CS 173, Spring 19 Examlet 4, white

Name: ____

NetID: ______________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

$$\begin{tikzcd} A \arrow[r] \arrow[d] & C \arrow[d] & E \\ B \arrow[r] & D \arrow[ul] & F \end{tikzcd}$$

Reflexive:	☐	Irreflexive:	√
Symmetric:	☐	Antisymmetric:	√
Transitive:	☐		

2. (5 points) Suppose that S is the set of all binary strings (i.e. finite sequences of 1's and 0's). Suppose that $\sim$ is the relation on S where $a \sim b$ if and only if a and b contain the same number of 1's. For example, `0101` $\sim$ `1000001`. List three members of [`111`].

   Solution: For example, `111`, `1101`, and `01110`.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ or $y \leq q$. Is T antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution:

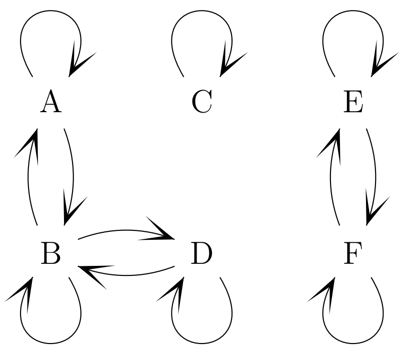
   This relation is not antisymmetric. We have $(0, 1)T(1, 0)$ and $(1, 0)T(0, 1)$, but $(0, 1) \neq (1, 0)$.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☑ $\surd$	Irreflexive:	☐
Symmetric:	☑ $\surd$	Antisymmetric:	☐
Transitive:	☐		

2. (5 points) A relation is a partial order if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: reflexive, antisymmetric, transitive

3. (5 points) Suppose that R is a relation on the integers such that xRy for all integers x and y. Is R an equivalence relation?

   Solution: Yes, R is an equivalence relation. All three properties of an equivalence relation (reflexive, symmetric, transitive) are requiring that certain pairs be related (e.g. unlike antisymmetry which requires that certain pairs not be related). So they have to be true because all pairs are related.

CS 173, Spring 19 Examlet 4, white

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

$$A \longrightarrow C \longleftarrow E$$

$$B \longrightarrow D \longleftarrow F$$

Reflexive:	☐	Irreflexive:	$\boxed{\surd}$
Symmetric:	☐	Antisymmetric:	$\boxed{\surd}$
Transitive:	$\boxed{\surd}$		

2. (5 points) Let's define the ~~equivalence~~ relation $\sim$ on $\mathbb{N}^3$ such that $(x, y, z) \sim (p, q, r)$ if and only $(x, y, z) = \alpha(p, q, r)$ for some integer α. ~~List three members of [(1, 2, 3)]~~ List three elements that are related to $(1, 2, 3)$ in either direction.

   Solution: For example $(1, 2, 3)$, $(-1, -2, -3)$, $(6, 12, 18)$.

3. (5 points) Suppose that R is the relation on the set of integers such that aRb if and only if $|a - b| \leq 13$ Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: No, it is not transitive. Consider $a = 0$, $b = 13$, $c = 26$. Then aRb and bRc. 'However, $|a - c| = 26$, so $a \not R c$.

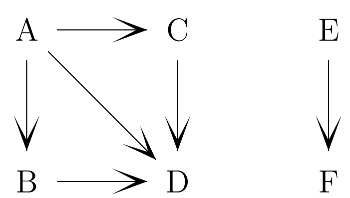
CS 173, Spring 19 Examlet 4, white

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: $\boxed{\surd}$

Symmetric: ☐ Antisymmetric: $\boxed{\surd}$

Transitive: $\boxed{\surd}$

2. (5 points) Let R be the relation on the integers such that xRy if and only if $\lfloor x/4 \rfloor = \lfloor y/4 \rfloor$. List the values in [8].

