CS 3100, Fall 2017, Assignment 1
150 pts total (will be scaled to 30)

Note: Note: We provide TA names so that you know whom to approach for grading-related matters for each question. As for seeking help, *any* TA will help you with *any* of the questions.

Note: Please see the change-list on Page 8 to see the changes we have made to this assignment since it was issued. This way you know what typos got fixed and why (full-transparency).

1 Asg-1 Questions

We will consider these languages in the following questions.

- $L_0 = \emptyset$
- $L_1 = \{\varepsilon\}$
- $L_{Ex1} = \{a, c, ab, \varepsilon\}$.
- L_{Univ} = all strings over a,b,c including the empty string. This is of course $\{a, b, c\}^*$.

1. **10 points:** (Arnab)

   Which of the aforesaid languages must $L_{Ex1}L_{Univ}$ equal to, and why? Give your solution in neat bulleted steps.

   Solution:

   - Concatenating these two languages gives {xy : x in L_{Ex1} and y in L_{Univ}}
   - Because both alphabets contain the empty string, the new language will contain everything that was already in both languages
   - Additionally, L_{Ex1} is made of strings already in L_{Univ}
   - Therefore, $L_{Ex1}L_{Univ}$ is equal to L_{Univ}

2. **20 points:** (Paridhi)

   (a) **4 points:** What is the powerset of L_{Ex1} Write out as a set.

   (b) **4 points:**What is L_{Ex1}^3 ? Write out as a concatenation of sets, and then present the final simplified set.

   (c) **4 points:**Is the cardinality of $\{a, \varepsilon\}^n$ going to be 2^n? Check for a few n and explain why language exponentiation need not obey this familiar formula that set cartesian products obey. A few crisp sentences for your answer please.

   (d) **4 points:**What is $\overline{L_{Ex1}}$, assuming that the universe (which you need for performing the complementation of a language) is L_{Univ} ?

   (e) **4 points:**What is L_{Ex1}^R where the superscript R denotes *reverse* ? Write your answer out as a set.

Solution:

(a) Powerset of $L_{Ex1} = \{\emptyset, \{a\}, \{ab\}, \{c\}, \{ac\}, \{aab\}, \{cab\}\}$

(b) $L^3_{Ex1} = L_{Ex1}L_{Ex1}L_{Ex1} = \{\varepsilon, a, aa, aaa, aaab, aab, aaba, aabab, aabc, aac, ab, aba, abaa, abaab, abab, ababa, ababab, ababc, abac, abc, abca, abcab, abcc, ac, aca, acab, acc, c, ca, caa, caab, cab, caba, cabab, cabc, cac, cc, cca, ccab, ccc\}$

(c) The cardinality of $\{a, \varepsilon\}^n$ will not be 2^n.
$\{a, \varepsilon\}^2 = \{\varepsilon, a, aa\} = 3$
$\{a, \varepsilon\}^3 = \{\varepsilon, a, aa, aaa\} = 4$
Because ε is the empty string, concatenating multiple of them together does not produce anything different. If ε was instead something like b, then the cardinality would be 2^n.

(d) $\overline{L_{Ex1}}$ ={x : x in L_{Univ} and x not in L_{Ex1}}

(e) $L^R_{Ex1} = \{\varepsilon, a, c, ba\}$

3. **20 points:** (Paul)

(a) **4 points:**Simplify the string concatenation $\varepsilon a \varepsilon c \varepsilon \varepsilon d$

(b) **4 points:**What is $star(L_{Ex1}, 2)$? Write it out as a set.

(c) **4 points:**What is the symmetric difference between L_{Ex1} and L_1? Write the answer as a set.

(d) **4 points:**

i. What is an example of an infinite set containing finite numbers?
ii. What is an example of an infinite set containing finite strings over $\Sigma = \{a\}$?

(e) **4 points:** Refer to Section 3.4. What is $lhomo(L_{Ex1}, f)$ where $f = \lambda x$: 'a'. Here we mean f takes any character and maps it to character 'a'. Write your answer out as a set.

