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Proof nets for first-order additive linear logic.
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In this talk we will present canonical proof nets for first-order additive linear logic, the fragment of linear logic with sum, product, and first-order universal and existential quantification. The central challenge is to combine the witnessing information to existential quantifiers with the behaviour of the additive conjunction. The latter creates multiple "slices", each containing a different version of the same quantifier, with a potentially different witness.

The challenge is met by upending the traditional evaluation of an existential quantifier by an immediate substitution: instead, the substitution is recorded separately, at each axiom link in the proof net. The result is a canonical notion of proof nets for this logic. The main thrust of the work resolves the technical consequences of the design. Efficient and intuitive correctness and sequentialization are given by "coalescence", an additive version of multiplicative contractibility; in essence, this is top-down sequentialization by simple graph rewriting. A main contribution is an intricate geometric correctness condition, which subtly combines "slicing" correctness for propositional additives with "dependency" correctness for quantification. Cut-elimination involves the composition of the witnessing substitutions in two proof nets by "composition + hiding" in the style of game semantics.

A further contribution is the observation — following recent work by Dominic Hughes for first-order multiplicative linear logic — that witnessing information can be omitted from first-order additive proof nets altogether, and reconstructed via unification. This yields a further, coarser notion of proof net that factors out any inessential choice in witness assignment.

Details can be found in the technical report [1].

[1] Willem B. Heijltjes, Dominic J. D. Hughes, and Lutz Straburger. *Proof nets for first-order additive linear logic.* Inria Research Report RR-9201, 2018