Entry and Employment Dynamics in the Presence of Market Power*

William Gamber†

December 9, 2022

Abstract

In this paper, I evaluate the role of fluctuations in business formation in amplifying business cycles. To do this, I study the response of aggregate employment to shocks in a general equilibrium model of producer dynamics with entry and exit. In the model, producers' markups rise with their size, so that, in response to a decline in entry, incumbents' market shares rise and they increase their markups and reduce employment. I find that entry fluctuations lead to economically meaningful amplification of business cycle shocks in the model.

*This paper was the first chapter of the author's dissertation and was previously circulated under the title "Entry, Variable Markups, and Business Cycles."

†Contact: will.gamber@frb.gov. I am very grateful to the editor, Greg Kaplan, and two anonymous referees for their comments. I am also very grateful to my advisors Simon Gilchrist, Ricardo Lagos, and Virgiliu Midrigan and committee members Corina Boar and Mark Gertler for their guidance and support throughout this project. I would also like to thank Jaroslav Borovička, Ryan Decker, Giuseppe Fiori, Sebastian Graves, James Graham, Andrew McCallum, Dino Palazzo, Erick Sager, Michael Siemer, Melinda Suveg (discussant), Venky Venkateswaran, and Joshua Weiss for their helpful comments, as well as seminar participants at NYU, the Federal Reserve Board, the Firm Dynamics, Market Structures and Productivity Workshop at the University of Kent and the Lisbon Macro Workshop at Católica Lisbon. The views expressed in this paper are solely those of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or any other person associated with the Federal Reserve System.

1 Introduction

During the Great Recession, the number of new businesses created each year declined by more than 35 percent relative to its peak in the mid 2000s and remained depressed through 2019, leading to a persistent decline in the number of operating businesses.[1] This fall in entry accompanied a decline in employment of over 6 percent relative to trend that only slowly returned to its pre-recession level. In this context, it is natural to ask: what is the role of entry in amplifying business cycles?

In this paper, I answer this question using a quantitative general equilibrium model of producer dynamics that is consistent with important features of the data. First, the model features producers that increase their markups as their market shares rise, so that a fall in entry leads incumbents to increase their markups and reduce employment. Second, these producers are heterogeneous in size, and the model replicates the lifecycle pattern of producer growth that we observe in US data. And lastly, producers in the data face adjustment frictions that inhibit the reallocation of inputs.

In the model, a shock to the cost of entry that leads business formation to fall as much as it did in the United States during the Great Recession leads the average markup to increase significantly and generates a decline in aggregate employment of nearly 2.5 percent. I show that the producer lifecycle, heterogeneity, adjustment frictions, and variable markups all play important roles in the size of this effect. I also quantify the extent to which this mechanism amplifies TFP shocks in the model, finding that the model with entry increases the effects of a fall in TFP on aggregate employment by about 70% relative to a model with no entry. All in all, these results suggest that endogenous fluctuations in entry could account for about 2.5 percentage points of the 6 percent decline in employment observed during the Great Recession.

An existing literature uses structural models to study the role of entry in amplifying business cycles. Depending on the assumptions underlying their models, papers in that literature come to very different conclusions about the importance of entry. On one hand, papers that study the role of entry fluctuations in business cycle models in which firms are homogeneous and in which markups vary with the number of competitors typically find large effects of entry on business cycle fluctuations. (Bilbiie, Ghironi and Melitz (2012) and Jaimovich and Floetotto (2008)) The homogeneity assumption, while key to tractability, is at odds with data; entering firms are significantly smaller than the average firm. (Midrigan (2008)) More recent work studies entry fluctuations in theoretical models with a realistic firm lifecycle but without variable markups. (Clementi and Palazzo (2016) and Lee and Mukoyama (2018)) These papers find a much more modest role for entry. Lastly, Edmond, Midrigan and Xu (2018)

[1] Source: US Census Bureau Business Dynamics Statistics Database. Here, I define a business as an establishment, but similar statistics hold for firms.

study entry in a model with heterogeneous firms and variable markups but no variable input adjustment costs and find almost no effect of entry on the aggregate markup.

This paper provides a framework for understanding these disparate results. The baseline model in this paper features heterogeneous producers, a realistic lifecycle, variable markups, and variable input adjustment frictions. I show that each of these elements is key to matching microdata evidence on producer behavior. This model also nests models similar to those from previous work. Using the full baseline model, I find that the effects of entry are economically meaningful but modest relative those without producer heterogeneity. Thus, the effects of entry on the business cycle are likely to be much smaller than the large effects implied by homogeneous-producer models but somewhat larger than the small effects in previous heterogeneous-producer models.

I begin the paper by presenting a general equilibrium Hopenhayn (1992) model with several features, including (1) a variable elasticity of demand, (2) labor adjustment costs, (3) a producer lifecycle, and (4) heterogeneity in size among producers, even after conditioning on age. Producers in the model have ex-ante heterogeneous, stochastic productivity. They are each the monopolistic supplier of a differentiated variety and face downward sloping demand with an elasticity that declines with relative size. The shape of these demand curves implies that producers have an incentive to increase their markups as their output rises relative to the overall market. Producers must pay a convex net hiring and firing cost, which slows their responses to idiosyncratic shocks and prevents inputs from rapidly reallocating across businesses. Lastly, businesses exit each period and are replaced, in steady state, by newly created businesses.

I then quantify a key mechanism in the model, namely, that markups rise with producer size. My approach is motivated by the "production function approach" (PFA) that has been popular in the recent macroeconomic literature on markups (see De Loecker, Eeckhout and Unger (2020), for example). The intuition behind this approach is that, under the assumption that producers can frictionlessly adjust their variable inputs, the wedge between variable input use and revenue is informative about the size of the markup. I show that this wedge in the data varies strongly with producer size; the typical producer in the sample increases its variable input bill much less than one-for-one with its sales. This finding suggests that markups vary strongly with producer size.

I use the quantified model to structurally interpret these regressions. As highlighted by Bond et al. (2020), the PFA requires the restrictive assumption that variable inputs can be costlessly adjusted. To relax this assumption, I use the reduced-form regression coefficients, along with data on employment reallocation, to discipline parameters in the model, including the degree of adjustment costs and the extent to which the elasticity of demand falls with producer size. I then simulate a panel of producers in the model and estimate the same PFA regressions on simulated data. I show that not accounting for adjustment costs leads to an overstatement of the relationship between producer size and markups but that large producers' markups do vary significantly with their size, with an elasticity of around 25% to relative sales.

To study the effects of fluctuations in entry on aggregate employment, I then introduce a shock to the cost of entry to the model. In the model, this shock leads to a temporary decline in entry that has economically meaningful and persistent effects on aggregate employment. The fall in entry increases the market shares of incumbent producers and leads them to increase their markups, produce less, and reduce employment. Love-for-variety effects mean that the decline in the number of operating producers reduces aggregate productivity. The movements in the markup and effective productivity are economically significant; in response to a shock that reduces entry by one-third, as much as the fall during the Great Recession, the aggregate markup rises 0.8 percent and aggregate productivity falls 0.6 percent. Because of these changes, employment declines almost 2.5 percent.

I next study the mechanisms in the model that generate these large fluctuations in employment in response to the fall in entry. I show that both adjustment costs and variable markups are key to generating this response, and a model missing either of these ingredients generates a much smaller increase in markups and decline in employment.

To study the role of competition, I compare the model to one with a constant elasticity of demand and no adjustment frictions. This model resembles those in recent papers with heterogeneous firms and constant markups, like Clementi and Palazzo (2016). I find that the effects of entry on aggregate employment are 75 percent larger in the variable markups economy than in the constant elasticity model. The difference between the two models arises primarily because falling entry leads incumbent producers to increase their markups, leading to a decline in the labor share. I conclude that the recent literature studying the role of entry in business cycle amplification in heterogeneous producer models understates the importance of entry because it ignores the effects of entry on the markups of incumbents.

To study the role of adjustment costs, I next study a model with variable elasticity of demand but no adjustment costs. This model is similar to the one in Edmond, Midrigan and Xu (2018). In the model with no adjustment costs, producers raise their markups in response to the shock to entry. This change in producer policy functions causes the unweighted average markup to rise. However, because small, low-markup producers face a higher elasticity of demand than large, high-markup producers, they benefit more from the fall in competition. This feature of demand implies that employment reallocates away from large producers to small producers, meaning that the employment-weighted average markup, the correct measure of the aggregate markup in this model, does not rise by much. Without adjustment costs, reallocation undoes 80 percent of the immediate rise in the markup. In the baseline model, adjustment costs prevent small producers from increasing their employment rapidly and inhibit this reallocation.

To understand the role of heterogeneity in the propagation of entry fluctuations to the real economy, I compare the baseline model to two alternative models that omit key aspects of producer heterogeneity. In the first alternative model, I assume there is no producer lifecycle: entering producers are the same size as incumbents, on average. TFP moves much more in response to the shock in that econnomy, leading the effects on aggregate employment to be roughly double in that economy relative to the baseline. I then compare the baseline to models with no heterogeneity and translog or CES demand, as in Bilbiie, Ghironi and Melitz (2012), finding that a shock to entry in those models has effects on employment that that are 1.5 to 2.5 times the size of those in the baseline economy.

Having studied the transmission of entry fluctuations to aggregate employment in isolation, I then study the response of the economy to a TFP shock that leads to endogenous movements in entry. A decline in TFP leads entry to fall, the markup to rise and employment to fall. I show that endogenous entry fluctuations lead aggregate employment to fall by 70 percent more relative to a no-entry baseline, driven largely by a rise in the markup. Entry thus plays an economically meaningful role in business cycles.

2 Quantitative Model

In this section, I present the general equilibrium producer dynamics model I use to study business cycle fluctuations in entry. The framework is a general equilibrium Hopenhayn (1992) model with a convex employment adjustment cost and variable elasticity of demand.

2.1 Environment

Time in the model is discrete and continues forever. There are three types of agents in this economy: (1) a representative household who consumes a final good, supplies labor, and holds a portfolio of all producers in the economy; (2) a final goods producer who uses a continuum of intermediate inputs to produce the final good; and (3) a variable measure of monopolistically competitive intermediate goods producers.

2.2 Household

A representative household chooses a state-contingent path for consumption of the final good $\{C_t\}$ and labor supplied $\{L_t\}$ to maximize the discounted sum of future utility:

$$\sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \tag{2.1}$$

The household receives wage W_t and profits Π_t from its ownership of a portfolio of all producers in the economy. I normalize the price of the final good to 1, and so the household's budget constraint is:

$$C_t \leqslant W_t L_t + \Pi_t. \tag{2.2}$$

The intratemporal first-order condition of an optimal solution to the household's problem implies a labor supply curve:

$$W_t = -\frac{u_{L,t}}{u_{C,t}}. \tag{2.3}$$

2.3 Final goods producer

A perfectly competitive representative producer assembles the final consumption good using a continuum of measure N_t intermediate goods as inputs. Each differentiated intermediate variety is indexed by ω. The final goods producer takes as given the prices of the intermediate goods and minimizes the cost of producing output. Its production function takes the following form:

$$\int_0^{N_t} \Upsilon\left(\frac{y_t(\omega)}{Y_t}\right) d\omega = 1, \tag{2.4}$$

where $\Upsilon(q)$ is a function that satisfies three conditions: it is increasing ($\Upsilon'(q) > 0$) and concave ($\Upsilon''(q) < 0$), and $\Upsilon(1) = 1$. Given quantities of each intermediate variety $\{y_t(\omega)\}$, aggregate output Y_t is defined as the solution to Equation (2.4).