   Solution: [8] contains (only) 8, 9, 10, and 11

3. (5 points) Let T be a reflexive relation defined on the integers. Let S be the relation on the integers such that aSb if and only if there is an integer k such that aTk and kTb. Is S reflexive? (I.e. is S reflexive for any reflexive relation T?) Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: S is always reflexive. Pick any integer x. xTx because T is reflexive. If we let $k = x$, then we also have xTk and kTx. So xSx.

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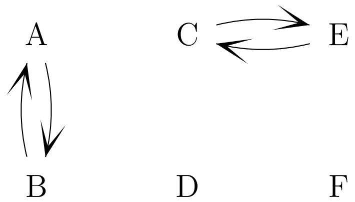
Name: ______________________________________

NetID: ____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	√
Symmetric:	√	Antisymmetric:	☐
Transitive:	☐		

2. (5 points) Can a relation be irreflexive, symmetric, and also transitive? Either give such a relation or briefly explain why it's not possible to construct one.

   Solution: Yes, this is possible. However, this must be the empty relation in which no elements are related. (No edges in the graph representation.) If we have aRb, then we must have bRa (by symmetry) and so aRa (by transitivity), which is inconsistent with the relation being irreflexive.

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $x = y$. Is R a partial order?

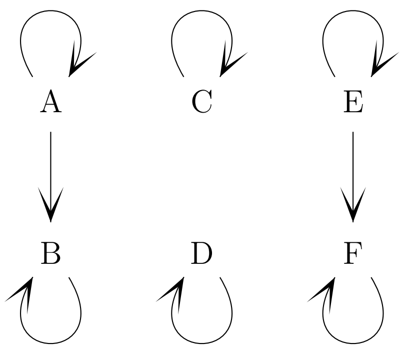
   Solution: Yes, R is a partial order. It's reflexive because elements are always related to themselves. The only way to match the hypothsis of transitive is for all three integers to be equal, which makes the conclusion true. So it's transitive. It is anti-symmetric by vacuous truth.

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	√	Irreflexive:	
Symmetric:		Antisymmetric:	√
Transitive:	√		

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be symmetric.

   Solution: For any $x, y \in A$, if xRy, then yRx.

3. (5 points) Let S be the relation defined on $\mathbb{N}$ such that aTb if and only if $a = b + 2k$ for some natural number k. Is T antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

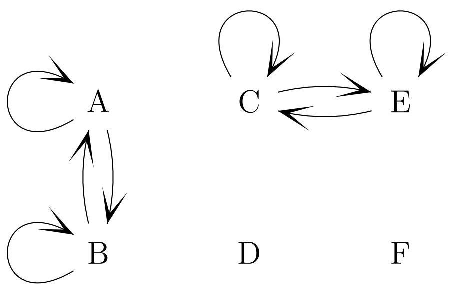
   Solution: This relation is antisymmetric. Notice that k cannot be negative, so if aTb then $a \leq b$. So two numbers can be related in both directions only if they are equal.

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☑ Antisymmetric: ☐

Transitive: ☑

2. (5 points) Let's define the equivalence relation $\sim$ on $\mathbb{N}^3$ such that $(x, y, z) \sim (p, q, r)$ if and only $x + y + z = p + q + r$. List three members of $[(1, 2, 3)]$.

   Solution: The three coordinates need to be non-negative integers that sum to 6. So some example members are $(1, 2, 3)$, $(3, 2, 1)$, and $(3, 0, 3)$.

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $xy = 1$ for all integers x and y. Is R a partial order?

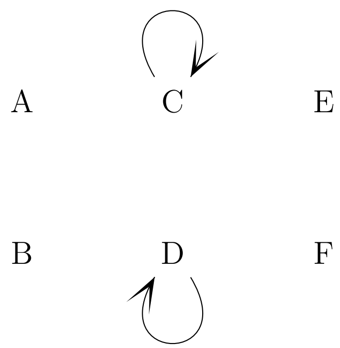
   Solution: No, R is not a partial order. Notice that the only relations are when $x = y = 1$ or $x = y = -1$. So it's transitive and antisymmetric, but not reflexive.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☑ √ Antisymmetric: ☑ √

Transitive: ☑ √

2. (5 points) A relation is an equivalence relation if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: reflexive, symmetric, transitive

3. (5 points) Suppose that $\succeq$ is the relation between subsets of the integers such that $A \succeq B$ if and only if $A - B \neq \emptyset$. (A and B are sets of integers, so $A - B$ is a set difference.) Is $\succeq$ transitive? Informally explain why it's true or give a concrete counter-example.

   Solution: $\succeq$ is not transitive. Consider $A = C = \{3\}$ and $B = \{4\}$. Then $A - B = \{3\}$ and $B - C = \{4\}$. So $A \succeq B$ and $B \succeq C$. But $A - C = \emptyset$, so we don't have $A \succeq C$.

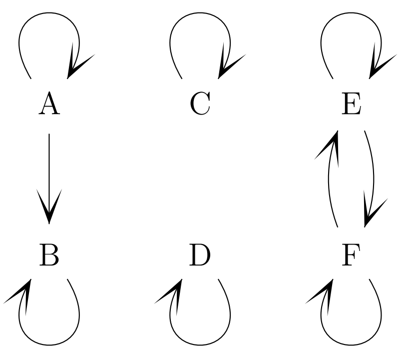
CS 173, Fall 18 Examlet 4, Part B

Name:____

NetID:____________________ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	√	Irreflexive:	
Symmetric:		Antisymmetric:	
Transitive:	√		

2. (5 points) Let R be the equivalence relation on the real numbers such that xRy if and only if $\lfloor x \rfloor = \lfloor y \rfloor$. Give three members of the equivalence class [13].

   Solution: 13, 13.1, 13.7

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ or $y \leq q$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not transitive. We have $(0, 0)T(-10, 10)$ (look at the second coordinate). We also have $(-10, 10)T(-5, -5)$ (look at the first coordinate). But it's not the case that $(0, 0)T(-5, -5)$.

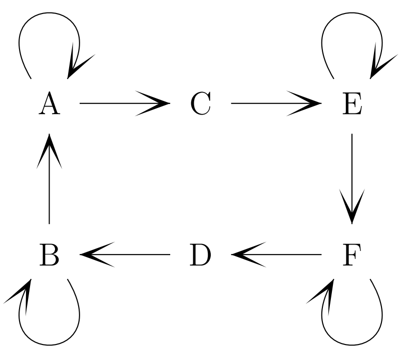
CS 173, Fall 18 Examlet 4, Part B

Name:____

NetID:____________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☐
Symmetric:	☐	Antisymmetric:	$\boxed{\surd}$
Transitive:	☐		

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be antisymmetric.

   Solution: For any $x, y \in A$, if xRy and yRx, then $x = y$. Or for any $x, y \in A$, if xRy and $x \neq y$, then $y \not R x$.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ and $y \leq q$. Is T antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is antisymmetric. Suppose that $(x, y)T(p, q)$ and $(p, q)T(x, y)$. Then $x \leq p$ and $y \leq q$, and also $p \leq x$ and $q \leq y$. So $x = y$ and $y = q$. So $(x, y) = (p, q)$.

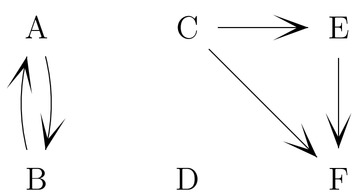
CS 173, Spring 18 Examlet 4, Part B

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☑ $\surd$
Symmetric:	☐	Antisymmetric:	☐
Transitive:	☐		

2. (5 points) Recall that $\mathbb{N}^2$ is the set of all pairs of natural numbers. Let's define the equivalence relation $\sim$ on $\mathbb{N}^2$ as follows: $(x, y) \sim (p, q)$ if and only $|x - y| = |p - q|$. List three members of $[(2, 3)]$.

   Solution: (2,3), (3,4), (14,13)

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $x = y$. Is R an equivalence relation?

