# CS 173, Spring 19 Examlet 9, white

**Name:** _____

**NetID:** _____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Consider the following grammar $G$, with start symbol $S$ and terminals $a$ and $b$.

$$S \rightarrow a\ S\ a \mid b\ S\ b \mid a\ S\ b \mid b\ S\ a \mid a \mid b$$

Amy claims that this generates all non-empty strings containing a's and/or b's. Is this correct? Justify your answer.

**Solution:**

Amy is wrong. This grammar only generates strings of odd length.

2. (4 points) Check the (single) box that best characterizes each item.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Total number of leaves in a full and complete 5-ary tree of height $h$ | $5^h$ | [√] | $\leq 5^h$ | [ ] | $\geq 5^h$ | [ ] | $5^{h+1}-1$ | [ ] |
| The level of a leaf node in a full and complete binary tree of height $h$. | 0 | [ ] | 1 | [ ] | $h-1$ | [ ] | $\leq h$ | [ ] | $h$ | [√] |

**Name:**_____

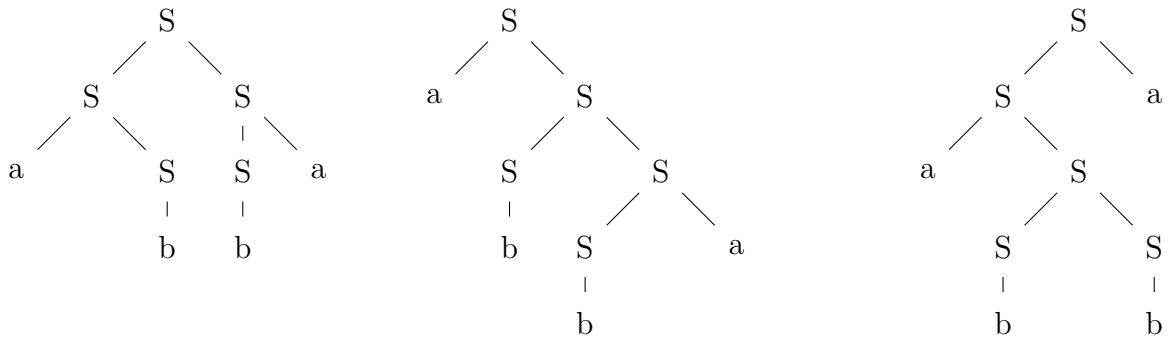
**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Here is a grammar with start symbol $S$ and terminal symbols $a$ and $b$. Draw three parse trees for the string `abba` that match this grammar.

$$S \rightarrow \quad S\ S \mid a\ S \mid S\ a \mid b$$

**Solution:**




2. (4 points) Check the (single) box that best characterizes each item.

A full $m$-ary tree with $i$ internal nodes has $mi + 1$ nodes total. always [√] sometimes [ ] never [ ]

A binary tree of height $h$ has at least $2^{h+1} - 1$ nodes. true [ ] false [√]

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Consider the following grammar $G$

   $S \to S\ b\ S \mid a \mid c\ d$

   $S$ is the only start symbol. The terminal symbols are $a$, $b$, $c$, and $d$.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why $G$ cannot generate this sequence of leaf labels.

   `aaacd`

   **Solution:** In grammar G, making strings with more than two leaves requires using the first rule (SbS) which produces a b. This string can't be generated by G because it is more than two characters long with no b in it.

   `bbbbb`

   **Solution:** Impossible. Since the only terminal in the string is b, the only rule we could be using is $S \to SbS$. But each time we use this rule, the count of S nodes without children increases by one. This is a problem, since S nodes can't be leaves.

2. (4 points) Check the (single) box that best characterizes each item.

   The mathematical symbol for an empty (zero-length) string — $\emptyset$ ☐ e ☐ $\epsilon$ ☑ NULL ☐

   Number of bit strings of length $\leq k$. — $2^k$ ☐ $2^k - 1$ ☐ $2^{k-1}$ ☐ $2^{k+1} - 1$ ☑

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Min's virus detection code needs to generate all strings of the form $a^n b^n$. That is, all strings that consist of a sequence of one or more a's followed by the same number of b's. Write a context-free grammar G that will do this.

   **Solution:**

   G has start symbol S, terminals a and b, and the following rules:

   $S \rightarrow\ a\ S\ b\ \mid\ a\ b$

2. (4 points) Check the (single) box that best characterizes each item.

   The number of nodes in a binary tree of height $h$

   | | | | |
   |---|---|---|---|
   | $\geq 2^h$ | ☐ | $2^{h+1} - 1$ | ☐ |
   | $\leq 2^{h+1} - 1$ | ☑ | $\geq 2^{h+1} - 1$ | ☐ |

   A tree node is a descendent of itself.

   always ☑ sometimes ☐ never ☐

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Consider the following grammar $G$

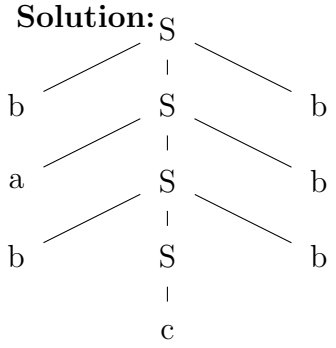
   $S \to a\ S\ b \mid b\ S\ b \mid c$

   $S$ is the only start symbol. The terminal symbols are $a$, $b$, and $c$.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why $G$ cannot generate this sequence of leaf labels.

   `ababb`
   **Solution:**
   This is impossible. In strings produced by G, the middle character must be a c.

