CS 173, Spring 19 Examlet 4, colored

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let's define a relation T between natural numbers follows:

aTb if and only if $a = b + 2k$, where k is a natural number

Working directly from this definition, prove that T is antisymmetric.

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

A closed interval of the real line can be represented as a pair (c, r), where c is the center of the interval and r is its radius. Let $X = \{(c, r) \mid c, r \in \mathbb{R}, r \geq 0\}$ be the set of closed intervals represented this way.

Now, let's define the interval containment $\preceq$ on X as follows

$(c, r) \preceq (d, q)$ if and only if $r \leq q$ and $|c - d| + r \leq q$.

Prove that $\preceq$ is transitive.

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

Suppose that T is a relation on the integers which is transitive. Let's define a relation R on the integers as follows:

xRy if and only if there is an integer k such that xTk and kTy.

Prove that R is transitive.

CS 173, Spring 19 Examlet 4, colored

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let T be the relation defined on $\mathbb{Z}^2$ by

$(x, y)T(p, q)$ if and only if $x < p$ or $(x = p$ and $y \leq q)$

Prove that T is antisymmetric.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Define the relation $\sim$ on $\mathbb{Z}$ by

$x \sim y$ if and only if $5 \mid (3x + 7y)$

Working directly from the definition of divides, prove that $\sim$ is transitive.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $A = \{(x, y, z) \in \mathbb{Z}^3 : x \leq y \leq z\}$. Let's define a relation R on A as follows:

$(a, b, c)R(x, y, z)$ if and only if $a \leq x$ and $z \leq b$.

Working directly from this definition, prove that R is antisymmetric.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers.

Define a relation $\gg$ on A as follows:

$(x, y) \gg (p, q)$ if and only if there exists an integer $n \geq 1$ such that $(x, y) = (np, nq)$.

Prove that $\gg$ is antisymmetric.

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

Let $A = \{(x, y, z) \in \mathbb{Z}^3 : x \leq y \leq z\}$. Let's define a relation R on A as follows:

$(a, b, c)R(x, y, z)$ if and only if $a \leq x$ and $z \leq b$.

Working directly from this definition, prove that R is transitive.

Name:____

NetID:______________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4**

Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(x, y)T(p, q)$ if and only if $(xy)(p + q) = (pq)(x + y)$

Prove that T is transitive.

Name:____

NetID:________________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4**

Suppose that T is a relation on the integers which is antisymmetric. Let's define a relation R on pairs of integers such that $(p, q)R(a, b)$ if and only if $(a + b)T(p + q)$ and bTq. Prove that R is antisymmetric.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(x, y)T(p, q)$ if and only if $(xy)(p + q) < (pq)(x + y)$

Prove that T is transitive.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Suppose that n is some integer ≥ 2. Let's define the relation R_n on the integers such that aR_nb if and only if $a \equiv b + 1 \pmod{n}$. Prove the following claim

Claim: If R_n is symmetric, then $n = 2$.

You must work directly from the definition of congruence mod k, using the following version of the definition: $x \equiv y \pmod{k}$ iff $x - y = mk$ for some integer m. You may use the following fact about divisibility: for any non-zero integers p and q, if $p \mid q$, then $|p| \leq |q|$.

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $A = \{(x, y) \in \mathbb{R}^2 \mid x + y = 10\}$. Consider the relation T on A defined by

$(a, b)T(p, q)$ if and only if $aq \geq bp$

Prove that T is antisymmetric.

CS 173, Spring 18 Examlet 4, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let T be the relation defined on $\mathbb{N}^2$ by

$(x, y)T(p, q)$ if and only if $x < p$ or $(x = p$ and $y \leq q)$

Prove that T is transitive.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let's define a relation T between natural numbers follows:

aTb if and only if $a = b + 2k$, where k is a natural number

Working directly from this definition, prove that T is antisymmetric.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Suppose that T is a relation on the integers which is antisymmetric. Let's define a relation R on pairs of integers such that $(p, q)R(a, b)$ if and only if $(a + b)T(p + q)$ and bTq. Prove that R is antisymmetric.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(a, b)T(p, q)$ if and only if $ab \mid p$

Working directly from the definition of divides, prove that T is transitive.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let's define a relation R on $\mathbb{Z}^3$ as follows:

$(a, b, c)R(x, y, z)$ if and only if $c = x$, $a = y$, and $b = z$.

Working directly from this definition, prove that R is antisymmetric.

CS 173, Fall 17 Examlet 4, Part A

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

Define the relation $\sim$ on $\mathbb{Z}$ by

$x \sim y$ if and only if $5 \mid (3x + 7y)$

Working directly from the definition of divides, prove that $\sim$ is transitive.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let's define the relation $\succeq$ on $\mathbb{N}^2$ by

$(x, y) \succeq (a, b)$ if and only if $x - a \geq 2$ and $y \geq b$.

Prove that $\succeq$ is transitive.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

A closed interval of the real line can be represented as a pair (c, r), where c is the center of the interval and r is its radius. Let $X = \{(c, r) \mid c, r \in \mathbb{R}, r \geq 0\}$ be the set of closed intervals represented this way.

Now, let's define the interval containment $\preceq$ on X as follows

$(c, r) \preceq (d, q)$ if and only if $r \leq q$ and $|c - d| + r \leq q$.

Prove that $\preceq$ is antisymmetric.

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

Let $A = \mathbb{N} \times \mathbb{N}$, i.e. pairs of natural numbers.

Define a relation $\gg$ on A as follows:

$(x, y) \gg (p, q)$ if and only if there exists an integer $n \geq 1$ such that $(x, y) = (np, nq)$.

Prove that $\gg$ is transitive.

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Suppose that T is a relation on the integers which is transitive. Let's define a relation R on pairs of integers such that $(p, q)R(a, b)$ if and only if $(a+b)T(p+q)$. Prove that R is transitive.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let T be the relation defined on $\mathbb{Z}^2$ by

$(x, y)T(p, q)$ if and only if $x < p$ or ($x = p$ and $y \leq q$)

Prove that T is antisymmetric.

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

Let $A = \{(a,b) \in \mathbb{R}^2 \ : \ a > 0, b > 0, \text{ and } a + b = 90\}$. That is, an element of A is a pair of positive reals that sum to 90 (e.g. the non-right angles in a right triangle). Now, let's define a relation $\sim$ on A as follows:

$(a,b) \sim (p,q)$ if and only if $a = p$ or $a = q$.

Prove that R is transitive.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

For any two real numbers with $a \leq b$, the closed interval $[a, b]$ is defined by $[a, b] = \{x \in \mathbb{R} \ : \ a \leq x \leq b\}$. Let J be the set containing all closed intervals $[a, b]$. Let's define the relation F on J as follows:

$[s, t]F[p, q]$ if and only if $q \leq s$

Prove that F is antisymmetric.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(x, y)T(p, q)$ if and only if $x \leq p$ and $xy \leq pq$

Prove that T is antisymmetric.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

A closed interval of the real line can be represented as a pair (c, r), where c is the center of the interval and r is its radius. Let $X = \{(c, r) \mid c, r \in \mathbb{R}, r \geq 0\}$ be the set of closed intervals represented this way.

Now, let's define the interval "starts" relation $\preceq$ on X as follows

$(c, r) \preceq (d, q)$ if and only if $c + q = d + r$ and $c + r \leq d + q$

Prove that $\preceq$ is transitive.