CS 173, Spring 19 Examlet 12, white

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $f : \mathbb{Z}_{12} \to \mathbb{P}(\mathbb{Z}_{12})$ be defined by $f(x) = \{y \in \mathbb{Z}_{12} \mid y^2 = x\}$. Let $S = \{f(x) \mid x \in \mathbb{Z}_{12}\}$.

(3 points) $S =$

Solution: $\{\{2,4,8,10\},\{0,6\},\{1,5,7,11\},\{3,9\},\emptyset\}$

(3 points) Is S a partition of $\mathbb{Z}_{12}$? Check the partition properties that are satisfied.

No Empty set $\square$ No Partial Overlap $\boxed{\surd}$ Covers base set $\boxed{\surd}$

(7 points) Suppose that $A_1, A_2, \ldots, A_n$ are non-empty subsets of A, and let $P = \{A_1, A_2, \ldots, A_n\}$. Also suppose that $A_1 \cap A_2 \cap \ldots \cap A_n = \emptyset$ and $A_1 \cup A_2 \cup \ldots \cup A_n = A$. Is P a partition of A? Explain why or why not.

Solution: P is not necessarily a partition of A. The issue is that $A_1 \cap A_2 \cap \ldots \cap A_n = \emptyset$ can be true even when some pairs of (distinct) subsets overlap. For example, $A_1 = \{1,2\}$, $A_2 = \{2,3\}$, and $A_3 = \{3,4\}$. Then $A_1 \cap A_2 \cap A_3 = \emptyset$ but A_1 and A_2 intersect.

(2 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is	an integer $\square$	a set of integers $\boxed{\surd}$	undefined $\square$
	a power set $\square$	one or more integers $\square$	

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(7 points) Suppose that A is a set and P is a collection of subsets of A. Using precise language and/or notation, state the conditions P must satisfy to be a partition of A.

Solution: P cannot contain the empty set. Every element of A must belong to exactly one element of P.

The second condition is frequently split into two sepaate conditions. That is, every element of A must belong some to element of P, and two distinct elements of P cannot overlap.

(2 points) $\{\{p, q\} \ : \ p \in \mathbb{Z}^+, q \in \mathbb{Z}^+, \text{ and } pq = 6\} \ =$

Solution: $\{\{1, 6\}, \{2, 3\}\}$

(6 points) Check the (single) box that best characterizes each item.

$\{\{a, b\}, c\} = \{a, b, c\}$ true $\square$ false $\boxed{\surd}$

If $f : \mathbb{N} \rightarrow \mathbb{P}(\mathbb{Q})$ then $f(3)$ is a rational $\square$ a set of rationals $\boxed{\surd}$ undefined $\square$ a power set $\square$ one or more rationals $\square$

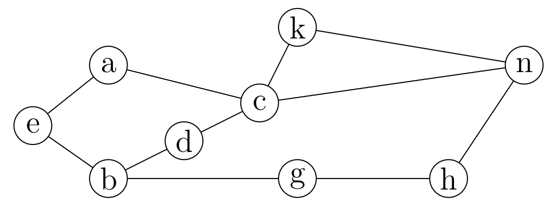
$\binom{k}{k-1}$ 1 $\square$ 2 $\square$ k-1 $\square$ k $\boxed{\surd}$ undefined $\square$

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is at right.
V is the set of nodes. E is the set of edges.



Let $M : (V, \mathbb{N}) \to \mathbb{P}(V)$ such that $M(x, n) = \{y \in V \mid \text{ there is a path of length } n \text{ from } x \text{ to } y\}$.
Let $P(x) = \{M(x, n) \mid n \in \mathbb{N}\}$.

(3 points) $M(c, 2) =$

Solution: $M(c, 2) = \{b, e, n, h, k\}$

(3 points) Is $P(c)$ a partition of V? Check the partition properties that are satisfied.

No Empty set ☐ No Partial Overlap ☐ Covers base set ☑ $\surd$

(7 points) Let $f : X \to Y$ be any function, and let A and B be subsets of X. For any subset S of X define its image $f(S)$ by $f(S) = \{f(s) \in Y \mid s \in S\}$. Is it the case that $f(A) \cap f(B) = f(A \cap B)$? Informally explain why this is true or give a concrete counter-example showing why it is not.

Solution: This is false. Let $X = \{a, b\}$ and $Y = \{c\}$. Define $f : X \to Y$ by $f(x) = c$ for all $x \in X$. Suppose that $A = \{a\}$ and $B = \{b\}$. Then $A \cap B = \emptyset$, $sof(A \cap B) = \emptyset$. But $f(A) \cap f(B) = \{c\}$

(2 points) Check the (single) box that best characterizes each item.

