Peter J. Taylor

Distance Transformation and Distance Decay Functions

In his comprehensive review of spatial interaction models Olsson [*37*, p. 51] states: "it is evident that a correct transformation of the data is crucial . . . It is therefore surprising that a good thorough discussion of the problems connected with selecting and making this transformation has not yet appeared." This paper attempts to rectify this omission by focusing attention on the transformation of distance in distance decay functions. As such it falls into four main sections.

(1) In the introductory section the system of distance decay functions employed in this study is described and the problem of describing distance decay is introduced by an examination of some published Pareto curves.

(2) The second section presents the results of experiments designed to investigate the role of distance transformation in decay functions.

(3) In the third section the concept of an optimum distance transformation is introduced and an algorithm for obtaining such a description is presented.

(4) Finally, the findings of this research are summarized in a concluding section by relating this study to previous interaction research and also to the general problem of data transformation.

The functions considered in this paper consist of the simple two variable form

$$I_z = f(d_z) \tag{1}$$

where I_z is some measure of interaction intensity over a distance d_z (that is, the ratio between observed interaction frequency and population or area at d_z) and $f(d_z)$ is a monotonically decreasing function of distance. There are obviously an infinite number of particular functions that can be specified as equation (1). Thus consideration is limited to a set of functions that are related to the Goux classification of distance decay functions based on distance transformation [*18*].

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The Goux Classification

Distance decay functions have been evolved in two separate research fields: spatial interaction studies in the social sciences and gene dispersal research in genetics and despite a common interest in marriage distances these two lines of enquiry have remained largely isolated from one another. The breakdown of this isolation dates from the 1962 Monaco symposium on "Human Displacements". Contributors to this conference were evenly divided between geneticists and social scientists [*43*, p. xv]. The Morrill and Pitts experiments [*36*] using models from both schools can be cited as evidence for the breakdown of this isolation. However, perhaps a more important indication is the paper by the French geneticist J.-M. Goux which presented a classification of distance decay functions from both sets of literature [*18*].

Goux starts with the general function

$$I_z = ke^{-bf(d_z)} \tag{2}$$

which in linear form is:

$$\log I_z = a - bf(d_z) \tag{3}$$

with $a = \log k$, using Napierian logarithms. Goux divides this general function into two types of model which are termed, by reference to their linear forms, 'double-log models' where $f(d_z) = \log d_z^m$ and 'single-log models' where $f(d_z) = d_z^m$. Special cases that occur in the double-log case are (1) the Pareto model [*20, 21*] when $m = 1$:

$$\log I_z = a - b \log d_z \tag{4}$$

and (2) the log-normal model [*30, 36*] when $m = 2$:

$$\log I_z = a - b\,(\log d_z)^2. \tag{5}$$

Special cases that occur in the single-log case are (1) the exponential model [*4, 9*] when $m = 1$:

$$\log I_z = a - b\, d_z \tag{6}$$

(2) the normal model [*4, 5, 6, 16, 23, 39, 45, 46*] when $m = 2$:

$$\log I_z = a - b\, d_z^2 \tag{7}$$

and (3) the square root exponential model [*4*] when $m = 0.5$:

$$\log I_z = a - b\sqrt{d_z}\,. \tag{8}$$

These special cases will be referred to below as the five standard models.

Hyman's [*26*, p. 109] recent 'generalized exponential model' has affinities with Goux's system but is more limited incorporating only the exponential, power (Pareto) and more complex Tanner model. The two systems are alike in that they identify transformation of distance as the essential difference between the various distance decay functions.

Pareto Residual Patterns

Although the Pareto model is undoubtably the most common function used in spatial interaction research it has also been widely criticized. For instance Hägerstrand [*20*, p. 117] admits that "As a kind of generally applicable 'law of migration' equation 1 (the Pareto function) cannot be accepted." The function is criticized not only because it is generally a poor predictor but more important is the fact that these errors have a consistent pattern. A linear description of a set of data is only satisfactory when the residuals do not themselves depart significantly from a random pattern [*19*, p. 100], and a number of published Pareto graphs are assessed in this light.

A fairly early example of the use of the Pareto function in spatial interaction research is the study of local travel in Chicago by Carroll and Bevis [*10*]. On the basis of a high correlation coefficient between distance and total person movement they conclude that the function is "sound." However, in their graph showing the fitted curve and observations, the residuals by no means suggest randomness. The residual sequence is as follows:

$$- - + + + + + + + - + + - - - - - -$$

This sequence can be tested for randomness using the simple Runs Test [*31*, pp. 322–5]. At the 5% significance level with nine positive and nine negative signs, six runs or less are required to indicate divergence from randomness (a run being a continuous sequence of common signs). In the sequence above there are only five runs which strongly suggest nonrandomness. Other residual patterns from published Pareto graphs are now considered.

