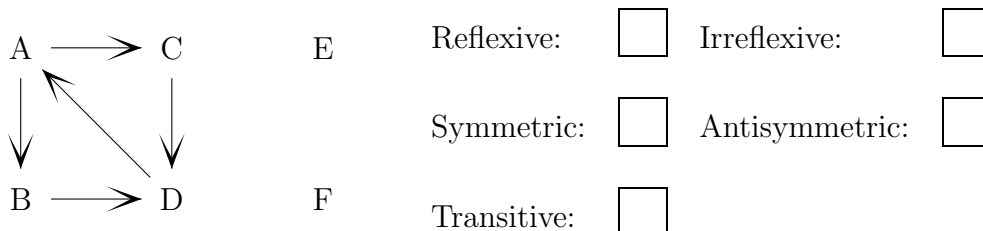


Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



2. (5 points) Suppose that S is the set of all binary strings (i.e. finite sequences of 1's and 0's). Suppose that \sim is the relation on S where $a \sim b$ if and only if a and b contain the same number of 1's. For example, $0101 \sim 1000001$. List three members of $[111]$.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ or $y \leq q$. Is T antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

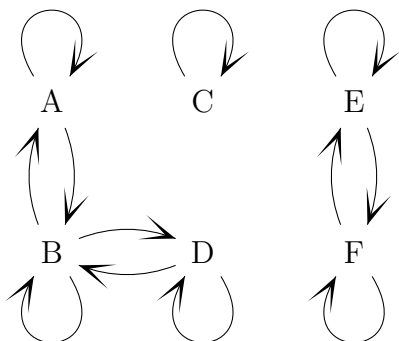
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) A relation is a partial order if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

3. (5 points) Suppose that R is a relation on the integers such that xRy for all integers x and y . Is R an equivalence relation?

Name: _____

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1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

$$A \longrightarrow C \longleftarrow E$$

 Reflexive: ☐ Irreflexive: ☐

 Symmetric: ☐ Antisymmetric: ☐

$$B \longrightarrow D \longleftarrow F$$

 Transitive: ☐

2. (5 points) Let's define the equivalence relation \sim on \mathbb{N}^3 such that $(x, y, z) \sim (p, q, r)$ if and only if $(x, y, z) = \alpha(p, q, r)$ for some integer α . ~~List three members of $[(1, 2, 3)]$~~ List three elements that are related to $(1, 2, 3)$ in either direction.

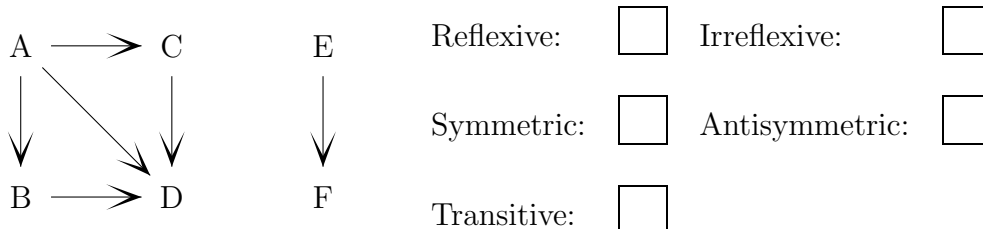
3. (5 points) Suppose that R is the relation on the set of integers such that aRb if and only if $|a - b| \leq 13$. Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



2. (5 points) Let R be the relation on the integers such that xRy if and only if $\lfloor x/4 \rfloor = \lfloor y/4 \rfloor$. List the values in $[8]$.

3. (5 points) Let T be a reflexive relation defined on the integers. Let S be the relation on the integers such that aSb if and only if there is an integer k such that aTk and kTb . Is S reflexive? (I.e. is S reflexive for any reflexive relation T ?) Informally explain why it is, or give a concrete counter-example showing that it is not.

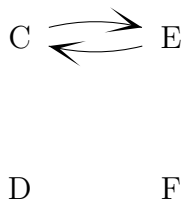
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



D F

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Can a relation be irreflexive, symmetric, and also transitive? Either give such a relation or briefly explain why it's not possible to construct one.

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $x = y$. Is R a partial order?

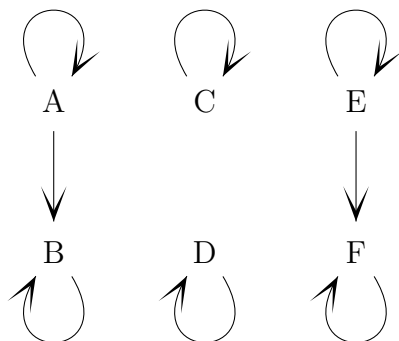
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Suppose that R is a relation on a set A . Using precise mathematical words and notation, define what it means for R to be symmetric.

3. (5 points) Let T be the relation defined on \mathbb{N} such that aTb if and only if $a = b + 2k$ for some natural number k . Is T antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

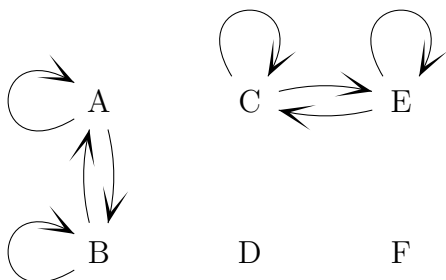
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Let's define the equivalence relation \sim on \mathbb{N}^3 such that $(x, y, z) \sim (p, q, r)$ if and only if $x + y + z = p + q + r$. List three members of $[(1, 2, 3)]$.

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $xy = 1$ for all integers x and y . Is R a partial order?

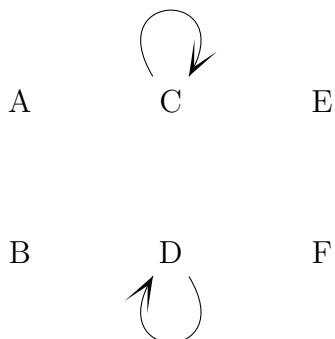
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) A relation is an equivalence relation if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

3. (5 points) Suppose that \succeq is the relation between subsets of the integers such that $A \succeq B$ if and only if $A - B \neq \emptyset$. (A and B are sets of integers, so $A - B$ is a set difference.) Is \succeq transitive? Informally explain why it's true or give a concrete counter-example.

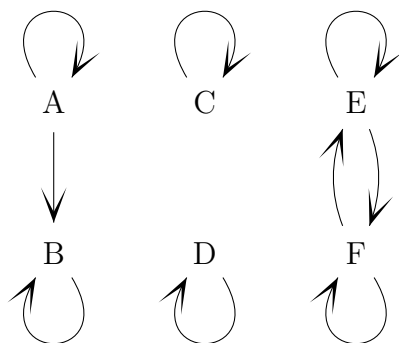
Name: _____

NetID: _____

Lecture: B

Discussion: Friday 11 12 1 2 3 4

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Let R be the equivalence relation on the real numbers such that xRy if and only if $\lfloor x \rfloor = \lfloor y \rfloor$. Give three members of the equivalence class $[13]$.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ or $y \leq q$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

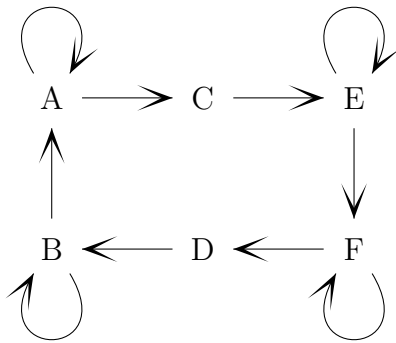
Name: _____

NetID: _____

Lecture: B

Discussion: Friday 11 12 1 2 3 4

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Suppose that R is a relation on a set A . Using precise mathematical words and notation, define what it means for R to be antisymmetric.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ and $y \leq q$. Is T antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

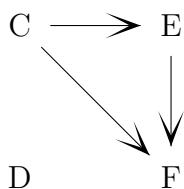
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Recall that \mathbb{N}^2 is the set of all pairs of natural numbers. Let's define the equivalence relation \sim on \mathbb{N}^2 as follows: $(x, y) \sim (p, q)$ if and only $|x - y| = |p - q|$. List three members of $[(2, 3)]$.

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $x = y$. Is R an equivalence relation?

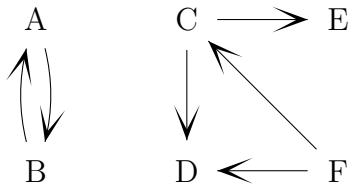
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Can a relation be symmetric and also antisymmetric? Either give such a relation or briefly explain why it's not possible to construct one.

3. (5 points) Let J be the set of open intervals of the real line, i.e. $J = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$. Let's define the "disjoint" relation D on J by $(a, b)D(c, d)$ if and only if $b \leq c$ or $d \leq a$. Is D transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Suppose that R is a partial order on a set A . What additional property is required for R to be a linear order (aka total order)? Give specific details of the property, not just its name.

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $2 \mid (x + y + 1)$. Is R transitive?

Name:_____

NetID:_____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

A C E

Reflexive: ☐ Irreflexive: ☐

(that is, 6 nodes
and no arrows
at all)

Symmetric: ☐ Antisymmetric: ☐

B D F

Transitive: ☐

2. (5 points) Let R be the equivalence relation on the real numbers such that xRy if and only if $\lfloor x \rfloor = \lfloor y \rfloor$. Give three members of the equivalence class $[13]$.

3. (5 points) Suppose that R is a relation on pairs of integers such that $(x, y)R(a, b)$ if and only if $x - a \geq 2$ and $y \geq b$. Is R a partial order?

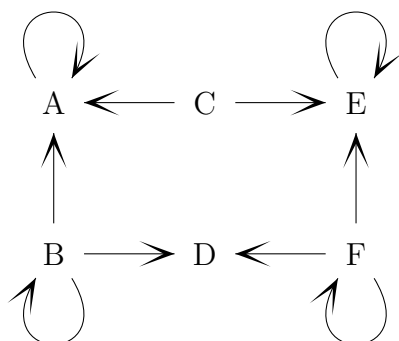
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) **Notice that this problem was corrected early in the exam. This is the corrected version.** Let's define the relation \sim on \mathbb{Z} such that $x \sim y$ if and only $|x - y| = 3$. List all elements related to 7.

3. (5 points) Let S be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)S(p, q)$ if and only if $x^2 + y^2 \leq p^2 + q^2$. Is S antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

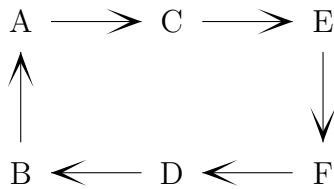
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Suppose that R is an equivalence relation on a set A . Using precise set notation, define $[x]_R$, i.e. the equivalence class of x under the relation R .

3. (5 points) Let J be the set of open intervals of the real line, i.e $J = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$. Let's define the "touches" relation T on J by $(a, b)T(c, d)$ if and only if $a = d$ or $b = c$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

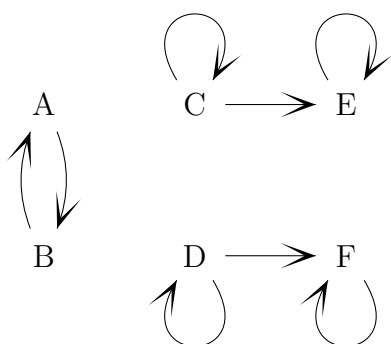
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Let \sim be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y) \sim (p, q)$ if and only if $x^2 + y^2 = p^2 + q^2$. Find three elements in the equivalence class $[(0, 1)]$

3. (5 points) Suppose that \preceq is the relation between subsets of the integers such that $A \preceq B$ if and only if $A - B = \emptyset$. (A and B are sets of integers, so $A - B$ is a set difference.) Is \preceq antisymmetric? Informally explain why it's true (e.g. use a Venn diagram) or give a concrete counter-example.

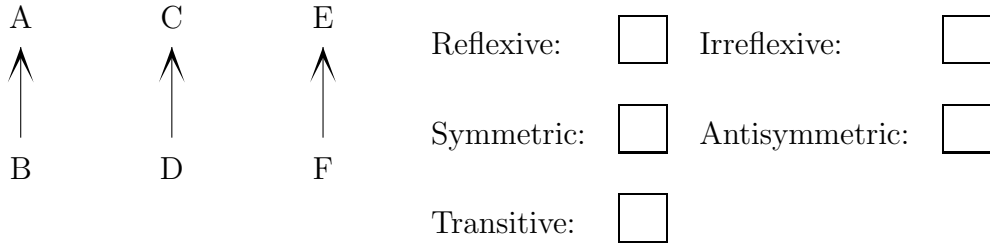
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



