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Tracking and Analyzing the Momentum of a Midair Collision Using a 3D Camera

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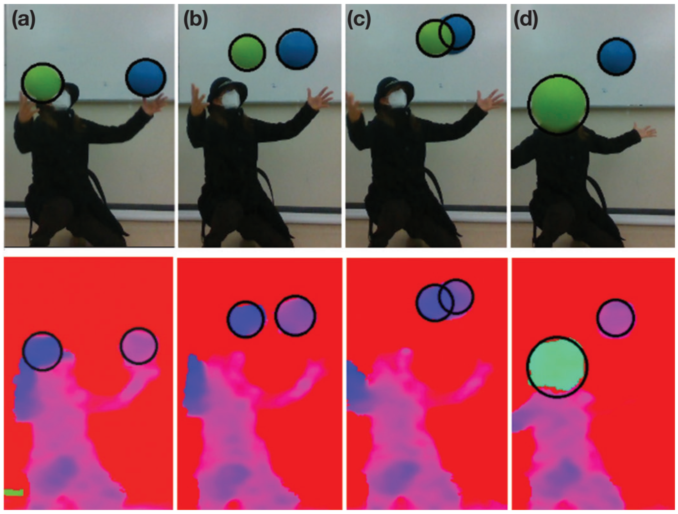

Fig. 1. Images of the tracked balls in flight. The black circles around each ball represent the location as detected by the software. The top row displays RGB camera images, while the bottom row displays colorized depth images that show light green, blue, magenta, and red in increasing distance from the camera. Each column represents an instance in time: (a) first image after the balls are released, (b) last image tracked before the collision, (c) first image tracked after the collision, and (d) last tracked image of the video sequence.

Video capture has been a common tool in physics instruction for several decades. Due to technological improvements in camera systems that are affordable and easy to use, students in advanced high school and introductory college physics courses can now analyze more complex systems than in the past. Here, we use a 3D camera system with a Python program to automatically track and investigate the momentum of a midair collision between two playground balls. The program is freely available for instructors and is open source.[1] The collision activity can be used in a student lab setting or as an instructor-led, in-classroom exercise. Students will be required to analyze a two-mass system synthesizing key concepts from mechanics (e.g., kinematics, dynamics, center of mass, conservation laws, momentum, and impulse) and will discover that the slope of the momentum as a function of time is the force acting along each axis.

Midair collisions of two balls rarely stay within the original plane of motion, making it difficult to use traditional cameras for tracking their locations quantitatively. However, a single 3D camera can provide depth tracking, enabling real-time motion capture and analysis of colliding projectiles along all three coordinate axes. Typically, during momentum labs, such as the 3D experiment done with two smartphones by Pereira et al.,[2] students determine whether the momentum of the system is conserved. Similarly, in our activity momentum conservation will be analyzed. However, an emphasis is placed on the momentum of the system's center of mass as a function of time in all three spatial dimensions.

3D tracking

In order to analyze and model motion in 3D, physics educators have developed labs that use multiple cameras,[2,3] local positioning systems,[4] 3D simulations,[5] and the Microsoft Xbox Kinect sensor.[6] Here, we present a novel approach using software that displays 3D position and momentum data for multiple objects that students can capture with a reasonably priced 3D camera. While video analysis software programs, such as Tracker,[7] are capable of tracking objects in videos by color once the videos are downloaded,[8] motion is limited to one spatial plane, and distortion may occur at close range.[9] Our solution is not inhibited by such restrictions, automatically tracking motion in all dimensions and without distortion at close range.

We used the Intel RealSense D435i,[10] composed of an RGB camera, two infrared cameras, and an infrared emitter. The two infrared cameras determine depth using stereoscopic vision and are enhanced at close range (depending on lighting conditions from about 0.2 to 3 m) by detecting a scattered pattern produced by the infrared laser emitter. Under ideal conditions, the camera system achieves a 1.5% depth accuracy. The camera is capable of tracking objects using color at a resolution of 1280 × 720 at 60 frames per second (fps) for prerecorded scenes or 30 fps in real time. We found the RealSense camera well suited for this task, but other 3D camera systems like the Microsoft Kinect[6] and modern smartphones or tablets, which increasingly have depth-sensing capabilities, could be used as well.

We wrote a Python program to collect and analyze the data from the 3D camera system.[1] Familiarity with Python is helpful for teachers to use the program, but we have released the software such that it can be run as an executable without any required programming experience. While the program can be used for motion tracking in more general cases, in this particular case the software automatically tracks the motion of the color balls. Additionally, it displays the position as a function of time, as well as calculates and displays the momentum of the balls. For the activity, it is left as an exercise for the student to calculate the center of mass of the system. That data is added back to the program to display the position and momentum of the center of mass along with the individual balls.

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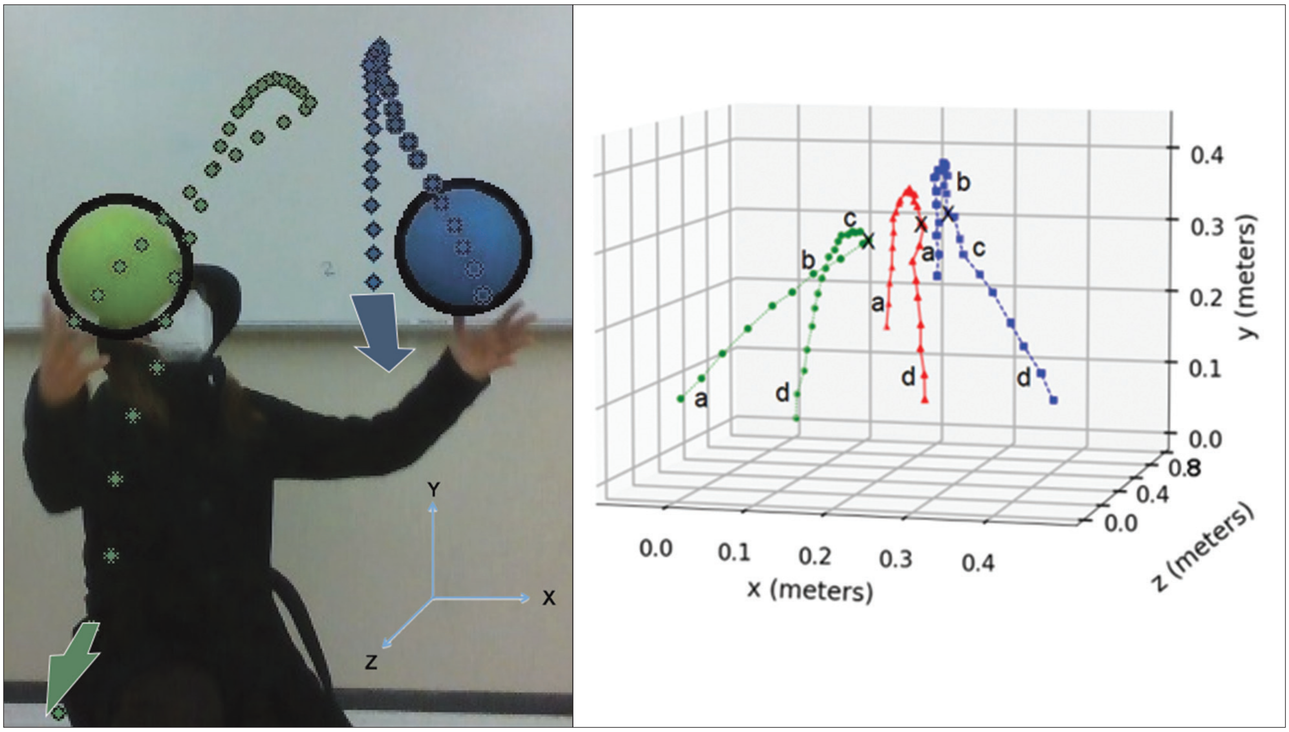

Fig. 2. Left: An image from the video showing the two colored playground balls. Overlayed on the image are dots representing the paths of the balls after they are tossed. Right: Three-dimensional graph of the corresponding position data as displayed by the program (with modified axis for legibility and overlaid letters to indicate the significant moments in time). The green circles, blue squares, and red triangles represent the green ball, blue ball, and the center of mass's position respectively. The letters a–d represent the ball at the locations noted in Fig. 1.

Activity

The objective of the activity is to study how momentum changes as a function of time for a midair collision along all three planes of motion. Positional data along with video of the scene are recorded while the balls are in flight (Figs. 1 and 2).

