Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use proof by contrapositive to prove the following claim, using your best mathematical style and working directly from the definitions of "odd" and "even." (You may assume that odd and even are opposites.)

For all integers p and q, if $p^2(q^2 - 4)$ is odd, then p and q are odd.

You must begin by explicitly stating the contrapositive of the claim:

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $x \equiv y \pmod{k}$ if and only if $x = y + nk$ for some integer n.

> For all integers x, y, p, q and m, with $m > 0$, if $x \equiv p \pmod{m}$ and $y \equiv q \pmod{m}$, then $x^2 + xy \equiv p^2 + pq \pmod{m}$.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) For any two real numbers x and y, the harmonic mean is $H(x,y) = \frac{2xy}{x+y}$. Using this definition and your best mathematical style, prove the following claim:

For any real numbers x and y, if $0 < x < y$, then $H(x,y) < y$.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim by contrapositive.

> For all real numbers x and y, if x is not rational, then $2x + 3y$ is not rational or y is not rational.

You must begin by explicitly stating the contrapositive of the claim:

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) A triple (a, b, c) of positive integers is Pythagorean if $a^2 + b^2 = c^2$. Use proof by contrapositive to prove the following claim, using your best mathematical style and working directly from the definitions of "odd" and "even." (You may assume that odd and even are opposites.)

For any Pythagorean triple (a, b, c), if c^2 is odd, then a is even or b is even.

You must begin by explicitly stating the contrapositive of the claim:

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) For any two real numbers x and y, the harmonic mean is $H(x, y) = \frac{2xy}{x+y}$. Using this definition and your best mathematical style, prove the following claim:

For any real numbers x and y, if $0 < x < y$, then $x < H(x, y)$.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $x \equiv y \pmod{k}$ if and only if $x = y + nk$ for some integer n.

For all integers a, b, c, p and k (c positive), if $ap \equiv b \pmod{c}$ and $k \mid a$ and $k \mid c$, then $k \mid b$.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Prove the following claim, using your best mathematical style. Hint: look at remainders and use proof by cases.

For any integer n, $n^2 + 2$ is not divisible by 4.

Name:____

NetID:______________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4** **5**

(15 points) Use proof by contrapositive to prove the following claim, using your best mathematical style and working directly from the definitions of "odd" and "even." (You may assume that odd and even are opposites.)

For all integers x and y, if $3x + y^2 + 2$ is odd, then x is even or y is even.

You must begin by explicitly stating the contrapositive of the claim:

Name:____

NetID:____________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4** **5**

(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim:

For all rational numbers x, y and z, if y is non-zero, then $5(\frac{x}{y}) - 2z$ is rational.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Prove the following claim, using direct proof and your best mathematical style.

For any integers m and k, if $k \leq 7$ and $0 < m - 3 \leq \frac{k}{7}$, then $m^2 - 9 \leq k$.

Name:____

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $a \equiv b \pmod{k}$ if and only if $a - b = nk$ for some integer n.

Claim: For all integers a, b, c, d, j and k (j and k positive), if $a \equiv b \pmod{k}$ and $c \equiv d \pmod{k}$ and $j \mid k$, then $a + c \equiv b + d \pmod{j}$.

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use proof by contrapositive to prove the following claim, using your best mathematical style and working directly from the definition of "divides." ($p \nmid q$ is the negation of $p \mid q$.)

For all integers k, a, b, if $k \nmid ab$, then $k \nmid a$ and $k \nmid b$.

You must begin by explicitly stating the contrapositive of the claim.

CS 173, Spring 18 **Examlet 2, Part A**

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim:

For all real numbers p and q ($p \neq -1$), if $\frac{2}{p+1}$ and $p + q$ are rational, then q is rational.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) A natural number n is "snarky" if and only if $n = 3m + 1$, where m is a natural number. Use this definition and your best mathematical style to prove the following claim:

For all natural numbers x and y, if x and y are snarky, then $(x + y)^2$ is snarky.

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) A pair of positive integers (a, b) is defined to be a *partition* of a positive integer n if and only if $ab = n$. Using this definition and your best mathematical style, prove the following claim:

> For all positive integers a, b, and n, if (a, b) is a partition of n and $1 < a < \sqrt{n}$, then $\sqrt{n} < b < n$.

CS 173, Fall 17 Examlet 2, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Prove the following claim, using direct proof and your best mathematical style.

For any integers x, y, and z, if $100x + 10y + z$ is divisible by 9, then $x + y + z$ is divisible by 9.

Hint: analyze the difference between $100x + 10y + z$ and $x + y + z$.

Name:____

NetID:______________________ **Lecture:** A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Prove the following claim, working directly from the definitions of "remainder" and "divides", and using your best mathematical style.

For all real numbers k, m, n and r ($n \neq 0$), if $r = \text{remainder}(m, n)$, $k \mid m$, and $k \mid n$, then $k \mid r$.

Name:____

NetID:______________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim:

For all real numbers x and y, $x \neq 0$, if x and $\frac{y+1}{2}$ are rational, then $\frac{5}{x} + y$ is rational.

CS 173, Fall 17 Examlet 2, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Recall that $\gcd(m, n)$ is the largest integer that divides both m and n. Use this definition, the definition of divides, and your best mathematical style to prove the following claim by contrapositive.

For all integers p and q, if $p + 6q = 23$ then $\gcd(p, q) \neq 7$.

You must begin by explicitly stating the contrapositive of the claim:

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) An integer k is a perfect square if $k = n^2$ where n is a non-negative integer. Prove the following claim:

For any integer p, if $p \geq 8$ and $p + 1$ is a perfect square, then p is composite (aka not prime).

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $a \equiv b \pmod{k}$ if and only if $a = b + nk$ for some integer n.

Claim: For all integers a, b, c, d, and k (k positive), if $a \equiv b \pmod{k}$ and $c \equiv d \pmod{k}$ then $a^2 + c \equiv b^2 + d \pmod{k}$.

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Notice that, for any integer p, $\lfloor p \rfloor = \lfloor p + \frac{1}{2} \rfloor = p$. Using this fact and your best mathematical style, prove the following claim:

For any integer n, if n is odd, then $\left\lfloor \frac{n}{2} \right\rfloor^2 + \left\lfloor \frac{n}{2} \right\rfloor \geq \frac{1}{2} \left\lfloor \frac{n^2}{2} \right\rfloor$

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $a \equiv b \pmod{k}$ if and only if $a = b + nk$ for some integer n.

Claim: for all integers a, b, c, d, j, and k (j and k positive), if $a \equiv b \pmod{j}$, $c \equiv d \pmod{k}$, and $j \mid k$, then $a + c \equiv b + d \pmod{j}$.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim:

For all rational numbers x, y and z, if y is non-zero, then $5(\frac{x}{y}) - 2z$ is rational.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim by contrapositive.

> For all real numbers x and y, if x is not rational, then $2x + 3y$ is not rational or y is not rational.

You must begin by explicitly stating the contrapositive of the claim:

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Prove the following claim, using your best mathematical style. Hint: look at remainders and use proof by cases.

For any integer n, $n^2 + 2$ is not divisible by 4.

Name:____

NetID:________________________ **Lecture:** **A** **B**

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(15 points) A triple (a, b, c) of positive integers is Pythagorean if $a^2 + b^2 = c^2$. Use proof by contrapositive to prove the following claim, using your best mathematical style and working directly from the definitions of "odd" and "even." (You may assume that odd and even are opposites.)

For any Pythagorean triple (a, b, c), if c is odd, then a is even or b is even.

You must begin by explicitly stating the contrapositive of the claim: