2018-02-12_ASTR445_galaxyCountingOnSphere

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0.1 2018-01-12 ASTR 445 Galaxy counting on the sphere

In discussion of number counts of galaxies uniformly distributed on the sphere, the following expressions arise:

$$c = 2\pi R^2 n \left[1 - \cos\left(\frac{r}{R}\right)\right] \tag{1}$$

while for flat space,

$$c_f = \pi r^2 n \tag{2}$$

On reflection, it's not immediately obvious how to demonstrate which is bigger, c~or c_f. Here's the comparison:

Recognizing that $r = R\theta$ (using angles in radians), we see that the spherical version becomes

$$c = 2\pi r^2 n \left[\frac{1 - \cos\theta}{\theta^2}\right] \tag{3}$$

which of course becomes

$$c = \frac{c_f}{2} \left[\frac{1 - \cos\theta}{\theta^2}\right] \equiv c_f f(\theta) \tag{4}$$

So the question of whether the counts on the sphere are greater or smaller than on the plane, becomes a question of whether the $f(\theta)$ is bigger or smaller than 0.5.

It's not obvious to me analytically quite how $f(\theta)$ behaves, so I just do this numerically:

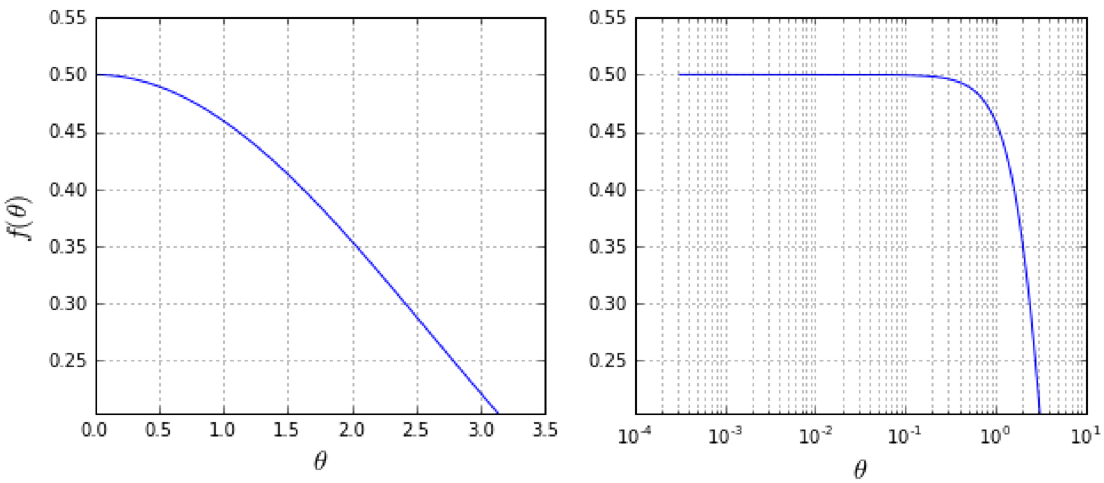
```
In [1]: import numpy as np
        import matplotlib.pylab as plt
        %matplotlib inline
```

```
In [2]: theta = np.logspace(-4,0., 1000) * np.pi
        fTheta = (1 - np.cos(theta))/theta**2
```

```
In [3]: fig1 = plt.figure(1, figsize=(10,4))
        ax1 = fig1.add_subplot(121)
        dum = ax1.plot(theta, fTheta)
        ax1.set_xlabel(r'$\theta$', fontsize=16)
        ax1.set_ylabel(r'$f(\theta)$', fontsize=16)
        ax1.grid(which='both')
```

```python
ax2 = fig1.add_subplot(122, sharey=ax1)
dum2 = ax2.semilogx(theta, fTheta)
ax2.set_xlabel(r'$\theta$', fontsize=16)
#ax2.set_ylabel(r'$f(\theta)$', fontsize=16)
ax2.grid(which='both')
ax2.set_ylim(np.min(fTheta), 0.55)  # [sic]
```

Out[3]: (0.20264236728467555, 0.55)



And so we see that $f(\theta) \leq \frac{1}{2}$, for $0 \leq \theta \leq \pi$.

In [ ]: