

CMSC 57 Discrete Mathematical Structures in Computer Science II
Exer 4. IE Principle, Derangements, and Pigeonhole Principle

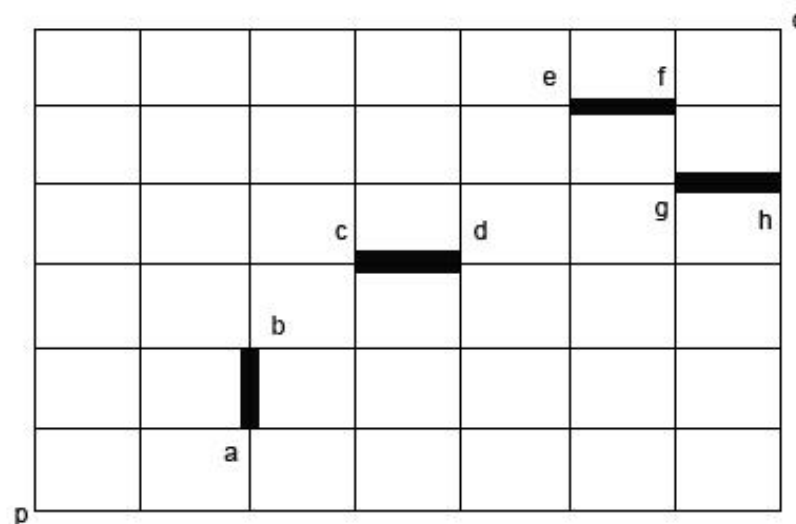
On your paper, write the exercise number and title, your name, student number, the drawing in your stomach if you were a care bear, section, and the date today. Read and discuss the problem and the possible solutions with your partner before writing on your papers. Answer each item as neatly as possible. Minimize erasures. Review your solutions before submitting your papers. Do not discuss your work with any other group. Show your complete solutions. Explain how you arrived at your answers.

- Find the number of integers from the set $\{1, 2, \dots, 500\}$ which are
(i) divisible by at least one of 2, 3 and 5;
(ii) divisible by none of 2, 3 and 5.
- Let S be the set of 3-digit numbers abc such that $a, b, c \in \{1, 2, \dots, 9\}$ and a, b, c are pairwise distinct. (Thus, $489 \in S$, but $313 \notin S$ and $507 \notin S$.) Find the number of members abc in S such that $a \neq 1$, $b \neq 4$ and $c \neq 8$.
- Find the number of integers from the set $\{600, 601, \dots, 1500\}$ which are multiples of 6 or 9.
- Three test – cloud pillows, A, B, and C were tried by the Care bears. Among those who took part in the test, there were 25 who liked at least one cloud pillow. Of all the Care bears who did not like pillow A, the number who liked pillow B was twice the number who liked pillow C. The number who liked only pillow A was one more the number who liked pillow A and at least one other pillow. Of all the Care bears who liked just one pillow, half did not like pillow A. How many liked just pillow B?
- In a survey of the cloud perfume preference of a group of Care bears, it was found that:
22 liked lavender;
25 liked chamomile;
39 like jasmine;
9 like both chamomile and lavender;
17 liked lavender and jasmine;
20 liked chamomile and jasmine;
6 liked all three;

Given that four liked none of the above, how many Care bears were surveyed?

6. In the town of Care-a-Lot, there are Care bears. Each can be described as cute or cuddly, warm or cool, female or male. Four are cute warm females, 17 are females, 14 are cute Care bears, 4 are cute females, 11 are cute warm Care bears, 5 are warm females, 3 are cuddly cool males, 17 are cute or warm Care bears. How many Care bears are there in the town of Care-a-lot?

7. The 6 by 7 grid below represents a map regarding a street representation of a district in Care-a-Lot.



(i) Find in the grid the number of shortest p – q routes;

(ii) A monster known as Barney attacks Care-a-Lot. Tenderheart Bear leads a rescue team and they need to get from point p to q fast (thus, they need to take a shortest route). They have determined that they need to pass through street a – b to pick up the wounded Cheer Bear, Wish Bear, and Funshine Bear; **not** pass through c – d as the monster Barney is lurking there; and pass through **either** street e – f **or** street g – h to alert the more powerful adult Care Bears. How many shortest p – q routes are there given the conditions above?



8. Laugh-a-Lot Bear plays a prank to his companions Daydream Bear, Friend Bear, Surprise Bear, Secret Bear, and Share Bear by exchanging the contents of their lunchboxes. In how many ways can he distribute the wrong lunches to his friends?

9. There are 15 Care Bears seated in a classroom. The teacher is not satisfied with the seating arrangement since all Care bears are always daydreaming or dozing off. He demands that everyone move to a new seat except for Polite Panda and Perfect Panda (yes, they are Care bears!) since they are the only ones who are paying attention. How many new seating arrangements are possible?

10. At the annual Care bear dance, 20 Care bear couples are dancing peacefully. Of the 20 couples, 10 are “warm” couples. The principal arrives and decides that Care bears, both warm and cool, should mingle. He asks that everyone switch to a new partner. Of course the warm again end up with the warm, and the cool with the cool. Given this, how many new pairings are possible?

11. For their field trip, 28 different clouds are given to 28 Care bears. But since they are already late for the trip, they hurriedly packed their clouds with Champ Care Bear’s cloud being the only one marked. When they arrived at the site, Champ Care Bear’s cloud was easily handed to him while the 27 others exchanged clouds until they felt that the cloud they were sitting on was theirs. How many possible arrangements are there if only 13 of the 28 clouds (including Champ Care Bear’s) belonged rightfully to their masters?

12. How many numbers should you pick from the first eight counting numbers so that you are sure that at least any pair from those picked numbers will add up to 9? Show your solution via Pigeonhole Principle.

13. Given 8 different natural numbers, none greater than 15, show that at least three pairs of them have the same positive difference.

14. University of Plushies, Land of Bears (UPLB) has a total of 783 different “large class mode” lecture sections in a semester. Among these sections, no two sections can meet at the same time in the same room. There are twelve different time slots at which classes can occur, but can have the option of WF or TTh scheduling, making the total to 24 different time slots. How many specialized “large class mode” classrooms are needed to accommodate all of the 783 lecture sections?

15. For the upcoming Care-a-Lot anniversary, 124 Care bears signed up for a contest ala “Amazing Race”. The organizing committee plans to group the participants into 29 separate teams (and there are no limits in the number of people per team). Seventy five of the Care bears have already ganged themselves to 18 teams. The remaining 11 teams will come from the remaining ungrouped Care bears. At least how many students will one of the 29 teams have?