The optimal solution to the cost minimization of the final goods producer implies a demand curve for each intermediate good:

$$p_t(\omega) = \Upsilon'\left(\frac{y_t(\omega)}{Y_t}\right) D_t, \tag{2.5}$$

where the D_t is the demand index, defined as

$$D_t \equiv \left(\int_0^{N_t} \Upsilon'\left(\frac{y_t(\omega)}{Y_t}\right) \frac{y_t(\omega)}{Y_t} d\omega\right)^{-1}. \tag{2.6}$$

For the main exercises in this paper, I use the Klenow and Willis (2016) specification of $\Upsilon(q)$:

$$\Upsilon(q) = 1 + (\sigma - 1)\exp\left(\frac{1}{\epsilon}\right)\epsilon^{\frac{\sigma}{\epsilon}-1}\left[\Gamma\left(\frac{\sigma}{\epsilon}, \frac{1}{\epsilon}\right) - \Gamma\left(\frac{\sigma}{\epsilon}, \frac{q^{\epsilon/\sigma}}{\epsilon}\right)\right] \tag{2.7}$$

where $\sigma > 1$, $\epsilon \geqslant 0$ and $\Gamma(s, x)$ denotes the upper incomplete Gamma function:

$$\Gamma(s, x) = \int_x^\infty t^{s-1}\epsilon^{-t} dt. \tag{2.8}$$

This specification of Υ generates an elasticity of demand for each variety that is decreasing in its relative quantity $q_t \equiv y_t/Y_t$ so that large producers set higher markups than small producers.[2] Under the Klenow and Willis (2016) specification,

$$\Upsilon'(q) = \frac{\sigma - 1}{\sigma}\exp\left(\frac{1 - q^{\frac{\epsilon}{\sigma}}}{\epsilon}\right) \tag{2.9}$$

In this case, the elasticity of demand is $\sigma q^{-\frac{\epsilon}{\sigma}}$. The demand elasticity declines with the relative quantity demanded of that intermediate good, and the elasticity of the elasticity of demand to quantity produced (the "superelasticity of demand") is the ratio $-\epsilon/\sigma$.

2.4 Intermediate goods producers

At each date t, there is a mass N_t of intermediate goods producers, each of whom is the monopolistic supplier of a differentiated variety ω. Each hires labor in a perfectly competitive market at wage W_t, produces its variety using a constant returns production function, and sells it to the final goods producer, taking as given the demand schedule.

Timing works as follows: in each period, each producer observes its idiosyncratic productivity z_t and the state of the aggregate economy, Λ_t. It then hires workers, produces output, and sells its differentiated variety to the final goods producer. Producers face labor adjustment costs $\phi(\ell, \ell')$ as a function of last period's employment ℓ and their current employment ℓ'. After selling their output and paying adjustment costs, each producer draws an i.i.d. fixed cost $\phi_F \sim G_F$ to operate in the following period. If it chooses not to pay the random fixed cost, it exits. The value of exit is normalized to 0. They discount future streams of profits using the discount factor m.[3]

[2] Similar forces exist in models of oligopolistic competition with a finite number of producers, such as Atkeson and Burstein (2008). However, this specification accommodates a continuum of producers and is a tractable way to model variable markups in a dynamic model without concerns about the existence of multiple equilibria in a dynamic game.

[3] In the deterministic steady state, the producer discounts future steams of profit at rate β, regardless of the household's stochastic discount factor. Later in the paper, I study deterministic dynamics. For my baseline results, I assume that producers discount future streams of profits using the risk neutral discount

Let Λ summarize aggregate states that are relevant to each producer. The recursive problem of an incumbent producer that employed ℓ workers last period and has drawn productivity z is :

$$V(\ell, z; \Lambda) = \max_{p,\ell'} \pi(z, \ell', p; \Lambda) - \phi(\ell, \ell') + \int \max\left\{0, \tilde{V}(\ell', z, c_F; \Lambda)\right\} dJ(c_F), \quad (2.10)$$

$$\tilde{V}(\ell, z, c_F; \Lambda) = -c_F + \beta\mathbb{E}\left[m' V(\ell, z'; \Lambda) \middle| z\right], \quad (2.11)$$

$$\pi(z, \ell', p; \Lambda) = \left(p - \frac{W}{\ell'}\right) d(p; \Lambda), \quad (2.12)$$

$$y \leqslant z\ell'. \quad (2.13)$$

Equation (2.10) shows that the value of a producer is its period profits $\pi(\cdot)$, less the adjustment costs it pays $c(\cdot)$ and plus its continuation value. Its continuation value is the integral over fixed cost draws c_F of the value of continuing to operate next period. Equation (2.11) describes the value of continuing to operate, which equals the expected value of operating next period, discounted using the household's stochastic discount factor and the exogenous producer destruction rate, less the fixed cost of operation. Equations (2.12) and (2.13) describe the production function and demand system that producers face.

2.5 Entrants

There is free entry in the model. Each period, an unlimited mass of potential entrants considers whether to begin producing. Each potential entrant observes the aggregate state of the economy and decides whether to pay a sunk cost c_E to enter. After paying the sunk cost, each entrant draws a value for idiosyncratic productivity from a distribution $H(z)$, freely hires labor, and immediately produces and sells output.[4]

factor β. This assumption is equivalent to assuming either (1) the economy is small and open so its interest rate is fixed or (2) all producers are owned by a measure zero, risk-neutral mutual fund that distributes profits to households. The reason that I choose a risk-neutral discount rate is that the preference specification I use counterfactually implies that interest rates rise in recessions. As emphasized in Winberry (2020), interest rates are pro–cyclical, consistent with a countercyclical discount factor. In this paper, as in Winberry (2020), the interest rate affects producer dynamics. To avoid mischaracterizing the effect of falling entry on aggregate employment, I fix the discount rate and thus the interest rate.

In appendix F, I study the response of the economy to aggregate shocks when producers price streams of profit using the household's stochastic discount factor. In response to the decline in entry, consumption initially falls and returns to its steady state. Under the household preferences that I use, this movement leads the discount factor to fall. The decline in the discount factor has two effects that amplify the response of the economy to entry shocks: (1) it decreases the value of entry further and thus deepens and prolongs the fall in entry and (2) it makes producers more hesitant to hire.

[4] An alternative model of entry would be the selection model presented in Clementi and Palazzo (2016). In that model, each potential entrant observes a signal of its productivity after entry and then decides whether

The value of an entrant who has paid the sunk entry cost is:

$$V_E = \int_z \max_\ell V(\ell, z) dH(z). \tag{2.14}$$

The optimal policy of the potential entrant is to enter if and only if $c_E \leqslant V_E$. In equilibrium, potential producers will enter until the sunk cost of entry equals the value of entry – at which point potential entrants are indifferent between entering and not.

2.6 Aggregation

There are useful aggregation results for this economy.[5] Consider the aggregate production function, where Z_t denotes aggregate productivity:

$$Y_t = Z_t L_t. \tag{2.15}$$

Some algebra shows that aggregate productivity is the inverse quantity–weighted mean of producer–level inverse productivities:

$$Z_t = \left(\int \int \frac{y_t(z, \ell)}{Y z} d\Lambda_t(z, \ell) \right)^{-1}. \tag{2.16}$$

This quantity grows with the number of producers (love of variety) and with the extent to which output is produced primarily by high–productivity producers. The superelasticity of demand is one source of misallocation, since it implies that large, high productivity producers reduce their output relative to a constant-elasticity benchmark.

The aggregate markup is implicitly defined as the inverse labor share:

$$\mathcal{M}_t = \frac{Y_t}{W_t L_t}. \tag{2.17}$$

A rise in the aggregate markup implies a fall in the share of revenue paid to labor. One can show that the aggregate markup is the cost–weighted average of producer–level markups:

$$\mathcal{M}_t = \int \int \mu_t(z, L) \frac{\ell_t(z, L)}{L_t} d\Lambda_t(z, L). \tag{2.18}$$

to enter. As I show in Appendix C, this model has two important counterfactual implications: (1) the entry rate exhibits significantly less volatility than it does in the data and (2) the share of employment among entrants and young firms varies too little relative to the US data.

[5] Note that solving the model still requires approximating the value function of the producers. See Appendix B for details.

3 Markups and firm size in data

In this section, I quantify a key feature of the theoretical environment that I study: namely, the extent to which large producers increase their markups significantly as their market shares rise.

3.1 Data and sample

The data I use are a panel of publicly listed, US-based firms in Compustat. I restrict the sample to observations between 1985 and 2018, exclude financial firms and utilities, and for my baseline results classify firms using the Fama-French-49 industry definition.[6]

This sample, while not representative of the average firm in the economy, accounts for a large portion of US output and employment. Firms in this sample are only 1 percent of firms in the United States, but their sales equal roughly 75 percent of nominal gross national income and their total employment accounts for 30 percent of nonfarm payrolls. In my baseline results, I use the cost-of-goods-sold (COGS) as a measure of variable input costs. COGS includes materials and intermediate inputs, labor costs, energy, and other expenses associated with the production of the firm.[7]

3.2 Markups and firm size

The measurement framework I use is motivated by the production function approach, recently popularized by De Loecker and Warzynski (2012) and De Loecker, Eeckhout and Unger (2020). Consider a producer with a production function in a variable input L and a static input K.[8] The distinction between variable and static inputs is that the producer can costlessly adjust its variable input use, whereas its static inputs may be subject to adjustment costs. The producer's ability to produce and sell output might depend on conditions out of the its control, such as productivity and demand, which I summarize with A. The production function can be expressed as

$$Y = Q(A; K, L). \tag{3.1}$$

Denote by α the elasticity of output with respect to the variable input L. This coefficient might vary over time or across firms and industries. A first-order condition

[6] This classification groups NAICS-4 industries by activity so that each group has roughly the same number of firms. The results that follow are not sensitive to the definition of industry – in Appendix A, I show that similar results hold using SIC and NAICS definitions at various levels of granularity.

[7] See De Loecker, Eeckhout and Unger (2020) for a more detailed discussion. In Appendix A, I explore results using alternative measures of variable input costs.

[8] It is easy to extend this framework to the case with many variable and static inputs. In that case, the first-order condition that I derive below holds for *any* of the variable inputs.

with respect to L gives a relationship between total variable input cost WL, revenue PY, the markup μ, and the output elasticity.

$$WL = \alpha \frac{PY}{\mu}. \tag{3.2}$$

To estimate the relationship between the markup μ and revenue PY, I will then estimate how variable costs WL covary with revenue. Taking logs of this first order condition gives the following equation:

$$\log WL = \log \alpha + \log PY - \log \mu. \tag{3.3}$$

The specification I estimate in data is, for each firm f in industry i at date t:

$$\Delta \log(COGS)_{ift} = \log \alpha_{g(ift)} + \beta \Delta \log(PY)_{ift} + \epsilon_{ift}. \tag{3.4}$$

Comparing equations (3.3) and (3.4), it is clear that a larger covariance between markups and revenues at the firm level generates a lower value for β. If markups do not covary at all with revenues, then $\beta = 1$, and the more that markups covary with revenue, the more that this coefficient deviates from 1. To quantify how much firms increase their markups when their market shares rise, I estimate the following regression:

$$\Delta \log COGS_{ift} = \alpha_{it} + \underset{(0.002)}{0.654} \times \Delta \log PY_{ift} + \epsilon_{ift} \tag{3.5}$$

where ift denotes the observation for firm f in industry i at date t.[9,10] Figure 1 depicts a binscatter plot of this regression. Consistent with the hypothesis that firms increase their markups as they grow, the estimated coefficient of 0.654 is statistically significantly less than one. Under the assumptions underlying the production function approach the estimated coefficient implies an elasticity of the markup to firm size of about 35%.

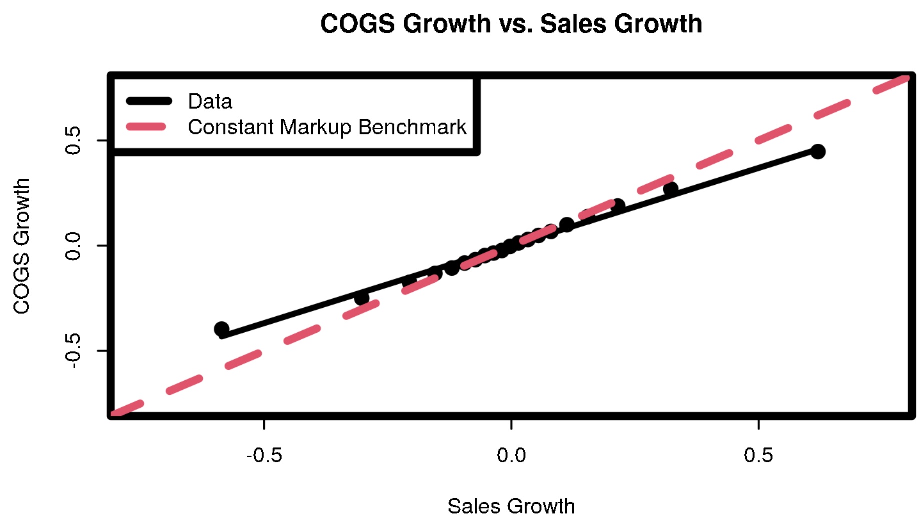
3.3 The frictionless assumption

The PFA attributes the entirety of the less than one–for–one relationship between revenue and variable input use to markup variation. An alternative explanation for

[9] In parentheses, I show standard errors. See Appendix A for more details on the regression, as well as estimates for a variety of specifications for the variable cost WL and choices of fixed effects.