We collected data for a collision of two playground balls, a blue ball ($m_1 = 240 \pm 1$ g) and a green ball ($m_2 = 118 \pm 1$ g). All other green and blue objects were removed from the scene to assist the software in tracking the balls and reduce noise in the data. The camera was level and about 2 m from the experimenter. Care must be taken that balls are tossed lightly to reduce camera blur. The *x*- and *y*- positions were tracked from the RGB camera pixel location while the depth (*z*-position) was provided by the camera system and determined by parallax using the two infrared cameras. Collecting the video before tracking, as opposed to analyzing it in real time, optimized the data rate to 60 fps, providing at least five data points on either side of the collision. The measured location of the ball's center is less accurate when obscured by the other ball as is seen in Fig. 1(c), where the tracking circle on the blue ball does not encompass the entirety of the ball. The center of mass of the system was calculated by the students with guidance from a provided spreadsheet, then fed back into the program and displayed alongside the data for the individual masses. Using the video of the collision as seen in Fig. 1, we separated the

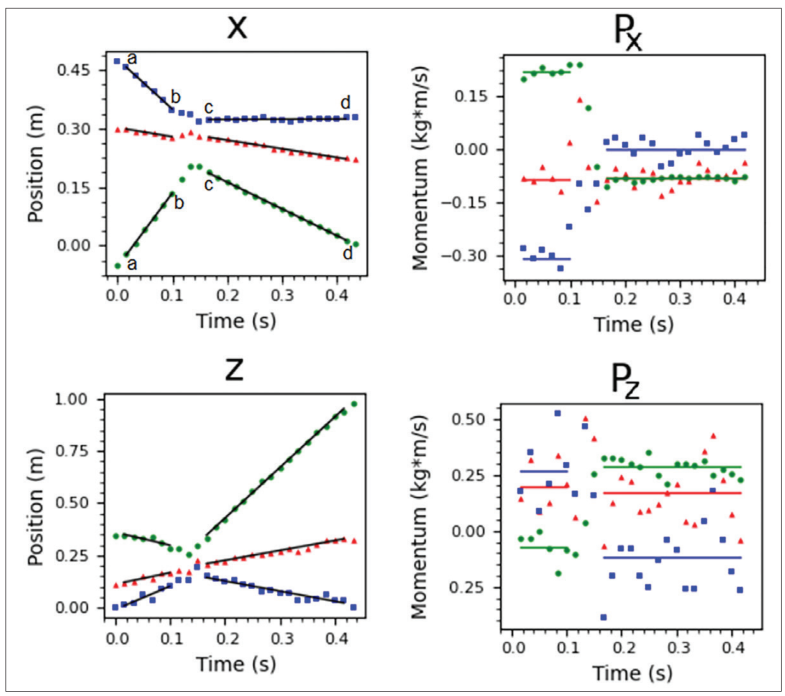

Fig. 3. Position and momentum as a function of time for the horizontal directions as displayed by the program (with letters overlaid on the *x*-position graph to indicate the analyzed moments in time). The green circles, blue squares, and red triangles represent the green ball, blue ball, and the center of mass of the two-ball system, respectively. The black lines represent linear fits to the position data calculated by the program. The solid color horizontal trendlines for the momentum data were calculated and graphed by the program from the derivative of the linear fits for the respective position graphs before and after the collision. Note that the line for momentum of the center of mass in the *x*-direction is hidden by the line for the green ball after the collision.

collision into three time domains: before the collision (a–b), overlapping and colliding balls (b–c), and after the collision (c–d). For each time interval, trendlines and graphs are automatically generated by the program. Gathering data along all three coordinate planes provides the opportunity to analyze horizontal and vertical motion separately.

