Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, bc) = 1$, then $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$.

   Solution: This is true. If $\gcd(a, bc) = 1$, then a doesn't share any prime factors with bc. Since the prime factors of b are a subset of these, they also can't overlap with the prime factors of a. Similarly for c.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1568, 546)$. Show your work.

   Solution:

   $1568 - 546 \times 2 = 1568 - 1092 = 476$

   $546 - 476 = 70$

   $476 - 70 \times 6 = 476 - 420 = 56$

   $70 - 56 = 14$

   $56 - 14 \times 3 = 0$

   So the GCD is 14.

3. (4 points) Check the (single) box that best characterizes each item.

   For any positive integers p and q, if $\text{lcm}(p, q) = pq$, then p and q are relatively prime. true [√] false [ ]

   Zero is a factor of 7. true [ ] false [√]

CS 173, Spring 19 Examlet 2, white

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, $\gcd(ca, cb) = c \cdot \gcd(a, b)$

   Solution: This is true.

   c divides both ca and cb. So $\gcd(ca, cb)$ must have the form cm, where m is an integer. But then cm is the largest integer that divides both ca and cb if and only if m is the largest integer that divides both a and b.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2015, 837)$. Show your work.

   Solution:

   $2015 - 837 \times 2 = 2015 - 1674 = 341$

   $837 - 341 \times 2 = 837 - 682 = 155$

   $341 - 155 \times 2 = 341 - 310 = 31$

   $155 - 31 \times 5 = 0$

   So the GCD is 31.

3. (4 points) Check the (single) box that best characterizes each item.

   $\gcd(0, 0)$ 0 ☐ k ☐ undefined ☑

   $25 \equiv 4 \pmod 7$ true ☑ false ☐

CS 173, Spring 19 Examlet 2, white

Name:______________________________

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integer k, $(k-1)^2 \equiv 1 \pmod{k}$.

   Solution: This is true. Notice that $(k-1) - (-1) = k$. So $k - 1 \equiv (-1) \pmod{k}$. Therefore $(k-1)^2 \equiv (-1)^2 \equiv 1 \pmod{k}$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1183, 351)$. Show your work.

   Solution:

   $1183 - 3 \times 351 = 1183 - 1053 = 130$

   $351 - 2 \times 130 = 351 - 260 = 91$

   $130 - 91 = 39$

   $91 - 3 \times 39 = 91 - 78 = 13$

   $39 - 3 \times 13 = 0$

   So the GCD is 13.

3. (4 points) Check the (single) box that best characterizes each item.

   If a and b are positive and $r = \text{remainder}(a, b)$, then $\gcd(b, r) = \gcd(b, a)$ true [√] false [ ]

   $7 \mid -7$ true [√] false [ ]

CS 173, Spring 19 Examlet 2, white

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, b) = 1$ and $\gcd(b, c) = 1$, then $\gcd(a, c) = 1$.

   Solution: This is false. Consider $a = c = 3$ and $b = 2$. Then a and b have no common factors, i.e. $\gcd(a, b) = 1$. Also b and c have no common factors, i.e. $\gcd(b, c) = 1$. But $\gcd(a, c) = 3$.

2. (6 points) Write pseudocode (iterative or recursive) for a function gcd(a,b) that implements the Euclidean algorithm. Assume both inputs are positive.

   Solution:

   gcd(a,b)

   x=a

   y=b

   while ($b > 0$)

   r = remainder(a,b)
   a = b
   b = r

   return a

3. (4 points) Check the (single) box that best characterizes each item.

   $\gcd(p, q) = \dfrac{pq}{\text{lcm}(p, q)}$ (p and q positive integers) always ☑ sometimes ☐ never ☐

   $-7 \equiv 13 \pmod 6$ true ☐ false ☑

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers a, b, and c, if $a \mid bc$, then $a \mid b$ or $a \mid c$

   Solution: This is false. Consider $a = 6$, $b = 3$, $c = 2$. Then $a|bc$, but a doesn't divide either b or c.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1702, 1221)$. Show your work.

   Solution: $1702 - 1221 = 481$

   $1221 - 481 \times 2 = 1221 - 962 = 259$

   $481 - 259 = 222$

   $259 - 222 = 37$

   $222 - 6 \times 37 = 0$

   So the GCD is 37.

