CMSC 57 Discrete Mathematical Structures in Computer Science II
Exercise 7: Independent Events and Conditional Probability.

On your paper, write the exercise number and title, your name, student number, the caricature of your favorite ICS instructor/professor, section, and the date today. Read and discuss the problem and the possible solutions with your partner before writing on your papers. Answer each item as neatly as possible. Minimize erasures. Review your solutions before submitting your papers. Do not discuss your work with any other group. Show your complete solutions

1. Suppose that we toss a die. The sample space is: S= {1, 2, 3, 4, 5, 6}. Each of the six numbers is a sample point and is assigned probability 1/6. Define the events E and F as follows:
    E={1, 3, 4}
    F={3, 4, 5, 6}
    Prove that E and F are independent.

2. A firm undertakes two projects, A and B. The probabilities of having a successful outcome are 2/3 for project A and 4/5 for project B. What is the probability that neither of the two projects will have a successful outcome if their outcomes are independent?

3. A coin is tossed three times. What is the probability that exactly two heads occur, given that
    (a) the first outcome was a head?
    (b) the first outcome was a tail?
    (c) the first two outcomes were heads?
    (d) the first two outcomes were tails?
    (e) the first outcome was a head and the third outcome was a head?

4. A die is rolled twice. What is the probability that the sum of the faces is greater than 7, given that
    (a) the first outcome was a 4?
    (b) the first outcome was greater than 3?

(c) the first outcome was a 1?
(d) the first outcome was less than 5?

5. A card is drawn at random from a deck of cards. What is the probability that
    (a) it is a heart, given that it is red?
    (b) it is a jack, given that it is red?

6. From a deck of five cards numbered 2, 4, 6, 8, and 10, respectively, a card is drawn at random and replaced. This is done three times. What is the probability that the card numbered 2 was drawn exactly two times, given that the sum of the numbers on the three draws is 12?

7. A student is applying to Harvard and Dartmouth. He estimates that he has a probability of .5 of being accepted at Dartmouth and .3 of being accepted at Harvard. He further estimates the probability that he will be accepted by both is .2. What is the probability that he is accepted by Dartmouth if he is accepted by Harvard? Is the event "accepted at Harvard" independent of the event "accepted at Dartmouth"?