1 Hyperspectral imaging and blind source separation

1. Describe the general principle of hyperspectral imaging. What is hyperspectral unmixing (also more generally called blind source separation) ? Why is it useful ?

2. Why does blind source separation require additional priors on the sought-after factors ? Name two examples of such priors and explain them.

3. Let us consider a linear source separation problem $\mathbf{X} = \mathbf{A}^*\mathbf{S}^* + \mathbf{N}$, in which we only want to learn the mixing matrix $\mathbf{A}^* \in \mathbb{R}^{m \times n}$ from $\mathbf{X}$. Thus, in the following, the sources $\mathbf{S}^* \in \mathbb{R}^{n \times t}$ are assumed to be known [1].
Let us further assume [2] the coefficients of $\mathbf{A}^*$ to lie within the interval $[0, 1]$. We aim at estimating $\mathbf{A}^*$ through the minimization of the following cost function :

$$\arg\min_{\mathbf{A} \in \mathcal{C}} \frac{1}{2}\|\mathbf{X} - \mathbf{A}\mathbf{S}\|_F^2, \tag{1}$$

with $\mathcal{C} = \{\mathbf{A} \in \mathbb{R}^{m \times n} | 0 \leq a_{ij} \leq 1 \text{ for all } 0 \leq i \leq m \text{ and } 0 \leq j \leq n\}$ and $\|.\|_F$ the Frobenius norm.

a) Compute the gradient over $\mathbf{A}$ of the $\frac{1}{2}\|\mathbf{X} - \mathbf{A}\mathbf{S}\|_F^2$ term (the full derivation is required, not only the result).

1. In hyperspectral imaging, it would correspond to assume known abundancies : it might look to be a strong assumption, but it is actually useful in the context of minimization algorithms using block coordinates techniques.

2. In hyperspectral imaging, this is realistic if the endmember signatures are normalized.

b) Recall the definition of an indicator function, and rewrite the cost function (1) with such an indicator function.

c) Is the problem (1) convex ? Briefly justify your answer. What does it mean about the number of minima of (1) ?

d) Recall the definition of a proximal operator of a closed convex function f. Give an explanation / interpretation.

e) What is the proximal operator of an indicator function of a convex closed non-empty set ?

f) From your above answers, write the pseudo-code of an algorithm enabling to minimize the cost function (1). Detail the algorithms parameter(s) and how to choose them.