Claeson [*11*] in a study of cinema visists in Sweden presents seven sets of interaction data fitted by Pareto equations. The residual sequences are as follows:

$$(- + + - - - -), (- + + + - - -), (- + + + + - -),$$
$$(- - + + + + -), (- + + -), (- - + + - -),$$
$$(- + + + + + - -).$$

Unfortunately there are not enough observations in these sequences to test for randomness but an overall pattern can be identified. All the sequences have the same three components: (i) at least one negative residual followed by (ii) a run of positive residuals and then (iii) negative residuals again. The pattern is common to both Claeson's and Carroll and Bevis's residual sequences and is repeated again in Hägerstrand's Monaco paper [*21*] in regard to migration data at both the scales of 'macro-observations' (up to 100 km), and the 'micro-observations' for Asby and Kisa.

Although three general components have been identified in the above residual patterns, it is the first component, the over-estimation at the shorter distances, which has attracted most attention. Thus, when Hägerstrand uses Pareto predictions of migration as a surrogate measure of private information fields he uses observed migrations over the shortest distance because of the model's over-estimation but does not attempt to correct for middle distance under-estimation and over-estimation again at the longer distances [*22*, p. 244]. The reason for the emphasis on the shorter distances is simply that although these residuals are comparable to the others in terms of log. values, conversion back to actual interaction predictions results in very large discrepancies.[1]

The most explicit recognition of the problem of Pareto over-estimation of short distances has been made by Claeson [11] who obtained the poor fits illustrated by the residuals above. He used an alternative curve-fitting approach by calibrating new Pareto functions from a point $1 + \Delta d$ to the right of the origin. He then wrote the function:

$$\text{Log } I_s = a' - b \log d_s \tag{9}$$

where a' denotes that the base constant should not be defined as the intercept with the y axis but with another axis Δd to the right.

Claeson's purpose in developing equation 9 seems to be to obtain more satisfactory base constants to test his hypotheses. However, the theoretical implications of the procedure have been discussed by Olsson [*38*, p. 17]. He suggests that "the curve in the distance intervals Δd . . . is probably parallel to the x axis. If this is so, interaction intensity within distance Δd is of course independent of distance." Olsson goes on to relate what he calls "the plateau effect" to the work of Garrison [*17*] and Marble [*32*], who both found that distance had little influence on short-distance shopping behavior. Morrill seems to have come to a similar conclusion. In discussing the Pareto function he writes:

> Finally the function universally exaggerates the very important close-in migration. Two possible reasons for the poor fit at close distances are (1) lack of statistical information within small administrative units and (2) the fact of overcoming heavy threshold expenses of moving; it may not be worth the effort to move only a very short distance. Research needs to be done in this field. [*34*, pp. 217–18].

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[1] Problems related to relative and absolute errors in fitting least squares curves when the dependent variable is logged (as in all models considered here) are discussed by Anderson [*2*] and Taylor [*44*].

Experiments

The research that developed directly from the above considerations consists of a series of experiments using various published sets of interaction-distance tables as test data.[2] Each experiment consists of systematically varying the distance transformation so that changes in other elements of the regression models can be observed. The first experiment is a simple test of Olsson's "plateau effect."

Olsson's "Plateau-Effect" as a Function of the Pareto Model

Since the Pareto function is a special case of the double-log. type model it is one of an infinite number of possibilities that result once the exponent is allowed to vary. From this standpoint there is no reason to assume that an exponent of unity is any more natural than any other exponent. Thus, it is of interest to see whether the plateau effect occurs generally with all models or if it is simply a property of the Pareto equation.

Graphs relating the first residual to the transformation exponent in the double-log case are plotted on Figure 1 for eight sets of test data. Beginning with the familiar Pareto model in all eight diagrams the residual is positive at $+1$ indicating Olsson's plateau effect in the test data. However, on every diagram as the exponent increases the size of the residual decreases until a negative residual occurs indicating under-estimation—the opposite of Olsson's plateau effect. In fact, these graphs clearly illustrate the point that the magnitude of the first residual is a function of the distance transformation, which means that, in terms of the problem identified above, the plateau effect is a property of the Pareto model and not of the interaction data. This point seems to have been missed by Claeson in his detailed discussion of Olsson's work [*12*]. The conclusion is emphasized by simply observing the pattern of the untransformed data, two examples of which are shown in Figure 2. These patterns are concave upwards in an extreme manner and the models in Goux's classification can be interpreted as attempts to transform the pattern into a linear one to allow simple linear regression description.

Distance Transformation and Linearity

The conclusion concerning the plateau effect should not really be surprising in view of the fact that the importance of distance transformations to distance

[2] Specifically the eight sets of test data and their sources are as follows:
(i) Perry's Dorset marriage distances data (1837–86) [*40*]
(ii) Perry's Dorset marriage distances data (1887–1936) [*40*]
(iii) Stouffer's Cleveland migration data [*33, 36*]
(iv) Hagerstrand's Asby migration data [*21, 30, 33, 36*]
(v) Hagerstrand's Kisa migration data [*21*]
(vi) The Traffic Audit Bureau's Cedar Rapids contact data [*33, 36*]
(vii) Pitt's Kagawa marriage distances data [*36*]
(viii) Henderson's Seattle marriage distances data [*36*].