2. (5 points) A relation is a partial order if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

3. (5 points) Suppose that T is the relation on the set of integers such that aTb if and only if $\gcd(a, b) = 3$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

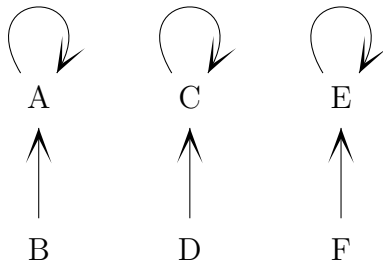
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Suppose that R is a relation on a set A . Using precise mathematical words and notation, define what it means for R to be symmetric.

3. (5 points) Suppose that R is the relation on \mathbb{Z}^4 such that $(a, b, c, d)R(w, x, y, z)$ if and only if $c = w$, $d = x$, $a = y$, and $b = z$. Is R symmetric? Informally explain why it's true or give a concrete counter-example.

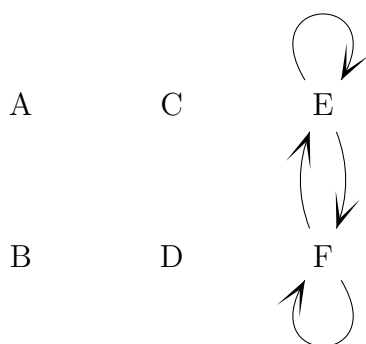
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Let R be the relation on the integers such that aRb if and only if $2a \equiv -3b \pmod{5}$. Find three elements in the equivalence class $[7]$.

3. (5 points) Suppose that R is the relation on \mathbb{Z}^3 such that $(a, b, c)R(x, y, z)$ if and only if $c = x$, $a = y$, and $b = z$. Is R transitive? Informally explain why it's true or give a concrete counter-example.

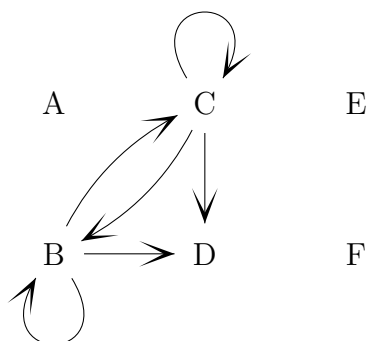
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Suppose that S is the set of all binary strings (i.e. finite sequences of 1's and 0's). Suppose that \sim is the relation on S where $a \sim b$ if and only if a and b are the same length. For example, $01011 \sim 00010$. List three members of $[1111]$.

3. (5 points) Let T be the relation on \mathbb{R}^2 such that $(x, y)T(p, q)$ if and only if $(x, y) = \alpha(p, q)$ for some real number α . Is T symmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

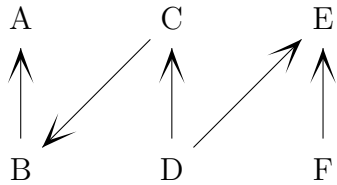
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

2. (5 points) Can a relation with at least one related pair (i.e. at least one arrow in a diagram) be irreflexive, symmetric, and also transitive? Either give such a relation or briefly explain why it's not possible to construct one.

3. (5 points) Suppose that \succeq is the relation between subsets of the integers such that $A \succeq B$ if and only if $A - B \neq \emptyset$. (A and B are sets of integers, so $A - B$ is a set difference.) Is \succeq transitive? Informally explain why it's true or give a concrete counter-example.

Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

C \longrightarrow ED \longleftarrow FReflexive: ☐Irreflexive: ☐Symmetric: ☐Antisymmetric: ☐Transitive: ☐

2. (5 points) Recall that \mathbb{Z}^2 is the set of all pairs of integers. Let's define the equivalence relation \sim on \mathbb{Z}^2 as follows: $(a, b) \sim (p, q)$ if and only if $ab = pq$. List three members of $[(5, 6)]$.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ and $y \leq q$. Is T antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