   `babcbbb`
   **Solution:**




2. (4 points) Check the (single) box that best characterizes each item.

   The level of the root node in a tree of height $h$. 0 [√] 1 [ ] $h-1$ [ ] $h$ [ ] $h+1$ [ ]

   A tree node is a proper ancestor of itself. always [ ] sometimes [ ] never [√]

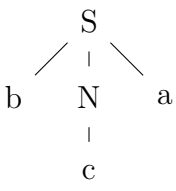
**Name:**_____

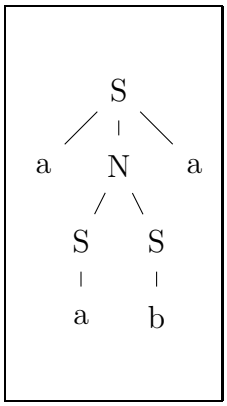
**NetID:**_____ **Lecture: A B**

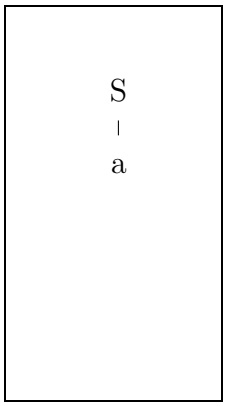
**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

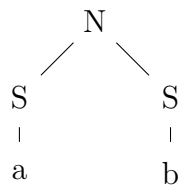
1. (8 points) Here is a grammar with start symbol $S$ and terminal symbols $a$, $b$, and $c$. Circle the trees that match the grammar.

$$S \rightarrow a\ N\ a \mid b\ N\ b \mid a \mid b$$
$$N \rightarrow S\ S \mid c$$




2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height $h$ has at least $2^h - 1$ nodes. true ☐ false ☑

A full $m$-ary tree with $i$ internal nodes has ____ nodes total.

$mi - 1$ ☐ $mi$ ☐

$mi + 1$ ☑ $\leq mi + 1$ ☐

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

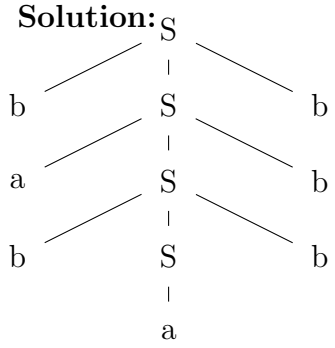
1. (8 points) Consider the following grammar $G$

   $S \to a\ S\ b \mid b\ S\ b \mid a \mid b$

   $S$ is the only start symbol. The terminal symbols are $a$ and $b$.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why $G$ cannot generate this sequence of leaf labels.

   `bababbb`
   **Solution:**




   `aaaab`
   **Solution:**
   This is impossible. In a string produced by grammar G, all characters after the middle of the string must be b's.

2. (4 points) Check the (single) box that best characterizes each item.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| The number of leaves in a binary tree of height $h$ | $2^h$ | ☐ | $2^{h+1}-1$ | ☐ | $\geq 2^h$ | ☐ | $\leq 2^h$ | ☑ |
| The number of paths between two distinct nodes in an $n$-node tree. Paths in opposite directions count as the same. | $n$ | ☐ | $2n$ | ☐ | $\frac{n(n-1)}{2}$ | ☑ | | |
| | $n(n-1)$ | ☐ | $n^2$ | ☐ | $\frac{n(n+1)}{2}$ | ☐ | | |

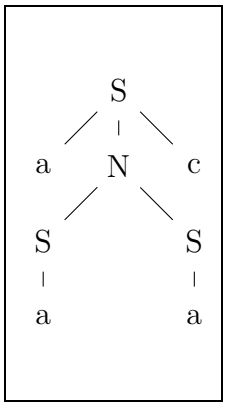
**Name:**_____

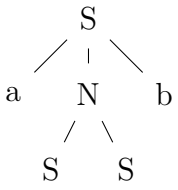
**NetID:**_____ **Lecture: A B**

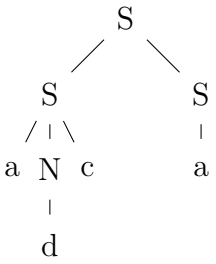
**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

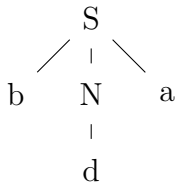
1. (8 points) Here is a grammar with start symbol $S$ and terminal symbols $a$, $b$, $c$, and $d$. Circle the trees that match the grammar.

$$\begin{aligned} S &\rightarrow a\ N\ b \mid a\ N\ c \mid a \\ N &\rightarrow S\ S \mid d \end{aligned}$$




2. (4 points) Check the (single) box that best characterizes each item.

The diameter of a tree of height $h$.

| | | | | | |
|---|---|---|---|---|---|
| $\leq h$ | ☐ | $h$ | ☐ | $h+1$ | ☐ |
| $2h$ | ☐ | $\leq 2h$ | ☑ | | |

The number of nodes in a full complete binary tree of height $h$

| | | | |
|---|---|---|---|
| $\geq 2^h$ | ☐ | $2^{h+1}-1$ | ☑ |
| $\leq 2^{h+1}-1$ | ☐ | $\geq 2^{h+1}-1$ | ☐ |

