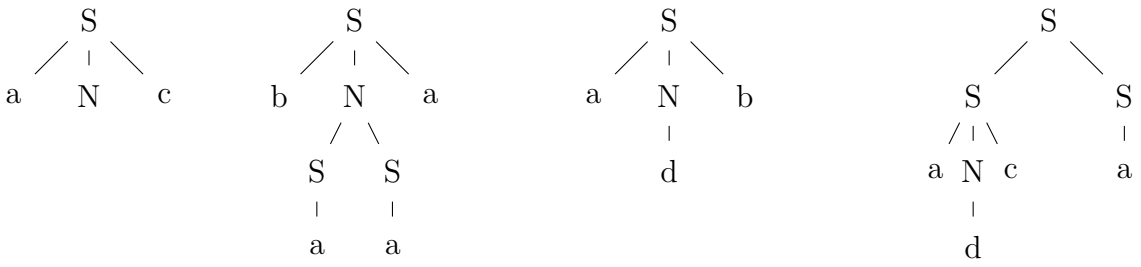
Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$\begin{aligned} S &\rightarrow b\ N\ a \mid a\ N\ c \mid a \\ N &\rightarrow S\ S \mid d \end{aligned}$$



2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n} 2^k$ $2^{n+1}-1$ ☐ $2^{n+1}-2$ ☐ $2^{n+1}-3$ ☐ 2^n-1 ☐

2^h is ____ the number of leaves in a binary tree of height h.

an upper bound on ☐ exactly ☐

a lower bound on ☐ not a bound on ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string $a\ b\ a\ b\ a\ b\ a$ that match this grammar.

$$S \ \rightarrow \ S\ b\ S \ \mid \ a$$

2. (4 points) Check the (single) box that best characterizes each item.

An m-ary tree with i internal nodes has $mi + 1$ nodes total. always ☐ sometimes ☐ never ☐

Total number of leaves in a 3-ary tree of height h 3^h ☐ $\leq 3^h$ ☐ $\frac{1}{2}(3^{h+1} - 1)$ ☐ $3^{h+1} - 1$ ☐

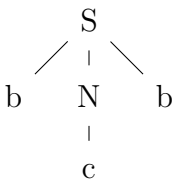
Name:_____

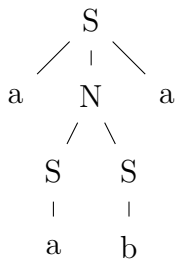
NetID:_____ **Lecture: A B**

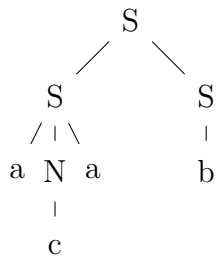
Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

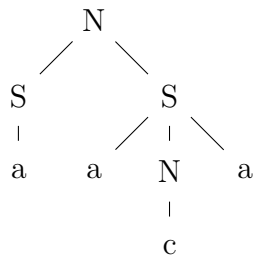
1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, and c. Circle the trees that match the grammar.

$$\begin{aligned} S &\rightarrow a\ N\ a \mid b\ N\ b \mid a \mid b \\ N &\rightarrow S\ S \mid c \end{aligned}$$









2. (4 points) Check the (single) box that best characterizes each item.

The number of paths between two distinct nodes in an n-node tree. Paths in opposite directions count as the same.

n	☐	$2n$	☐	$\frac{n(n-1)}{2}$	☐
$n(n-1)$	☐	n^2	☐	$\frac{n(n+1)}{2}$	☐

$\sum_{k=0}^{n} 2^k$ $2^n - 2$ ☐ $2^n - 1$ ☐ $2^{n-1} - 1$ ☐ $2^{n+1} - 1$ ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (8 points) Consider the following grammar G

   $S \to S\ b\ S \mid a \mid c\ d$

   S is the only start symbol. The terminal symbols are a, b, c, and d.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   $a\ b\ a\ c\ a$ $b\ b\ b\ b\ b$

2. (4 points) Check the (single) box that best characterizes each item.

   The level of a leaf node in a full and complete binary tree of height h. 0 ☐ 1 ☐ $h-1$ ☐ $\leq h$ ☐ h ☐

   Height of a binary tree with 2^n nodes. $\leq n-1$ ☐ $\leq n$ ☐ $\leq 2^n$ ☐ $\leq 2^n - 1$ ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string $a\ a\ b$ that match this grammar.

$$\begin{aligned} S &\rightarrow a\ N \mid N \\ N &\rightarrow a\ S \mid S \mid b \end{aligned}$$

2. (4 points) Check the (single) box that best characterizes each item.

The number of paths between two nodes in an n-node tree. Paths in opposite directions count as different.

n ☐ $2n$ ☐ $\frac{n(n-1)}{2}$ ☐

$n(n-1)$ ☐ n^2 ☐ $\frac{n(n+1)}{2}$ ☐

A tree node is an ancestor of itself.

always ☐ sometimes ☐ never ☐