William Frank
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HW8 – Bayesian Networks and Independence

1. Independences from Probability Tables

P(A, B, C) = P(A) * P(B | A) * P(C | AB)
f_1 = P(A), f_2 = P(B | A), f_3 = P(C | AB).
f_1 and f_2 can be combined into f_4 = P(A, B), giving P(A, B, C) = f_3 * f_4.

A	B	P(A, B)
0	0	0.13541666666
0	1	0.35416666666
1	0	0.11458333333
1	1	0.39583333333

A	B	C	P(C | AB)
0	0	0	0.307692307692309
0	0	1	0.692307692307691
0	1	0	0.470588235294118
0	1	1	0.529411764705882
1	0	0	0.727272727272727
1	0	1	0.272727272727273
1	1	0	0.842105263157895
1	1	1	0.157894736842105

2. Please answer the following conditional independence questions from the model.
   1. A is not always independent of H. As there are arrows from A to H then it is possible to construct conditional probabilities such that A and H are dependent.
   2. A and C both affect F, which in turn affects H. Even if C is fixed, A and H can still be dependent.
   3. With a fixed C and F the model does not have any way for A to affect H. Therefore A is independent of H given C and F.
   4. This is a common cause with a fixed cause, therefore E and B are always independent given A.
   5. By observing F we fix the common effect of B and E, meaning that B and E are not always independent.
   6. Because A is given B and E will be independent, even though C and F are fixed.

3. Using inference by enumeration, compute the following probabilities.
   1. $p(+b, -e|+a, +j, +m) = \alpha\, p(+b)\, p(-e)\, p(+j \mid +a)\, p(+m \mid +a)\, p(+a \mid +b, -e)$
      $= \alpha\, (.001)\, (.998)\, (.9)\, (.7)\, (.94) = \alpha\, 0.0005910156$

      $\alpha = 1 / (p(+b, -e|+a, +j, +m) + p(+b, +e|+a, +j, +m) + p(-b, -e|+a, +j, +m) + p(-b, +e|+a, +j, +m)) = 1 / (0.00059 + 0.0000012 + 0.00063 + 0.00037) = 1 / (0.00159) = 628.46$

      $p(+b, -e|+a, +j, +m) = 628.46 * 0.0005910156 = 0.371$

   2. $p(+b \mid +a) = \alpha\, p(+b)\, \Sigma\_e\, p(e)\, p(+a \mid +b, e)\, \Sigma\_m\, p(m \mid +a)\, \Sigma\_j\, p(j \mid +a)$
      $\Sigma\_m\, p(m \mid +a)\, \Sigma\_j\, p(j \mid +a) = 1$, therefore
      $p(+b \mid +a) = \alpha\, p(+b)\, \Sigma\_e\, p(e)\, p(+a \mid +b, e)$
      $= \alpha * 0.001 * ((0.998 * 0.94) + (0.002 * 0.95) = \alpha\, 0.00094$

      $\alpha = 1 / (p(+b \mid +a) + p(-b \mid +a)) = 1 / (0.00094 + 0.0016) = 393.7$

      $p(+b \mid +a) = 393.7 * 0.00094 = 0.370$

   3. $p(+b \mid +e, +a) = \alpha\, p(+b)\, p(+e)\, p(+a \mid +b, +e)\, \Sigma\_m\, p(m \mid +a)\, \Sigma\_j\, p(j \mid +a)$
      $= \alpha\, p(+b)\, p(+e)\, p(+a \mid +b, +e)$
      $= \alpha\, (.001)\, (.002)\, (.95) = \alpha\, 0.0000019$

      $\alpha = 1 / (p(+b \mid +e, +a) + p(-b \mid +e, +a)) – 1 / (0.0000019 + 0.00058) = 1720.22$

      $p(+b \mid +e, +a) = 1720.22 * 0.0000019 = 0.003$

Given that the alarm is going off and there is/was an earthquake, it is very unlikely that there was also a burglary. As opposed to the previous answer, which told us if earthquake is unknown then the alarm going off means a 37.1% chance of a burglary.

   4. $p(+a \mid +j, -m) = \alpha\, p(+j \mid +a)\, p(-m \mid +a)\ \ \Sigma\_e\, p(e)\, \Sigma\_b\, p(b)\, p(+a \mid b, e)$
      $= \alpha\, (.9)\, (.3)\, ((.998)\, ((.999)\, (.001))\, ((.001)\, (.94)))\, ((.002)\, ((.999)\, (.29))\, ((.001)\, (.95)))$
      $= \alpha\, 1.34e\text{-}13$

      $\alpha = 1 / (p(+a \mid +j, -m) + p(-a \mid +j, -m)) = 1 / (1.34e\text{-}13 + 2.10e\text{-}13) = 2.86e12$

      $p(+a \mid +j, -m) = 1.34e\text{-}13 * 2.86e12 = 0.398$

   Using variable elimination:

   5. $p(+b \mid +a) = p(+b)\, p(e)\, p(+a \mid +b, e)\, p(m \mid +a)\, p(j \mid +a)$
      Eliminating e:
      $p(e)\, p(+a \mid +b, e) \Rightarrow p(+a, e \mid +b) \Rightarrow p(+a \mid +b) = 0.94002$

e	$p(+a, e \mid +b)$
0	0.0019
1	0.93812

      $p(+b \mid +a) = p(+b)\, p(+a \mid +b)\, p(m \mid +a)\, p(j \mid +a)$

Eliminating m:
p(m | +a) => 1
p(+b | +a) = p(+b) p(+a | +b) p(j | +a)

Eliminating j:
p(j | +a) => 1
p(+b | +a) = p(+b) p(+a | +b)

p(+b | +a) = p(+b) p(+a | +b) = 0.001 * 0.94002 = 0.00094
0.00094 * α = 0.370

6. p(+a | +j, -m) = α p(+j | +a) p(-m | +a) p(e) p(b) p(+a | b, e)
Eliminating b:
p(b) p(+a | b, e) => p(+a, b | e) => p(+a | e)

b	e	p(+a, b | e)
0	0	0.000999
0	1	0.28971
1	0	0.00094
1	1	0.00095

e	p(+a | e)
0	0.001939
1	0.29066

Eliminating e:
p(e) p(+a | e) => p(+a, e) => p(+a) = 0.290082558

e	p(+a, e)
0	0.000003878
1	0.29007868

p(+a | +j, -m) = α p(+j | +a) p(-m | +a) p(+a) = α * 0.9 * 0.3 * 0.290082558 = α * 0.078.

α = 1 / (p(+a | +j, -m) + p(-a | +j, -m)) = 1 / (0.078 + 0.191) = 3.703

p(+a | +j, -m) = 3.703 * 0.078 = 0.290