[10] In contrast to De Loecker, Eeckhout and Unger (2020), I do not estimate the elasticity of output with respect to COGS, instead allowing fixed effects to pick up variation in α across firms and over time. This avoids two issues with their approach. First, I do not need to compute a measure of real output for each firm. As Bond et al. (2020) show, this is challenging in Compustat, where we only observe revenue. And second, my approach allows for more heterogeneity in production functions across firms.

Figure 1: COGS and Sales Growth in Compustat



Note: The figure a binscatter of COGS and revenue growth, residualized by industry time fixed effects. The solid black line depicts the fitted values from estimating Equation 3.5. The constant markup benchmark (dashed red line) is a 45-degree line. Source: author's calculations

this regression coefficient is the presence of variable input adjustment costs. These could be hiring and firing costs, long–term contracts in variable inputs markets, or other rigidities that inhibit a firm from increasing its variable input use when it faces a productivity shock. To understand how adjustment costs could lead to a less than one–for–one relationship between revenue and variable input use, consider a firm with an infinite labor adjustment cost. In response to an increase in productivity, the firm would increase its revenue without changing its employment at all, implying a regression coefficient of 0. Under the assumptions of the production function approach, one would mistakenly conclude that this firm increases its markups one-for-one with its relative size.

A key assumption of the PFA is that the variable input can be costlessly adjusted. In particular, for a firm facing adjustment costs on its variable input (that is, it was not truly variable), the static first order condition in the PFA does not hold. In that case, the quantity μ would represent any wedge distorting the firm's production choices away from their static optimums – not just the markup.

To avoid misattributing variation in this wedge to variation in the markup, I will use the model to discipline my interpretation of these regression coefficients. When I calibrate the model, I will jointly choose both the superelasticity of demand, which determines how market power varies with market share, and the degree of adjustment costs to match both the estimated coefficient in this regression and external data on firm–level labor adjustment dynamics. This strategy allows me to interpret these regressions in a structural model with adjustment costs. I will return to these regression estimates in the next section.

4 Steady state

In the steady state of the model, producers are heterogeneous along a number of dimensions. Each producer's idiosyncratic state variables are its productivity and employment. Producers have a lifecycle, beginning small and slowly hiring workers and becoming more productive. Moreover, producers face labor adjustment costs, and so their output and pricing decisions are history dependent. In addition, producers differ in the elasticity of demand they face and thus in the markups they set.

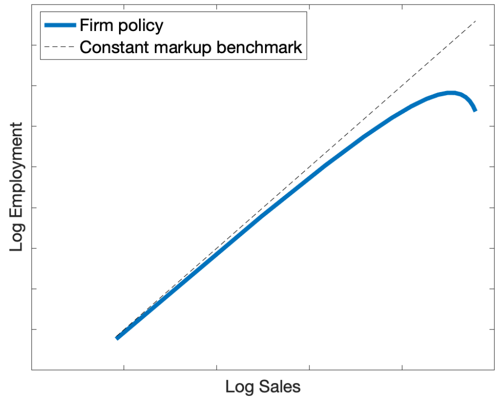
4.1 Markups and producer size

A key mechanism in the model is that the elasticity of demand falls with relative size, such that producers have an incentive to increase their markups as they grow relative to the market. To measure the strength of this mechanism, my calibration targets the regression coefficients I presented in section 3. Two mechanisms in the model directly affect this regression coefficient: (1) the elasticity of demand falls with producer size, leading producers to increase their markups as they grow, and (2) adjustment costs prevent producers from adjusting their variable input use in response to productivity shocks.

To understand the role of mechanism (1), whose strength is dictated by the superelasticity, consider the model without adjustment costs. In that case, each producer 's only idiosyncratic state variable is its productivity. As its productivity rises, it produces more and its elasticity of demand falls. In response, it increases its markups. The increase in markups means that the producer increases its employment less than one-for-one with its sales. Figure 2 depicts the relationship between sales and employment in this model in blue, and the same relationship in a model with constant markups as the black dashed line. Producers in the variable markup model increase their markups as their sales grow, which implies that the slope of the sales-employment relationship is always less than one. Because larger producers increase their markups more with sales than small producers do, this relationship is also concave. For the largest producers, markups increase so much with sales that their employment actually falls as they gain market share.

While I estimate a linear regression of variable input growth on sales growth in the data, that relationship between employment and revenue is not linear in the model.

Figure 2: Employment and sales in the frictionless model



Note: The figure depicts the relationship between employment and sales in a version of the model with no adjustment costs. The constant markup benchmark (dashed black line) is a 45-degree line. Source: author's calculations

This result presents a challenge in calibrating the model, as the average Compustat producer is larger than the average producer in the economy, which might lead me to overstate the extent to which markups rise with market share for the average producer. To calibrate the model, I choose parameters so that equation (3.5) estimated on a sample of the 1% largest producers in the model matches the regression from data. This procedure generates a comparable subsample to estimate the super-elasticity.[11]

Adjustment costs also affect this regression coefficient: a higher adjustment cost leads input use to vary less with revenue. To help identify the size of adjustment frictions, I also require the model to match the autocorrelation of within-firm employment growth in Compustat. A higher value of the adjustment cost leads producers in the model to gradually respond to idiosyncratic shocks, increasing the autocorrelation.

4.2 Calibration

Functional forms. I use Greenwood, Hercowitz and Huffman (1988) preferences:

$$u(C_t, L_t) = \frac{1}{1-\gamma}\left(C_t - \psi\frac{L_t^{1+\nu}}{1+\nu}\right)^{1-\gamma}. \tag{4.1}$$

I also impose a quadratic form for the labor adjustment cost, with a fixed employment depreciation rate δ. The size of the adjustment cost is determined by the parameter ϕ_ℓ.

[11]In the model, these producers account for 14% of sales and 11% of employment.

Table 1: Pre-set parameters

Parameter	Description	Value	Source/Target
β	Discount factor	0.96	Annual model
$\mathbb{P}(\text{exit})$	Probability of exit	0.11	Annual entry rate
ν	Inverse Frisch elasticity	0.5	Clementi and Palazzo (2016)
δ	Employment depreciation rate	0.19	Siemer (2014)

Note: This table summarizes part of the parameterization of the model. These parameter values were each chosen without targeting a particular moment in model simulations. Siemer (2014) estimates the employment depreciation rate as the average quit rate in JOLTS. Source: author's calculations.

$$\phi(L, L_{-1}) = \phi_\ell \left(\frac{L - (1-\delta)L_{-1}}{(1-\delta)L_{-1}} \right)^2 L_{-1}. \tag{4.2}$$

I assume that productivity follows an AR(1) process in logs, with persistence ρ_z and innovation variance σ_z^2. Entering producers draw their initial productivity value from a shifted version of the stationary distribution implied by the law of motion for incumbent productivity, $G(\log(z))$. In particular, entering producers draw their initial value of log productivity from the distribution $H(log(z)) = G(log(z) + d_E)$. I choose the parameter d_E to match the average employment of entering establishments relative to the overall average in the BDS.[12]

Calibration strategy. I fix five parameters and then jointly choose the remaining parameters to ensure that the model is consistent with salient features of the data. The pre-set parameter choices are summarized in table 1. I then simultaneously choose productivity innovation persistence and dispersion ρ_z and σ_z, the adjustment cost parameter ϕ_ℓ, the demand parameters σ and ϵ, and the productivity disadvantage for new entrants d_E. To simplify the calibration procedure, I set the sunk cost of entry to 1 and the fixed cost of production to ∞ with probability $\mathbb{P}(\text{exit}) = 0.11$ and 0 with probability 0.89, so the only exit in the model is exogenous.

While the value of each of these parameters affects several moments in the model, each intuitively corresponds to one or two moments. The persistence of productivity and the dispersion of its innovations affect the cross–sectional variance of producer–level log sales growth and the distribution of relative sales. The productivity differential affects the relative size of entering producers. I identify the degree of adjustment costs with the auto-correlation of producer-level log employment growth. A rise

[12] An alternative way to generate a lifecycle of producer size is by assuming that entering producers face the same productivity distribution as incumbents but must pay adjustment frictions to grow to their initial optimal size. In Appendix E, I explore this alternative.

Table 2: Calibrated parameters

Parameter	Description	Value	Targeted moment
ρ_s	TFP persistence	0.85	Frac. rel. sales below 1
σ_s	TFP innovation dispersion	0.15	Var. emp. growth
ϕ_ℓ	Adjustment cost	0.05	Autocorr. emp. growth
ϵ/σ	Superelasticity	0.67	Labor–sales regression
d_E	Prod. disadvantage of entrants	0.44	Average size entering producer
σ	Elasticity parameter	50	Average markup

Note: Table summarizes part of the parameterization of the model. These parameter values were jointly chosen to match the 6 targeted moments. The variance and autocorrelation of employment growth and the regression coefficient were computed on a sample of the 1% largest producers in the simulated model economy. Source: author's calculations.

in the adjustment cost increases this auto-correlation; without the adjustment cost, the model generates a counterfactually negative auto-correlation. The superelasticity affects the relationship between producer size and the markup and so affects the within–producer regression coefficient of employment on sales. Table 2 summarizes the parameter choices.

The model performs well along a number of targeted and untargeted moments. Table 3 summarizes the model's fit. As in the data, the model generates a wedge between the variance of labor growth and the variance of sales growth. The wedge between these two numbers is in line with its value in the data. The model also fits the share of employment at entrant and young establishments that I estimate in the BDS. Fitting these variables is key to ensuring that the model accurately measures the aggregate importance of entrants. The model matches the distribution of producer size reasonably well, matching facts established in Edmond, Midrigan and Xu (2018) using U.S. Census data. The model slightly overstates the fraction of producers with relative size below 1 and slightly understates the fraction with relative size below 10. The model misses the far right tail of producers with relative sales above 50, as shown in the last line of table 3.

Superelasticity estimate. My estimate of the superelasticity is consistent with estimates from a broad literature that uses producer–level data. Estimates of the superelasticity using microdata tend to be below 1. My estimates are close to Amiti, Itskhoki and Konings (2019), Berger and Vavra (2019), and Gopinath, Itskhoki and Rigobon (2010), who estimate the superelasticity using within-producer price responses to marginal cost shocks.

Consistent with other studies that use microdata to estimate the superelasticity,

Table 3: Calibration targets and model fit

Moment	Target	Source	Model moment
$\text{Var}(\Delta \log L))$	0.06	Compustat	0.06
$\rho(\Delta \log L_t, \Delta \log L_{t-1})$	0.13	Compustat	0.14
Labor–sales regression	0.654	Compustat	0.654
Average size of entering producer	50 percent	CP	52 percent
Frac. rel. sales. below 1	87 percent	EMX	88 percent
Cost–weighted average markup	1.25	DLE	1.26
$\text{Var}(\Delta \log PY))$	0.14	Compustat	0.14
$\rho(\Delta \log P_tY_t, \Delta \log P_{t-1}Y_{t-1})$	0.12	Compustat	0.13
Frac. rel. sales below 10	99 percent	EMX	97 percent
Frac. rel. sales below 50	99.9 percent	EMX	100 percent

Note:DLE: De Loecker, Eeckhout and Unger (2020), CP: Clementi and Palazzo (2016), EMX: Edmond, Midrigan and Xu (2018)
Untargeted moments below line

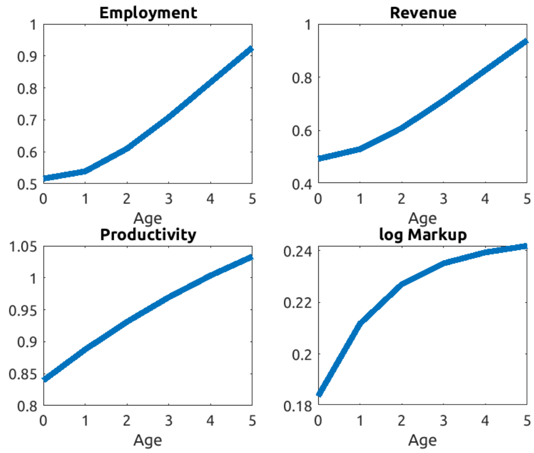
Note: The table summarizes the model's fit of the data. It shows the targeted value and model moment. Explicitly targeted moments are above the single line. The variance and autocorrelation of employment and sales growth and the regression coefficient were computed on a sample of the 1% largest producers in the simulated model economy. Source: author's calculations.

my value of $\epsilon/\sigma = 0.67$ is nearly two orders of magnitude smaller than estimates using macroeconomic data. As noted by Klenow and Willis (2016), the large estimates of the superelasticity needed to account for macroeconomic persistence are inconsistent with micro–level evidence. In this model, setting the superelasticity near the estimates in Lindé and Trabandt (2019) and Smets and Wouters (2007) would imply a counterfactually large markup-size relationship.

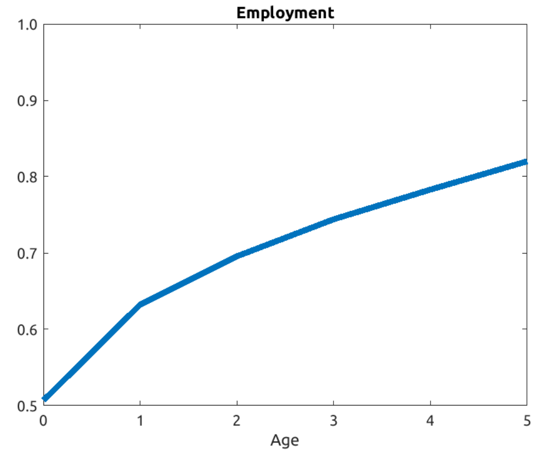
Market power versus labor adjustment. As discussed earlier, the within-producer regression coefficient of employment growth on sales growth (the slope coefficient in Equation 3.5, denoted here by β_L) could be less than 1 for many reasons. In the model, the two forces that generate the less-than-one-for-one regression coefficient are the positive superelasticity of demand and labor adjustment costs. The model allows me to decompose the reduced-form regression coefficient into each component.

The regression coefficient in the model is 0.655. When I set $\phi_\ell = 0$, re-solve the model, simulate a panel of producers in the new model, and estimate the regression coefficient, I find $\hat{\beta}_L = 0.695$. When $\phi_\ell = 0.05$ (as in the baseline model) and the superelasticity of demand is 0, the regression coefficient rises to $\hat{\beta}_L = 0.941$. This decomposition suggests that labor adjustment costs account for between 12 percent and 17 percent of the deviation of the regression coefficient from 1.

Figure 3: Life cycle of the producer in the quantitative model and data



(a) Producer lifecycle in the model



(b) Producer lifecycle in data

Note: The figure summarizes the lifecycle of an establishment in the model and in data. Each panel in subfigure (a) shows the path of the average of a particular establishment-level variable for producers of a particular age relative to its average for all incumbents. Subfigure (b) shows average employment relative to the population average in the BDS data. Source: U.S. Census Bureau, author's calculations.

This decomposition shows that ignoring variable input adjustment costs would lead an econometrician to overstate the relationship between firm size and market power. However, it also shows that, even accounting for variable adjustment costs, large firms' markups rise significantly with with their market shares.

Aggregate parameters. There are two parameters whose values do not affect the steady state of the economy, only its response to aggregate shocks. These parameters are the inverse Frisch elasticity, which I set to be $\nu = 1/2$, and the disutility of labor parameter, ψ, which I set so that the steady state wage is 1.

The lifecycle of the producer. Producers in the model, as in the data, start their lives small and grow slowly. Figure 3 shows that the average entering producer in the model employs around 50 percent of the labor force of the average incumbent producer, as in the data. In the model, they reach 90% of the size of the average producer by around age five. The model achieves this outcome in two ways: (1) the average productivity of entering producers is lower than that of incumbents and slowly reverts to the mean and (2) labor adjustment costs further slow the growth of new producers.

Producers' markups in the model also follow a life-cycle pattern, beginning low and slowly increasing.[13] The desire to set high markups derives from a demand elastic-

[13] Peters (2019) presents evidence for the lifecycle pattern of markups.

ity that decreases with relative size. Because young producers' market shares slowly grow, their markups also increase slowly with age. The cost–weighted average markup increases by around 6 log points over the first five years of a producer's life in the model.

Discussion. In this paper, I study a model with a collection of mechanisms, each of which is motivated by a feature of microdata. Tables 4 and 5 summarize these key mechanisms, whether they are present in other papers in the literature, and the features of the data that motivate those mechanisms. These mechanisms are: (1) variable input adjustment costs, (2) variable markups, and (3) heterogeneity, including the producer lifecycle. This paper is the only study that combines all of these ingredients.

As Table 5 shows, a model missing any of these ingredients is at odds with the data. The adjustment cost in the model is key to generating the positive auto-correlation of net hiring present in microdata. Without any adjustment cost, the auto-correlation is negative, reflecting reversion to the mean in the productivity process. The variable markup is key to generating the less than one-for-one relationship between sales and variable input growth that is present in the data. Without variable markups, variable input use varies nearly one-for-one with sales. And lastly, the producer lifecycle is key to ensuring that entering producers represent a realistic share of aggregate employment.

The combination of these mechanisms differs from existing papers, as shown in Table 4. In the next section, I explore the role of each of these mechanisms for the propagation of entry fluctuations to the rest of the economy.

Table 4: Mechanisms present in quantitative theories of entry

	This paper	Edmond, Midrigan and Xu (2018)	Clementi and Palazzo (2016)	Bilbiie, Ghironi and Melitz (2012)
Variable input adjustment cost	✓	✗	✗	✗
Variable markup	✓	✓	✗	✓
Producer lifecycle	✓	✗	✓	✗
Heterogeneous producers	✓	✓	✓	✗

Table 5: Key mechanisms and their identifying moments

	Relevant Moment	Data value	Model value	Counterfactual value
Variable input adjustment cost	Auto-correlation of net hiring	0.13	0.14	-0.23
Variable markup	Regression of sales growth on variable input bill growth	0.654	0.655	0.941
Producer lifecycle	Avg. relative size entering producer	0.50	0.51	1.00

5 Market structure and entry fluctuations

The goal of this paper is to assess the role of entry fluctuations in amplifying business cycles. So far, I have described a quantitative structural framework to use to make this assessment and presented a quantification of that framework. In this section, I study the response of the model economy to two different shocks: first, a shock to the cost of entry, and second, a shock to aggregate TFP. First studying the shock to the cost of entry isolates the effects of entry on the rest of the economy and allows me to analyze the mechanisms that impact these effects. After discussing these mechanisms, I then proceed to analyze the role of entry in amplifying a traditional business cycle shock.

5.1 An entry shock

I first solve for the response of the model economy to an unexpected shock to the cost of entry. The size of the shock is chosen so that the resulting decline in the number of operating producers in the model matches the decline in the number of establishments during the Great Recession, and it reverts back to its steady-state value with persistence 0.685.[14] After the initial shock is realized, all agents in the economy have perfect foresight of all aggregate variables going forwards as the economy returns to its steady state. I describe the solution method in detail in appendix B.

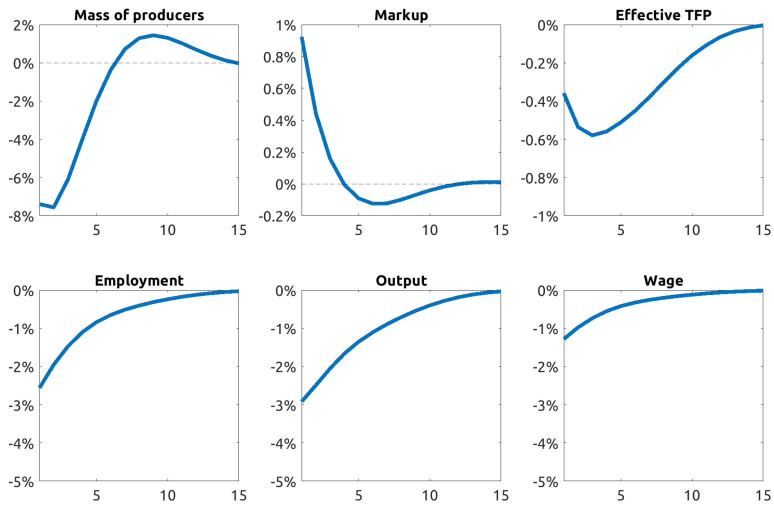
Figure 4 depicts the response of the model to a shock to the cost of entry. The shock causes a fall in entry that leads the mass of establishments to decline by a little over 7 percent and the market shares of incumbents to rise. In response, incumbents increase their markups, and the cost–weighted average markup rises by 0.9 percent. Effective TFP, equal to the ratio of output to aggregate employment, falls gradually by about 0.6 percent. Employment falls over 2 percent on impact, and output falls nearly 3 percent. The wage satisfies the household labor supply equation and falls around 1 percent.

In response to the shock, the entry rate and share of employment among entrants and young producers fall. Figure 5 depicts the role of entrants following the shock. The entry rate falls by around 5 percentage points. It recovers quickly, with some overshooting, in part because the mass of entering producers (the numerator) recovers quickly while the mass of operating producers (the denominator) only gradually returns to its steady state level. The employment share among entering producers falls from 6 percent to around 3 percent. These fluctuations are all in line with those that the US experienced during the Great Recession.[15]

[14]This value of the persistence is chosen to match the persistence of TFP shocks in Clementi and Palazzo (2016). This is also the persistence of the TFP shock I study later in the paper.

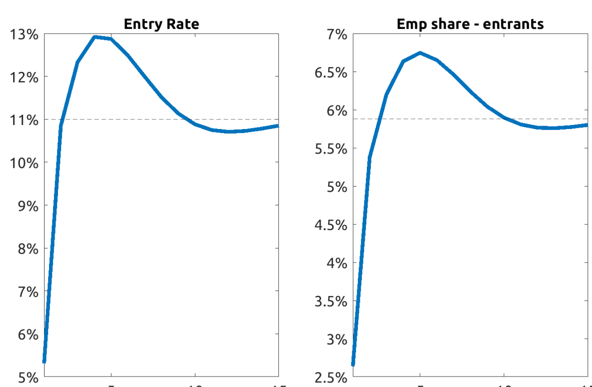
[15]According to the U.S. Census Bureau's BDS, the establishment entry rate fell from 13% to 9%, and the share of employment among entering establishments fell from 5.5 percent to 3.5 percent.

Figure 4: Response of the baseline quantitative model to an MIT shock



Note: The figure depicts the response of several aggregate variables to a persistent, unexpected shock to the cost of entry. Each line depicts the percent deviation of the variable from its steady state value. The size of the shock is chosen to match the fall in the number of establishments per capita during the Great Recession. Following the shock, the economy follows a perfect foresight path back to steady state. Source: author's calculations.

Figure 5: Entry following the shock



Note: The figure depicts the path of the entry rate and employment share at entrants following the shock. Source: author's calculations.

5.2 Markups and productivity

To understand the roles of the average markup μ_t and aggregate TFP Z_t in generating the contraction in employment, it is useful to study the aggregated version of the model. This aggregated model is summarized by an aggregate production function (equation 5.1), the definition of the markup as the inverse labor share (equation 5.2), and the labor supply equation (equation 5.3).[16]

$$Y_t = Z_t L_t, \tag{5.1}$$

$$\mu_t = \frac{Y_t}{W_t L_t}, \tag{5.2}$$

$$W_t = \psi L_t^{\nu}. \tag{5.3}$$

Given paths for the cost–weighted markup μ_t and aggregate effective productivity Z_t, equations (5.1) to (5.3) imply paths for output Y_t, employment L_t, and the wage W_t. While changing the paths of μ_t or Z_t and recomputing these aggregate quantities does not necessarily constitute an equilibrium of this economy, this representation allows for a decomposition of the response of aggregate variables to a shock.

Figure 6 depicts the paths of output, employment, and the wage under different paths for the markup and productivity. In blue, I allow both to follow their equilibrium paths. In red, I hold the markup fixed, and, in yellow, I hold TFP fixed. As they show, the rising markup generates a fall of 1.8 percent in employment, which represents most of the immediate decline in employment. As the markup gradually returns to its steady state value (with some overshooting), the decline in TFP accounts for all of the fall in employment.

