1

2. $g \in H_1 \cap H_2$,

4. Def^n. $o(g) = \min\{n > 0 \mid g^n = e\}$.

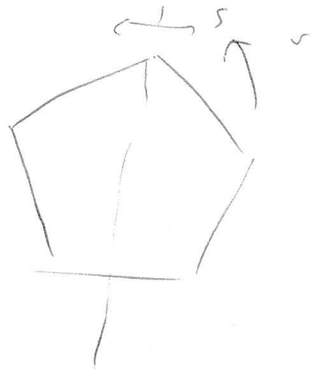
[Fact: If $g^k = e$ then $o(g) \mid k$.]

Let $\mathbb{N} \longrightarrow G$
$n \longmapsto g^n$.

G finite $\Rightarrow \exists\, m \neq n$ s.t. $g^m = g^n$.

wlog $m > n$.
Then $g^{m-n} = e$ so $o(g) < \infty$.

5. follows from division algorithm.

7. 

$rsr = s \quad (*)$

Let $\theta : D_{2n} \longrightarrow C_n$ be a homomorphism.

Then applying θ to $(*)$:
$\theta(r)\,\theta(s)\,\theta(r) = \theta(s)$
$= \theta(r)^2\,\theta(s)$ because C_n abelian.

②

$\therefore \; \theta(r)^2 = e.$

(q. 6. $\Rightarrow$ $o(\theta(r)) = 1$ or 2)

~~$\Rightarrow 2\theta(r) \equiv 0 \mod n$~~

$\therefore \; \theta(r) = e$ because n odd.

~~Similarly, $\theta(s)$~~

Also, we know $s^2 = e$ in D_{2n}

$\Rightarrow \; \theta(s)^2 = e$ in C_n

$\Rightarrow \; \theta(s) = e$ because n odd as above.

Since every $g \in G$ is either r^i or sr^i, it follows that $\theta(g) = e$ $\forall g \in G$, as required.

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8. $H \leq C_n = \langle \xi \rangle$

If $H \neq \{e\}$, choose $k > 0$ minimal s.t. $\xi^k \in H$.

If $\xi^l \in H$ then, by the division algorithm,

$l = kq + r$

for $0 \leq r < k$, but

$\xi^r = \xi^{l-kq} = \xi^l (\xi^k)^{-q} \in H$

$\Rightarrow r = 0$ & $\xi^l = (\xi^k)^q$. $\therefore H = \langle \xi^k \rangle$.

so is cyclic.

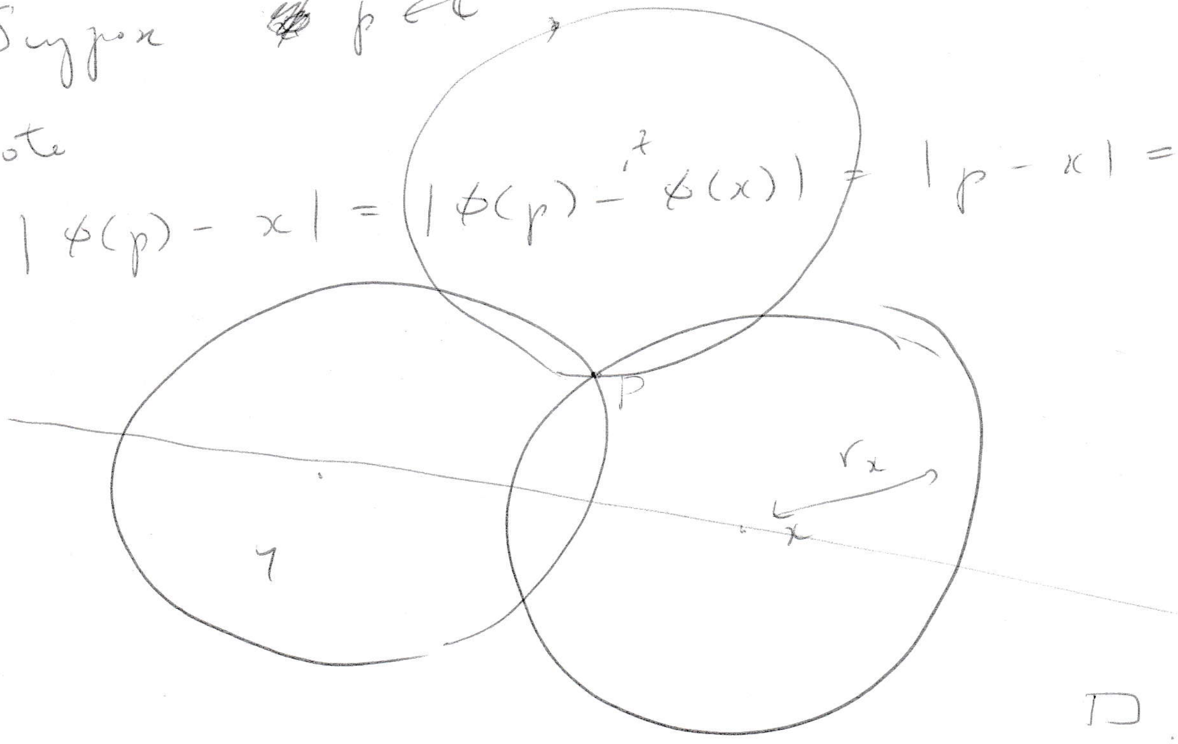
12. **Fact:**

If $\phi : \mathbb{C} \longrightarrow \mathbb{C}$ is an isometry, $x, y, z \in \mathbb{C}$ are not colinear & $\phi(x) = x$, $\phi(y) = y$, $\phi(z) = z$ then $\phi = \mathrm{id}_{\mathbb{C}}$.

Suppose $p \in \mathbb{C}$

Note

$$|\phi(p) - x| = |\phi(p) - \phi(x)| = |p - x| =: r_x$$



$\square$.

Let $\phi \in \mathrm{Isom}\,\mathbb{C}$.

Claim: $\exists\, f : z \longmapsto az + b$

or $z \longmapsto a\bar{z} + b$

s.t. $f \circ \phi$ fixes $0, 1, i$.

Then fact $\Rightarrow$ $\phi = f^{-1}$. Consider

$$g : z \longmapsto z - \phi(0) \longmapsto a^{-1}(z - \phi(0))$$

where $a = \phi(1) - \phi(0)$

(4)

Since $|g(i) - g(0)| = |i - 0|$

& $|g(i) - 1| = |i - 1|$

$\to g(i) = \pm i$.

If $g(i) = i$, set $f(z) = g(z)$.

If $g(i) = -i$, set $f(z) = g(\bar{z})$.

Now we can check that

$f \circ \phi$

fixes $0, 1, i$ as required. □

13. <u>Diff eq</u>: $xy = \text{const}$.

$xy' + y = 0$.

14.