

LINEARITY

General Form

$$\underline{Lu = 0}$$

Properties

$$L(au) = aLu, \quad L(u_1 + u_2) = Lu_1 + Lu_2$$

$$Lu = 0 \rightarrow aLu = 0 \rightarrow L(au) = 0$$

$$\rightarrow Lu_1 + Lu_2 = 0 \rightarrow L(u_1 + u_2) = 0$$

$$L(\alpha u_1 + \beta u_2) = \alpha Lu_1 + \beta Lu_2$$

Show, $\frac{du}{dt} + \frac{1}{\tau}u = 0$ is a linear inhomogeneous eqⁿ.

$$Lu \equiv \frac{du}{dt} + \frac{1}{\tau}u \quad (\text{since } Lu = 0)$$

$$\forall L(au) = \frac{d(au)}{dt} + \frac{1}{\tau}au = a \left(\frac{du}{dt} + \frac{1}{\tau}u \right) = \underline{aLu}$$

AND

$$\forall L(u_1 + u_2) = \frac{d(u_1 + u_2)}{dt} + \frac{1}{\tau}(u_1 + u_2)$$

$$= \frac{du_1}{dt} + \frac{du_2}{dt} + \frac{u_1}{\tau} + \frac{u_2}{\tau}$$

$$= \frac{du_1}{dt} + \frac{u_1}{\tau} + \frac{du_2}{dt} + \frac{u_2}{\tau}$$

$$= \underline{Lu_1} + Lu_2$$