Solution:

(a) $\varepsilon a \varepsilon c \varepsilon \varepsilon d = acd$

(b) star$(L_{Ex1}, 2) = \{\varepsilon, ab, cab, aba, abc, aa, ac, cc, ca, c, a, abab, aab\}$

(c) lsymdiff$(L_{Ex1}, L_1) = \{a, ab, c\}$

(d) i. $\{1, 2, 3, 4, ...\}$
ii. $\{\varepsilon, a, aa, aaa, ...\}$, or $\{a\}^*$

(e) lhomo$(L_{Ex1}, f) = \{\varepsilon, a, aa\}$

4. **10 points:** (Harshitha)

   (a) **3 points:** Peek at Chapter 11 for a simple HTML example. What regular patterns can you see in this example? Answer in 2-3 sentences saying why these are regular patterns. (Also read Ch1 for answering this.)

   (b) **3 points:** What context-free patterns can you see in the same example? Answer in 2-3 sentences saying why these are context-free patterns. (Also read Ch1 for answering this.)

   (c) **4 points:** Do the exercise concerning powersets from Appendix A, Section A.1.2 ("Powerset"). Take x to be ≥ 0.

Solution:

(a) The body type tags are regular patterns. They must be surrounded with angle brackets and start with a backslash or nothing followed by an allowed string of fixed length. These tags are finite and fixed-size, making them regular patterns.

(b) The body type tags are also a context free pattern. An opening tag cannot come before a closing tag, and any tag opened inside a tag must be closed inside that tag. Because these follow the rules of a nesting pattern it is context free.

(c) i. Because x is even and less than 5 it is restricted to 0, 2, and 4. Similarly, y is restricted to 4 options. By knowing this we know the cardinality of S is 12.

ii. (0, {1}), (0, {2}), (0, {1, 2}), (2, {1}), (2, {2}), (2, {1, 2})

iii. The cardinality of the powerset of N with cardinality n is 2^n if n > 0, if n = 0 then cardinality is 2.
pow($\emptyset$) = $\{\emptyset, \{\emptyset\}\}$
pow({1}) = $\{\emptyset, \{1\}\}$
pow({1, 2}) = $\{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

5. **20 points:** (Maryam)

   (a) **8 points:** Summarize the proof that $L^* = L^{**}$ in about eight neat bulletted steps, going through the key parts of the proof.

   (b) **6 points:** List the first 7 strings formable over alphabet $\{a, b, c\}$ in lexicographic order. Be sure to include ε.

   (c) **6 points:** List the first 7 strings formable over alphabet $\{a, b, c\}$ in numeric order. Be sure to include ε.

Solution:

(a)
- $L^* = \{x : x \text{ in } L^k \text{ for some } k \text{ in } Nat\}$, therefore
- $L^{*^*} = \{x : x \text{ in } L^{*^k} \text{ for some } k \text{ in } Nat\}$
- $L^{*^k} = L_1 L_2 L_3 ... L_M)$, where $L_i = \{x : x \text{ in } L^m \text{ for some } m \text{ in } Nat\}$
- A string x is in L^* if x is in L^k for some k in Nat
- Such an x is also in L^{*^*} because we can set m from line 3 equal to k, therefore every x in L^* is also in L^{*^*}
- Given x in L^{*^*} by choosing $m = m_1, m = m_2, ..., m = m_k$, then choose $k = m_1 + m_2 + ... + m_k$
- For this k, x is in L^k, therefore every x in L^{*^*} is also in L^*
- Therefore, $L^* = L^{**}$

(b) $\varepsilon, a, aa, aaa, aaaa, aaaaa, aaaaaa$

(c) $\varepsilon, a, b, c, aa, ab, ac$

6. **20 points:** (Arnab)

Let $L_E = \{0^{2i} \ : \ i \geq 0\}$.

(a) **(10 points):** Show that $L_E = \{(00)^i \ : \ i \geq 0\}$ (the parentheses are used to group the two 0s and are not part of the alphabet). Use Algebra to write one form to the other.

Let $L_O = \{0^{2i+1} \ : \ i \geq 0\}$.

Let $A = \{0\}^*$ and $B = L_O \cup L_E$.

