

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: A

Discussion: Monday & Wednesday 1:30 2:30

(15 points) Use (strong) induction to prove the following claim:

Claim:  $\frac{(2n)!}{n!n!} < 4^n$ , for all integers  $n \geq 2$

Proof by induction on  $n$ .

Base case(s):

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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1. (7 points) You found the following claim on a hallway whiteboard. Suppose that  $f$  and  $g$  are increasing functions from the reals to the reals, for which all output values are  $> 1$ . If  $f(x)$  is  $O(g(x))$ , then  $\log(f(x))$  is  $O(\log(g(x)))$ . Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$	$\Theta(\log n)$	<input type="checkbox"/>	$\Theta(\sqrt{n})$	<input type="checkbox"/>	$\Theta(n)$	<input type="checkbox"/>	$\Theta(n \log n)$	<input type="checkbox"/>
$T(n) = 2T(n - 1) + c$	$\Theta(n^2)$	<input type="checkbox"/>	$\Theta(n^3)$	<input type="checkbox"/>	$\Theta(2^n)$	<input type="checkbox"/>	$\Theta(3^n)$	<input type="checkbox"/>

$T(1) = d$	$\Theta(\log n)$	<input type="checkbox"/>	$\Theta(\sqrt{n})$	<input type="checkbox"/>	$\Theta(n)$	<input type="checkbox"/>	$\Theta(n \log n)$	<input type="checkbox"/>
$T(n) = T(n - 1) + n$	$\Theta(n^2)$	<input type="checkbox"/>	$\Theta(n^3)$	<input type="checkbox"/>	$\Theta(2^n)$	<input type="checkbox"/>	$\Theta(3^n)$	<input type="checkbox"/>

$n^{1.5}$ is	$\Theta(n^{1.414})$	<input type="checkbox"/>	$O(n^{1.414})$	<input type="checkbox"/>	neither of these	<input type="checkbox"/>
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Suppose  $f(n)$  is  $\Theta(g(n))$ .  
 Will  $g(n)$  be  $O(f(n))$ ?      no       sometimes       yes