

RULES of INFERENCE and EQUIVALENCE

An **argument** is a sequence of propositions or sentences written as:

$$\begin{array}{l} F_1 \\ F_2 \quad \text{OR} \quad (F_1 \wedge F_2 \wedge \dots \wedge F_n) \rightarrow Q \\ \vdots \\ \hline F_n \\ \hline \therefore Q \end{array}$$

The argument is said to be **valid** provided that if all the premises are true, then Q must also be true.

PROPOSITIONAL RULES OF INFERENCE

	Name	Rule	Tautological Form
1.	Addition	$\frac{P}{\therefore P \vee Q}$	$P \rightarrow (P \vee Q)$
2.	Simplification	$\frac{P \wedge Q}{\therefore P}$	$(P \wedge Q) \rightarrow P$
3.	Conjunction	$\frac{P}{\therefore P \wedge Q}$	
4.	Modus Ponens	$\frac{P \rightarrow Q}{\therefore Q}$	$[(P \rightarrow Q) \wedge P] \rightarrow Q$
5.	Modus Tollens	$\frac{P \rightarrow Q}{\therefore \sim P}$	$[(P \rightarrow Q) \wedge \sim Q] \rightarrow \sim P$
6.	Disjunctive Syllogism	$\frac{P \vee Q}{\therefore Q}$	$[(P \vee Q) \wedge \sim P] \rightarrow Q$
7.	Hypothetical Syllogism	$\frac{P \rightarrow Q}{\therefore P \rightarrow R}$	$[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightarrow (P \rightarrow R)$
8.	Constructive Dilemma	$\frac{P \rightarrow Q}{\therefore Q \vee S}$	$[(P \rightarrow Q) \wedge (R \rightarrow S) \wedge (P \vee R)] \rightarrow (Q \vee S)$
9.	Destructive Dilemma	$\frac{P \rightarrow Q}{\therefore \sim P \vee \sim R}$	$[(P \rightarrow Q) \wedge (R \rightarrow S) \wedge (\sim Q \vee \sim S)] \rightarrow (\sim P \vee \sim R)$

THE FUNDAMENTAL RULES OF LOGIC (LAWS of EQUIVALENCE or TAUTOLOGICAL BICONDITIONALS)

	Name	Fundamental Rule
1.	Reflexivity	$P \leftrightarrow P$
2.	Double Negation	$\sim(\sim P) \leftrightarrow P$
3.	Commutativity	$(P \wedge Q) \leftrightarrow (Q \wedge P)$ $(P \vee Q) \leftrightarrow (Q \vee P)$ $(P \leftrightarrow Q) \leftrightarrow (Q \leftrightarrow P)$
4.	Associativity	$[(P \wedge Q) \wedge R] \leftrightarrow [P \wedge (Q \wedge R)]$ $[(P \vee Q) \vee R] \leftrightarrow [P \vee (Q \vee R)]$ $[(P \leftrightarrow Q) \leftrightarrow R] \leftrightarrow [P \leftrightarrow (Q \leftrightarrow R)]$
5.	Distributivity	$[P \wedge (Q \vee R)] \leftrightarrow [(P \wedge Q) \vee (P \wedge R)]$ $[P \vee (Q \wedge R)] \leftrightarrow [(P \vee Q) \wedge (P \vee R)]$
6.	Idempotency	$(P \wedge P) \leftrightarrow P$ $(P \vee P) \leftrightarrow P$
7.	Identity	$(P \wedge T) \leftrightarrow P$ $(P \vee F) \leftrightarrow P$
8.	Inverse	$(P \wedge \sim P) \leftrightarrow F$ $(P \vee \sim P) \leftrightarrow T$
9.	Dominance	$(P \wedge F) \leftrightarrow F$ $(P \vee T) \leftrightarrow T$
10.	Absorption	$[P \wedge (P \vee Q)] \leftrightarrow P$ $[P \vee (P \wedge Q)] \leftrightarrow P$
11.	De Morgan's Laws	$\sim(P \wedge Q) \leftrightarrow (\sim P \vee \sim Q)$ $\sim(P \vee Q) \leftrightarrow (\sim P \wedge \sim Q)$
12.	Contrapositive	$(P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P)$
13.	Material Implication	$(P \rightarrow Q) \leftrightarrow (\sim P \vee Q)$
14.	Material Equivalence	$(P \leftrightarrow Q) \leftrightarrow [(P \rightarrow Q) \wedge (Q \rightarrow P)]$ $(P \leftrightarrow Q) \leftrightarrow [(P \wedge Q) \vee (\sim P \wedge \sim Q)]$
15.	Exportation	$[(P \wedge Q) \rightarrow R] \leftrightarrow [P \rightarrow (Q \rightarrow R)]$