# CS 173, Fall 18 Examlet 9, Part B

**Name:**_____

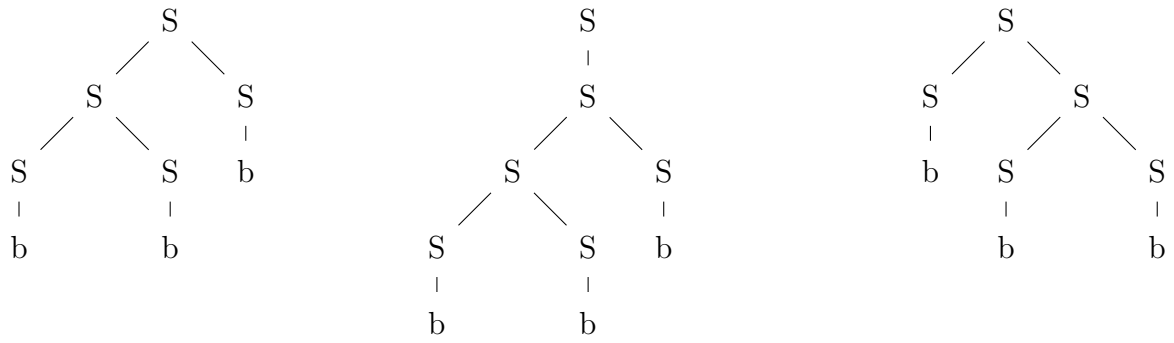
**NetID:**_____ **Lecture: B**

**Discussion: Friday 11 12 1 2 3 4**

1. (8 points) Here is a grammar with start symbol $S$ and terminal symbol $b$. Draw three parse trees for the string `bbb` that match this grammar.

$$S \rightarrow S\ S \mid S \mid b$$

**Solution:**




2. (4 points) Check the (single) box that best characterizes each item.

Number of bit strings of length $k$.

$2^k$ ☑ $2^k - 1$ ☐ $2^{k-1}$ ☐ $k$ ☐

The chromatic number of a full 3-ary tree

1 ☐ 2 ☐ $\leq 2$ ☑

3 ☐ $\leq 3$ ☐ can't tell ☐

**Name:**_____

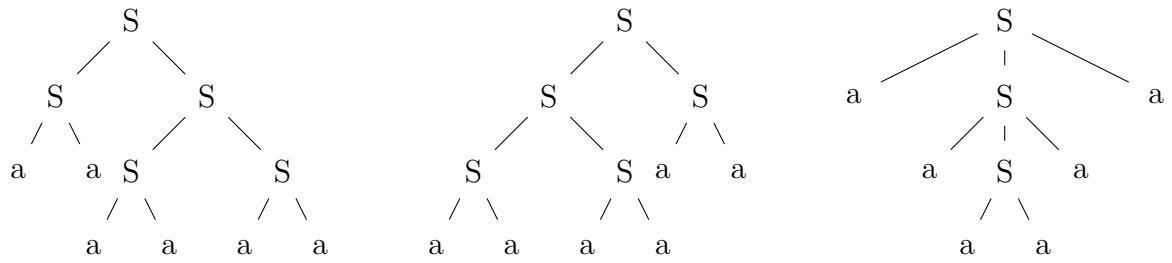
**NetID:**_____ **Lecture: B**

**Discussion: Friday 11 12 1 2 3 4**

1. (8 points) Here is a grammar with start symbol $S$ and terminal symbol $a$. Draw three parse trees for the string `aaaaaa` that match this grammar.

$$S \rightarrow S\ S \mid a\ S\ a \mid a\ a$$

**Solution:**




2. (4 points) Check the (single) box that best characterizes each item.

The chromatic number of a full 3-ary tree

| | | | | | |
|---|---|---|---|---|---|
| 1 | ☐ | 2 | ☐ | $\leq 2$ | ☑ |
| 3 | ☐ | $\leq 3$ | ☐ | can't tell | ☐ |

Number of bit strings of length $k$.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $2^k$ | ☑ | $2^k - 1$ | ☐ | $2^{k-1}$ | ☐ | $k$ | ☐ |

# CS 173, Spring 18 Examlet 9, Part B

**Name:**_____

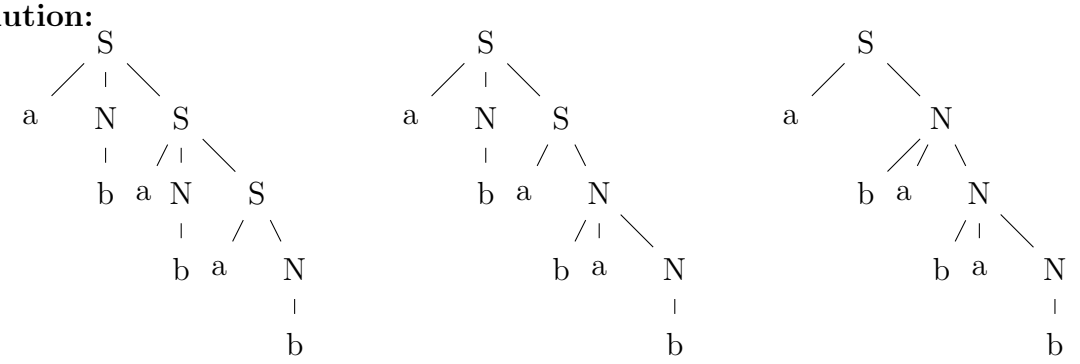
**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Here is a grammar with start symbol $S$ and terminal symbols $a$ and $b$. Draw three parse trees for the string `abababa` that match this grammar.

$$\begin{aligned} S &\rightarrow a\ N \mid a\ N\ S \\ N &\rightarrow b\ a\ N \mid b \end{aligned}$$

**Solution:**




2. (4 points) Check the (single) box that best characterizes each item.

A tree node is a descendent of itself.

always [√] sometimes [ ] never [ ]

The number of nodes in a full complete binary tree of height $h$

$\geq 2^h$ [ ] $2^{h+1} - 1$ [√]

$\leq 2^{h+1} - 1$ [ ] $\geq 2^{h+1} - 1$ [ ]

# CS 173, Spring 18 Examlet 9, Part B

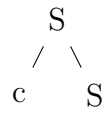
**Name:**_____

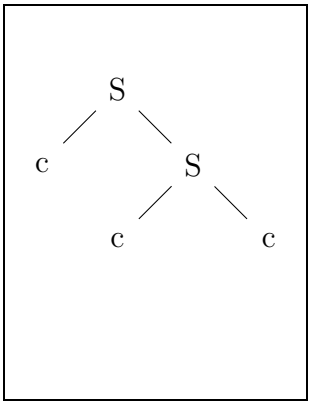
**NetID:**_____ **Lecture: A B**

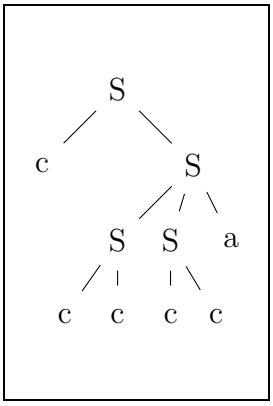
**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

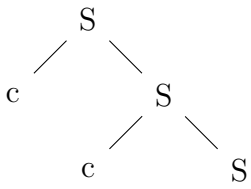
1. (8 points) Here is a grammar, with start variable $S$ and terminals $a$ and $c$. Circle the trees that match the grammar.

$$S \rightarrow S\ S\ a \mid c\ S \mid c\ c$$




2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height $h$ has at most $2^{h+1} - 1$ nodes. true [√] false [ ]

A tree with $n$ edges has ____ nodes.

$n-1$ [ ] $n$ [ ]

$n+1$ [√] $n/2$ [ ]

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Consider the following grammar $G$

   $S \to a\ S\ b \mid b\ S\ b \mid a \mid b$

   $S$ is the only start symbol. The terminal symbols are $a$ and $b$.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why $G$ cannot generate this sequence of leaf labels.

   `aabaaba`
   **Solution:**
   This is impossible. Strings produced by G have to end in a b, unless they are length 1.

   `aababaa`
   **Solution:**
   This is impossible. Strings produced by G have to end in a b, unless they are length 1.

2. (4 points) Check the (single) box that best characterizes each item.

   The root node of a tree is an internal node — always ☐ sometimes ☑ never ☐

   The level of the root node in a tree of height $h$. — $-1$ ☐ $0$ ☑ $1$ ☐ $h-1$ ☐ $h$ ☐

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Give a context-free grammar that generates all strings of the form $a^+b^+$. That is, all strings that consist of a sequence of one or more a's followed by a sequence of one or more b's.

   **Solution:**

   $S \to A\ B$

   $A \to a\ A \mid a$

   $B \to b\ B \mid b$

2. (4 points) Check the (single) box that best characterizes each item.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Number of non-empty bit strings of length $k$. | $2^k$ | √ | $2^k - 1$ | ☐ | $2^{k-1}$ | ☐ | $k$ | ☐ |
| The number of paths between two distinct nodes in an $n$-node tree. Paths in opposite directions count as the same. | $n$ | ☐ | $2n$ | ☐ | $\frac{n(n-1)}{2}$ | √ | | |
| | $n(n-1)$ | ☐ | $n^2$ | ☐ | $\frac{n(n+1)}{2}$ | ☐ | | |

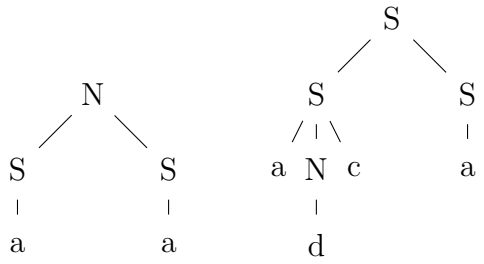
**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

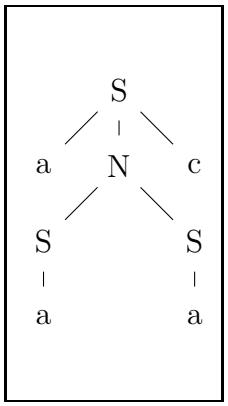
1. (8 points) Here is a grammar with start symbol $S$ and terminal symbols $a$, $b$, $c$, and $d$. Circle the trees that match the grammar.

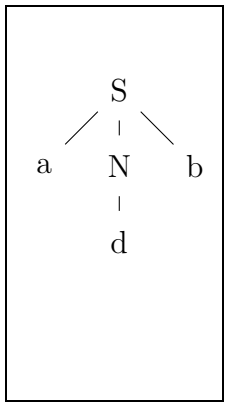
$$S \rightarrow a\ N\ b \mid a\ N\ c \mid a$$
$$N \rightarrow S\ S \mid d$$










2. (4 points) Check the (single) box that best characterizes each item.

A full $m$-ary tree with $i$ internal nodes has ____ nodes total.

| | | | |
|---|---|---|---|
| $mi - 1$ | ☐ | $mi$ | ☐ |
| $mi + 1$ | ☑ | $\leq mi + 1$ | ☐ |

Height of a binary tree with $2^n$ nodes.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\leq n - 1$ | ☐ | $\leq n$ | ☐ | $\leq 2^n$ | ☐ | $\leq 2^n - 1$ | ☑ |

**Name:**_____

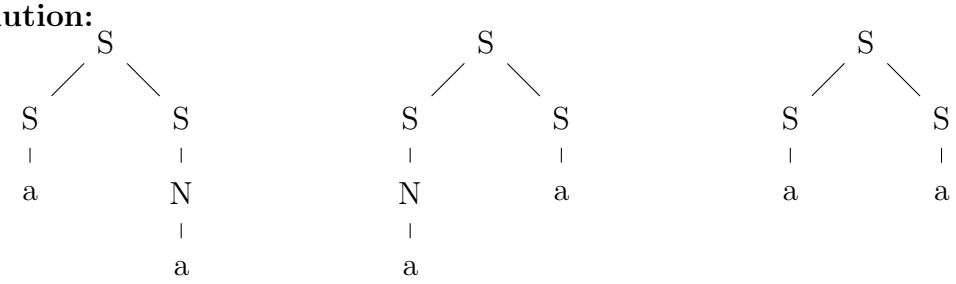
**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Here is a grammar with start symbol $S$ and terminal symbol $a$. Draw three parse trees for the string `aa` that match this grammar.

$$\begin{aligned} S &\rightarrow S\ S \mid N \mid a \\ N &\rightarrow a \end{aligned}$$

**Solution:**




2. (4 points) Check the (single) box that best characterizes each item.

| | | | | | |
|---|---|---|---|---|---|
| A tree node is a proper ancestor of itself. | always | ☐ | sometimes | ☐ | never | ☑ |
| Removing an edge from a tree (with at least one edge) produces two trees. | always | ☑ | sometimes | ☐ | never | ☐ |

# CS 173, Fall 17 Examlet 9, Part B

**Name:**_____

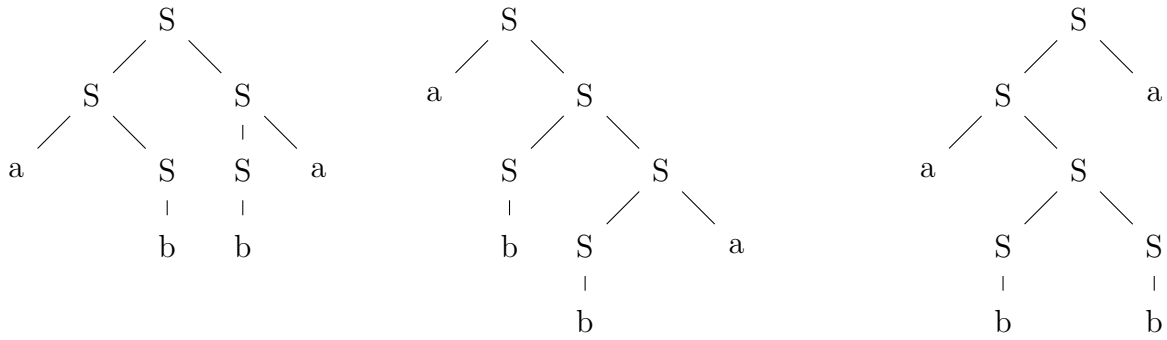
**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Here is a grammar with start symbol $S$ and terminal symbols $a$ and $b$. Draw three parse trees for the string `abba` that match this grammar.

$$S \rightarrow S\,S \mid a\,S \mid S\,a \mid b$$

**Solution:**




2. (4 points) Check the (single) box that best characterizes each item.

The mathematical symbol for an empty (zero-length) string

$\emptyset$ ☐ e ☐ $\epsilon$ ☑ NULL ☐

Number of nodes at level $k$ in a full complete binary tree.

$2^k$ ☑ $2^k - 1$ ☐ $2^{k+1} - 1$ ☐ $2^{k-1}$ ☐

# CS 173, Fall 17 Examlet 9, Part B

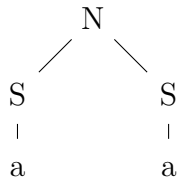
**Name:**_____

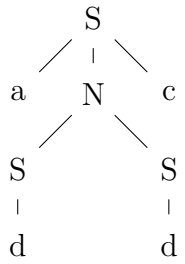
**NetID:**_____ **Lecture: A B**

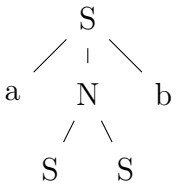
**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

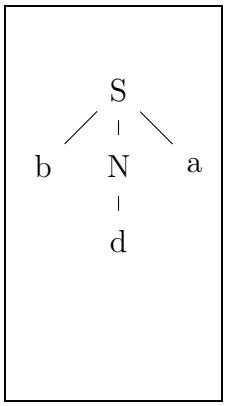
1. (8 points) Here is a grammar with start symbol $S$ and terminal symbols $a$, $b$, $c$, and $d$. Circle the trees that match the grammar.

$$S \rightarrow b\ N\ a \mid a\ N\ c \mid a$$
$$N \rightarrow S\ S \mid d$$




2. (4 points) Check the (single) box that best characterizes each item.

Number of bit strings of length $k$. $2^k$ [√] $2^k - 1$ [ ] $2^{k-1}$ [ ] $k$ [ ]

A full $m$-ary tree with $i$ internal nodes has $mi + 1$ nodes total. always [√] sometimes [ ] never [ ]

# CS 173, Fall 17 Examlet 9, Part B

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Consider the following grammar $G$

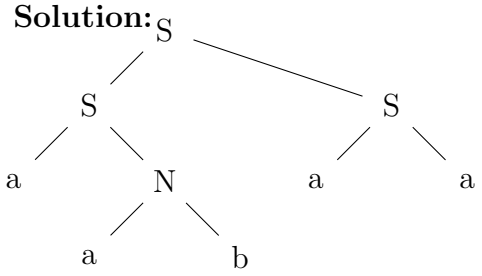
$$\begin{aligned} S &\rightarrow S\ S \mid a\ N \mid a\ a \\ N &\rightarrow S\ a \mid a\ b \end{aligned}$$

$S$ is the only start symbol. The terminal symbols are $a$ and $b$

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why $G$ cannot generate this sequence of leaf labels.

`aabaa`

**Solution:**




`ab`

**Solution:** This is impossible. An ab sequence must come from the rule $N \rightarrow a\ b$. But $N$ isn't a start symbol and getting to this rule from $S$ would require adding something else to the string.

2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height $h$ has at least $2^{h+1} - 1$ nodes. true ☐ false ☑

The level of the root node in a tree of height $h$. 0 ☑ 1 ☐ $h-1$ ☐ $h$ ☐ $h+1$ ☐

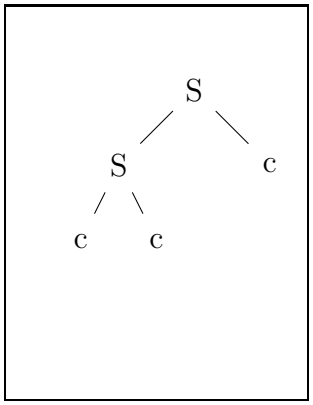
**Name:**_____

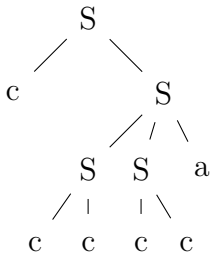
**NetID:**_____ **Lecture: A B**

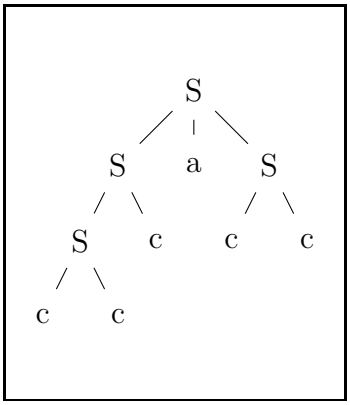
**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

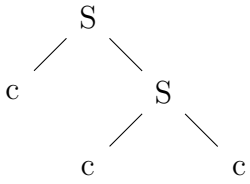
1. (8 points) Here is a grammar, with start variable $S$ and terminals $a$ and $c$. Circle the trees that match the grammar.

$$S \rightarrow S\ a\ S \mid S\ c \mid c\ c$$




2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height $h$ has at least $2^h - 1$ nodes. true ☐ false ☑

Number of bit strings of length $\leq k$. $2^k$ ☐ $2^k - 1$ ☐ $2^{k-1}$ ☐ $2^{k+1} - 1$ ☑

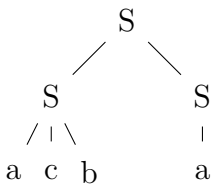
**Name:**_____

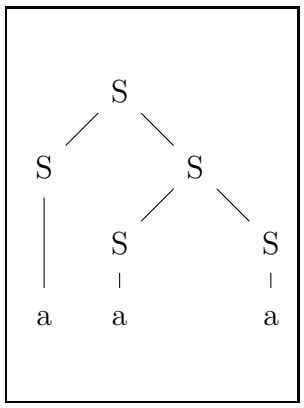
**NetID:**_____ **Lecture: A B**

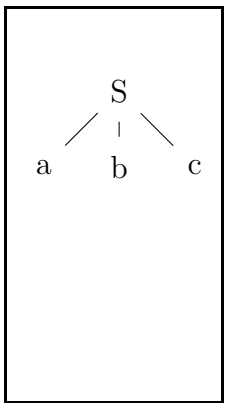
**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

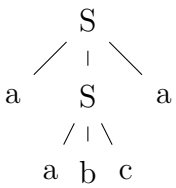
1. (8 points) Here is a grammar with start symbol $S$ and terminals symbols $a$,$b$, and $c$. Circle the trees that match the grammar.

$$S \rightarrow S\ S \mid a\ b\ c \mid a$$




2. (4 points) Check the (single) box that best characterizes each item.

The number of nodes in a binary tree of height $h$

| | | | |
|---|---|---|---|
| $\geq 2^h$ | ☐ | $2^{h+1} - 1$ | ☐ |
| $\leq 2^{h+1} - 1$ | ☑ | $\geq 2^{h+1} - 1$ | ☐ |

The diameter of a tree of height $h$.

| | | | | | |
|---|---|---|---|---|---|
| $\leq h$ | ☐ | $h$ | ☐ | $h + 1$ | ☐ |
| $2h$ | ☐ | $\leq 2h$ | ☑ | | |

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6**

1. (8 points) Consider the following grammar $G$

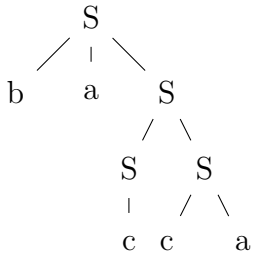
   $S \to b\ a\ S \mid S\ S \mid c \mid c\ a$

   $S$ is the only start symbol. The terminal symbol are $a$, $b$, and $c$.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why $G$ cannot generate this sequence of leaf labels.

   `baba`
   **Solution:** This is impossible. The only rule generating `ba` is $S \to b\ a\ S$. So `baba` requires two applications of this rule. But that will leave us with an extra S at the end and S isn't a terminal.

   `bacca`
   **Solution:**




2. (4 points) Check the (single) box that best characterizes each item.

   The number of leaves in a binary tree of height $h$

   $2^h$ ☐ $2^{h+1}-1$ ☐ $\geq 2^h$ ☐ $\leq 2^h$ ☑

   The diameter of a full, complete 7-ary tree of height $h$.

   $\leq h$ ☐ $h$ ☐ $h+1$ ☐

   $2h$ ☑ $7h$ ☐ $7h+1$ ☐

**Name:**_____

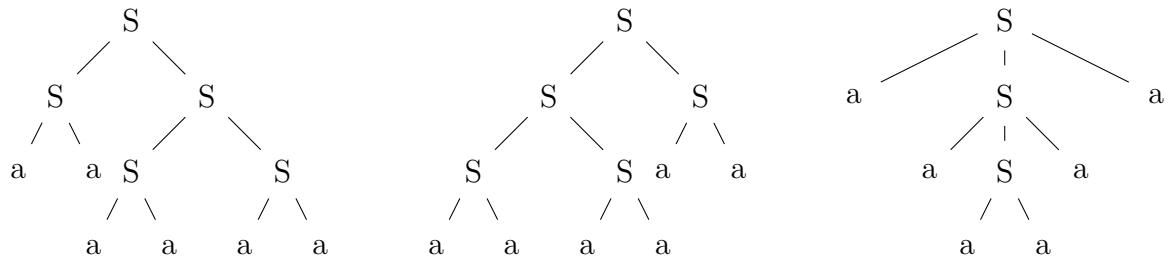
**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

1. (8 points) Here is a grammar with start symbol $S$ and terminal symbol $a$. Draw three parse trees for the string $a\ a\ a\ a\ a\ a$ that match this grammar.

$$S \ \rightarrow \ \ S\ S \ \mid \ a\ S\ a \ \mid \ a\ a$$

**Solution:**




2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height $h$ has at most $2^{h+1} - 1$ nodes. true [√] false [ ]

The root node of a tree is a leaf. always [ ] sometimes [√] never [ ]

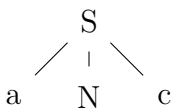
**Name:**_____

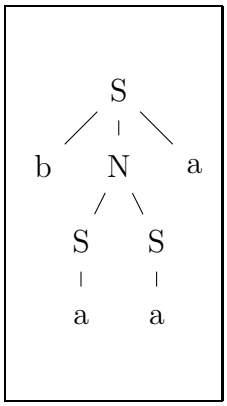
**NetID:**_____ **Lecture: A B**

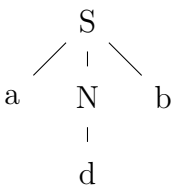
**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

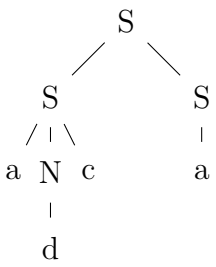
1. (8 points) Here is a grammar with start symbol $S$ and terminal symbols $a$, $b$, $c$, and $d$. Circle the trees that match the grammar.

$$S \rightarrow b\ N\ a \mid a\ N\ c \mid a$$
$$N \rightarrow S\ S \mid d$$




2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n} 2^k$ $2^{n+1}-1$ ☐ $2^{n+1}-2$ ☑ $2^{n+1}-3$ ☐ $2^n-1$ ☐

$2^h$ is ____ the number of leaves in a binary tree of height $h$.

an upper bound on ☑ exactly ☐

a lower bound on ☐ not a bound on ☐

**Name:**_____

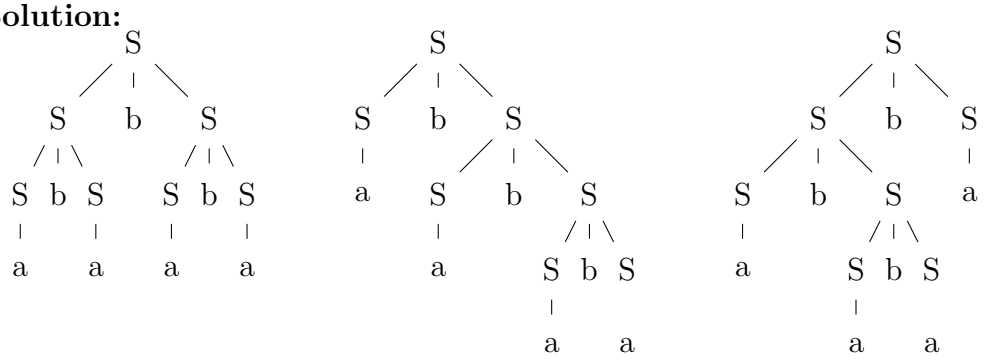
**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

1. (8 points) Here is a grammar with start symbol $S$ and terminal symbols $a$ and $b$. Draw three parse trees for the string $a\ b\ a\ b\ a\ b\ a$ that match this grammar.

$$S \ \rightarrow \ S\ b\ S \ \mid \ a$$

**Solution:**




2. (4 points) Check the (single) box that best characterizes each item.