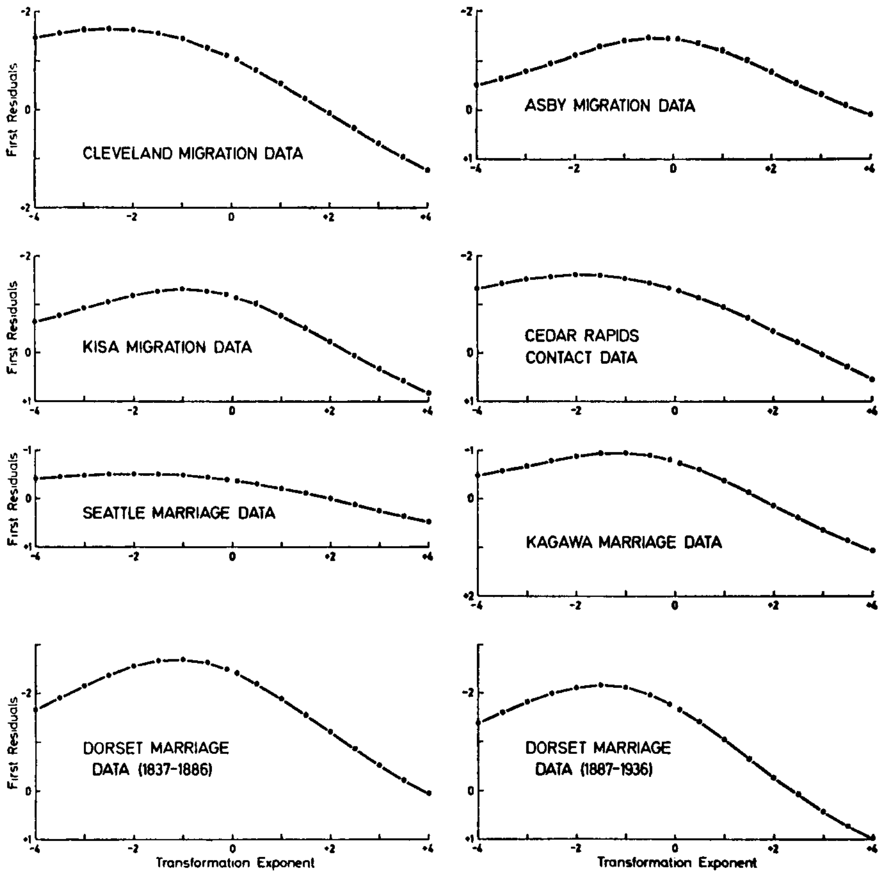

FIG. 1. First Residual-Transformation Patterns: the Double-Log. Case.

decay functions was recognized in genetics research a generation ago. Like the Pareto model the normal distribution was found to give a poor fit to interaction data but in this case the model consistently under-estimated near the origin as universally as the Pareto over-estimated. To produce more accurate leptokurtic models Bateman [4] compared different transformations and derived his square root exponential model. The pattern of the first residuals with transformation exponents for the eight sets of test data in the single log. case is shown in Figure 3 and clearly illustrates Bateman's arguments. As with the double-log. case (Figure 1) a functional relationship is apparent. The normal model is represented by the exponent at $+2$ and, as expected, the residuals are positive confirming leptokurtosis in every case. The absolute size of the residual decreases as we move below an exponent of $+2$ towards the exponential model at $+1$ where the residual is much smaller, finally changing to a negative residual in the vicinity of $+0.5$, the square root exponential

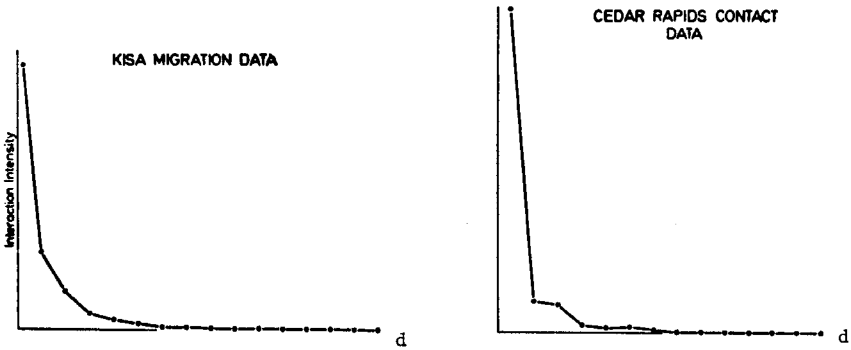

FIG. 2. Two Examples of the Test Data.

model in each case. The fact that this change from positive to negative residual agrees with the transformation that Bateman [4] found better is not a coincidence but is related to the degree of linearity achieved with that particular transformation.

The Asby migration data were selected for plotting using the five different distance scales each corresponding to a transformation required by one of the standard models (Figure 4). In every case the y axis, 'standardized interaction,' uses a logarithmic scale as required by all the standard models and the least squares regression line is also shown on each graph. Beginning with the exponential model, where distance is not distorted, the data are reasonably linear but a slight degree of the original concave upwards pattern of the data remains. This shows that a logarithmic transformation of the interaction alone is not sufficient to make the data linear and, thus, the regression line will under-estimate at short and long distances. If distances are squared, as for the normal model, the data pattern reverts towards the original concave upwards form and the regression line gives a particularly poor fit. This suggests that the opposite type of distortion of data, that is one that compresses distance intervals at longer distances, is required. The three remaining standard models all incorporate this type of transformation and with the square root exponential particularly, the concave upwards pattern all but disappears and a close-fitting linear regression curve is noted. These relationships between the single-log. models extend Bateman's [4] conclusions into a social science context.

Turning to double-log. models the Pareto function, which incorporates distance as logarithmic scale, is considered first. Here a new 'concave downwards' pattern is found which also appears in the log-normal case but to a lesser extent, the squaring of the log. distance having counteracted the full distortion of the logarthmic scale.

The relationships between the five models illustrated by the Asby data can be considered typical for all the data considered in this investigation. These findings are translated into a general schematic representation of the effect of the model's distance transformations on interaction data (Figure 5). This

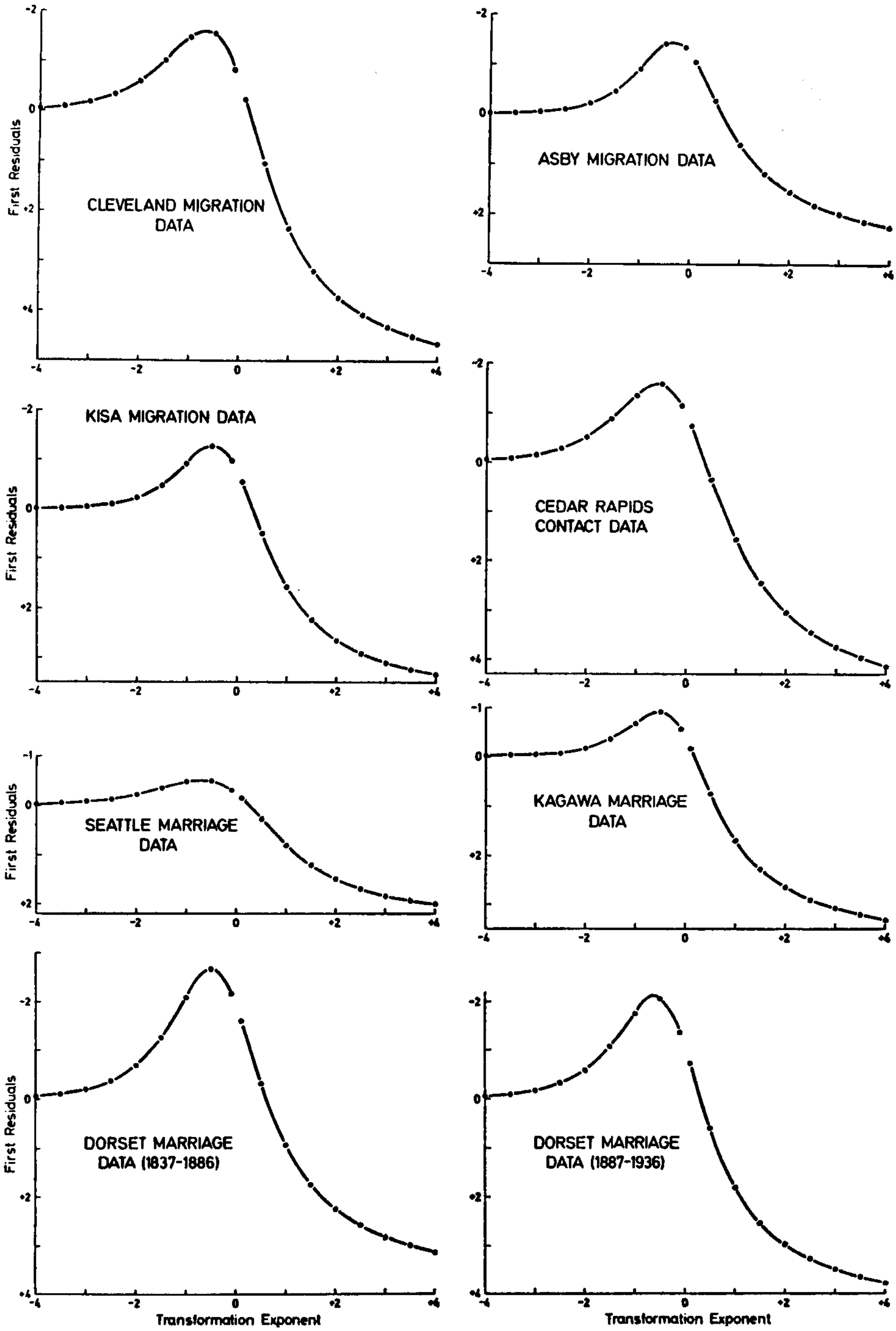

FIG. 3. First Residual-Transformation Patterns: the Single-Log. Case.