(b) **(10 points):** Show that these sets are equal, *i.e.*, $A = B$.

Solution:

(a) $(00)^i = 0^i 0^i = 0^{2i}$

(b) $A = \{\varepsilon, 0, 00, 000, ...\}$
$B = \{\varepsilon, 00, 0000, 000000, ...\}\{0, 000, 00000, 0000000, ...\}$
$B = \{\varepsilon, 0, 00, 000, ...\}$
$A = B$

7. **10 points:** (Paridhi)

Is $L_O = L_O^*$? Why or why not? Clearly explain. Write the answer steps in neat bullets. L_O is defined in Question 6 Solution:

- $L_O = \{0, 000, 00000, 0000000, ...\}$
- $L_O^* = L_O^0 L_O^1 L_O^2 ...$

- $L_O^0 = \{\varepsilon\}$
- L_O does not have ε in it, but L_O^* does, therefore L_O != L_O^*

8. **10 points: JOVE solution alone is required:** (Harshitha) Let L_E and L_O be as defined in Question 6. Enter L_E into Jove, limiting i to be ≤ 5. Enter L_O into Jove, limiting i to be ≤ 4. Now take the star of $\{0\}$ using the *star* function in Jove, limiting n to 10. Show you can establish that $\{0\}^* = L_O \cup L_E$ using Jove, and Python tests you write. Note that you can obtain all the language definitions introduced in Chapters 2 and 3 (lunion, lint, star, nthnumeric, etc.) by declaring

```
from jove.LangDef import *
```

Solution:

Write your Jove solution in the Jove file submission.

9. **10 points: JOVE solution alone is required:** (Maryam)

   (a) (**5 points**) Using the function `nthnumeric` in the book, demonstrate within Jove that you can list the first 12 strings over $\{'a', 'b'\}$ in numeric order. Hint: It is natural to use a list comprehension here. Note that you can obtain all the language definitions introduced in Chapters 2 and 3 (lunion, lint, star, nthnumeric, etc.) by

   ```
   from jove.LangDef import *
   ```

   (b) (**5 points**) Define alphabets $\Sigma_1 = \{a, b, c\}$ and $\Sigma_2 = \{0, 1\}$. Define languages $L_1 = star(\Sigma_1, 8)$ and $L_2 = star(\Sigma_2, 8)$. We want to generate the language concatenation of L_1 and L_2, and determine the number of elements in it. Express these actions within Jove and demonstrate that you can generate $L_1 L_2$ and the count of the number of elements in it. **You must include** a simple test that checks the answer by comparing the length of $L_1 L_2$ against the lengths of L_1 and L_2.

Solution:

Write your Jove solution in the Jove file submission.

10. **20 points: JOVE solution alone is required:** (Harshitha)
Let the alphabet be $\{0, 1, 2\}$. Design a DFA that accepts just these classes of strings:

    - all the odd-length strings that start with a 1, or
    - all the even-length strings that start with a 2.

    (a) **(10 pts):** Enter your design in Jove using the markdown syntax for DFA. **You must print** your solution's DFA with the black-hole state shown (using `dotObj_dfa_w_bh`) and with multiple edges going from one state to another fused (using the `FuseEdges=True` option).

(b) **(5 pts):** Generate all the strings from $star(\{0,1,2\},4)$. Show that your DFA accepts only the specified set of strings. Submit your Jove notebook with your solution.

(c) **(5 pts):** **Express** the language of this DFA using the language operations $\cup$, $star$, and others as needed in as compact a way as you can. Let $A = \{0,1,2\}$, and in your solution use A in lieu of $\{0,1,2\}$ to make your solution compact and readable.

Solution: $\{1 + A^{2i+1} : i \geq 0\} \cup \{2 + A^{2i} : i \geq 0\}$

Write your Jove solution in the Jove file submission.

2 Change List

- (8/21/18, 2pm): Changed `dot_dfa...` to `dotObj_dfa...` (the former does not draw a picture)
- Question 2(b): Changed union to concatenation.
- Added this: *take x to be ≥ 0 in Question 4(c).*
- Clarified in Questions 7 and 8 that L_E and L_O came from Question 6