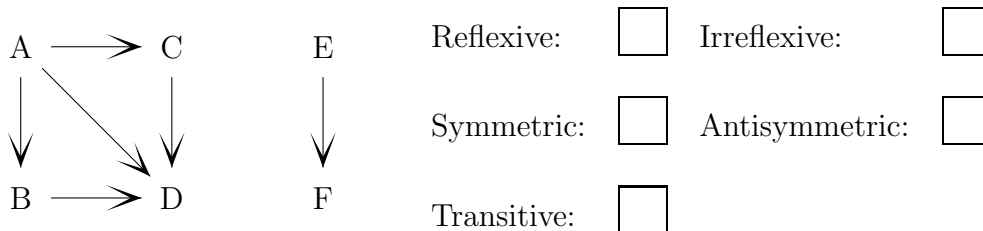
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



2. (5 points) Suppose that R is a relation on a set A . Using precise mathematical words and notation, define what it means for R to be antisymmetric.

3. (5 points) Suppose that R is an equivalence relation on the integers. Is it true that $y \in [x]_R$ if and only if $x \in [y]_R$, for any integers x and y ? Informally explain why it's true or give a concrete counter-example.

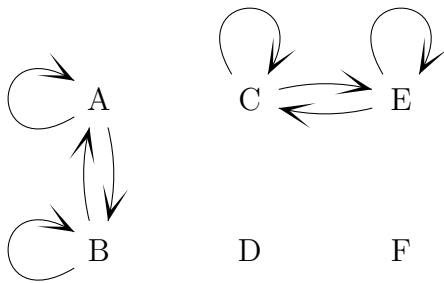
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Suppose that R is a relation on a set A . Using precise mathematical words and notation, define what it means for R to be symmetric.

3. (5 points) Suppose that R is the relation on the set of integers such that aRb if and only if $|a - b| \leq 13$. Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

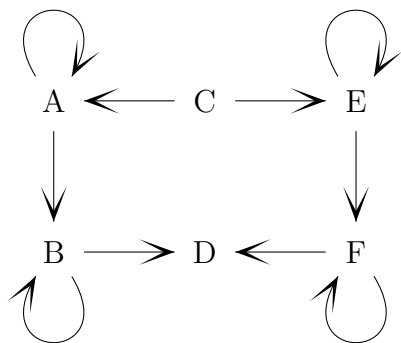
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:

☐

Irreflexive:

☐

Symmetric:

☐

Antisymmetric:

☐

Transitive:

☐

2. (5 points) Suppose that S is the set of all binary strings (i.e. finite sequences of 1's and 0's). Suppose that \sim is the relation on S where $a \sim b$ if and only if a and b contain the same number of 1's. For example, $0101 \sim 1000001$. List three members of $[111]$.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x - p \leq y - q$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

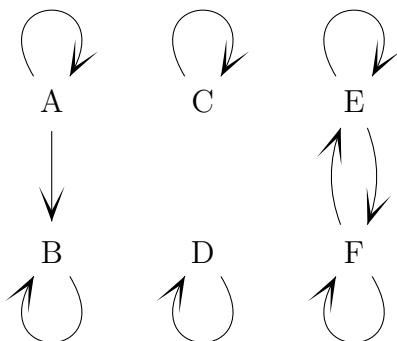
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Let's define the equivalence relation \sim on \mathbb{R} such that $x \sim y$ if and only $|x - y| \in \mathbb{Z}$. List three members of $[1.7]$.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $xp + yq = 0$. Is T irreflexive? Informally explain why it is, or give a concrete counter-example showing that it is not.

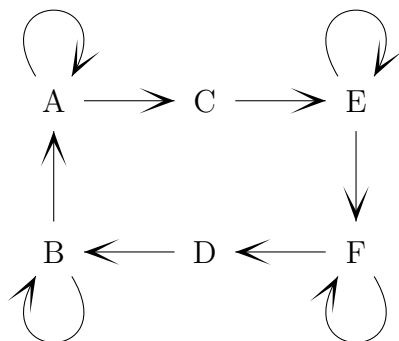
Name: _____

NetID: _____

Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

Reflexive: ☐ Irreflexive: ☐Symmetric: ☐ Antisymmetric: ☐Transitive: ☐

2. (5 points) Can a relation be reflexive, symmetric, and also antisymmetric? Either give such a relation or briefly explain why it's not possible to construct one.

3. (5 points) Let R be the relation on \mathbb{Z} such that xRy if and only if $|x| + |y| = 2$

Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.