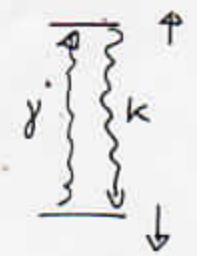
classical master / rate equation

prob. to be in excited state

$$\dot{n} = -kn + \gamma p$$

prob. to be in ground state

$$\dot{p} = kn - \gamma p$$



assume : $\quad k = \gamma$

$$\partial_t \begin{pmatrix} n \\ p \end{pmatrix} = k \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} n \\ p \end{pmatrix}$$

$$\left| \begin{pmatrix} -1-\lambda & 1 \\ 1 & -1-\lambda \end{pmatrix} \right| = 0$$

$$(1+\lambda)^2 - 1 = 0 \qquad (1+\lambda)^2 = 1 \qquad \lambda_1 = 0$$

$$\lambda_2 = -2$$

$$v_1 = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} , \quad v_2 = \frac{1}{2}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} = -2\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$v(t) = c_0 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_1 e^{-2tk} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$t=0$: $\quad v(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

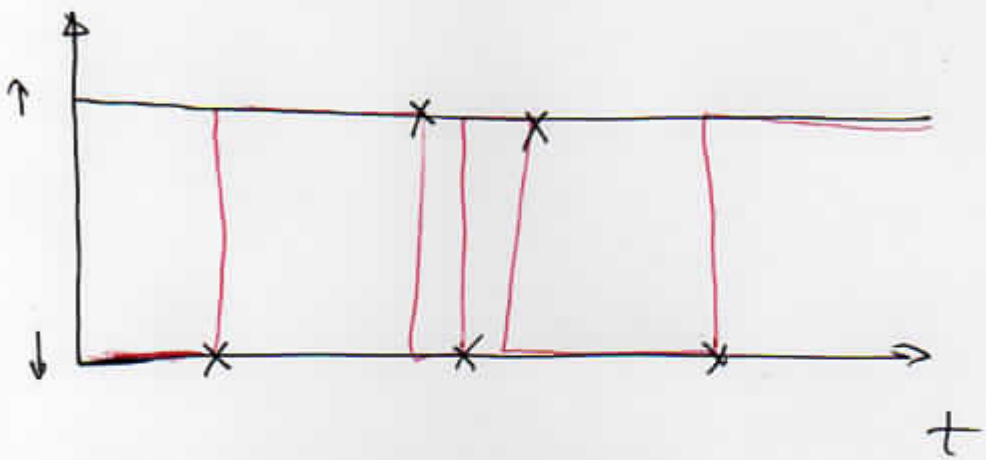
$$v(0) = c_0 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} c_0 - c_1 \\ c_0 + c_1 \end{pmatrix} \qquad c_0 = c_1 = \frac{1}{2}$$