$\{4, 5, 6\} \cap \{6, 7\}$ 6 ☐ $\{6\}$ ☑ $\surd$ $\{\{6\}\}$ ☐

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(7 points) Suppose that $f : A \to B$ is a function. Let's define $T : B \to \mathbb{P}(A)$ by $T(m) = \{x \in A \mid f(x) = m\}$. Then let $P = \{T(m) \mid m \in B\}$. Under what conditions is P a partition of A? Briefly justify your answer.

Solution: $T(m)$ is the set of pre-images of m. Every element $x \in A$ has exactly one image in B. So it belongs to exactly one set $T(m)$. That covers two of the partition properties.

However, P will contain the empty set if f is not onto. So P is a partition if and only if f is onto.

(2 points) $\{p + q^2 \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ 1 \leq p \leq 2 \text{ and } 1 \leq q \leq 3\} =$

Solution: $\{2, 3, 5, 6, 10, 11\}$

(6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$ — always ☐ sometimes ☐ never ☑ $\surd$

Set B is a partition of a finite set A. Then

$|B| \leq 2^{|A|}$ ☐ $|B| \leq |A|$ ☑ $\surd$

$|B| = 2^{|A|}$ ☐ $|B| \leq |A + 1|$ ☐

Pascal's identity states that $\binom{n}{k}$ is equal to

$\binom{n-1}{k} + \binom{n-1}{k-1}$ ☑ $\surd$ $\binom{n-1}{k} + \binom{n-1}{k+1}$ ☐ $\binom{n-1}{k} + \binom{n-2}{k}$ ☐

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $f : \mathbb{R} \to \mathbb{P}(\mathbb{R})$ such that $f(x) = \{p \in \mathbb{R} \mid \lfloor x \rfloor = \lfloor p \rfloor\}$. Let $T = \{f(x) \mid x \in \mathbb{R}\}$.

(3 points) Describe (at a high level) the elements of $f(7)$:

Solution: All the real numbers whose floor is 7.

(3 points) Is T a partition of $\mathbb{R}$? Check the partition properties that are satisfied.

No Empty set $\boxed{\surd}$ No Partial Overlap $\boxed{\surd}$ Covers base set $\boxed{\surd}$

(7 points) Define $f : \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z})$ by $f(x, k) = \{y \in \mathbb{Z} : x = y + kn \text{ for some } n \in \mathbb{Z}\}$. Suppose that $k|p$. Compare $f(r, k)$ and $f(r, p)$. Justify your answer.

Solution: $f(r, p)$ is a subset of $f(r, k)$. Because k divides p, two numbers that differ by a multiple of p must also differ by a multiple of k, but not vice versa. So each equivalence class mod k is the union of several equivalence classes mod p.

(2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(\emptyset)$ $\quad \emptyset$ $\boxed{}$ $\quad \{\emptyset\}$ $\boxed{\surd}$ $\quad \{\{\emptyset\}\}$ $\boxed{}$ $\quad \{\emptyset, \{\emptyset\}\}$ $\boxed{}$

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Let $f : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ be defined by $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$. Suppose that $f(a) = f(b) \cap f(c)$. Express a in terms of b and c. Briefly justify your answer.

Solution: Every element of $f(b)$ contains all multiples of b and $f(c)$ contains multiples of c. So $f(a)$ must contain all numbers that are multiples of both b and c. a is the smallest element of $f(a)$. So $a = lcm(b, c)$.

(2 points) $\{\{p\} \ | \ p \in \{2, 3, 4\} \} \ =$

Solution: $\{\{2\}, \{3\}, \{4\}\}$

(6 points) Check the (single) box that best characterizes each item.

$|\{A \subseteq \mathbb{Z}_4 \ : \ |A| \text{ is even}\}|$ 1 ☐ 6 ☐ 7 ☐ 8 ☑ infinite ☐

There is a set A such that $|\mathbb{P}(A)| \leq 2$. true ☑ false ☐

$\binom{n}{1}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☑ undefined ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Suppose that $A = \{2, 3, 5, 13, 17\}$. Define a function $F : A \to \mathbb{P}(A)$ and a set S by $F(x) = \{y \in A \mid y \text{ is a factor of } x\}$ $\quad S = \{F(x) \mid x \in A\}$

(3 points) $S =$

Solution: $\{\{2\}, \{3\}, \{5\}, \{13\}, \{17\}\}$.

(3 points) Is S a partition of A? Check the partition properties that are satisfied.

