

## CMSC 57: Discrete Mathematical Structures in Computer Science 2

### Exercise 9: Algebraic Structures

On your papers, write your name, student number, section, the date today and your animal totem if you were an Indian (totem = guiding animal spirit).



A. For every given below, determine which kind of algebraic structure it is, and **give at least one proof** of why it is not of the higher structure (e.g. It is a semigroup but not a monoid because it doesn't have an identity element, etc).

If it is a semigroup, **show associativity**.

If it is a commutative semigroup, **show commutativity**.

If it is a monoid, **identify the identity element**.

If it is a group, **identify the inverse element**.

if it is an abelian group, **show commutativity**.

*		b				
		2	4	6	8	10
a	2	2	4	6	8	10
	4	4	6	8	10	2
	6	6	8	10	2	4
	8	8	10	2	4	6
	10	10	2	4	6	8

1.  $A = \{2, 4, 6, 8, 10\}$ ,  $(a*b)$  is illustrated by the table on the right
2.  $A = \mathbb{Z}^+$ ,  $(a*b) = a+b$
3.  $A = \mathbb{Z}^+ + \{0\}$ ,  $(a*b) = a \cdot b$
4.  $A = \mathbb{Q}^+$ ,  $(a*b) = ((a \cdot b)/11) + 3$
5.  $A = \mathbb{Q}^+$ ,  $(a*b) = (a \cdot b)/11$
6.  $A = \{1, 2, \dots, 100\}$ ,  $(a*b) = |a-b|$
7.  $A = \{1, -1, i, -i\}$ ,  $(a*b) = a \cdot b$
8.  $A = \mathbb{Z}^+$ ,  $(a*b) = a+b$
9.  $A =$  set of positive even numbers,  $(a*b) = (a \cdot b)/2$
10.  $A =$  set of positive odd numbers,  $(a*b) = 2(a \cdot b) + 1$

B. For each **group structures** identified above, give a single cyclic subgroup generated by  $a \in A$  (pick only one element  $a$  from  $A$ ). Show at least 7 elements generated by the element.

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