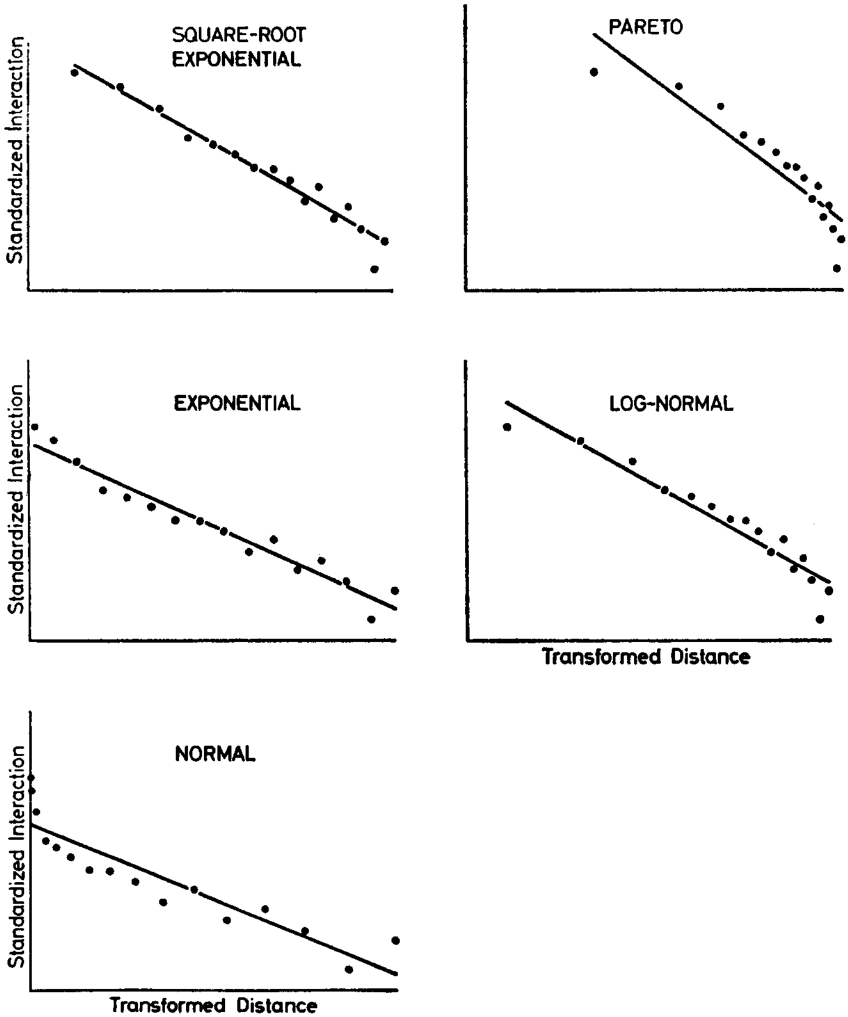

FIG. 4. The Five Standard Model Descriptions of the Asby Migration Data.

schematic representation of the relationships between the five standard models can be traced back to the querying of the plateau effect with the Pareto model. Another, similar example of an erroneous interpretation can be identified, this time using the exponential model.

Perry's "Four Mile Zone" as a Function of the Exponential Model

As part of his study of working-class isolation in Dorset in the nineteenth century, Perry [*40*] plots marriage distance data on single-log. graph paper

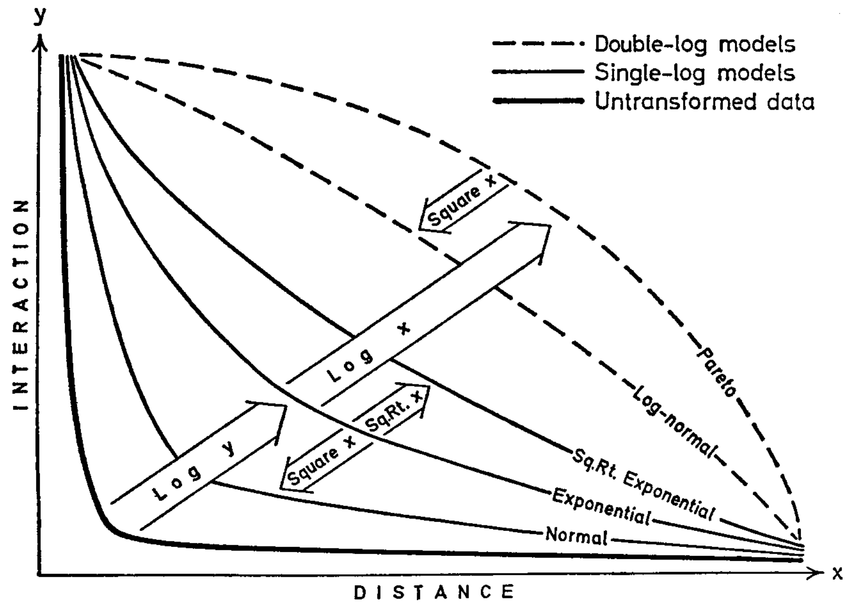

FIG. 5. A Schematic Representation of the Influence of Distance Transformations.

and describes it with two linear regression curves. Thus he has two exponential equations, one for up to four miles with a steep gradient and another for longer distances which has a gentle gradient. Perry interprets his analysis as follows:

> . . . it is suggested that four miles—an hour's walk for a young countryman—was perhaps the greatest distance that the average man was prepared to walk (and return) at fairly frequent intervals, and thus the greatest distance at which most of the working class could maintain a courtship. [*40*, p. 131.]