3. (4 points) Check the (single) box that best characterizes each item.

   If a and b are positive integers and $r = \text{remainder}(a, b)$, then $\gcd(b, r) = \gcd(r, a)$ — true ☐ false ☑

   $29 \equiv 2 \pmod 9$ — true ☑ false ☐

Name: ______________________________

NetID: ____________________ **Lecture:** A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a,b) = n$ and $\gcd(a,c) = p$, then $\gcd(a,bc) = np$.

   Solution: This is false. Consider $a = b = c = 3$. Then if $\gcd(a,b) = 3$ and $\gcd(a,c) = 3$, but $\gcd(a,bc)$ is 3, not 9.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2380, 391)$. Show your work.

   Solution:

   $2380 - 391 \times 6 = 2380 - 2346 = 34$

   $391 - 34 \times 11 = 391 - 374 = 17$

   $34 - 17 \times 2 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   For any positive integers p, q, and k, if $p \equiv q \pmod{k}$, then $p^2 \equiv q^2 \pmod{k}$ true [$\surd$] false [ ]

   $2 \mid -4$ true [$\surd$] false [ ]

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For any positive integers p and q, $p \equiv q \pmod 1$.

   Solution: This is true. $p \equiv q \pmod 1$ is equivalent to $p - q = n \times 1 = n$ for some integer n. But we can always find an integer that will make this equation balance!

2. (6 points) Use the Euclidean algorithm to compute $\gcd(7917, 357)$. Show your work.

   Solution:

   $7917 - 22 \times 357 = 63$

   $357 - 5 \times 63 = 42$

   $63 - 42 = 21$

   $42 - 2 \times 21 = 0$

   So the GCD is 21.

3. (4 points) Check the (single) box that best characterizes each item.

   If a and b are positive integers and $r = \text{remainder}(a, b)$, then $\gcd(a, b) = \gcd(r, a)$ true ☐ false ☑ ($\surd$)

   $-2 \equiv 2 \pmod 4$ true ☑ ($\surd$) false ☐

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is false, or give a concrete example showing that it is not.

   There is an integer n such that $n \equiv 5 \pmod 6$ and $n \equiv 6 \pmod 7$?

   Solution: This is true. Consider $n = 41$. $41 \equiv 5 \pmod 6$ and $41 \equiv 6 \pmod 7$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1224, 850)$. Show your work.

   Solution:

   $1224 - 850 = 374$

   $850 - 374 \times 2 = 850 - 748 = 102$

   $374 - 102 \times 3 = 374 - 306 = 68$

   $102 - 68 = 34$

   $68 - 34 \times 2 = 0$

   So the GCD is 34.

3. (4 points) Check the (single) box that best characterizes each item.

   Two positive integers p and q are relatively prime if and only if $\gcd(p, q) = 1$. true $\boxed{\surd}$ false $\boxed{}$

   $0 \mid 0$ true $\boxed{\surd}$ false $\boxed{}$

Name:____

NetID:____________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4** **5**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, bc) > 1$, then $\gcd(a, b) > 1$ and $\gcd(a, c) > 1$.

   Solution: This is false. Consider $a = b = 3$ and $c = 2$. Then $bc = 6$. So $\gcd(a, bc) = 3 > 1$ but $\gcd(a, c) = 1$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1012, 299)$. Show your work.

   Solution: $1012 - 3 \times 299 = 1012 - 897 = 115$

   $299 - 2 \times 115 = 299 - 230 = 69$

   $115 - 69 = 46$

   $69 - 46 = 23$

   $46 - 2 \times 23 = 0$

   So the GCD is 23.

3. (4 points) Check the (single) box that best characterizes each item.

   $7 \mid 0$ true [√] false [ ]

   $k \equiv -k \pmod{k}$ always [√] sometimes [ ] never [ ]

Name:____

NetID:____________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4** **5**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all non-zero integers a and b, if $a \mid b$ and $b \mid a$, then $a = b$.

   Solution: This is false. Consider $a = 3$ and $b = -3$. Then $a \mid b$ and $b \mid a$, but $a \neq b$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2737, 2040)$. Show your work.