No Empty set $\boxed{\surd}$ No Partial Overlap $\boxed{\surd}$ Covers base set $\boxed{\surd}$

(7 points) Let $f : X \to Y$ be any function, and let A and B be subsets of X. For any subset S of X define its image $f(S)$ by $f(S) = \{f(s) \in Y \mid s \in S\}$. Is it the case that $f(A) \cup f(B) = f(A \cup B)$? Informally explain why this is true or give a concrete counter-example showing why it is not.

Solution: This is true.

y is in $f(A \cup B)$ if and only if y is the image of a value in $A \cup B$. But this is true exactly when y is the image of a value in A or y is the image of a value in B. That is y is in $f(A) \cup f(B)$.

(2 points) Check the (single) box that best characterizes each item.

A partition of a set A contains $\emptyset$ always $\boxed{}$ sometimes $\boxed{}$ never $\boxed{\surd}$

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Give an example of a partition P of $\mathbb{N}$ where the set P is infinite. Be specific.

Solution: Suppose that each natural number is in its own partition set. That is $P = \{\{x\} \mid x \in \mathbb{N}\}$. Then P is a partition of $\mathbb{N}$ and P is infinite.

(2 points) $\{pq \mid p \in \mathbb{N},\ q \in \mathbb{N},\ p + q = 6\} =$

Solution: $\{0, 5, 8, 9\}$

(6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$ always ☐ sometimes ☑ never ☐

$|\{\emptyset\}|$ 0 ☐ 1 ☑ 2 ☐ 3 ☐ 4 ☐ undefined ☐

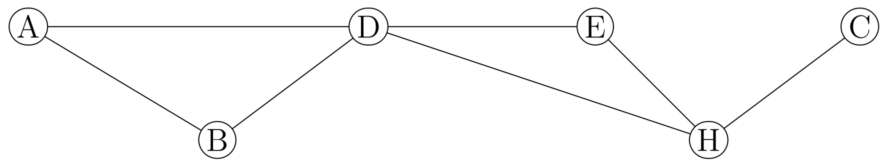
$\{4, 5\} \cap \{6, 7\}$ $\emptyset$ ☑ $\{\emptyset\}$ ☐ nothing ☐ undefined ☐

Name:____

NetID:________________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4**

Graph G is at right.
V is the set of nodes in G.
$M = \{0, 1, 2, 3, 4\}$



Define $f : M \to \mathbb{P}(V)$ by $f(n) = \{p \in V \ : \ d(p, E) = n\}$, where $d(a, b)$ is the (shortest-path) distance between a and b. Let $P = \{f(n) \mid n \in M\}$.

(6 points) Fill in the following values:

f(0) =

Solution: $\{E\}$

f(1) =

Solution: $\{D, H\}$

P =

Solution: $\{\emptyset, \{E\}, \{D, H\}, \{C, B, A\}\}$

(7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

Solution: No, P is not a partition of V. The subsets cover all of V with no partial overlap. However, P contains the empty set, since $f(3) = f(4) = \emptyset$.

(2 points) Check the (single) box that best characterizes each item.

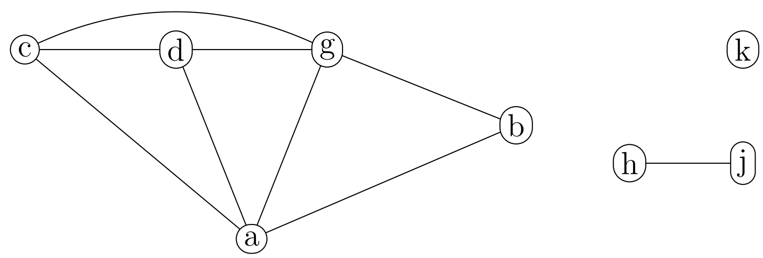
$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always [√] sometimes [ ] never [ ]

Name:____

NetID:________________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4**

Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(E)$ be defined by $f(n) = \{e \in E \mid n \text{ is an endpoint of } e\}$. And let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|V| =$ **Solution:** 8

$f(d) =$ **Solution:** $\{cd, ad, dg\}$

$f(h) =$ **Solution:** $\{hj\}$

(7 points) Is T a partition of E? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: No, T is not a partition of E. T contains all edges in E. However, $f(k)$ is the empty set, so T contains the empty set. Also, there is partial overlap between the subsets, e.g. $f(d)$ and $f(a)$ are different but share the edge ad.

(2 points) State the definition of $\binom{n}{k}$, i.e. express $\binom{n}{k}$ in terms of more basic arithmetic operations.