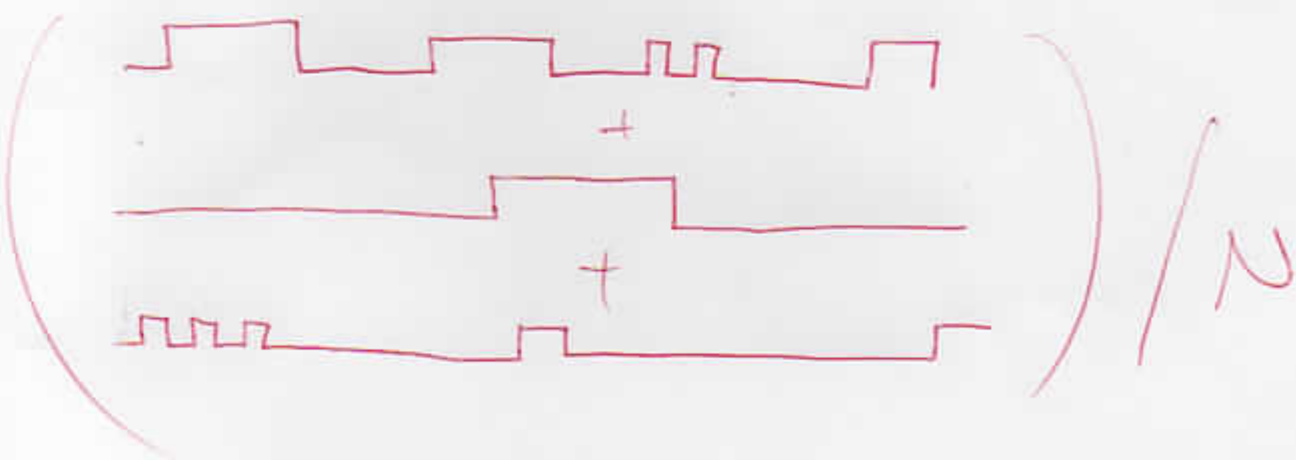
average behaviour

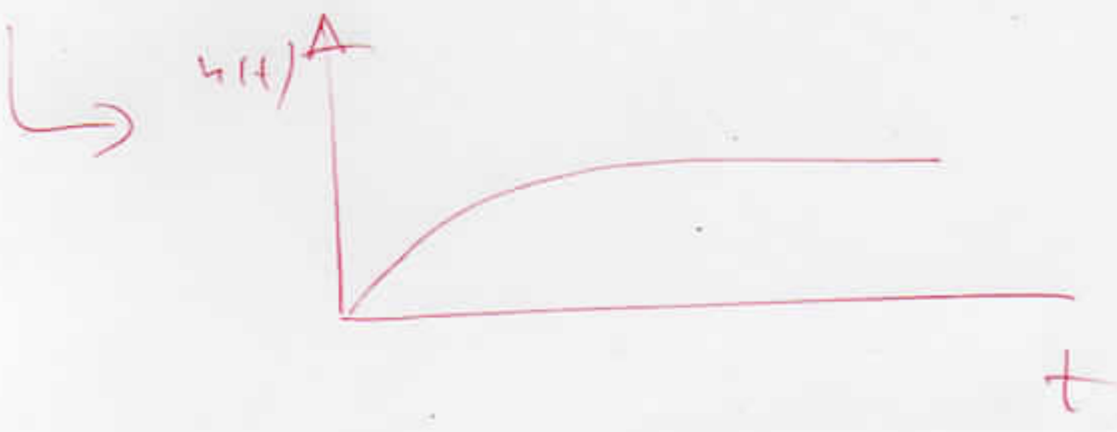
$$\hookrightarrow \quad v(t) = \frac{1}{2}\begin{pmatrix} 1 - e^{-2tk} \\ 1 + e^{-2tk} \end{pmatrix}$$

$$\hookrightarrow \quad n(t) = \frac{1}{2}\left(1 - e^{-2tk}\right)$$

microscopic dynamics





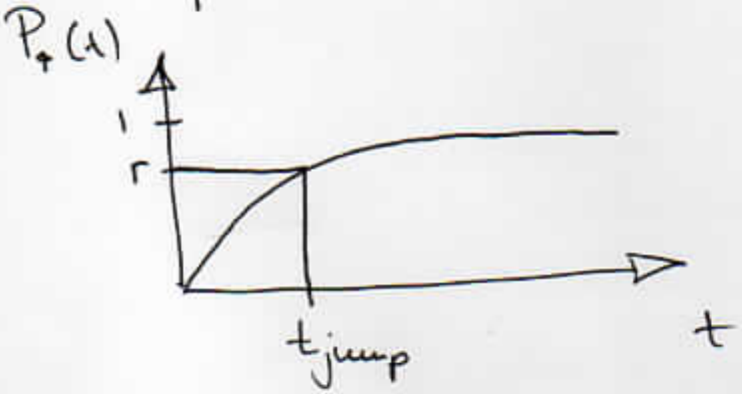


kinetic Monte Carlo

probability for state changes

$$\uparrow \xrightarrow{k} \downarrow \qquad 1 - e^{-kt} = P_\uparrow(t)$$

$$\downarrow \xrightarrow{\gamma} \uparrow \qquad 1 - e^{-\gamma t} = P_\downarrow(t)$$



draw random number r

<u>TASK:</u> calculate $n(t)$ & $p(t)$ via master eq. and kinetic Monte-Carlo for single spin

from quantum to classical

quantum master equation

ϱ .. density matrix

$$\frac{\partial}{\partial t}\varrho = \partial_t \varrho = \underbrace{\boxed{\dot{\varrho} = -i\,[H,\varrho]}}_{\text{von Neumann equation}} + \underbrace{\gamma L \varrho L^{+} - \frac{\gamma}{2}\{L^{+}L, \varrho\}}_{\text{noise}}$$

dephasing: $L = n \equiv |\uparrow\rangle\langle\uparrow| \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\uparrow\rangle\langle\uparrow| \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Hamiltonian

$$H = \begin{pmatrix} \Delta & \Omega \\ \Omega & 0 \end{pmatrix}, \quad \psi = \begin{pmatrix} p_\uparrow \\ p_\downarrow \end{pmatrix}$$

$$\hookrightarrow \; i \begin{pmatrix} \dot{p}_\uparrow \\ \dot{p}_\downarrow \end{pmatrix} = \begin{pmatrix} \Delta & \Omega \\ \Omega & 0 \end{pmatrix} \begin{pmatrix} p_\uparrow \\ p_\downarrow \end{pmatrix}$$

$$i \dot{p}_\uparrow = \Delta p_\uparrow + \Omega p_\downarrow$$

$$i \dot{p}_\downarrow = \Omega p_\uparrow$$

$$\rho = \begin{pmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{pmatrix} \qquad \left.\begin{matrix} \rho_{\uparrow\uparrow} \leftrightarrow n \\ \rho_{\downarrow\downarrow} \leftrightarrow p \end{matrix}\right\} \text{classical probabilities}$$

$$\dot{\rho} = -i \begin{pmatrix} \Delta & \Omega \\ \Omega & 0 \end{pmatrix} \begin{pmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{pmatrix} + i \begin{pmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{pmatrix} \begin{pmatrix} \Delta & \Omega \\ \Omega & 0 \end{pmatrix}$$

$$+ \gamma \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \frac{\gamma}{2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{pmatrix}$$

$$- \frac{\gamma}{2} \begin{pmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= -i \begin{pmatrix} \cancel{\Delta\rho_{\uparrow\uparrow}} + \Omega\rho_{\downarrow\uparrow} & \Delta\rho_{\uparrow\downarrow} + \Omega\rho_{\downarrow\downarrow} \\ \Omega\rho_{\uparrow\uparrow} & \Omega\rho_{\uparrow\downarrow} \end{pmatrix} + i \begin{pmatrix} \cancel{\Delta\rho_{\uparrow\uparrow}} + \Omega\rho_{\uparrow\downarrow} & \Omega\rho_{\uparrow\uparrow} \\ \Delta\rho_{\downarrow\uparrow} + \Omega\rho_{\downarrow\downarrow} & \Omega\rho_{\downarrow\uparrow} \end{pmatrix}$$

$$+ \gamma \begin{pmatrix} \cancel{\rho_{\uparrow\uparrow}} & 0 \\ 0 & 0 \end{pmatrix} - \frac{\gamma}{2} \begin{pmatrix} \cancel{\rho_{\uparrow\uparrow}} & \rho_{\uparrow\downarrow} \\ 0 & 0 \end{pmatrix} - \frac{\gamma}{2} \begin{pmatrix} \cancel{\rho_{\uparrow\uparrow}} & 0 \\ \rho_{\downarrow\uparrow} & 0 \end{pmatrix}$$

coherences

$$= i \begin{pmatrix} \Omega(-\rho_{\downarrow\uparrow} + \rho_{\uparrow\downarrow}) & \boxed{-\Delta\rho_{\uparrow\downarrow} + \Omega(-\rho_{\downarrow\downarrow} + \rho_{\uparrow\uparrow})} \\ \Delta\rho_{\downarrow\uparrow} + \Omega(-\rho_{\uparrow\uparrow} + \rho_{\downarrow\downarrow}) & \Omega(-\rho_{\uparrow\downarrow} + \rho_{\downarrow\uparrow}) \end{pmatrix} - \frac{\gamma}{2} \begin{pmatrix} 0 & \boxed{\rho_{\uparrow\downarrow}} \\ \rho_{\downarrow\uparrow} & 0 \end{pmatrix}$$

adiabatic elimination

$$\dot{\rho}_{\uparrow\downarrow} = i\left(-\Delta\rho_{\uparrow\downarrow} + \Omega(\rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow})\right) - \frac{\gamma}{2}\rho_{\uparrow\downarrow}$$

≈ 0

$$+(i\Delta + \gamma/2)\,\rho_{\uparrow\downarrow} = i\Omega(\rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow})$$

$$\rho_{\uparrow\downarrow} = \frac{i\Omega}{i\Delta + \gamma/2}(\rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow})$$

(VI)

$$\dot{\rho}_{\uparrow\uparrow} = i\Omega(\rho_{\uparrow\downarrow} - \rho_{\downarrow\uparrow})$$

$$= i\Omega\left(\frac{i\Omega}{i\Delta+\gamma/2}(\rho_{\uparrow\uparrow}-\rho_{\downarrow\downarrow}) - \frac{-i\Omega}{-i\Delta+\gamma/2}(\rho_{\uparrow\uparrow}-\rho_{\downarrow\downarrow})\right)$$

$$= i\Omega\left[\frac{i\Omega}{i\Delta+\gamma/2} - \frac{-i\Omega}{-i\Delta+\gamma/2}\right](\rho_{\uparrow\uparrow}-\rho_{\downarrow\downarrow})$$

$$= i\Omega\left[\frac{\cancel{\Omega\Delta}+i\frac{\Omega}{2}\gamma - \cancel{\Omega\Delta}+i\frac{\Omega\gamma}{2}}{\Delta^2+(\gamma/2)^2}\right](\rho_{\uparrow\uparrow}-\rho_{\downarrow\downarrow})$$

$$= -\frac{\Omega^2}{2}\gamma\,\frac{1}{\Delta^2+(\gamma/2)^2}(\rho_{\uparrow\uparrow}-\rho_{\downarrow\downarrow})$$

$$\dot{\rho}_{\downarrow\downarrow} = -\dot{\rho}_{\uparrow\uparrow} \longrightarrow \rho_{\downarrow\downarrow}+\rho_{\uparrow\uparrow} = 1$$

$$\dot{\rho}_{\uparrow\uparrow} = -\frac{\Omega^2}{2}\gamma\,\frac{1}{\Delta^2+(\gamma/2)^2}(\rho_{\uparrow\uparrow}-\rho_{\downarrow\downarrow})$$

$$\dot{\rho}_{\downarrow\downarrow} = -\frac{\Omega^2}{2}\,\frac{1}{\Delta^2+(\gamma/2)^2}(\rho_{\downarrow\downarrow}-\rho_{\uparrow\uparrow})$$

$$\boxed{\begin{array}{ll} \rho_{\uparrow\uparrow} \to n & K = \frac{\Omega^2}{2}\gamma\,\frac{1}{\Delta^2+(\gamma/2)^2} \\ \rho_{\downarrow\downarrow} \to p & \gamma_c = K \end{array}}$$