   Solution:

   $2737 - 2040 = 697$

   $2040 - 697 \times 2 = 2040 - 1394 = 646$

   $697 - 646 = 51$

   $646 - 51 \times 12 = 646 - 612 = 34$

   $51 - 34 = 17$

   $34 - 17 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   $29 \equiv 2 \pmod 9$ true $\boxed{\surd}$ false $\boxed{}$

   Two positive integers p and q are relatively prime if and only if $\gcd(p, q) > 1$. true $\boxed{}$ false $\boxed{\surd}$

Name: ____

NetID: ______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Let a and b be integers, $b > 0$. The formula $a = bq + r$ partially defines the quotient q and the remainder r of a divided by b. What other constraint must we add to completely determine q and r?

   Solution: $0 \leq r < b$

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2262, 546)$. Show your work.

   Solution:

   $2262 - 546 \times 4 = 2262 - 2184 = 78$

   $546 - 7 \times 78 = 0$

   So the GCD is 78.

3. (4 points) Check the (single) box that best characterizes each item.

   $7 \mid 0$ true [√] false [ ]

   $7 \equiv -7 \pmod{k}$ always [ ] sometimes [√] never [ ]

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, bc) > 1$, then $\gcd(a, b) > 1$ or $\gcd(a, c) > 1$.

   Solution: This is true. If $\gcd(a, bc) > 1$, then there is some prime $p > 1$ that divides both a and bc. Since p is prime, p must divide a or b. So $\gcd(a, b) > 1$ or $\gcd(a, c) > 1$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2079, 759)$. Show your work.

   Solution:

   $2079 - 759 \times 2 = 2079 - 1518 = 561$

   $759 - 561 = 198$

   $561 - 198 \times 2 = 561 - 396 = 165$

   $198 - 165 = 33$

   $165 - 33 \times 5 = 0$

   So the GCD is 33.

3. (4 points) Check the (single) box that best characterizes each item.

$-2 \equiv 2 \pmod 9$	true ☐	false ☑
If a and b are positive and $r = \text{remainder}(a, b)$, then $\gcd(b, r) = \gcd(b, a)$	true ☑	false ☐

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all natural numbers a and b, if $a \mid b$ and $b \mid a$, then $a = b$.

   Solution: This is true. If $a \mid b$ and $b \mid a$, then $b = pa$ and $a = qb$, where p and q are integers. Since a and b are natural numbers and therefore not negative, p and q cannot be negative. So $b = pqb$, so $pq = 1$. So $p = q = 1$ and therefore $a = b$. [This is more detail than you'd need for full credit.]

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2385, 636)$. Show your work.

   Solution:

   $2385 - 3 \times 636 = 2385 - 1908 = 477$

   $636 - 477 = 159$

   $477 - 3 \times 159 = 0$

   So the GCD is 159.

3. (4 points) Check the (single) box that best characterizes each item.

   $-7 \equiv 13 \pmod 5$ true [√] false [ ]

   For any integers p and q, if $p \mid q$ then $p \leq q$. true [ ] false [√]

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers s, t, p, q, if $s \equiv t \pmod{p}$ and $p \mid q$, then $s \equiv t \pmod{q}$.

   Solution: This is false.

   Informally, since q is larger than p, congruence mod q makes finer distinctions among numbers than p does.

   More formally, consider $s = 1$, $t = 4$, $p = 3$ and $q = 6$. Then $3 \mid 6$ and s and t are congruent mod 3, but but s and t aren't congruent mod 6.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(221, 1224)$. Show your work.

   Solution:

   $1224 - 5 \times 221 = 1224 - 1105 = 119$

   $221 - 119 = 102$

   $119 - 102 = 17$

   $102 - 17 \times 6 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   $-7 \mid 0$ true [$\surd$] false [ ]

   $k \equiv -k \pmod{7}$ always [ ] sometimes [$\surd$] never [ ]

CS 173, Spring 18 Examlet 2, Part B

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers a, b, and c, if $a \mid c$ and $b \mid c$, then $ab \mid c$

   Solution: This is not true. Consider $a = b = c = 5$. Then $a \mid c$ and $b \mid c$. But $ab = 25$ and $c = 5$. So ab does not divide c.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(7839, 1474)$. Show your work.