Solution: $$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

CS 173, Spring 18 Examlet 12, Part B

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Let $f : X \to Y$ be any function, and let A and B be subsets of X. For any subset S of X define its image $f(S)$ by $f(S) = \{f(s) \in Y \mid s \in S\}$. Is it the case that $f(A \cap B) \subseteq f(A) \cap f(B)$? Informally explain why this is true or give a concrete counter-example showing why it is not.

Solution: This is true. Suppose that y is in $f(A \cap B)$. Then (by the definition of image), there is an x in $A \cap B$ such that $f(x) = y$. But since x is in $A \cap B$, $x \in A$ and $x \in B$. So then $f(x) = y$ must be in both $f(A)$ and $f(B)$.

(8 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A \cap B) \subseteq \mathbb{P}(A \cup B)$ always $\boxed{\surd}$ sometimes $\square$ never $\square$

Pascal's identity states that $\binom{n}{k}$ is equal to $\binom{n-1}{k} + \binom{n-1}{k-1}$ $\boxed{\surd}$ $\binom{n-1}{k} + \binom{n-1}{k+1}$ $\square$ $\binom{n-1}{k} + \binom{n-2}{k}$ $\square$

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(1.73)$ is a rational $\square$ a set of rationals $\square$ undefined $\boxed{\surd}$ one or more rationals $\square$ a power set $\square$

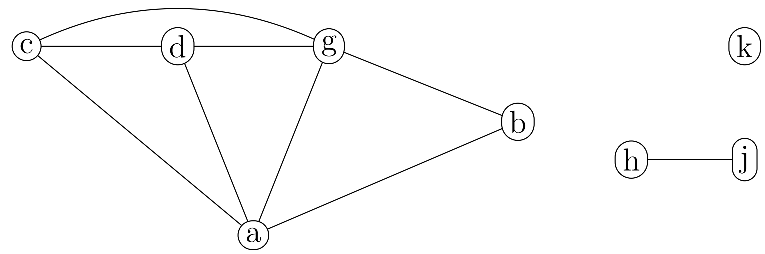
Set B is a partition of a finite set A. Then $|A| = |B|$. always $\square$ sometimes $\boxed{\surd}$ never $\square$

Name:____

NetID:__________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(E)$ be defined by $f(n) = \{e \in E \mid n \text{ is an endpoint of } e\}$. And let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|V| =$ **Solution:** 8

$f(d) =$ **Solution:** $\{cd, ad, dg\}$

$f(h) =$ **Solution:** $\{hj\}$

(7 points) Is T a partition of E? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: No, T is not a partition of E. T contains all edges in E. However, $f(k)$ is the empty set, so T contains the empty set. Also, there is partial overlap between the subsets, e.g. $f(d)$ and $f(a)$ are different but share the edge ad.

(2 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{P}(\mathbb{Q}) \to \mathbb{N}$ then $f(\{3\})$ is

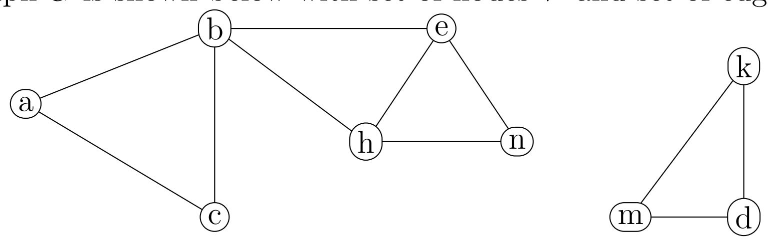
an integer ☑	a set of integers ☐	undefined ☐
one or more integers ☐	a power set ☐	

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is shown below with set of nodes V and set of edges E.



Let $F : V \to \mathbb{P}(V)$ such that $F(n) = \{v \in V \mid \text{ there is a cycle containing } n \text{ and } v\}$.
Let $T = \{F(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$F(k) =$

Solution: $\{m, d, k\}$

$F(b) =$

Solution: $\{a, b, c, e, n, h\}$

$|T| =$

Solution: 4

(7 points) Is T a partition of V? For each of the three conditions required to be a partition, explain why T does or doesn't satisfy that condition.

Solution: No, it is not a partition of V. There is partial overlap between $F(c)$ and $F(h)$. But T doesn't contain the empty set and covers all of V.

(2 points) State Pascal's identity.

Solution:

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(7 points) Suppose that R is a relation on $\mathbb{Z}$ which is reflexive and symmetric, but not transitive. Let's define $T(n) = \{a \in \mathbb{Z} \mid aRn\}$. Notice that $n \in T(n)$ for any integer n. The collection of all sets $T(n)$ does not form a partition of $\mathbb{Z}$. Explain (informally but clearly) why the fact that R is not transitive can cause one of the partition properties to fail.