Perry [*41*] has subsequently furnished further qualitative evidence for the existence of this four mile zone. This finding, like Olsson's plateau effect, is intuitively sensible and in this case ties in well with spatial behavior deduced from central place patterns. In his study of market areas in East Anglia, Dickinson [*15*] makes the point that "the distribution of medieval markets was such that all places were within two to four miles of one or more markets." Thus, Dickinson mentions the same threshold distance, 4 miles, as Perry when people had to walk to market, and this suggests the existence of some general critical threshold distance, at least for rural areas, before the advent of modern transport facilities.

Given that distance transformation can be a variable element however, a quite different interpretation of Perry's analysis can be made. In using the exponential model, linearity is dependent upon the single logarithmic transformation of interaction with no distortion of distance. The schematic diagram (Figure 5) predicts that this model will only reduce the concave upwards pattern of the data and will not produce a linear pattern. From this viewpoint Perry's initial steep gradient is quite compatible with the exponential model's position within the schematic framework presented above. Using a Pareto model with his data still produces a "plateau effect" (Figure 1). What has happened is that Perry has used a model which has not made his data linear so he has described the resulting curve by two straight lines. This is legitimate [*19*] but it does lead to dangers when interpretation is attempted with data that has been transformed. Perry's four-mile zone, like Olsson's plateau effect, results from the model used, in this particular case the exponential model.

Optimum Distance Transformation

Given a set of non-linear data there are three possible approaches to its description. The first is to describe it in terms of a polynomial function, such as the quadratic function used by Claeson [*11*] and Helvig [*24*], although if these models are used, it is not always obvious what role distance has as it occurs at least twice in the equation. The second approach, illustrated above, involves dividing the data into subsets and describing each subset separately by a linear equation. This method is particularly useful where it suggests 'threshold' values about which some functional relationships change. For instance, Chorley [*13, 14*] finds a threshold height at 720 feet around which the mean slope-altitude relationship changes in the Heddon basin, North Devon. However, this threshold value is derived from regression lines fitted to untransformed data whereas threshold values identified in spatial interaction by Claeson [*11*] and Perry [*40*] are the results of an approach which combines double-curve fitting with data transformation. As shown above, since there is no 'naturally' acceptable transformation, almost any required pattern of double straight line curves can be produced by manipulating the transformation and the solutions obtained using this method in spatial interaction studies are of little theoretical interest. The third approach involves finding the best transformation for a single linear description of the data. It is this approach that is pursued here.

The objective function used is the standard error of prediction using solutions of the least-squares type. The optimum transformation involves that exponent for which the standard error is minimized and it is applied with or without a logarithmic distance base so that both the double-log. and single-log. cases can be considered.

The Algorithm.—Drawing upon the information produced by the above experiments, standard errors of prediction are plotted against transformation exponents in Figures 6 and 7.

In the case of the double-log models, (Figure 6) the eight sets of data show a standard error monotonically declining through the Pareto and log-normal models to minima generally somewhere between the second and fourth power the rising again in the same manner. In the single-log. models (Figure 7) the minima are more pronounced, between zero and one, with the standard errors

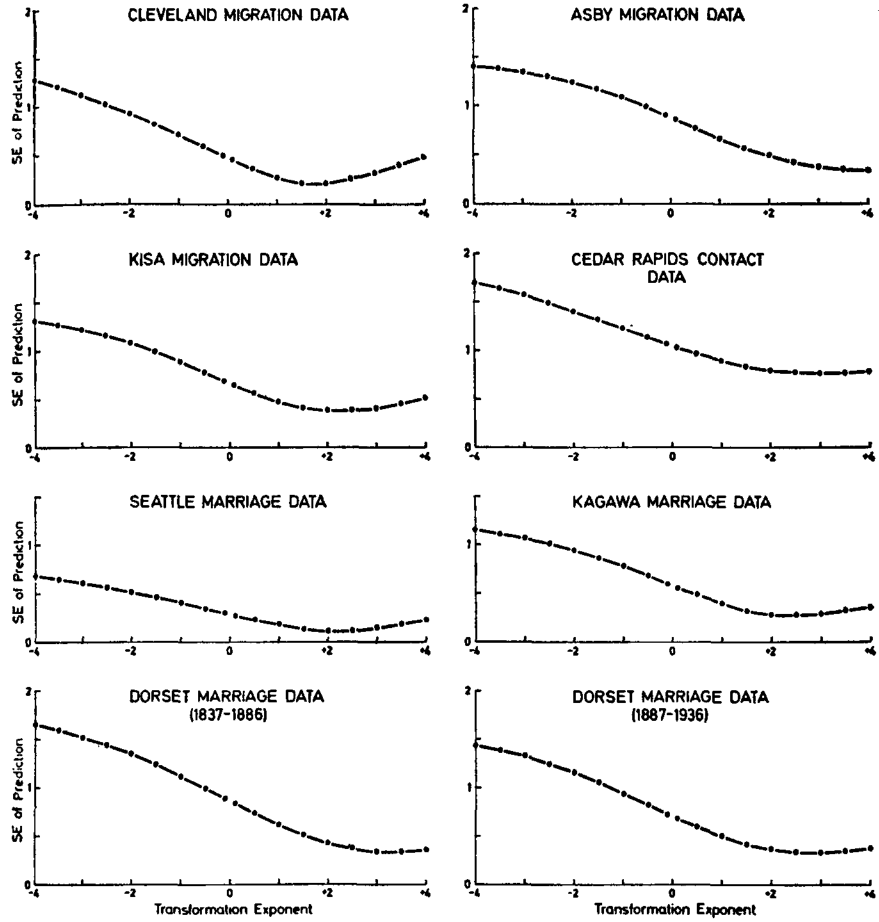

FIG. 6. Standard Error-Transformation Patterns: the Double-Log. Case.

rapidly rising to the right of these points. For all eight sets of data the function graphed is U-shaped and asymmetrical. This allows for a simple convergence algorithm to be devised to find the minimum point.