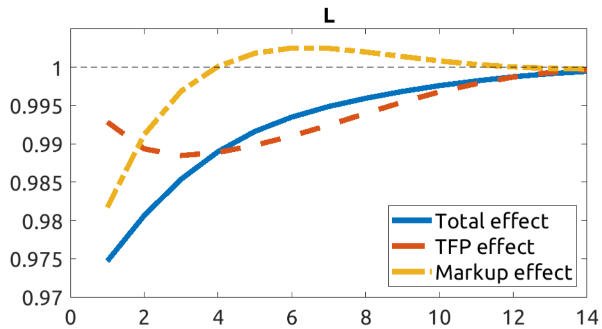
The cost–weighted markup. The increase in the aggregate markup accounts for around three quarters of the initial contraction in employment. As discussed earlier, the relevant measure of the aggregate markup in this economy is the cost–weighted markup:

$$\mathcal{M}_t = \int\int \mu_t(z,\ell)\frac{\ell_t(z,\ell)}{L_t}d\Lambda_t(z,\ell) \tag{5.4}$$

The shock to entry affects the markups of individual producers $\mu_t(z)$ and the distribution of employment across producers $\frac{\ell_t(z,\ell)}{L_t}d\Lambda_t(z,\ell)$. Two opposing forces affect the cost–weighted markup: (1) large producers raise their markups in response to the fall in entry and (2) there is a reallocation of employment from high–markup to low–markup

[16] These can be further simplified into log-affine labor supply and labor demand equations: $\log W_t = \log\psi + \nu\log L_t$ and $\log W_t = \log A_t - \log\mu_t$.

Figure 6: Decomposition of entry shock



Note: The figure depicts a decomposition of the effects of the shock on aggregate employment into the effects of TFP and the effects of the markup. The dot-dashed line depicts the effect of the markup, holding aggregate TFP fixed. Each line depicts the percent deviation of the variable from its steady state value. The dashed line depicts the effects of TFP, holding the markup fixed. Source: author's calculations.

producers.

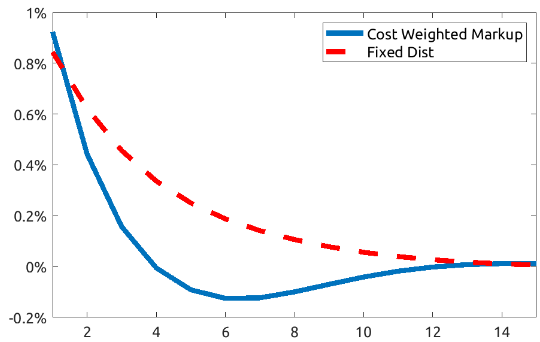
What drives force (2), the reallocation toward low-markup producers? In the model, variable markups are generated by a variable elasticity of demand; small producers set lower markups because they face a higher elasticity of demand than large producers. This feature also means that small producers are more exposed to competition from new entrants, and so they benefit more from the reduction in entry. Thus, the decline in entry leads to a reallocation of employment to low-markup, high-elasticity producers.

Adjustment costs in the model slow this reallocation to low-markup producers. One way to see this phenomenon is to compare the path of the markup holding $\ell_t(z,\ell)/L \times d\Lambda_t(z,\ell)$ fixed to the path of the cost-weighted average markup. Figure 7 depicts this comparison. In red, I allow producer-level markups to vary but hold the distribution of employment fixed. This plot shows that the average producer raises its markups persistently in response to the shock. The blue solid line shows the path of the markup in the baseline model and exhibits a more rapid return to its steady–state level.

Love of variety. There is love-of-variety in this model; productivity increases with the number of differentiated varieties. There is no closed-form expression for the love-of-variety effects in a model with heterogeneous firms and Kimball demand, so I instead compare the fall in aggregate TFP to the decline implied by the same reduction in the number of producers in a model of symmetric producers and CES demand. In that case, aggregate productivity is a function of the number of producers N and the elasticity of substitution σ_{CES}: $Z(N) = N^{\frac{1}{\sigma_{CES}-1}}$.[17]

[17] I set σ_{CES} so that $\frac{\sigma_{CES}}{\sigma_{CES}-1}$ equals the cost-weighted markup in the benchmark model.

Figure 7: Weighted and unweighted markups



Note: This figure depicts the path of the cost-weighted average markup in response to the shock to the mass of potential entrants. The solid line depicts the path of the cost-weighted markup, allowing both the policy function of producers and the distribution of employment across producers to vary. The red dashed line shows the markup, holding fixed the distribution of employment across producers. Source: author's calculations.

This calibration implies $\sigma_{CES} \approx 5$, so that the love-of-variety effects imply a decline in effective TFP of around one-quarter of the decline in the number of producers, or almost 2 percent. This decline is much larger than the actual decline in effective TFP. I return to this discrepancy in my discussion of the role of the producer life-cycle in Section 5.4.

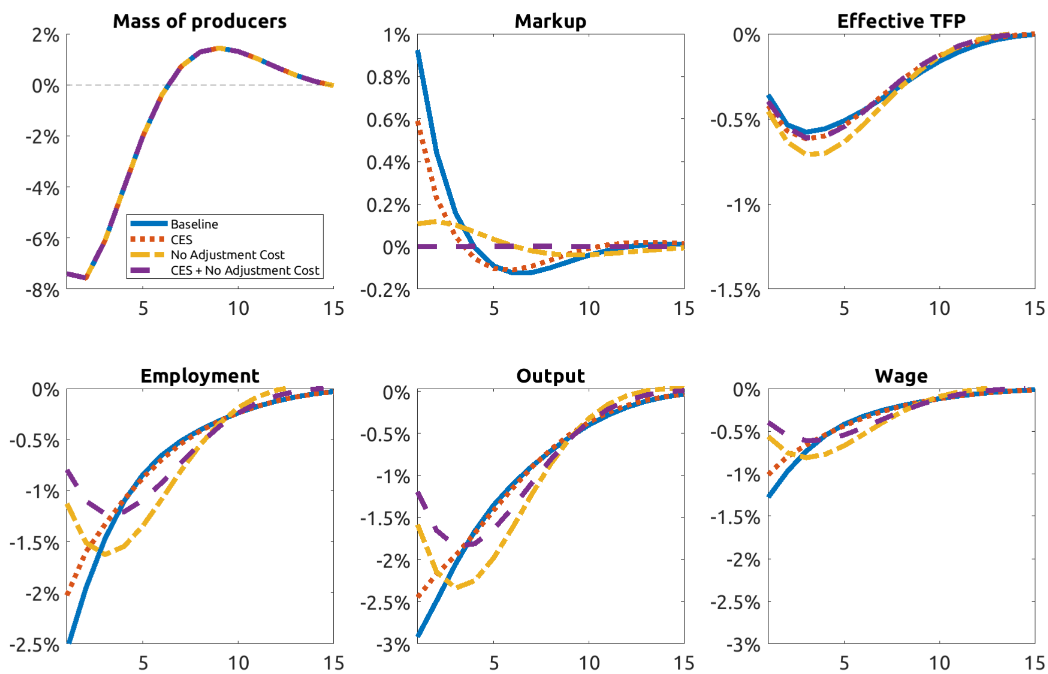
5.3 The roles of adjustment costs and variable markups

Two of the key mechanisms identified in Table 5 are variable markups and the variable input adjustment cost. Figure 8 shows the response of the economy under different sets of assumptions concerning these two mechanisms. As the figure shows, while the dynamics of the markup varies significantly across these exercises, effective TFP moves very similarly in each model. I show below that the interaction between variable adjustment costs and variable markups is key to generating the rise in the markup in this framework.

The role of adjustment costs. As discussed above, variable markups lead to a reallocation of employment to low-markup producers, but adjustment costs slow this reallocation. To quantify this mechanism, I compare the response in the baseline economy to the response in an economy without adjustment costs. In the line denoted "No Adjustment Cost" in figure 8, I plot the path of the cost-weighted average markup in each economy. As shown, the markup rises by almost 90 percent less under this assumption than in the model with adjustment costs.

The model without adjustment costs closely resembles the model in Edmond, Midri-

Figure 8: Response to an entry cost shock in four economies



Note: This figure depicts the response of several aggregate variables to a persistent unexpected shock to the cost of entry in four different models. The solid blue line depicts the "baseline model" as described in Section 2. The red dotted line depicts a model identical to the baseline except that it features a CES final goods production function, rather than Kimball. The yellow dot-dashed line depicts a model that is identical to the baseline except that producers face no adjustment costs. Lastly, the purple dashed line depicts a model with CES production and no adjustment costs. Source: author's calculations.

gan and Xu (2018), who find that fluctuations in entry have little effect on the aggregate markup in a model with variable markups, Pareto-distributed productivity, and frictionless adjustment of variable inputs. They find that an optimal entry subsidy, which increases the mass of operating firms by over 20%, has almost no effect on the aggregate markup.

The role of variable markups. To quantify the role of variable markups in the propagation of entry shocks to aggregate employment and output, I compare the Kimball model to one in which producers' demand elasticities do not vary with their market shares. This comparison model features constant elasticity of substitution (CES) preferences. To ensure that the models are comparable, I choose the elasticity of substitution in the CES model so that the cost–weighted markup in each model is identical. I keep all other parameters the same.

I subject each economy to the same entry shock as before. The line labeled "CES" in Figure 8 depicts the results of this experiment. These impulse response functions show that the variable elasticity of demand generates a significant fall in employment and amplifies the effects of an entry shock. The markup rises somewhat (by a about 60 basis points) in the CES model because adjustment costs push producers away from their frictionless optimal solution.[18] However, the rise in the markup is only about half of the rise in the Kimball model, meaning that employment in the CES model does not fall as sharply as in the Kimball model. The additional rise in the markup in the Kimball economy generates a nearly 20 percent extra fall in employment on impact in the model with variable markups. This difference disappears after around five years.

Model with constant markups and no adjustment costs. Markups in the CES economy move in response to the shock because of the labor adjustment friction. The lines labelled "CES + No Adjustment Cost" in Figure 8 depict the response of the economy with CES demand and no adjustment costs to the entry cost shock. Without either of these mechanisms, the markup does not vary in response to the shock, leading employment to fall by only about 40% as much as in the baseline economy.

This model closely resembles that in Clementi and Palazzo (2016), who study the role of entry in amplifying aggregate TFP shocks in an industry equilibrium model with perfect competition, a firm lifecycle, and flexible labor adjustment. They find that entry fluctuations play a relatively small role in the short-run dynamics of output and employment following a TFP shock but a larger role after several year. The reason

[18]Recall that the markup is price divided by marginal cost. The wage falls in response to the shock, giving producers an incentive to increase their employment. In a frictionless economy, firms would hire more workers and lower their price to keep their markups fixed. However, with an adjustment cost, not all producers will fully adjust in response to the shock, leading them produce less than they would absent the adjustment friction. Firms are constrained to be on their demand curves, and so this lower level of production is associated with higher prices.

for this growing effect is the *missing cohort effect*; entrants are small but eventually grow large, and so a missing cohort reduces employment demand many years after it would have started. Consistent with their findings, I find that entry fluctuations have quite persistent effects on aggregate employment and output. However, the model with variable markups also implies that a shock to entry has a larger effect on aggregate employment *on impact*, due to the response of incumbent producers' markups to missing competition from entrants.