Solution: For full credit, it's enough to give a well-explained specific example (e.g. using the relation $|a - b| \leq 10$) showing how partial overlap can occur.

Here's a more general (but still informal) argument. Since R is not transitive, there's three integers a, b, and c such that aRb and bRc but not aRc. Since aRb, $T(a)$ must contain a and b. Since bRc, $T(c)$ must contain c and b. So $T(a)$ and $T(c)$ overlap. But they can't be the same set because it's not true that aRc. So there is partial overlap.

Making the above argument full formal would require using the reflexive and symmetric properties of R.

(8 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{P}(\mathbb{Q}) \to \mathbb{N}$ then $f(3)$ is

- an integer ☐
- a set of integers ☐
- undefined ☑ ($\surd$)
- one or more integers ☐
- a power set ☐

$\{\mathbb{N}\}$ is a partition of $\mathbb{N}$.

- true ☑ ($\surd$)
- false ☐

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$

- always ☐
- sometimes ☑ ($\surd$)
- never ☐

$\binom{n}{0}$

- -1 ☐
- 0 ☐
- 1 ☑ ($\surd$)
- 2 ☐
- n ☐
- undefined ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Suppose that $g : A \to B$ is an onto function. Let's define $F(y) = \{x \in A \mid g(x) = y\}$. Then define $P = \{F(y) \mid y \in B\}$. Is P a partition of A? Briefly justify your answer.

Solution: Yes, P is a partition of A. Notice that F produces all the pre-images of an output value y. So x is in $F(g(x))$ and x can't be in any other set produced by F. Because g is onto, any output value y has at least one pre-image. So $F(y)$ can never be empty.

(2 points) State the binomial theorem.

Solution:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

(6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always ☑ sometimes ☐ never ☐

$\binom{0}{0}$ -1 ☐ 0 ☐ 1 ☑ 2 ☐ n ☐ undefined ☐

$|\ \mathbb{P}(\{4,5,6,7,8\} \times \emptyset)\ |$ $\emptyset$ ☐ $\{\emptyset\}$ ☐ 0 ☐ 1 ☑ 25 ☐ 2^5 ☐

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $f : \mathbb{R}^2 \to \mathbb{P}(\mathbb{R}^2)$ be defined by $f(x, y) = \{(p, q) \in \mathbb{R}^2 \mid \exists \alpha \in \mathbb{R}, (p, q) = \alpha(x, y)\}$.
Let $T = \{f(x, y) \mid (x, y) \in \mathbb{R}^2\}$.

(6 points) Answer the following questions:

$f(0, 0) =$

Solution: $\{(0, 0)\}$

Describe (at a high level) the elements of $f(0, 36)$:

Solution: $f(0, 36)$ is the line passing through the origin and $(0, 36)$.

Give an element of $\mathbb{P}(\mathbb{R}^2) - T$:

Solution: Many possible answers here. For example, $\emptyset$, or any finite set or any circle.

(7 points) Is T a partition of $\mathbb{R}^2$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: This is not a partition of $\mathbb{R}^2$. It doesn't contain the empty set (good). And the elements of T do cover all of the plane (good). However, all the lines contain the origin, so there is partial overlap (bad).

(2 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Q} \to \mathbb{P}(\mathbb{Q})$ then $f(1.73)$ is	a rational ☐	a set of rationals ☑	undefined ☐
	one or more rationals ☐	a power set ☐	

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Let $f : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ be defined by $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$. Suppose that $m = ab$, where a and b are two different primes. Express $f(m)$ in terms of $f(a)$ and $f(b)$. Briefly justify your answer.

Solution: $f(m)$ contains all multiples of m. Since $m = ab$, and a and b are distinct primes, this means that $f(m)$ contains all numbers that are multiples of both a and b. In other words, these are numbers that are in both $f(a)$ and $f(b)$. So $f(m) = f(p) \cap f(q)$.

(8 points) Check the (single) box that best characterizes each item.