The specific problem is to find the minimum standard error of prediction (y) and the corresponding optimum transformation of distance (x). The algorithm [*44*] starts with two outer x values about the turning point then defines two inner points at one third and two thirds of the way between them on the X axis. One x value is discarded with each iteration so that three x values are left containing the turning point. The algorithm does not necessarily simply discard the x value with the largest y but rather finds the x value with the lowest y and keeps two x values on either side of it. The iteration then proceeds by defining these two x values as new outer points, two new inner points are found, and the procedure is repeated. Iterating with four x values allows

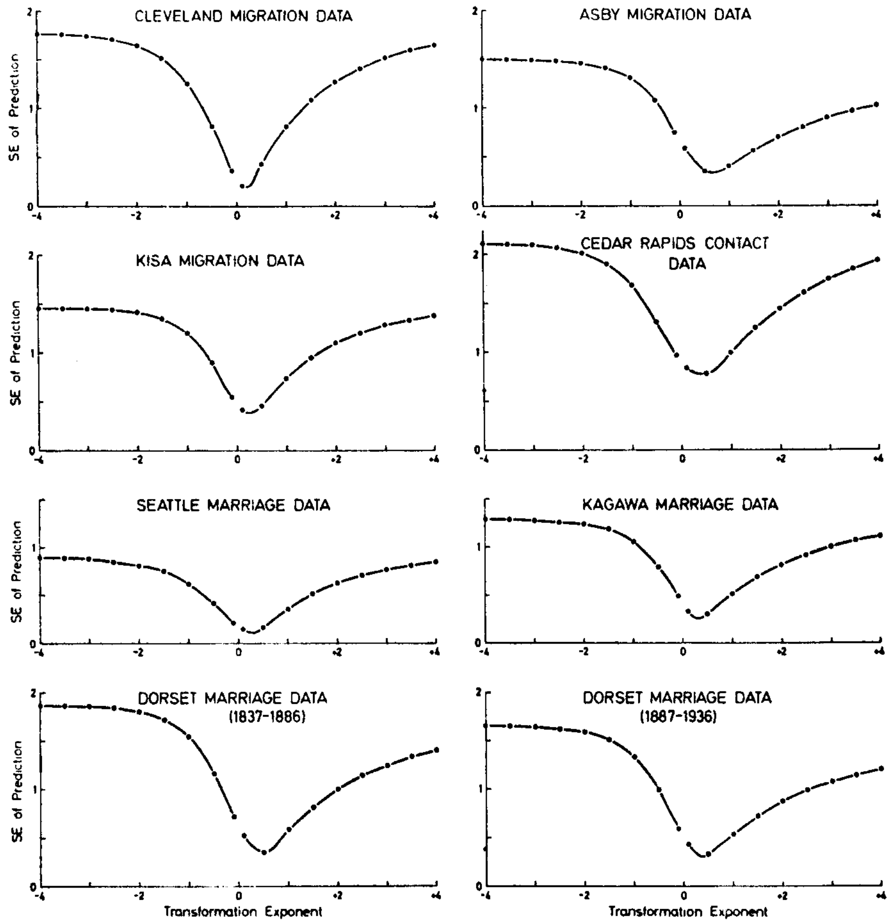

FIG. 7. Standard Error-Transformation Patterns: the Single-Log. Case.

convergence on the turning point of an asymmetric U-shaped curve. The iteration is terminated when a minimum value of y is found to a specified degree of accuracy and an optimum solution for x produced.

Optimum Solutions.—Table 1 shows the optimum transformation exponents and their corresponding minimum standard errors in both the single-log. and double-log. cases for the eight sets of test data. These results in general collaborate much of the previous evidence concerning distance transformation. For instance all the optimum solutions occur on the exponent axis at positions previously suggested by the first residuals (Figures 1 and 3), the standard error pattern (Figures 6 and 7) and the schematic diagram (Figure 5). However, although they are to some extent predictable, these optimum solutions do allow more specific discussion of the various sets of data in the two model situations.