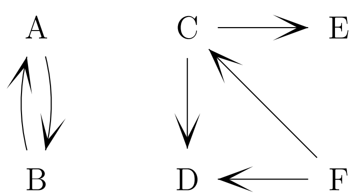
   Solution: Yes, R is an equivalence relation. It's reflexive because elements are always related to themselves. Since there aren't any relations between distinct elements, it's also symmetric and transitive.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



- Reflexive: ☐
- Irreflexive: ☑ $\surd$
- Symmetric: ☐
- Antisymmetric: ☐
- Transitive: ☐

2. (5 points) Can a relation be symetric and also antisymmetric? Either give such a relation or briefly explain why it's not possible to construct one.

   Solution: Yes, this is possible. Consider the relation R on the integers such that aRb if and only if $a = b$. This is both symmetric and antisymmetric.

3. (5 points) Let J be the set of open intervals of the real line, i.e $J = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$. Let's define the "disjoint" relation D on J by $(a, b)D(c, d)$ if and only if $b \leq c$ or $d \leq a$. Is D transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: D is not transitive. Consider $(1, 2)$, $(3, 5)$, and $(4, 6)$. Then $(4, 6)D(1, 2)$. $(1, 2)D(3, 5)$ and But not $(4, 6)D(3, 5)$.

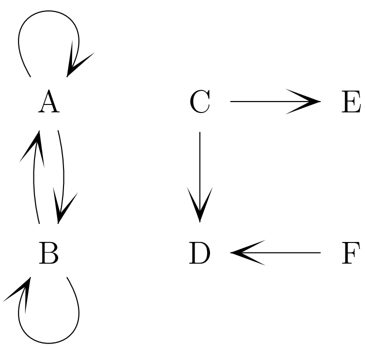
CS 173, Spring 18 Examlet 4, Part B

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☑ $\surd$

2. (5 points) Suppose that R is a partial order on a set A. What additional property is required for R to be a linear order (aka total order)? Give specific details of the property, not just its name.

   Solution: All pairs of elements must be comparable. That is, for any elements x and y in A, either xRy or yRx.

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $2 \mid (x + y + 1)$. Is R transitive?

   Solution: No, R is not transitive. For example, $2R3$ and $3R4$ but it's not the case that $2R4$.

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Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

A	C	E	
			(that is, 6 nodes and no arrows at all)
B	D	F	

Reflexive:	$\square$	Irreflexive:	$\surd$
Symmetric:	$\surd$	Antisymmetric:	$\surd$
Transitive:	$\surd$		

2. (5 points) Let R be the equivalence relation on the real numbers such that xRy if and only if $\lfloor x \rfloor = \lfloor y \rfloor$. Give three members of the equivalence class [13].

   Solution: 13, 13.1, 13.7

3. (5 points) Suppose that R is a relation on pairs of integers such that $(x, y)R(a, b)$ if and only if $x - a \geq 2$ and $y \geq b$. Is R a partial order?

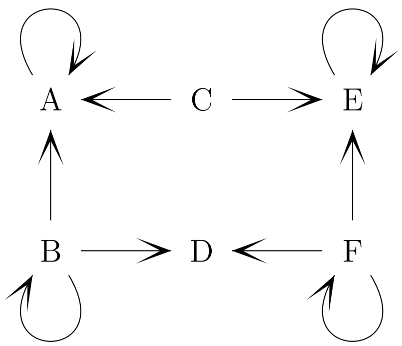
   Solution: No, R is not a partial order. It's transitive and antisymmetric. However it's impossible for an element (x, y) to be related to itself, because that would require $x - x \geq 2$. So it's not reflexive.

Name: ____

NetID: ______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: [ ] Irreflexive: [ ]

Symmetric: [ ] Antisymmetric: [$\surd$]

Transitive: [$\surd$]

2. (5 points) **Notice that this problem was corrected early in the exam. This is the corrected version.** Let's define the relation $\sim$ on $\mathbb{Z}$ sucn that $x \sim y$ if and only $|x - y| = 3$. List all elements related to 7.