$\lvert\{A \subseteq \mathbb{Z}_4 \ : \ \lvert A\rvert \text{ is even}\}\rvert$	1 ☐	6 ☐	7 ☐	8 ☑ √	infinite ☐

$\binom{n}{1}$	-1 ☐	0 ☐	1 ☐	2 ☐	n ☑ √	undefined ☐

There is a set A such that $|\mathbb{P}(A)| \leq 2$. true ☑ √ false ☐

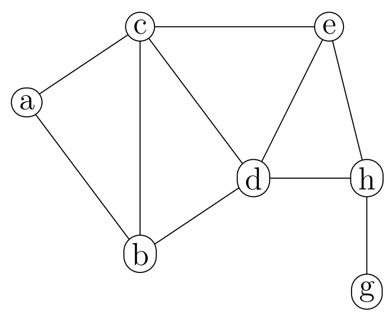
If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is	an integer ☐	a set of integers ☑ √	undefined ☐
	one or more integers ☐	a power set ☐	

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G with set of nodes V is shown below. Recall that $deg(n)$ is the degree of node n. Let's define $f : \mathbb{N} \to \mathbb{P}(V)$ by $f(k) = \{n \in V : \deg(n) = k\}$. Also let $T = \{f(k) \mid k \in \mathbb{N}\}$.



(6 points) Fill in the following values:

$f(4) =$
Solution: $\{c, d\}$
$f(1) =$
Solution: $\{g\}$
$|T| =$
Solution: 5. (The distinct members are f(0), f(1), f(2), f(3), and f(4).)

(7 points) Is T a partition of V? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: No, it is not a partition. There is no partial overlap between the sets in T and they cover all nodes in V. However, T contains the empty set (e.g. as the value of $f(17)$).

(2 points) Check the (single) box that best characterizes each item.

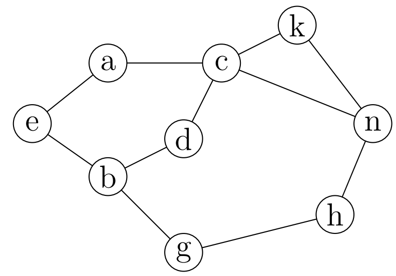
$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always ☑ sometimes ☐ never ☐

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is shown at right with set of nodes V and set of edges E. Let $M : (V, \mathbb{N}) \to \mathbb{P}(V)$ be defined by $M(x, n) = \{y \in V \mid \text{ there is a path of length } n \text{ from } x \text{ to } y\}$. Let $P(x) = \{M(x, n) \mid n \in \mathbb{N}\}$.



(6 points) Give the value of $M(c, n)$, for all values of n from 0 to 3.

Solution: $M(c, 0) = \{c\}$ $M(c, 1) = \{a, d, n, k\}$ $M(c, 2) = \{b, e, n, h, k\}$ $M(c, 3) = \{b, e, g, h\}$

(7 points) Is $P(c)$ a partition of V? For each of the three conditions required to be a partition, explain why $P(c)$ does or doesn't satisfy that condition.

Solution: $P(c)$ is not a partition of V. $P(c)$ does cover all of V. However, some of its elements have partial overlap, e.g. $M(c, 2)$ and $M(c, 3)$. Also, since there are only 9 nodes in the graph, no path has length greater than 8. So $M(c, 9) = \emptyset$ and therefore $P(c)$ contains the empty set.

(2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$ always ☐ sometimes ☐ never ☑

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Let $f : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ be defined by $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$. For which positive integers a and b is $f(a)$ a subset of $f(b)$? Briefly justify your answer.

Solution: $f(a) \subseteq f(b)$ is true if and only if every multiple of a is also be a multiple of b. This occurs exactly when a is a multiple of b.

(8 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(3)$ is

a rational ☐ a set of rationals ☑ undefined ☐
one or more rationals ☐ a power set ☐

$\{\{a, b\}, c\} = \{a, b, c\}$

true ☐ false ☑

Set B is a partition of a finite set A. Then

$|B| \leq 2^{|A|}$ ☐ $|B| \leq |A|$ ☑
$|B| = 2^{|A|}$ ☐ $|B| \leq |A + 1|$ ☐

$\binom{n}{1}$

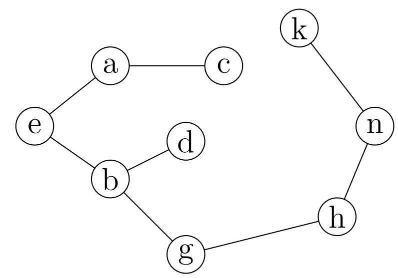
-1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☑ undefined ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is shown at right with set of nodes V and set of edges E. Let $M : (V, \mathbb{N}) \to \mathbb{P}(V)$ be defined by $M(x, n) = \{y \in V \mid \text{ there is a path of length } n \text{ from } x \text{ to } y\}$. Let $P(x) = \{M(x, n) \mid n \in \mathbb{N}\}$.



