

Name: _____

NetID: _____ Lecture: A

Discussion: Monday & Wednesday 1:30 2:30

(20 points) Recall that F_n is the nth Fibonacci number, and these start with $F_0 = 0$, $F_1 = 1$. Use (strong) induction to prove the following claim:

Claim: $F_n < (5/3)^n$ for any natural number n .

Proof by induction on n .

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name: _____

NetID: _____ Lecture: A

Discussion: Monday & Wednesday 1:30 2:30

(10 points) Suppose we have a function F defined (for n a power of 2) by

$$\begin{aligned} F(2) &= c \\ F(n) &= F(n/2) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$F(n) = F(n/2^k) + \sum_{i=0}^{k-1} n \frac{1}{2^i}$$

Finish finding the closed form for F . Show your work and simplify your answer.