   Solution: 4 and 10

3. (5 points) Let S be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)S(p, q)$ if and only if $x^2 + y^2 \leq p^2 + q^2$. Is S antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not antisymmetric. We have $(0, 1)S(1, 0)$ and $(1, 0)S(0, 1)$, but $(0, 1) \neq (1, 0)$.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

$$\begin{tikzcd} A \arrow[r] & C \arrow[r] & E \arrow[d] \\ B \arrow[u] & D \arrow[l] & F \arrow[l] \end{tikzcd}$$

Reflexive:	☐	Irreflexive:	$\surd$
Symmetric:	☐	Antisymmetric:	$\surd$
Transitive:	☐		

2. (5 points) Suppose that R is an equivalence relation on a set A. Using precise set notation, define $[x]_R$, i.e. the equivalence class of x under the relation R.

   Solution: $[x]_R = \{y \in A \mid xRy\}$

3. (5 points) Let J be the set of open intervals of the real line, i.e $J = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$. Let's define the "touches" relation T on J by $(a, b)T(c, d)$ if and only if $a = d$ or $b = c$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not transitive. Consider $(1, 2)$, $(2, 3)$, and $(3, 4)$. Then $(1, 2)T(2, 3)$ and $(2, 3)T(3, 4)$, but not $(1, 2)T(3, 4)$.

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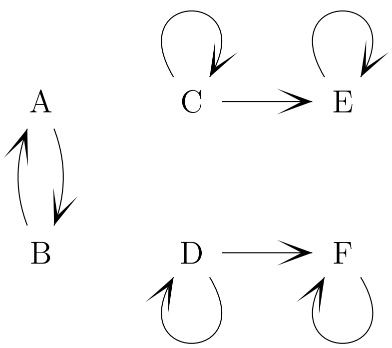
NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

(That is, no boxes checked.)

2. (5 points) Let $\sim$ be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y) \sim (p, q)$ if and only if $x^2 + y^2 = p^2 + q^2$. Find three elements in the equivalence class $[(0, 1)]$

Solution:

$(0, 1)$, $(1, 0))$, $(-1, 0)$ (for example)

3. (5 points) Suppose that $\preceq$ is the relation between subsets of the integers such that $A \preceq B$ if and only if $A - B = \emptyset$. (A and B are sets of integers, so $A - B$ is a set difference.) Is $\preceq$ antisymmetric? Informally explain why it's true (e.g. use a Venn diagram) or give a concrete counter-example.

Solution: $\preceq$ is antisymmetric. Suppose that $X - Y = \emptyset$ and $Y - X = \emptyset$.

Notice that $X = (X \cap Y) \cup (X - Y)$. Draw a Venn diagram if this isn't clear. Since $X - Y = \emptyset$, we have $X = (X \cap Y)$.

Similarly, $Y - X = \emptyset$ implies that $Y = (X \cap Y)$.

So $X = X \cap Y = Y$.

[An annotated Venn diagram would work fine as an informal explanation.]

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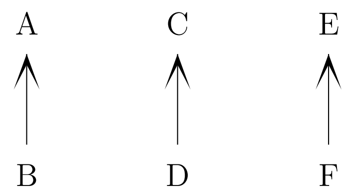
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☑ $\surd$
Symmetric:	☐	Antisymmetric:	☑ $\surd$
Transitive:	☑ $\surd$		

2. (5 points) A relation is a partial order if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: reflexive, antisymmetric, transitive

3. (5 points) Suppose that T is the relation on the set of integers such that aTb if and only if $\gcd(a, b) = 3$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

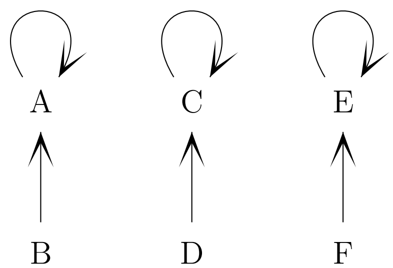
   Solution: This relation is not transitive. Suppose $a = 6$, $b = 15$, and $c = 12$. Then $\gcd(a, b) = 3$ and $\gcd(b, c) = 3$, but $\gcd(a, c) = 6$.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: $\square$ Irreflexive: $\square$

Symmetric: $\square$ Antisymmetric: $\boxed{\surd}$

Transitive: $\boxed{\surd}$

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be symmetric.

   Solution: For any $x, y \in A$, if xRy, then yRx.

3. (5 points) Suppose that R is the relation on $\mathbb{Z}^4$ such that $(a, b, c, d)R(w, x, y, z)$ if and only if $c = w$, $d = x$, $a = y$, and $b = z$. Is R symmetric? Informally explain why it's true or give a concrete counter-example.

   Solution: R is symmetric Suppose we have $(a, b, c, d)R(w, x, y, z)$. Then $c = w$, $d = x$, $a = y$, and $b = z$. Rewriting these equations gives us $y = a$, $z = b$, $w = c$, and $x = d$. This means that $(w, x, y, z)R(a, b, c, d)$.

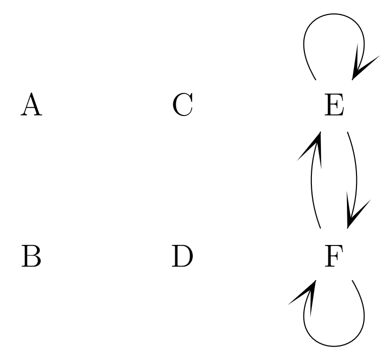
CS 173, Fall 17 Examlet 4, Part B

Name:______________________________________

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☐
Symmetric:	$\surd$	Antisymmetric:	☐
Transitive:	$\surd$		

2. (5 points) Let R be the relation on the integers such that aRb if and only if $2a \equiv -3b \pmod 5$. Find three elements in the equivalence class [7].

Solution:

-3, 2, 7 (for example)

3. (5 points) Suppose that R is the relation on $\mathbb{Z}^3$ such that $(a, b, c)R(x, y, z)$ if and only if $c = x$, $a = y$, and $b = z$. Is R transitive? Informally explain why it's true or give a concrete counter-example.

Solution: R is not transitive. We have

$$(1, 2, 0)R(0, 1, 2) \text{ and } (0, 1, 2)R(2, 0, 1)$$

but not

$$(1, 2, 0)R(2, 0, 1)$$

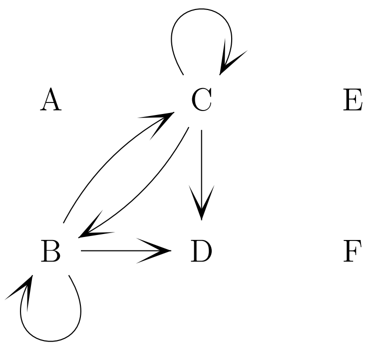
.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☐
Symmetric:	☐	Antisymmetric:	☐
Transitive:	☑ $\surd$		

2. (5 points) Suppose that S is the set of all binary strings (i.e. finite sequences of 1's and 0's). Suppose that $\sim$ is the relation on S where $a \sim b$ if and only if a and b are the same length. For example, `01011` $\sim$ `00010`. List three members of [`1111`].

   Solution: For example, `0101`, `1101`, and `0000`.

3. (5 points) Let T be the relation on $\mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $(x, y) = \alpha(p, q)$ for some real number α. Is T symmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: T is not symmetric. We have $(0, 0)T(p, q)$ by setting α to zero but not $(3, 4)T(0, 0)$.

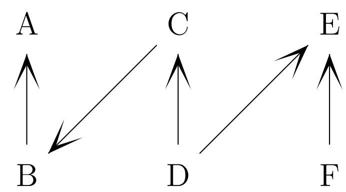
CS 173, Fall 17 Examlet 4, Part B

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	$\surd$
Symmetric:	☐	Antisymmetric:	$\surd$
Transitive:	☐		

2. (5 points) Can a relation with at least one related pair (i.e. at least one arrow in a diagram) be irreflexive, symmetric, and also transitive? Either give such a relation or briefly explain why it's not possible to construct one.