(6 points) Give the value of $M(g, n)$, for all values of n from 0 to 3.

Solution: $M(g, 0) = \{g\}$ $M(g, 1) = \{b, h\}$ $M(g, 2) = \{e, d, n\}$ $M(g, 3) = \{a, k\}$

(7 points) Is $P(g)$ a partition of V? For each of the three conditions required to be a partition, explain why $P(g)$ does or doesn't satisfy that condition.

Solution: $P(g)$ is not a partition of V. $P(g)$ does cover all of V. Because the graph has no cycles, each node is in exactly one of the subsets in $P(g)$, so no partial overlap. However, no path has length greater than 4. So $M(g, 5) = \emptyset$ and therefore $P(g)$ contains the empty set.

(2 points) Check the (single) box that best characterizes each item.

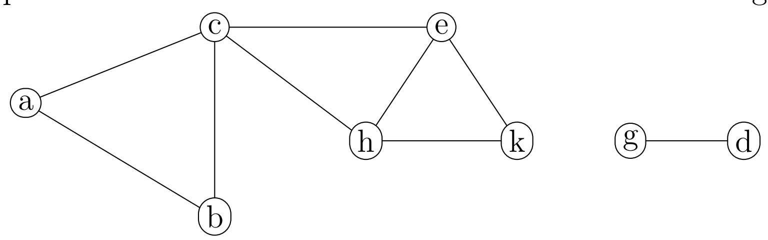
$\binom{n}{n}$ -1 ☐ 0 ☐ 1 ☑ 2 ☐ n ☐ undefined ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is shown below with set of nodes V and set of edges E.



Let $F : V \to \mathbb{P}(V)$ such that $F(n) = \{v \in V \mid \text{ there is a cycle containing } n \text{ and } v\}$.
Let $T = \{F(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$F(g) =$

Solution: $\emptyset$

$F(b) =$

Solution: $\{a, b, c\}$

$F(k) =$

Solution: $\{c, e, k, h\}$

(7 points) Is T a partition of V? For each of the three conditions required to be a partition, explain why T does or doesn't satisfy that condition.

Solution: No, it is not a partition of V. There is partial overlap between $F(b)$ and $F(k)$. T contains the empty set because $f(g) = \emptyset$. And some vertices (e.g. g) do not belong to any cycles and therefore aren't in any elements of T.

(2 points) Check the (single) box that best characterizes each item.

$\binom{n}{0}$ -1 ☐ 0 ☐ 1 ☑ 2 ☐ n ☐ undefined ☐

Name:____

NetID:__________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Can we create a set C such that C is a partition of $\mathbb{R}$ but $|C|$ is finite? Give a specific set C that works or briefly explain why it's impossible.

Solution: Yes. Suppose that C contains exactly two sets: the negative reals and the non-negative reals. Then $|C| = 2$, which is finite.

(8 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always ☑ sometimes ☐ never ☐

If $n \geq k \geq 0$, then $\binom{n}{k} = \binom{n}{n-k}$ true ☑ true for some n and k ☐ false ☐

$\binom{n}{0}$ -1 ☐ 0 ☐ 1 ☑ 2 ☐ n ☐ undefined ☐

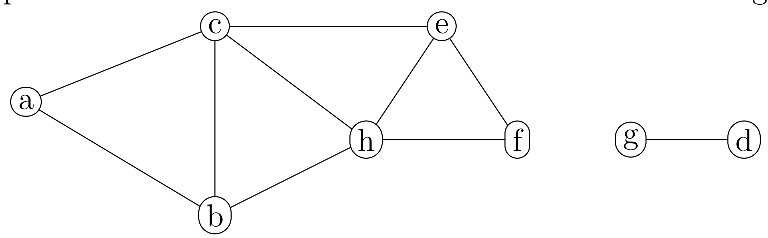
$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$ always ☐ sometimes ☐ never ☑

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is shown below with set of nodes V and set of edges E.



Let $f : V \to \mathbb{P}(V)$ such that $f(n) = \{v \in V \mid \text{ there is a cycle containing } n \text{ and } v\}$.
Let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|E| =$

Solution: 10

$f(b) =$

Solution: $\{a, b, c, e, f, h\}$

$f(h) =$

Solution: $\{a, b, c, e, f, h\}$

(7 points) Is T a partition of V? For each of the three conditions required to be a partition, explain why T does or doesn't satisfy that condition.

Solution: No, it is not a partition of V. There is no partial overlap (good). However, T contains the empty set because $f(g) = \emptyset$. And some vertices (e.g. k) do not belong to any cycles and therefore aren't in any elements of T.