Position and momentum along the horizontal x- and z-axes for the center of mass as well as for each ball are shown in Fig. 3. As would be expected for a system with no external forces, comparing the slopes for each ball on the position graphs before and after the collision immediately shows the greater change in velocity for the smaller mass after the collision. The momentum graphs are calculated by the program with Eq. (1) using the mass the students enter for the two balls, as well as the change in position from one data point to the next:

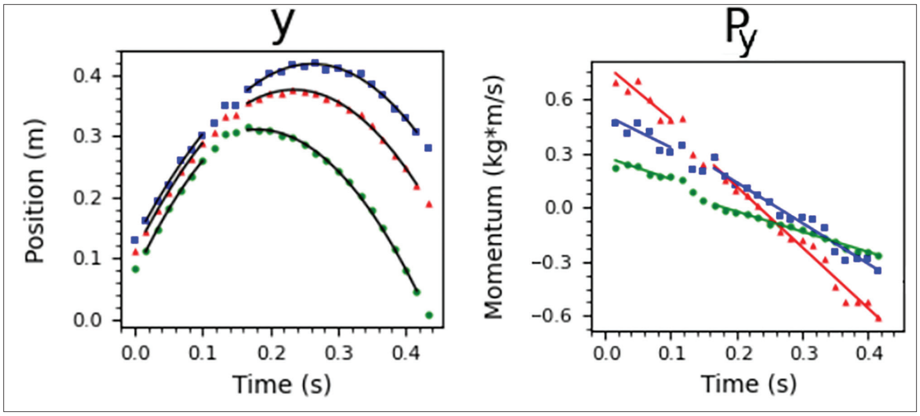

Fig. 4. Vertical position and momentum as a function of time. The black lines represent quadratic fits to the position data calculated by the program. The solid color linear trendlines for the momentum data were calculated and graphed by the program from the derivative of the quadratic fits for the respective position graphs before and after the collision.

$$\boldsymbol{p}_x(t) = m \cdot \frac{d\boldsymbol{x}(t)}{dt} \qquad \boldsymbol{p}_y(t) = m \cdot \frac{d\boldsymbol{y}(t)}{dt} \qquad (1)$$
$$\boldsymbol{p}_z(t) = m \cdot \frac{d\boldsymbol{z}(t)}{dt}.$$

The momentum can be written as a function of time as described in Eq. (2):

$$\boldsymbol{p}(t) = \boldsymbol{F}_{\text{avg}} t + \boldsymbol{p}_i. \qquad (2)$$

We found presenting the relationship between external forces and the momentum in this way was most instructive to students. Air resistance was small at the velocities involved; thus, one can reasonably neglect forces acting in the horizontal directions. As such, consistent with Eq. (2), the momentum of each ball stays the same in the horizontal plane (see Fig. 3) except for the time of the collision. The momentum for the center of mass in this plane is observed to stay the same for the full motion as there are no external horizontal forces acting on the system (see red triangles in the momentum graphs in Fig. 3). The change in the horizontal momentum of each ball from the collision is approximately equal and opposite of that of the other ball. Teachers and students can use this data to discuss Newton's third law, conservation of momentum, as well as external and internal forces on a closed system.

Figure 4 shows the vertical position and momentum vs. time data. Due to the influence of gravity, the position vs. time paths are parabolic before and after the collision. The slope of the momentum graph for each ball is the same before and after the collision and is only offset due to momentum transfer during the collision. Due to its smaller mass, the offset is larger for the green ball. If air resistance is negligible, gravity is the only force acting on the system during these periods, and the slope of the momentum as a function of time represents the weight. The momentum transfer during the collision is internal to the system, and the plot of momentum vs. time for the center of mass could be fitted with a linear line for the whole motion. This allows for the slope, now representing the weight of the two-mass system, to be calculated and compared before and after the collision or for the entire duration. Calculating the weight of the system from Eq. (2) with a linear fit to the point-by-point center of mass momentum (p_y) vs. t data and estimating the uncertainty using the Linest function results in a force of 3.38 ± 0.18 N. This can be compared to the 3.51 ± 0.01 N that one obtains using $(m_1 + m_2)g$. By dividing the force by the mass, students can also get the measured value for the gravitational acceleration, which was in our case 9.43 ± 0.50 m/s^2.

Conclusion

This activity provides an opportunity for students to connect previously acquired concepts of weight, projectile motion, and kinematic graphs to newly acquired knowledge of impulse and momentum. It also provides practice working with concepts in three dimensions, which can be challenging for students throughout the physics curriculum.

The use of 3D cameras and accompanying software expands the number of possible experiments for high school and undergraduate students, as well as providing kinesthetic/visual assistance in the students' conceptualization and quantification of physics phenomena. We expect the 3D camera and tracking software to become more prominent in physics classrooms as it provides the opportunity to reduce processing time, leaving more time to analyze complex systems.

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