   Solution:

   $7839 - 5 \times 1474 = 7839 - 7370 = 469$

   $1474 - 3 \times 469 = 1474 - 1407 = 67$

   $469 - 7 \times 67 = 0$

   So the GCD is 67.

3. (4 points) Check the (single) box that best characterizes each item.

   $-11 \equiv 4 \pmod 7$ true ☐ false ☑

   For any positive integers p, q, and k, if $p \equiv q \pmod k$, then $p^2 \equiv q^2 \pmod k$ true ☑ false ☐

CS 173, Spring 18 **Examlet 2, Part B**

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers s, t, p, q, if $s \equiv t \pmod{p}$ and $q \mid p$, then $s \equiv t \pmod{q}$.

   Solution: This is true.

   Informally, since q is smaller, congruence mod q makes coarser distinctions than congruence mod q. So this is in the right direction and the relationship $q \mid p$ ensures that the details work out.

   More formally, from $s \equiv t \pmod{p}$ and $q \mid p$, we get that $s = t + pk$ and $p = qj$, where k and j are integers. So $s = t + q(jk)$, which means that $s \equiv t \pmod{q}$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(4340, 1155)$. Show your work.

   Solution:

   $4340 - 1155 \times 3 = 4340 - 3465 = 875$

   $1155 - 875 = 280$

   $875 - 280 \times 3 = 875 - 840 = 35$

   $280 - 35 \times 24 = 0$

   So the GCD is 35.

3. (4 points) Check the (single) box that best characterizes each item.

   $-7 \equiv 13 \pmod{5}$ true [√] false [ ]

   $\gcd(k, 0)$ for k positive 0 [ ] k [√] undefined [ ]

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all non-zero integers a and b, if $a \mid b$ and $b \mid a$, then $a = b$.

   Solution: This is false. Consider $a = 3$ and $b = -3$. Then $a \mid b$ and $b \mid a$, but $a \neq b$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2015, 837)$. Show your work.

   Solution:

   $2015 - 837 \times 2 = 2015 - 1674 = 341$

   $837 - 341 \times 2 = 837 - 682 = 155$

   $341 - 155 \times 2 = 341 - 310 = 31$

   $155 - 31 \times 5 = 0$

   So the GCD is 31.

3. (4 points) Check the (single) box that best characterizes each item.

   $0 \mid 0$ true ☑ false ☐

   Two positive integers p and q are relatively prime if and only if $\gcd(p, q) > 1$. true ☐ false ☑

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a,b) = n$ and $\gcd(a,c) = p$, then $\gcd(a,bc) = np$.

   Solution: This is false. Consider $a = b = c = 3$. Then if $\gcd(a,b) = 3$ and $\gcd(a,c) = 3$, but $\gcd(a,bc)$ is 3, not 9.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1609, 563)$. Show your work.

   Solution:

   $1609 - 2 \times 563 = 1609 - 1126 = 483$

   $563 - 483 = 80$

   $483 - 6 \times 80 = 3$

   $80 - 26 \times 3 = 80 - 78 = 2$

   $3 - 2 = 1$

   $2 - 2 \times 1 = 0$

   So the GCD is 1.

3. (4 points) Check the (single) box that best characterizes each item.

   For any positive integers p and q, if $\text{lcm}(p,q) = pq$, then p and q are relatively prime. true [√] false [ ]

   $(5 \times 5) \equiv 1 \pmod 6$ true [√] false [ ]

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, $\gcd(ca, cb) = c \cdot \gcd(a, b)$

   Solution: This is true.

   c divides both ca and cb. So $\gcd(ca, cb)$ must have the form cm, where m is an integer. But then cm is the largest integer that divides both ca and cb if and only if m is the largest integer that divides both a and b.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2380, 391)$. Show your work.

   Solution:

   $2380 - 391 \times 6 = 2380 - 2346 = 34$

   $391 - 34 \times 11 = 391 - 374 = 17$

   $34 - 17 \times 2 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   $25 \equiv 4 \pmod 7$ true $\boxed{\surd}$ false $\boxed{}$

   Two positive integers p and q are relatively prime if and only if $\gcd(p, q) = 1$. true $\boxed{\surd}$ false $\boxed{}$

CS 173, Fall 17 Examlet 2, Part B

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers a, b, and c, if $a \mid bc$, then $a \mid b$ or $a \mid c$

   Solution: This is false. Consider $a = 6$, $b = 3$, $c = 2$. Then $a|bc$, but a doesn't divide either b or c.

2. (6 points) Write pseudocode (iterative or recursive) for a function gcd(a,b) that implements the Euclidean algorithm. Assume both inputs are positive.

   Solution:

```
gcd(a,b)
    x=a
    y=b
    while (b > 0)
        r = remainder(a,b)
        a = b
        b = r
    return a
```

3. (4 points) Check the (single) box that best characterizes each item.

$2 \mid -4$	true $\boxed{\surd}$	false $\boxed{}$
If a and b are positive and $r = \text{remainder}(a, b)$, then $\gcd(a, b) = \gcd(r, a)$	true $\boxed{}$	false $\boxed{\surd}$

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Explain how to use the Euclidean algorithm to test whether two positive integers p and q are relatively prime.

   Solution: Use the Euclidean algorithm to compute $\gcd(p, q)$. If the output is 1, p, and q are relatively prime.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1702, 1221)$. Show your work.

   Solution: $1702 - 1221 = 481$

   $1221 - 481 \times 2 = 1221 - 962 = 259$

   $481 - 259 = 222$

   $259 - 222 = 37$

   $222 - 6 \times 37 = 0$

   So $\gcd(1702, 1221) = 37$

3. (4 points) Check the (single) box that best characterizes each item.

If p, q, and k are primes, then $gcd(pq, qk) =$	q ☐	pq ☐	pqk ☐	$q\gcd(p, k)$ ☑
$29 \equiv 2 \pmod 9$	true ☑	false ☐		

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) For any real numbers x and y, let's define the operation $\oslash$ by the equation $x \oslash y = x^2 + y^2$. Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For any real numbers x,y, and z, $(x \oslash y) \oslash z = x \oslash (y \oslash z)$

   Solution: This is not true. Consider $x = y = 1$ and $z = 2$. Then $(x \oslash y) \oslash z = (x^2 + y^2)^2 + z^2 = (1 + 1)^2 + 2^2 = 8$. But $x \oslash (y \oslash z) = x^2 + (y^2 + z^2)^2 = 1^2 + (1^2 + 2^2)^2 = 1 + 5^2 = 26$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2737, 2040)$. Show your work.

   Solution:

   $2737 - 2040 = 697$

   $2040 - 697 \times 2 = 2040 - 1394 = 646$

   $697 - 646 = 51$

   $646 - 51 \times 12 = 646 - 612 = 34$

   $51 - 34 = 17$

   $34 - 17 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   $\gcd(p, q) = \frac{pq}{\text{lcm}(p, q)}$ (p and q positive integers) always [√] sometimes [ ] never [ ]

   $-3 \equiv 3 \pmod 4$ true [ ] false [√]

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any real numbers x and y, if x or y is irrational, then xy is irrational.

   Solution: This is not true. Consider $x = y = \sqrt{2}$. Then x or y is irrational (because they are both irrational). But $xy = 2$ which is rational.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1012, 299)$. Show your work.

   Solution: $1012 - 3 \times 299 = 1012 - 897 = 115$

   $299 - 2 \times 115 = 299 - 230 = 69$

   $115 - 69 = 46$

   $69 - 46 = 23$

   $46 - 2 \times 23 = 0$

   So $\gcd(1012, 299) = 23$

3. (4 points) Check the (single) box that best characterizes each item.

   $7 \mid -7$ true $\boxed{\surd}$ false $\square$

   $k \equiv -k \pmod{k}$ always $\boxed{\surd}$ sometimes $\square$ never $\square$

Name:____

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, b) = 1$ and $\gcd(b, c) = 1$, then $\gcd(a, c) = 1$.

   Solution: This is false. Consider $a = c = 3$ and $b = 2$. Then a and b have no common factors, i.e. $\gcd(a, b) = 1$. Also b and c have no common factors, i.e. $\gcd(b, c) = 1$. But $\gcd(a, c) = 3$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(3927, 637)$. Show your work.