(2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$ always ☐ sometimes ☑ never ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $f : \mathbb{R}^2 \to \mathbb{P}(\mathbb{R}^2)$ be defined by $f(x, y) = \{(p, q) \in \mathbb{R}^2 \mid x^2 + y^2 = p^2 + q^2\}$
Let $T = \{f(x, y) \mid (x, y) \in \mathbb{R}^2\}$.

(6 points) Answer the following questions:

$f(0, 0) =$

Solution: $\{(0, 0)\}$

Describe (at a high level) the elements of $f(0, 36)$:

Solution: The circle centered on the origin with radius 36.

The cardinality of (aka the number of elements in) T is:

Solution: infinite

(7 points) Is T a partition of $\mathbb{R}^2$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: Yes. The output of f is never the empty set. None of these circles (plus the dot at the origin) overlaps any of the others. And jointly they cover all of the plane.

(2 points) Check the (single) box that best characterizes each item.

Let A be a non-empty set, $\{A\}$ is a partition of A.	always $\boxed{\surd}$	sometimes $\square$	never $\square$

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(7 points) Suppose that A and B are disjoint sets, C_A is a partition of A and C_B is a partition of B. Is $C_A \cup C_B$ a partition of $A \cup B$? Briefly justify your answer.

Solution: Yes. Each element of A belongs to exactly one element of C_A. Each element of B belongs to exactly one element of C_B. Since A and B are disjoint, there can't be any partial overlap between the elements of C_A and C_B. Since neither C_A nor C_B contains the empty set, their union can't contain it either.

(8 points) Check the (single) box that best characterizes each item.

$|\mathbb{P}(\mathbb{P}(\emptyset))|$ 0 ☐ 1 ☐ 2 ☑ $\surd$ 3 ☐ 4 ☐ undefined ☐

If $f : \mathbb{P}(\mathbb{Q}) \to \mathbb{N}$ then $f(3)$ is a rational ☐ a power set of rationals ☐ a set of rationals ☐ undefined ☑ $\surd$

$|\{\emptyset\}|$ 0 ☐ 1 ☑ $\surd$ 2 ☐ 3 ☐ 4 ☐ undefined ☐

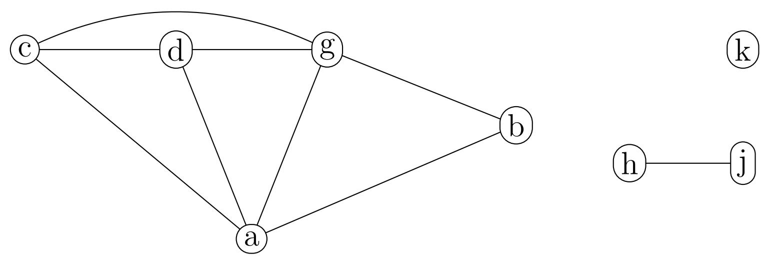
$\mathbb{P}(A \cup B) = \mathbb{P}(A) \cup \mathbb{P}(B)$ always ☐ sometimes ☑ $\surd$ never ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(V)$ be defined by $f(n) = \{v \in V \mid \text{there is a path from n to v}\}$. And let $T = \{f(n) \mid n \in V\}$.

6 points) Fill in the following values:

$f(k) =$ **Solution:** $\{k\}$

$f(d) =$ **Solution:** $\{a, b, c, d, g\}$

$T =$ **Solution:** $\{\{a, b, c, d, g\}\{h, j\}\{k\}\}$

(7 points) Is T a partition of V? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: Yes, T is a partition of V. T does not contain the empty set. The members of T contain all nodes in V, with no partial overlap

(2 points) Check the (single) box that best characterizes each item.

$\binom{n}{1}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☑ undefined ☐

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Can a set A be a partition of the empty set? Briefly justify your answer.

Solution: Yes. Suppose that A is the empty set. Then each member of the empty set is in exactly one member of A. Also, the empty set is not an element of A.

(8 points) Check the (single) box that best characterizes each item.

Pascal's identity states that $\binom{n+1}{k}$ is equal to $\binom{n}{k} + \binom{n}{k+1}$ ☐ $\binom{n}{k} + \binom{n-1}{k}$ ☐ $\binom{n}{k} + \binom{n}{k-1}$ ☑

$\mathbb{P}(A \cap B) \subseteq \mathbb{P}(A \cup B)$ always ☑ sometimes ☐ never ☐

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is an integer ☐ a set of integers ☑ one or more integers ☐ a power set ☐

A partition of a set A contains A always ☐ sometimes ☑ never ☐