Exercise 8: Random Variables and Probability Distribution

A. Identify whether the described Random variable is valid or not. If the random variable is **invalid**, explain why is it invalid.

1. Consider the random experiment of tossing a fair die twice. Let X be a random variable such that $X = 1$ if the sum of the rolls is even and $X = 2$ if the sum is odd.

   VALID

2. In the same random experiment mentioned above, let us define another random experiment Y such that $Y = 1$ if the first roll is greater than the second roll and $Y = -1$ if the first roll is less than the second roll.

   INVALID. No specific value is assigned for rolls where the first is equal to the second.

3. Consider the random experiment of counting the number of yawns that you make during the course of the day. Let C be the random variable defined as $C = 0$ if the number of yawns is between 1 and 100 and $C = 1$ if the number of yawns exceed 100.

   INVALID. No specific value is assigned for zero yawns.

B. Compute the probability of each specific value and show the probability distribution for the random variable. And answer the additional questions and requirements.

1. Given the random experiment of tossing a coin up to 10 times or until a tail shows up. Define the random variable H for counting the number of heads in the outcome. List the elements of each h of H. What is the probability of getting at least 3 heads?

   Let H be a random variable for counting the number of heads in the given random experiment.

$H = 0$	$\{T\}$	$H = 6$	$\{HHHHHHT\}$
$H = 1$	$\{HT\}$	$H = 7$	$\{HHHHHHHT\}$
$H = 2$	$\{HHT\}$	$H = 8$	$\{HHHHHHHHT\}$
$H = 3$	$\{HHHT\}$	$H = 9$	$\{HHHHHHHHHT\}$
$H = 4$	$\{HHHHT\}$	$H = 10$	$\{HHHHHHHHHH\}$
$H = 5$	$\{HHHHHT\}$		

h	0	1	2	3	4	5	6	7	8	9	10
$P(H = h)$	$1/11$	$1/11$	$1/11$	$1/11$	$1/11$	$1/11$	$1/11$	$1/11$	$1/11$	$1/11$	$1/11$

$$P(H \geq 3) = \sum_{i=3}^{10} P(H = i) = \frac{8}{11}$$

2. Consider the random experiment of rolling two distinct dice and the random variable N for counting the number of 1s or 6s in the rolls. List the elements of each specific value n of N. What is the probability of
   a. $N = 0$ and
   b. $N \geq 1$

$N = 0$	{(2,2), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4), (3,5),(4,2), (4,2), (4,3), (4,4), (4,5), (5,2), (5,3), (5,4), (5,2)}	$\|N = 0\| = 16$
$N = 1$	{(1,2), (1,3), (1,4), (1,5), (2,1), (3,1), (4,1), (5,1), (6,2), (6,3), (6,4), (6,5), (2,6), (3,6), (4,6), (5,6)}	$\|N = 1\| = 16$
$N = 2$	{(1,1),(1,6),(6,1),(6,6)}	$\|N = 2\| = 4$

n	0	1	2
$P(N = n)$	$16/36$	$16/36$	$4/36$

$$P(N = 0) = \frac{16}{36} \qquad P(N \geq 1) = \frac{20}{36}$$

3. In a random experiment of randomly answering a ten-question multiple choice exam with four choices per question. We define a random variable C for counting the number of correct answers. Which c has the greatest chance of occurring? What is the probability of
    a. getting at least seven **incorrect** answers
    b. getting three , five or six correct answers

c	0	1	2	3	4	5
$P(C=c)$	$\frac{3^{10}}{4^{10}}$	$\binom{10}{1}\frac{3^9}{4^{10}}$	$\binom{10}{2}\frac{3^8}{4^{10}}$	$\binom{10}{3}\frac{3^7}{4^{10}}$	$\binom{10}{4}\frac{3^6}{4^{10}}$	$\binom{10}{5}\frac{3^5}{4^{10}}$
	0.056314	0.187712	0.281568	0.250282	0.145998	0.058399

c	6	7	8	9	10
$P(C=c)$	$\binom{10}{6}\frac{3^4}{4^{10}}$	$\binom{10}{7}\frac{3^3}{4^{10}}$	$\binom{10}{8}\frac{3^2}{4^{10}}$	$\binom{10}{9}\frac{3^1}{4^{10}}$	$\binom{10}{10}\frac{3^0}{4^{10}}$
	0.016222	0.0030899048	0.0003862381	0.0000286102	0.0000009537

$P(C=c)$ has the largest chance of occurring is when $c=2$.

$P(C \leq 3) =$ 0.775875

$P(C = 3,5,6) =$ 0.324903