   Solution:

   $3927 - 6 \times 637 = 3927 - 3822 = 105$

   $637 - 6 \times 105 = 7$

   $105 - 15 \times 7 = 0$

   So the GCD is 7.

3. (4 points) Check the (single) box that best characterizes each item.

$\gcd(k, 0)$ for k non-zero	0 ☐	k ☑	undefined ☐
$7 \equiv 5 \pmod 1$	true ☑	false ☐	

Name:____

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Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Let a and b be integers, $b > 0$. We used two formulas to define the quotient q and the remainder r of a divided by b. One of these is $a = bq + r$. What is the other?

   Solution: $0 \leq r < b$

2. (6 points) Use the Euclidean algorithm to compute $\gcd(4263, 667)$. Show your work.

   Solution:

   $4263 - 6 \times 667 = 261$

   $667 - 2 \times 261 = 145$

   $261 - 145 = 116$

   $145 - 116 = 29$

   $116 - 4 \times 29 = 0$

   So the GCD is 29.

3. (4 points) Check the (single) box that best characterizes each item.

   For all prime numbers p, there are exactly two natural numbers q such that $q \mid p$. true $\boxed{\surd}$ false $\boxed{}$

   Zero is a factor of 7. true $\boxed{}$ false $\boxed{\surd}$

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers a, b, q and r, if $a = bq + r$, then $\gcd(a, b) = \gcd(a, r)$.

   Solution: This is false. Consider $a = 18$, $b = 5$, $q = 3$, and $r = 3$. Then we have $18 = 5 \cdot 3 + 3$. $\gcd(a, b) = 1$, but $\gcd(a, r) = 3$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1568, 546)$. Show your work.

   Solution:

   $1568 - 546 \times 2 = 1568 - 1092 = 476$

   $546 - 476 = 70$

   $476 - 70 \times 6 = 476 - 420 = 56$

   $70 - 56 = 14$

   $56 - 14 \times 3 = 0$

   So the GCD is 14.

3. (4 points) Check the (single) box that best characterizes each item.

   If a and b are positive and $r = \text{remainder}(a, b)$, then $\gcd(b, r) = \gcd(b, a)$ true ☑ false ☐

   $-7 \equiv 13 \pmod 6$ true ☐ false ☑

Name:____

NetID:_____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is false, or give a concrete example showing that it is not.

   There is an integer n such that $n \equiv 5 \pmod 6$ and $n \equiv 2 \pmod{10}$?

   Solution: There is no such n. If $n \equiv 5 \pmod 6$ and $n \equiv 2 \pmod{10}$, then $n = 5 + 6k$ and $n = 2 + 10j$, where k and j are integers. So $5 + 6k = 2 + 10j$. This implies that $3 = 10j - 6k$ which is impossible because the right side is divisible by 2 and the left side isn't.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(7917, 357)$. Show your work.

   Solution:

   $7917 - 22 \times 357 = 63$

   $357 - 5 \times 63 = 42$

   $63 - 42 = 21$

   $42 - 2 \times 21 = 0$

   So the GCD is 21.

3. (4 points) Check the (single) box that best characterizes each item.

$\gcd(0,0)$	0	☐	k	☐	undefined	☑
$29 \equiv 2 \pmod 9$	true	☑	false	☐		

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integer k, $(k-1)^2 \equiv 1 \pmod{k}$.

   Solution: This is true. Notice that $(k-1)-(-1) = k$. So $k-1 \equiv (-1) \pmod{k}$. Therefore $(k-1)^2 \equiv (-1)^2 \equiv 1 \pmod{k}$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1183, 351)$. Show your work.

   Solution:

   $1183 - 3 \times 351 = 1183 - 1053 = 130$

   $351 - 2 \times 130 = 351 - 260 = 91$

   $130 - 91 = 39$

   $91 - 3 \times 39 = 91 - 78 = 13$

   $39 - 3 \times 13 = 0$

   So the GCD is 13.

3. (4 points) Check the (single) box that best characterizes each item.

   $k \equiv -k \pmod{7}$ always ☐ sometimes ☑ never ☐

   $-2 \equiv 2 \pmod{4}$ true ☑ false ☐