5.4 The role of heterogeneity

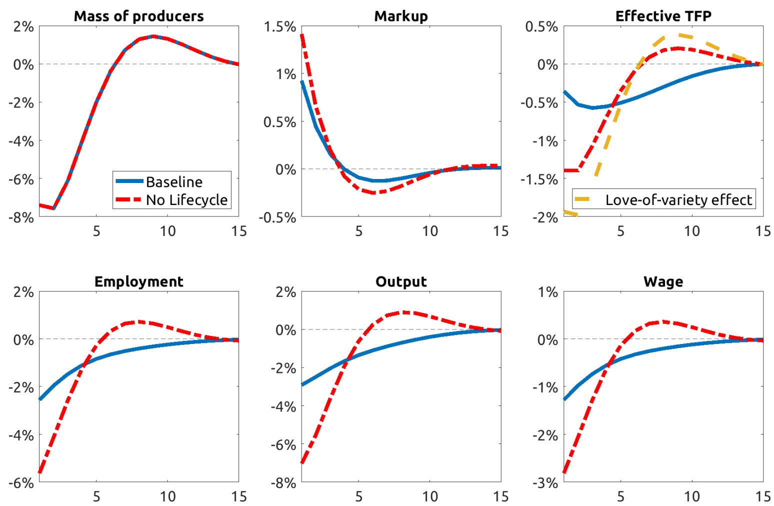
The firm lifecycle. Another key mechanism identified in Table 5 is the lifecycle of the producer. As shown in Table 3, entering establishments are roughly half the size (in terms of employment) of the average establishment in the economy. This fact is at odds with models that rely on the assumption of homogeneous producers, such as that in Bilbiie, Ghironi and Melitz (2012). To understand the role of the lifecycle in the propagation of entry fluctuations to the rest of the economy, I compare the baseline model to one in which entering producers have the same average size as incumbents.

The line marked "No Lifecycle" in Figure 9 depicts the results of this experiment. Employment initially falls by over twice as much in the economy with no lifecycle relative to the baseline, and it recovers much more quickly. The difference in the size of the impact and the speed of the recovery between these two impulses shows the role of the "missing cohort" effect; in the baseline model, because entering producers are small but grow over time, fluctuations in entry have a small but persistent effect on aggregate employment. Inspecting the paths for the aggregate markup and effective TFP shows that most of the difference in the two employment responses is due to TFP. TFP falls by about 3 times as much in the economy with no lifecycle.

The purple dashed line labeled "Love-of-variety effect" depicts the path of aggregate TFP under the symmetric CES formulation for the effects of love-of-variety. As it shows, the symmetric CES benchmark closely approximates the path of effective TFP in the model with no producer lifecycle. In the model with a lifecycle, love-of-variety effects are much smaller on impact because entrants are small and so contribute little to aggregate productivity.

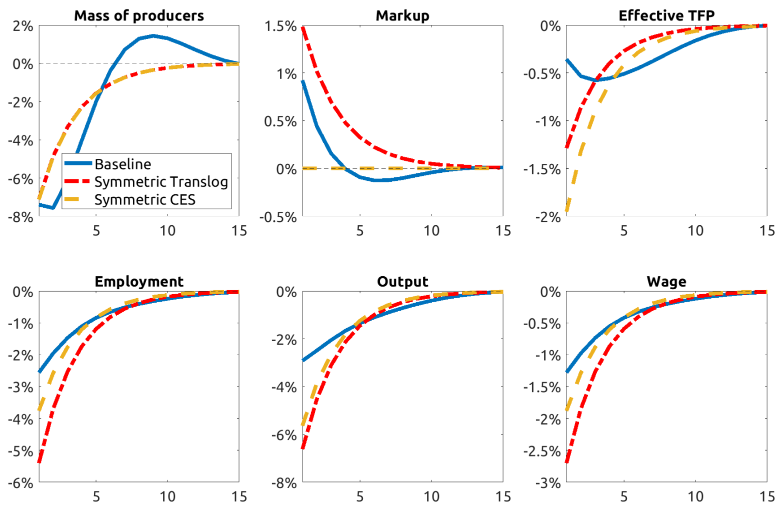
A homogeneous producer model. Bilbiie, Ghironi and Melitz (2012) study fluctuations in entry in a model with homogeneous producers and variable markups. To understand the relationship of my work to theirs, I compare the baseline model to two models in which all producers are identical as in Bilbiie, Ghironi and Melitz (2012). As in Bilbiie, Ghironi and Melitz (2012), I consider models with CES and Translog demand, and I set the labor adjustment cost $\phi_\ell = 0$. The model is otherwise

Figure 9: The role of the lifecycle in the effects of an entry shock



Note: The figure depicts the response of several aggregate variables to a persistent, unexpected shock to the cost of entry. Each line in each panel depicts the percent deviation of the variable from its steady state value. The size of the shock is chosen to match the fall in the number of establishments per capita during the Great Recession. Following the shock, the economy follows a perfect foresight path back to steady state. Source: author's calculations.

Figure 10: Entry and employment in symmetric models



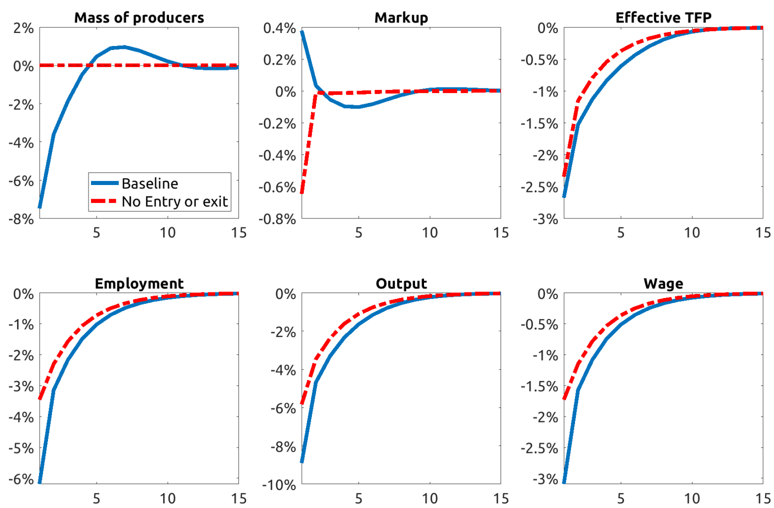
Note: The figure depicts the response of several aggregate variables to a persistent, unexpected shock to the cost of entry. Each line in each panel depicts the percent deviation of the variable from its steady state value. The size of the shock is chosen to match the fall in the number of establishments per capita during the Great Recession. Following the shock, the economy follows a perfect foresight path back to steady state. Source: author's calculations.

identical to the baseline model I study.

In the Translog specification that Bilbiie, Ghironi and Melitz (2012) use, the markup is a function of the number of producers: $\mu(N_t) = 1 + \frac{1}{\sigma N_t}$. There is a love-of-variety effect in the model, so that the price index is decreasing in the mass of producers. For more details, see appendix G.

I subject the baseline and both of the symmetric economies to a shock to the cost of entry. The persistence of the shock is 0.685 in each economy, and I choose the size of the initial shock to match the fall in the number of establishments during the Great Recession. The results of this experiment are depicted in Figure 10. Employment in the symmetric translog model falls by over twice as much as in the baseline model. The markup rises by 50% more and TFP falls by three times as much as in the model with no heterogeneity. The TFP effects are larger but that markup does not move in the CES model, resulting in employment effects that are between those in the baseline and symmetric translog model. These results show that accounting for heterogeneity greatly reduces the ultimate impact of a shock to the entry cost.

Figure 11: The role of entry in TFP shocks



Note: The figure depicts the path of the cost-weighted average markup in response to the shock to aggregate TFP in the baseline model and in one without entry and exit. Each panel depicts the percent deviation of the variable to its steady state value. Source: author's calculations.

5.5 Variable Markups and the amplification of business cycle shocks

I now subject the model economy to a shock to exogenous TFP, which I denote by A. I modify the economy so that the entry cost is denoted in units of labor and scales with exogenous TFP, as in Bilbiie, Ghironi and Melitz (2012): $c_E = f_E \frac{w}{A}$. The solid blue lines in figure 11 show the response of six model moments to a shock to TFP with a persistence of 0.685.[19] I choose the size of the shock so that aggregate employment falls by 6%, about how much it fell during the Great Recession.

In response to the exogenous decline in aggregate TFP, the mass of producers falls by around 8 percent, leading the markup to rise by 0.4% and effective TFP to fall by over 2.5%. Output falls by over 8 percent and the wage falls by roughly 3 percent. Applying the same decomposition as in Figure 6, the exogenous drop in TFP leads aggregate employment to fall by 4.7%, and the endogenous movement in the markup and TFP lead employment to fall by 1.3% on impact (relative to a counterfactual where neither change). The rise in the markup accounts for 0.8 percentage points of this initial drop, with endogenous TFP accounting for the remainder of the endogenous effect.

[19]I choose the persistence of the shock to follow Clementi and Palazzo (2016).

The role of entry. Figure 11 also shows the response of an economy without entry and exit to the same TFP shock.[20] As it shows, employment in the economy with entry and exit falls by around 75% more than in the economy without this margin of adjustment. Inspecting the other panels reveals that this difference is primarily due to the rise in the markup in the baseline model, which contrasts with a fall in the markup in the model without entry and exit. The rise in the markup in the baseline economy relative to the decline in the economy without entry and exit is roughly in line with the rise in the markup in response to the shock to the cost of entry discussed in Section 5.1.

The markup in the model without entry and exit falls because of adjustment costs, as discussed in Section 5.3: a fall in aggregate TFP reduces producers' optimal level of employment, but adjustment frictions prevent them from immediately adjusting their employment. Note that while the path of the markup accounts for most of the difference between the baseline economy and the model without entry or exit, it accounts for a small portion of the decomposition of the path of aggregate employment. This apparent discrepancy arises because the relevant counterfactual is one in which the markup *falls* in response to the decline in TFP, not one in which it stays flat, and thus, the decomposition is somewhat misleading.

6 Conclusion

In this paper, I assess the role of entry fluctuations in amplifying recessions in a general equilibrium model. The model features heterogeneous producers who face adjustment frictions and a life cycle profile for productivity, and set markups that vary with their relative size. My main finding is that entry plays an economically meaningful role in business cycle amplification, partly through its effects on markups, and partly through variety effects on TFP.

This paper follows a substantial literature studying the role of entry fluctuations in business cycle amplification. Papers in that literature come to very different conclusions, depending on the assumptions underlying those results. In this paper, I provide a framework for understanding those disparate results. In particular, I show that models that ignore heterogeneity likely overstate the role that entry plays in business cycles, and models with heterogeneity but that omit either variable markups or adjustment frictions likely understate the role that it plays.

There remain interesting avenues for future research. First, the countercyclical markups in the model may imply that inflation does not fall much in recessions. Fu-

[20] To keep the two models comparable, I set the mass of producers in the "no entry" economy equal to the steady state mass of producers in the baseline model.

ture research could incorporate nominal rigidities into this model and study inflation dynamics. Second, what does optimal policy look like in this model? Is there a role for entry subsidies? How should the government treat large producers in recessions? These questions are beyond the scope of this paper but are nonetheless relevant.

References

Amiti, Mary, Oleg Itskhoki, and Jozef Konings. 2019. "International Shocks, Variable Markups, and Domestic Prices." *Review of Economic Studies*, 86(6): 2356–2402.

Atkeson, Andrew, and Ariel Burstein. 2008. "Pricing-to-Market, Trade Costs, and International Relative Prices." *American Economic Review*, 98(5): 1998–2031.

Berger, David, and Joseph Vavra. 2019. "Shocks versus Responsiveness: What Drives Time-Varying Dispersion?" *Journal of Political Economy*, 127(5): 2104–2142.

Bilbiie, Florin O., Fabio Ghironi, and Marc J. Melitz. 2012. "Endogenous Entry, Product Variety, and Business Cycles." *Journal of Political Economy*, 120(2): 304–345.

Bond, Steve, Arshia Hashemi, Greg Kaplan, and Piotr Zoch. 2020. "Some Unpleasant Markup Arithmetic: Production Function Elasticities and their Estimation from Production Data." National Bureau of Economic Research, Inc NBER Working Papers 27002.

Clementi, Gian Luca, and Berardino Palazzo. 2016. "Entry, Exit, Firm Dynamics, and Aggregate Fluctuations." *American Economic Journal: Macroeconomics*, 8(3): 1–41.

De Loecker, Jan, and Frederic Warzynski. 2012. "Markups and Firm-Level Export Status." *American Economic Review*, 102(6): 2437–2471.

De Loecker, Jan, Jan Eeckhout, and Gabriel Unger. 2020. "The Rise of Market Power and the Macroeconomic Implications*." *The Quarterly Journal of Economics*, 135(2): 561–644.

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu. 2018. "How Costly Are Markups?" National Bureau of Economic Research, Inc NBER Working Papers 24800.

Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon. 2010. "Currency Choice and Exchange Rate Pass-Through." *American Economic Review*, 100(1): 304–336.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman. 1988. "Investment, Capacity Utilization, and the Real Business Cycle." *American Economic Review*, 78(3): 402–417.