An $m$-ary tree with $i$ internal nodes has $mi + 1$ nodes total. always ☐ sometimes ☑ never ☐

Total number of leaves in a 3-ary tree of height $h$ $3^h$ ☐ $\leq 3^h$ ☑ $\frac{1}{2}(3^{h+1} - 1)$ ☐ $3^{h+1} - 1$ ☐

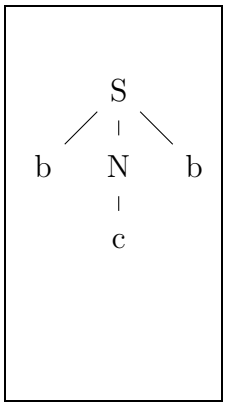
**Name:**_____

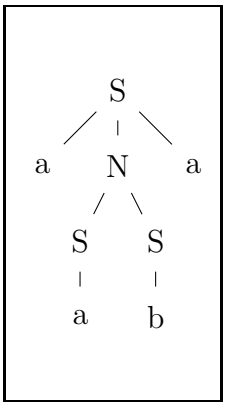
**NetID:**_____ **Lecture: A B**

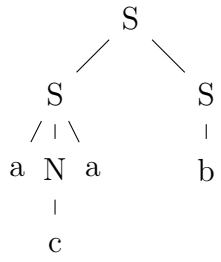
**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

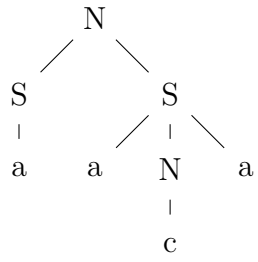
1. (8 points) Here is a grammar with start symbol $S$ and terminal symbols $a$, $b$, and $c$. Circle the trees that match the grammar.

$$S \rightarrow a\ N\ a \mid b\ N\ b \mid a \mid b$$
$$N \rightarrow S\ S \mid c$$




2. (4 points) Check the (single) box that best characterizes each item.

| | | | | | |
|---|---|---|---|---|---|
| The number of paths between two distinct nodes in an $n$-node tree. Paths in opposite directions count as the same. | $n$ ☐ | $2n$ ☐ | $\frac{n(n-1)}{2}$ ☑ |
| | $n(n-1)$ ☐ | $n^2$ ☐ | $\frac{n(n+1)}{2}$ ☐ |

| | | | | |
|---|---|---|---|---|
| $\sum_{k=0}^{n} 2^k$ | $2^n - 2$ ☐ | $2^n - 1$ ☐ | $2^{n-1} - 1$ ☐ | $2^{n+1} - 1$ ☑ |

**Name:**_____

**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

1. (8 points) Consider the following grammar $G$

   $S \to S\ b\ S \mid a \mid c\ d$

   $S$ is the only start symbol. The terminal symbols are $a$, $b$, $c$, and $d$.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why $G$ cannot generate this sequence of leaf labels.

   $a\ b\ a\ c\ a$

   **Solution:** Impossible. In this grammar, a c must always be followed by a d, and this one is followed by an a.

   $b\ b\ b\ b\ b$

   **Solution:** Impossible. Since the only terminal in the string is b, the only rule we could be using is $S \to SbS$. But each time we use this rule, the count of S nodes without children increases by one. This is a problem, since S nodes can't be leaves.

2. (4 points) Check the (single) box that best characterizes each item.

   The level of a leaf node in a full and complete binary tree of height $h$.

   $0$ ☐ $1$ ☐ $h-1$ ☐ $\leq h$ ☐ $h$ ☑

   Height of a binary tree with $2^n$ nodes.

   $\leq n-1$ ☐ $\leq n$ ☐ $\leq 2^n$ ☐ $\leq 2^n - 1$ ☑

**Name:**_____

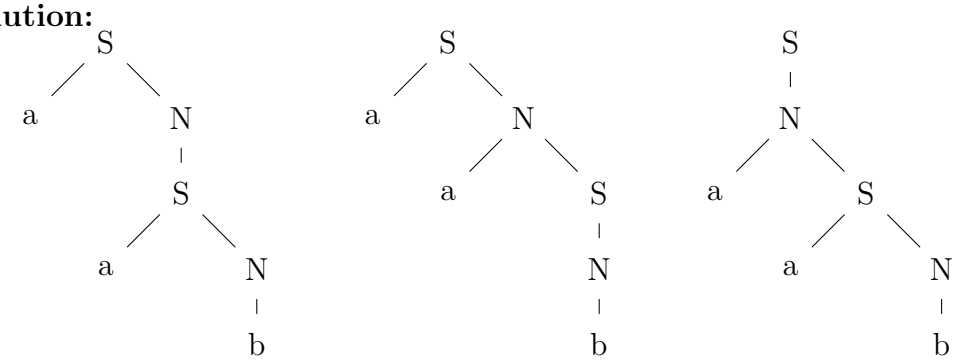
**NetID:**_____ **Lecture: A B**

**Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6**

1. (8 points) Here is a grammar with start symbol $S$ and terminal symbols $a$ and $b$. Draw three parse trees for the string $a\ a\ b$ that match this grammar.

$$\begin{aligned} S &\rightarrow a\ N \mid N \\ N &\rightarrow a\ S \mid S \mid b \end{aligned}$$

**Solution:**




2. (4 points) Check the (single) box that best characterizes each item.

The number of paths between two nodes in an $n$-node tree. Paths in opposite directions count as different.

| | | | | | |
|---|---|---|---|---|---|
| $n$ | ☐ | $2n$ | ☐ | $\frac{n(n-1)}{2}$ | ☐ |
| $n(n-1)$ | ☐ | $n^2$ | ☑ | $\frac{n(n+1)}{2}$ | ☐ |

A tree node is an ancestor of itself.

always ☑ sometimes ☐ never ☐