TABLE 1

OPTIMUM SOLUTIONS

Data	Double-Log Models: Optimum Exponent (m)	Double-Log Models: Standard Error	Single-Log Models: Optimum Exponent (m)	Single-Log Models: Standard Error
Cleveland	2.17	0.2137	0.14	0.2148
Asby	6.21	0.3452	0.65	0.3411
Kisa	2.95	0.3838	0.25	0.3879
Cedar Rapids	2.92	0.7714	0.37	0.7748
Seattle	1.74	0.1178	0.25	0.1145
Kagawa	2.72	0.2846	0.33	0.2684
Dorset (1837–86)	2.75	0.3355	0.45	0.3501
Dorset (1887–1936)	2.45	0.3408	0.39	0.3249

In the case of the double-log. models in Table 1, the log-normal model is much closer to the optimum than the Pareto in every case. The superiority of the former model has been previously suggested by Kulldorff [*30*], and more recently by Morrill and Pitts [*36*]. However, in all but one case (the Seattle data) even lessening the basic logarithmic over-transformation by squaring is insufficient and larger exponents are found to be optimal. In one case, the Asby data, a power as high as six is suggested. Thus, although the log-normal model is preferable to the Pareto, the conclusion is that this model still tends to over-transform distance.[3]

The relationship between optimal solutions and the standard models for the single-log case is almost identical. Once again the optimum solution, in all but one example, is not *among* the standard models but *beyond* them. Also, the model previously suggested as superior, the square root exponential [*4*] is closer to the optimum in every case. However, it would seem that this model undertransforms as most optimal solutions are between the third and fourth root.

One way in which the optimum exponent can be interpreted is as a measure of the degree of concavity in a set of data. Referring back to the schematic diagram in Figure 5, the double-log case measures the degree of concavity downwards from the Pareto model and the larger the exponent the more the overtransforming logarithmic base is reversed. In the single-log case, on the other hand, only interaction is logged so that the optimum exponent shows how far distance needs to be distorted to produce linearity so that the degree of concavity is measured upwards from the exponential model. Thus linearity is approached from two different directions: one exponent is measuring concavity downwards and the other upwards. Therefore, it is not surprising that the two sets of exponents do not agree in cases where differences between untransformed data are slight although it is encouraging that the exponents agree for the extreme cases (Seattle and Asby).

Where difference between relative sizes of exponents occur it is natural to

[3] It should be noted here that in subsequent use of this approach, using journey to work data between administrative areas, Gleave [*27*] found the Pareto model to be the most accurate of the five standard models in several instances.

inquire whether one set of optimum solutions tend generally to have lower standard errors so that they might be considered as overall optimum solutions. The evidence of Table 1 rejects this idea with the single-log. and double-log. cases sharing equally the optimum solutions at four each. This suggests that neither general model type is superior to the other.

Concluding Remarks

The final section consists of brief discussions on two themes. First, the optimum distance transformation approach is related to a sequence of research in spatial interaction studies and, second, the paper is concluded by relating the problems discussed above to some more general data transformation studies outside the distance decay context.

Historical Perspective

When presenting a new procedure in any field of research it is useful to consider where it fits into the overall pattern of development. The pattern identified here involves dropping two assumptions implicit in the original gravity model. The gravity model originally involved two particularly limiting restrictions, first, the distance transformation was specified as logarithmic distance, and second, the gradient of the transformed distance was the inverse square. This latter assumption was relaxed first. Anderson [*1*] in a paper comparing the gravity model with the intervening opportunity model tried three gradients: $-\frac{2}{3}$, -1 and -2, although an optimal alternative was not sought. The Pareto model of Ilke [*28, 29*] and Hagerstrand [*20*] introduced this next stage with a linear regression analysis derivation of the gradient giving an optimum solution in terms of least squares criteria.

However, the Pareto model maintained the distance transformation assumption of its predecessor. This was relaxed with the introduction of the work of geneticists into spatial interaction research after the 1962 Monaco conference [*43*]. As with the dropping of the gradient assumption by Anderson, the introduction of the geneticist's models led to trials of various transformations [*36*] prior to considerations of optimal solutions. The iterative algorithm outlined in this paper has addressed this question.

The General Transformation Problem

This paper has been solely concerned with the phenomena of spatial interaction although it would seem that the distance transformation problem is but one of a wide range of transformation problems in data analysis in general. At least two basic situations in which data transformation is carried out can be identified: (i) to convert data to fit some model assumptions (e.g. normality) prior to analysis [*3*] and (ii) to convert an apparently complex relationship into a simpler one so that the subsequent descriptive equation is easier to interpret.

The case presented here is obviously of the second type. A similar curve fitting procedure has been developed by Stevens [*24*, *41*] for describing growth curves in biology. However, these two specific approaches for decay and growth patterns are much more limited in scope than the work of statisticians, especially that of Box [*7*, *8*]. He presents procedures for transforming single independent variables in first- (linear) and second-order polynomial functions, for transformation of several independent variables in the multi-variate case, for the transformation of the dependent variable, and some discussion of the possible combination of joint transformations of both sides of an equation. In the case of dependent variable transformation [*7*], diagrams of standard error variations are presented which are similar to Figures 6 and 7 above. Thus it can be seen that the discussion and solution of the transformation problem in the spatial interaction context is clearly just one small aspect of a much more general topic of interest to statisticians. What the discussion in this paper does specifically contribute is to emphasize a basic need to appreciate the overall role of transforming data in a research situation: first to satisfy some legitimate research purpose and secondly to change the pattern of inter-relationships between the variables being examined. The spatial interaction research described above can thus be viewed as a specific case study that clearly illustrates the point that any interpretation of results must involve a careful consideration of this second 'spin-off' effect of the transformation process.

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