Haltiwanger, John, Ron S. Jarmin, and Javier Miranda. 2013. "Who Creates Jobs? Small versus Large versus Young." *The Review of Economics and Statistics*, 95(2): 347–361.

Hopenhayn, Hugo A. 1992. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium." *Econometrica*, 60(5): 1127–1150.

Jaimovich, Nir, and Max Floetotto. 2008. "Firm dynamics, markup variations, and the business cycle." *Journal of Monetary Economics*, 55(7): 1238–1252.

Klenow, Peter J., and Jonathan L. Willis. 2016. "Real Rigidities and Nominal Price Changes." *Economica*, 83(331): 443–472.

Lee, Yoonsoo, and Toshihiko Mukoyama. 2018. "A model of entry, exit, and plant-level dynamics over the business cycle." *Journal of Economic Dynamics and Control*, 96: 1–25.

Lindé, Jesper, and Mathias Trabandt. 2019. "Resolving the Missing Deflation Puzzle." C.E.P.R. Discussion Papers CEPR Discussion Papers 13690.

Midrigan, Virgiliu. 2008. "Comment on "Monetary Policy and Business Cycles with Endogenous Entry and Product Variety"." In *NBER Macroeconomics Annual 2007, Volume 22. NBER Chapters*, 355–364. National Bureau of Economic Research, Inc.

Peters, Michael. 2019. "Heterogeneous Markups, Growth and Endogenous Misallocation." *Econometrica*.

Siemer, Michael. 2014. "Firm Entry and Employment Dynamics in the Great Recession." Board of Governors of the Federal Reserve System (U.S.) Finance and Economics Discussion Series 2014-56.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586–606.

Winberry, Thomas. 2020. "Lumpy Investment, Business Cycles, and Stimulus Policy." Mimeo.

A Data

Cleaning procedure. I download a sample of Compsutat from WRDS. To clean the data, I use the following procedure:

- Keep only firms incorporated in the USA.
- Exclude utilities and financial firms – SIC codes 4900 - 4999 and 6900–6999.
- Exclude observations that are not in US dollars.
- Exclude observations with zero or negative values for SALE or EMP.

Other specifications. I estimate a variety of specifications for the variable cost WL and choices of fixed effects $g(ift)$. Table 6 summarizes the results. Each row contains results using a different measure of variable input cost, and in each column, I control for different levels of firm heterogeneity. I consider three measures of variable input use: total wage bill (XLR), total number of workers (EMP), and cost of goods sold (COGS). Data on wage bills are missing for many firms, and so I only have 17,501 observations of XLR, one-tenth the number of observations of COGS and EMP in the dataset.

Consistent with the hypothesis that firms increase their markups as their market shares grow, the estimated regression coefficient is statistically less than 1 across all nine specifications. My preferred specification is (3). In column (3), I estimate the regression using one-year growth rates.[21] This specification captures how, at a business cycle frequency, firms' variable input use varies when their revenues change relative to the whole industry. I find values well below 1 for these regressions, ranging from 0.356 for employment to 0.654 for COGS. These coefficients are interpretable as the amount by which a firm increases its variable input bill when its revenue growth is double that of the average firm in its industry.

[21] The results are robust to the definition of growth rate, but for my baseline results, I follow Haltiwanger, Jarmin and Miranda (2013) and use

$$g_{ift} = \frac{V_{if,t} - V_{if,t-1}}{\frac{1}{2}(V_{if,t} + V_{if,t-1})}.$$

Table 6: Variable input use and relative size over the whole sample

Dependent variable	(1)	(2)	(3)
$\log EMP$	0.8384	0.6275	0.356
	(0.0009)	(0.0016)	(0.0137)
$\log XLR$	0.8983	0.6716	0.4266
	(0.003)	(0.007)	(0.007)
$\log COGS$	0.9263	0.783	0.654
	(0.0007)	(0.002)	(0.002)
Specification	Log levels	Log levels	Growth rates
Fixed effects	Industry × Year	Firm + Industry × Year	Industry × Year

Note: This table depicts the results of estimating equation 3.5. Column (1) depicts the results using industry × year fixed effects. Column (2) depicts the results using firm + industry × year fixed effects. Column (3) depicts the results using growth rates. Source: Center for Research in Security Prices, CRSP/Compustat Merged Database, Wharton Research Data Services, http://www.whartonwrds.com/datasets/crsp/; author's calculations.

NAICS-4. Here, I document that the key regression that identifies the extent to which markups rise with firm size is robust to using NAICS-4 as the definition of an industry.

Table 7: Variable input use and relative size over the whole sample

Dependent variable	log PY		
	(1)	(2)	(3)
log EMP	0.8229186	0.623711	0.375305
	(0.0008742)	(0.001559)	(0.001798)
log XLR	0.885107	0.688669	0.469273
	(0.003)	(0.005639)	(0.006349)
log $COGS$	0.9164561	0.780266	0.651581
	(0.0007804)	(0.001595)	(0.001949)
Specification	Log levels	Log levels	Log difference
Fixed Effects	Industry × Year	Firm + Industry × Year	Industry × Year

NAICS-2. Similarly, for NAICS-2:

Table 8: Variable input use and relative size over the whole sample

Dependent variable	log PY		
	(1)	(2)	(3)
log EMP	0.8307641	0.632097	0.38278
	(0.0008417)	(0.001508)	(0.00174)
log XLR	0.891063	0.683225	0.459426
	(0.002387)	(0.005004)	(0.005529)
log $COGS$	0.9334514	0.79041	0.661271
	(0.0007165)	(0.00151)	(0.001869)
Specification	Log levels	Log levels	Log difference
Fixed Effects	Industry × Year	Firm + Industry × Year	Industry × Year

B Solution method

B.1 Free entry model

To find the initial steady state, I normalize aggregate output to 1 and the wage to 1. I approximate the value functions on a state space of a grid of 30 points for productivity and 50 points for labor. I discretize the productivity process using Rouwenhorst's method. Finding the steady state then involves finding a fixed point in the value of the demand index. The process is as follows:

1. Set D_L and D_U, the bounds on the values of the demand index.
2. Guess that $D_i = \frac{D_L + D_U}{2}$.
3. Given D_i, solve the value function of the incumbent firm. I solve this problem using value function iteration and the Howard Policy Improvement algorithm.
4. Given the value function of the incumbent firm, find the value of entry. This also implies policy functions of entering firms. If the value of entry is above the cost of entry, set $D_U = D_i$. Otherwise, set $D_L = D_i$. Return to step 2.
5. If the absolute difference between the value of entry and the cost of entry is less than 10^{-8}, this portion of algorithm is complete. Otherwise, continue.

Because any value for the mass of entering firms is consistent with the free-entry condition $V_E = c_E$, we can choose the mass of entering firms so that the implied demand index is consistent with the guessed demand index. To do this, we again use bisection.

After completing this process, we can then fix a value that the Kimball aggregator should integrate to (note, for expositional purposes I use 1, but it is irrelevant as long as it is fixed) and a value ω such that the intratemporal first order condition of the representative household holds.

B.2 Selection model

To find the initial steady state, I normalize aggregate output to 1 and the wage to 1. I approximate the value functions on a state space of a grid of 30 points for productivity and 50 points for labor. I discretize the productivity process using Rouwenhorst's method. Finding the steady state then involves finding a fixed point in the value of the demand index. The process is as follows:

1. Set D_L and D_U, the bounds on the values of the demand index.
2. Guess that $D_i = \frac{D_L + D_U}{2}$.

3. Given D_i, solve the value function of the incumbent firm. I solve this problem using value function iteration and the Howard Policy Improvement algorithm.
4. Given the value function of the incumbent firm, find the value of entry. This also implies policy functions of entering firms that depend on their productivity signal as well as entry decisions.
5. Given the policy functions of incumbent and entering firms, find the implied stationary distribution over the two state variables.
6. Compute the implied value of D_{out}. Define $diff = D_{out} - D_i$. If $|diff| < 10^{-8}$, the algorithm is complete. Otherwise, continue.
7. If $diff < 0$, then set $D_U = D_i$. Otherwise, set $D_L = D_i$. Return to step 2.

After completing this process, we can then fix a value that the Kimball aggregator should integrate to (note, for expositional purposes I use 1, but it is irrelevant as long as it is fixed) and a value ω such that the intratemporal first order condition of the representative household holds.

B.3 Dynamics

Solving for the response to an unexpected shock involves a shooting algorithm over W, C, and D.

C Alternative model: selection model of entry

An alternative to free entry is a *selection model* of entry. The key difference between free entry and selection is that in the latter model, potential entrants observe a signal about their future productivity before choosing whether to enter or not. This (1) makes entry less elastic to aggregate economic conditions and (2) means that the marginal potential entrant is low-productivity, and so fluctuations in the entry cost lead to small fluctuations in the employment share of new entrants.

The model. Each period, a mass M_t of potential entrants considers whether to begin producing. Each potential entrant draws an idiosyncratic signal of their future productivity $\phi \sim F$ and decides whether to enter. After paying the sunk cost, the entrant freely hires labor but cannot produce. Its productivity the following period is drawn from a distribution $H(z|\phi)$.

The value of an entrant who has drawn productivity signal ϕ and already paid the sunk entry cost is

$$V_E(\phi) = \int_z \max_L \beta(1-\phi)\mathbb{E}\Big[V(z,L)|\phi\Big]dH(z|\phi). \tag{C.1}$$

The optimal policy of the potential entrant is to enter if and only if $c_E \leqslant V_E(\phi)$. Under regularity conditions about $H(z|\phi)$, the value function $V_E(\phi)$ is monotonically increasing in ϕ, and so the policy of the entrant is to enter if and only if its signal exceeds a threshold $\hat{\phi}$.

Calibration. I assume that the transition from signal to realized productivity follows the same AR(1) process as underlying productivity. I assume that the distribution of signals is Pareto, and I choose the Pareto tail parameter to match the average employment at entering establishments relative to incumbents in the BDS.

Result 1: Entry is inelastic to aggregate TFP. In Figure 12, I show the response of entry to an aggregate TFP shock. I choose the size of the shock so that aggregate employment falls by 6 percent, about as much as it did in data relative to trend during the Great Recession. As the figure shows, the mass of establishments barely moves in response to the shock. During the Great Recession, the mass of operating establishments fell by over 7 percent relative to trend, and so the selection model generates too small of an endogenous movement to analyze this period.

Why is entry so inelastic in the selection model? In this model, the only potential entrants affected by a change in TFP are those on the margin of entering or not, whereas with a free entry condition, the mass of entrants changes to make the value of entry equal to the cost of entry. There are relatively few potential entrants at this margin, and so, entry is relatively inelastic.

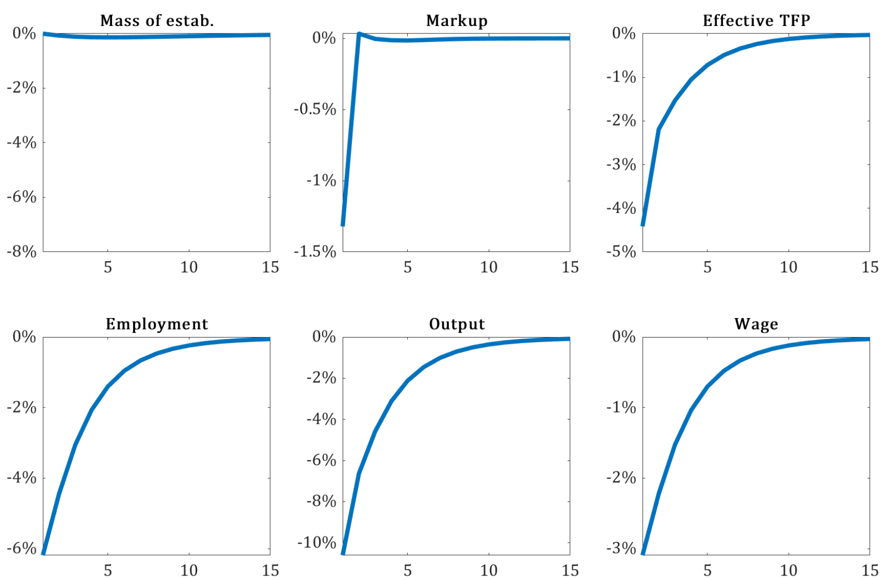


Figure 12: TFP shock in the selection model of entry

Result 2: The share of employment at entrants is inelastic to the cost of entry. A shock to the cost of entry has very little effect on the employment share among entering producers in the selection model economy. Figure 14 shows the paths of the entry rate and employment share at entrants following a shock to the cost of entry. As it shows, even though a sufficiently large shock to the cost of entry can generate a decline in the entry rate, it barely moves the employment share at entering establishments. This inelasticity is due to the fact that the producers on the margin between entering or not are relatively small and so do not account for much of aggregate employment.

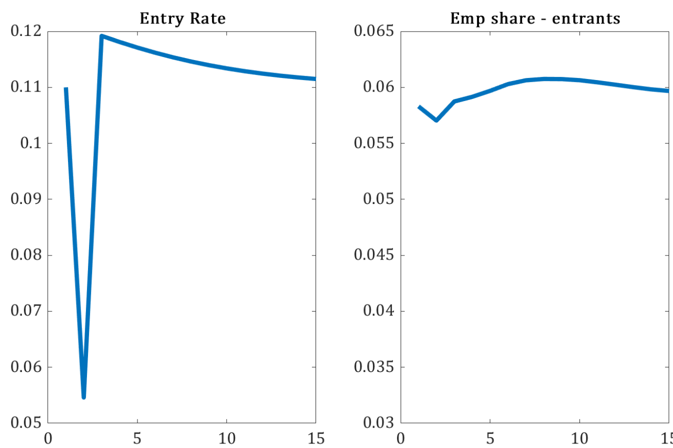


Figure 13: Entry cost shock in the selection model of entry

These tepid movements in the employment share at entrants are counterfactual relative to the Great Recession episode, when the employment share at new establishments fell by over 2 percentage points, from around 5.5 percent of total employment to below 3.5 percent.

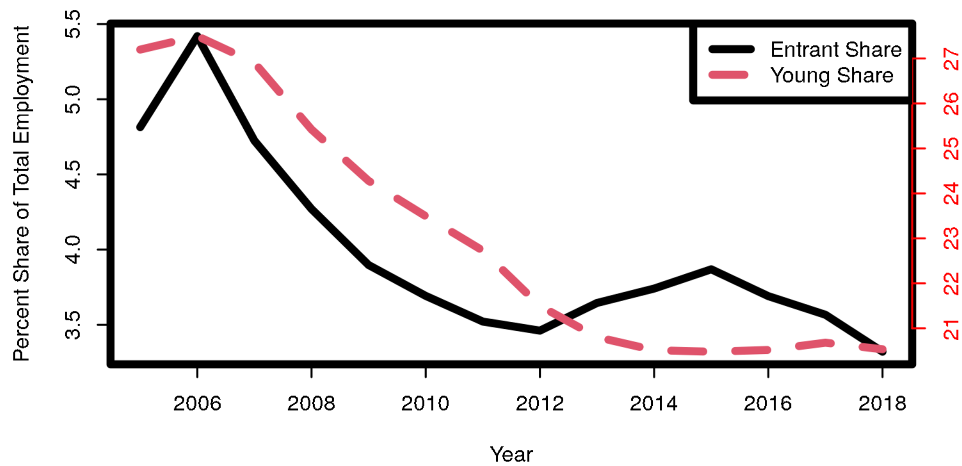


Figure 14: Employment shares in the data

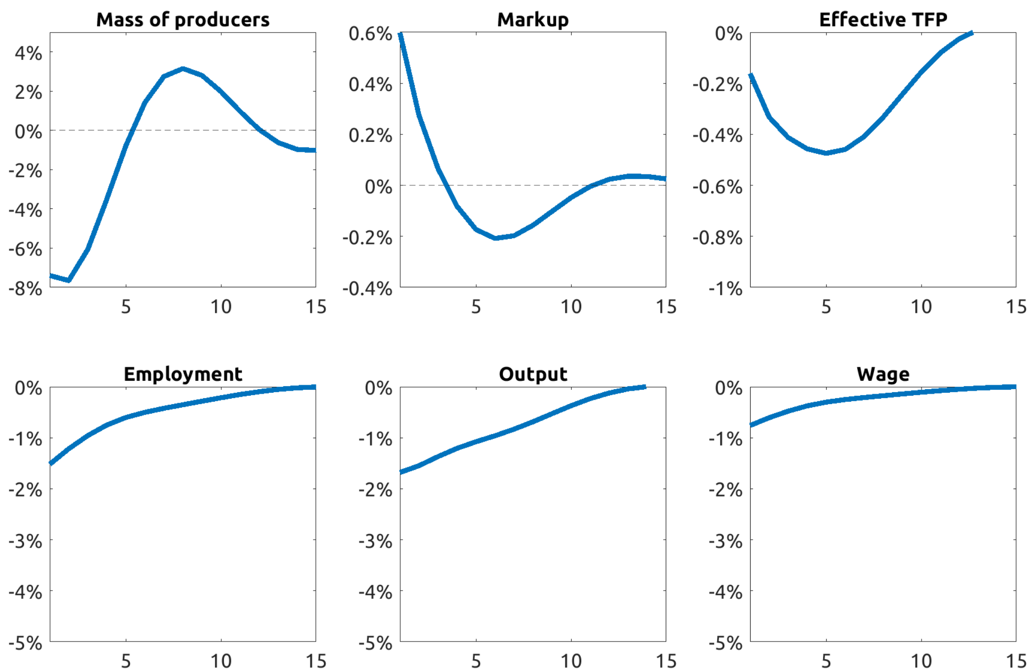
D Alternative calibration: firms

In this section, I study an alternative calibration in which the unit of analysis is the firm rather than the establishment. The key difference between the two calibrations is the average size of entrants. In the case of firms, entrants, on average, employ only 30% of the number of people as the average operating business. This reduces the effect of entry fluctuations.

It is worth noting that, in the case of the Great Recession, the mass of operating firms fell by more relative to trend than did the mass of operating establishments.

Model. To generate endogenous exit, I assume that producers must pay a fixed cost to operate each period.

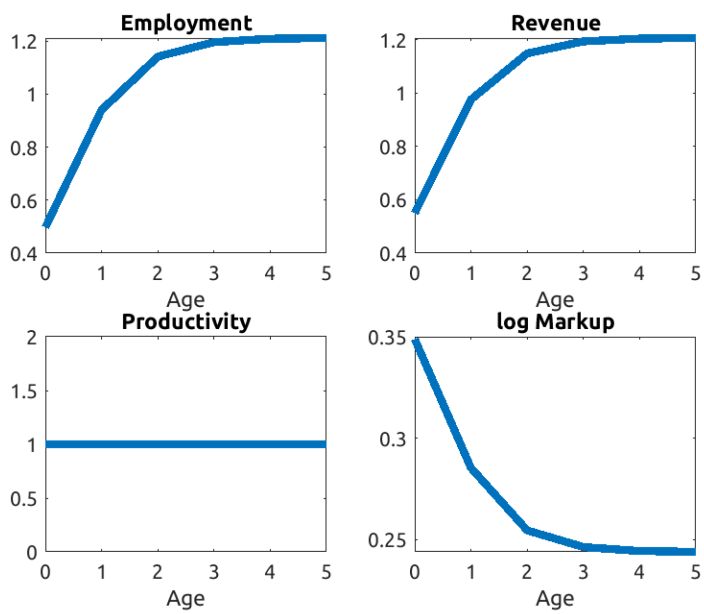
Figure 15: Entry cost shock in the firms model



E Alternative model: Lifecycle generated by adjustment frictions

In the baseline version of the model, I generate the lifecycle of the firm using productivity differences between incumbents and entrants. An alternative to that model is to have the lifecycle of the firm be generated entirely by adjustment frictions.

More precisely, I consider a version of the model where entrants draw their initial productivity from the stationary distribution implied by the law-of-motion for incumbent productivity. Their initial employment is ℓ_0. The figure below shows the lifecycle profile of this alternative model. As shown below, this model implies a much faster rise to average size. Counterfactually, it also implies that the markup falls with firm age. The reason for this pattern is that new firms are "too small" relative to a frictionless benchmark, and so their prices are too high.

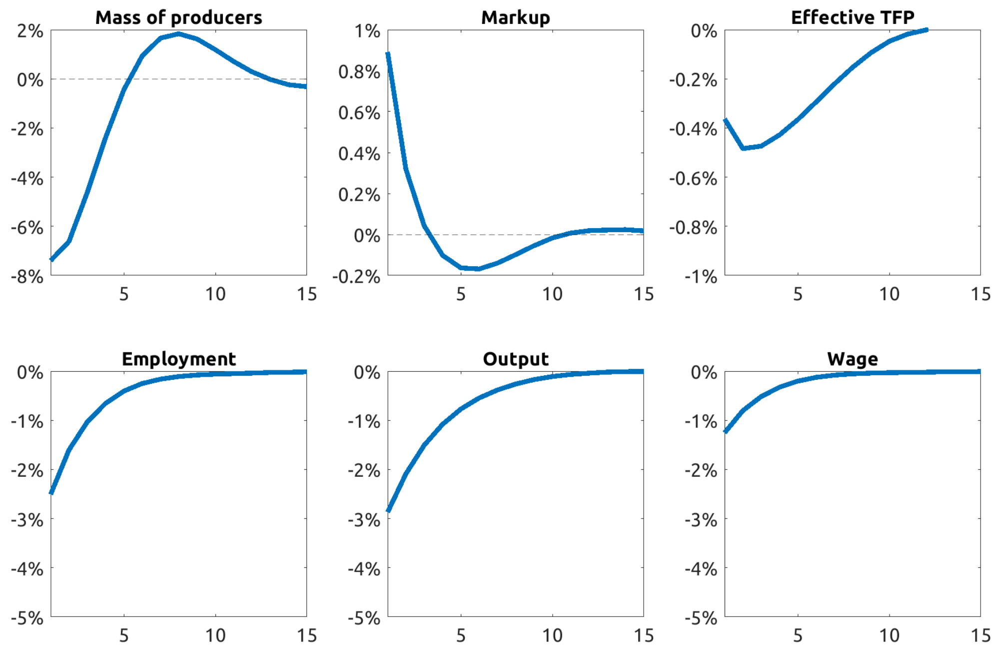


F Alternative model: stochastic discount factor

F.1 Shock to entry

In the case of Greenwood, Hercowitz and Huffman (1988) preferences, the stochastic discount factor is

Figure 16: Impulse response to an entry shock; variable stochastic discount factor



$$m_{t,t+1} = \beta \frac{\left(C_{t+1} - \psi \frac{L_{t+1}^{1+\nu}}{1+\nu}\right)^{-\gamma}}{\left(C_t - \psi \frac{L_t^{1+\nu}}{1+\nu}\right)^{-\gamma}}$$

Here, I show results using the stochastic discount factor for CRRA utility, with a risk aversion parameter of 1:

$$m_{t,t+1} = \beta \frac{C_{t+1}}{C_t}$$

The impulse response functions for the Kimball and CES economies to this shock are depicted in Figure 16. As they show, the dynamics are largely similar in the model with a varaible stochastic discount factor and in the model in which firms discount using a constant discount factor β.

G Alternative model: symmetric translog

I study a symmetric translog version of my baseline model. In that model, all producers are identical, and the representative household has translog preferences over differentiated varieties.

A solution to this economy is a solution to the following system of equations:

$$L = \left(\frac{w}{\psi}\right)^{\nu} \tag{G.1}$$

$$\mu_t = 1 + \frac{1}{\sigma N_t} \tag{G.2}$$

$$\pi_t = (1 - 1/\mu_t)(C_t/N_t) \tag{G.3}$$

$$v_t = \pi + \beta(1-\delta)v_{t+1} \tag{G.4}$$

$$N_t = (1-\delta)N_{t-1} + Ne_t \tag{G.5}$$

$$p_t = \mu_t \frac{w_t}{Z_t} \tag{G.6}$$

$$\rho_t = \exp\left(\frac{1}{2}\frac{\bar{N} - N_t}{2\bar{N}N_t}\right), \qquad \bar{N} = \text{mass of producers in steady state} \tag{G.7}$$

$$p_t = \rho_t \tag{G.8}$$

$$v_t = f_E \tag{G.9}$$

$$C_t = w_t L_t + N_t \pi_t \tag{G.10}$$

I solve the model using the sequence state jacobian python package, available here: https://github.com/shade-econ/sequence-jacobian.