   Solution: No, this is not possible. Suppose R is our relation and let x and y be two elements such that xRy. Then yRx because it's symmetric. Then xRx because it's transitive. But xRx means that R can't be irreflexive.

3. (5 points) Suppose that $\succeq$ is the relation between subsets of the integers such that $A \succeq B$ if and only if $A - B \neq \emptyset$. (A and B are sets of integers, so $A - B$ is a set difference.) Is $\succeq$ transitive? Informally explain why it's true or give a concrete counter-example.

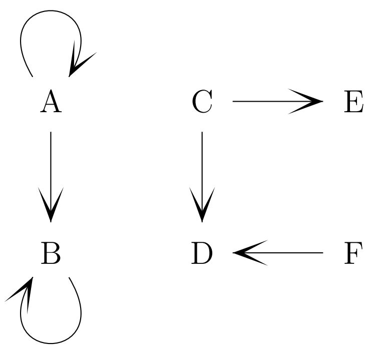
   Solution: $\succeq$ is not transitive. Consider $A = C = \{3\}$ and $B = \{4\}$. Then $A - B = \{3\}$ and $B - C = \{4\}$. So $A \succeq B$ and $B \succeq C$. But $A - C = \emptyset$, so we don't have $A \succeq C$.

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	$\square$	Irreflexive:	$\square$
Symmetric:	$\square$	Antisymmetric:	$\checkmark$
Transitive:	$\checkmark$		

2. (5 points) Recall that $\mathbb{Z}^2$ is the set of all pairs of integers. Let's define the equivalence relation $\sim$ on $\mathbb{Z}^2$ as follows: $(a, b) \sim (p, q)$ if and only $ab = pq$. List three members of $[(5, 6)]$.

   Solution: (5,6), (1,30), (-15,-2)

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ and $y \leq q$. Is T antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is antisymmetric. Suppose that $(x, y)T(p, q)$ and $(p, q)T(x, y)$. Then $x \leq p$ and $y \leq q$, and also $p \leq x$ and $q \leq y$. So $x = y$ and $y = q$. So $(x, y) = (p, q)$.

Name:____________________________________

NetID:______________________ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

$$\begin{tikzcd} A \arrow[r] \arrow[d] \arrow[dr] & C \arrow[d] & E \arrow[d] \\ B \arrow[r] & D & F \end{tikzcd}$$

Reflexive:	$\square$	Irreflexive:	$\boxed{\surd}$
Symmetric:	$\square$	Antisymmetric:	$\boxed{\surd}$
Transitive:	$\boxed{\surd}$		

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be antisymmetric.

   Solution: For any $x, y \in A$, if xRy and yRx, then $x = y$. Or for any $x, y \in A$, if xRy and $x \neq y$, then $y \not R x$.

3. (5 points) Suppose that R is an equivalence relation on the integers. Is it true that $y \in [x]_R$ if and only if $x \in [y]_R$, for any integers x and y? Informally explain why it's true or give a concrete counter-example.

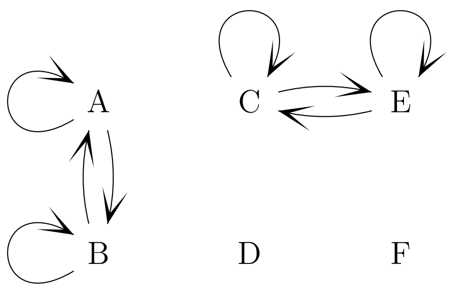
   Solution: This is true. If $y \in [x]_R$, then yRx. But equivalence relations are symmetric, so this implies that xRy, which means that $x \in [y]_R$. The same argument also works in the opposite direction.

Name:____

NetID:______________________ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☐
Symmetric:	√	Antisymmetric:	☐
Transitive:	√		

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be symmetric.

   Solution: For any $x, y \in A$, if xRy, then yRx.

3. (5 points) Suppose that R is the relation on the set of integers such that aRb if and only if $|a - b| \leq 13$ Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

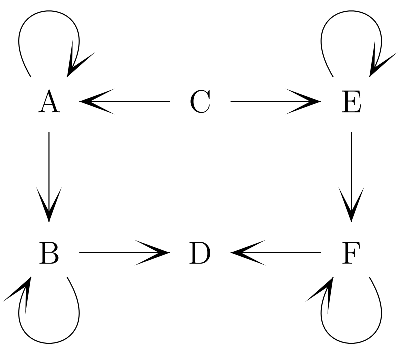
   Solution: No, it is not transitive. Consider $a = 0$, $b = 13$, $c = 26$. Then aRb and bRc. However, $|a - c| = 26$, so $a \not R c$.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☑ $\surd$

Transitive: ☐

2. (5 points) Suppose that S is the set of all binary strings (i.e. finite sequences of 1's and 0's). Suppose that $\sim$ is the relation on S where $a \sim b$ if and only if a and b contain the same number of 1's. For example, `0101` $\sim$ `1000001`. List three members of [`111`].

   Solution: For example, `111`, `1101`, and `01110`.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x - p \leq y - q$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

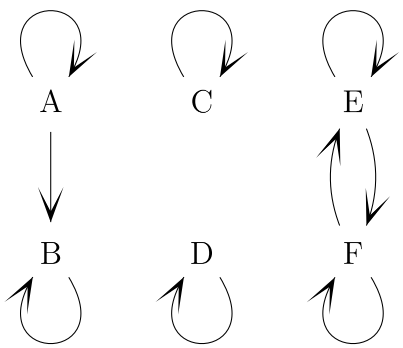
   Solution: This relation is transitive. Suppose that $(x, y)T(p, q)$ and $(p, q)T(m, n)$. Then $x - p \leq y - q$ and also $p - m \leq q - n$. Adding these two equations together, we get $(x - p) + (p - m) \leq (y - q) + (q - n)$. This simplifies to $x - m \leq y - n$. So $(x, y)T(m, n)$.

Name: ____

NetID: ______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	[√]	Irreflexive:	[ ]
Symmetric:	[ ]	Antisymmetric:	[ ]
Transitive:	[√]		

2. (5 points) Let's define the equivalence relation $\sim$ on $\mathbb{R}$ sucn that $x \sim y$ if and only $|x - y| \in \mathbb{Z}$. List three members of $[1.7]$.

   Solution: For example, 1.7, 2.7, and 1009.7.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $xp + yq = 0$. Is T irreflexive? Informally explain why it is, or give a concrete counter-example showing that it is not.

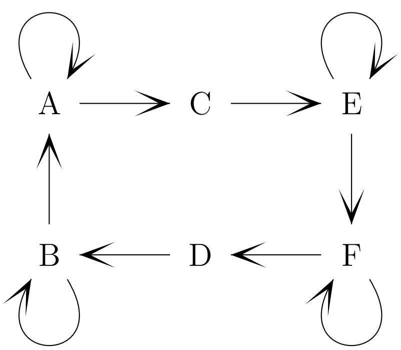
   Solution: This relation is not irreflexive, because $(0, 0)$ is related to itself.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☑

Transitive: ☐

2. (5 points) Can a relation be reflexive, symmetric, and also antisymmetric? Either give such a relation or briefly explain why it's not possible to construct one.

   Solution: Yes, this is possible. Suppose that our set contains three elements a, b, and c. And suppose that we have aRa, bRb, cRc and no other relationships. Then R is reflexive, symmetric and also antisymmetric.

3. (5 points) Let R be the relation on $\mathbb{Z}$ such that xRy if and only if $|x| + |y| = 2$

   Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: No, R is not transitive. We have $0R2$ and $2R0$, but not $0R0$.