Exam 3 Formula Sheet Heat Transfer

THERMAL RESISTANCE

	CONDUCTION	CONVECTION	RADIATION
WALL	$R_{t,cond} = \frac{L}{kA}$	$R_{t,conv} = \frac{1}{hA}$	$R_{t,rad} = \frac{1}{h_r A}$
CYLINDER	$R_{t,cond} = \frac{\ln(r/r)}{2\pi L k}$		

$$R = \frac{\Delta T}{q''}$$

CONVECTION

ASSUMPTIONS
- TEMPERATURE IN SOLID IS SPATIALLY UNIFORM AT INSTANT t

LUMPED CAPACITANCE (ZERO-D TRANSIENT PROBLEMS)

$$T(x,y,z,t) \rightarrow T(t)$$

$$t = \frac{\rho V c}{h A_s} \ln \frac{\theta}{\theta_i}$$

$\left.\begin{array}{l}\theta = T_i - T_\infty \\ \theta = T - T_i\end{array}\right\}$ USED TO MAKE STUFF ODES RATHER THAN PDES

A_s = AREA OF SURFACE

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = e^{-Bi\,Fo} = \exp\left[-\frac{hA_s}{\rho V c} t\right]$$

$$\tau_t = \frac{\rho V c}{h A_s}$$

τ = THERMAL TIME CONSTANT
ρ = DENSITY V = VOLUME
h = HEAT X-FER

CONDITIONS:

$$Bi = \frac{h L_c}{k} < 0.1 \qquad L_c = \frac{V}{A_s}$$

IF $Bi > 0.1$ TRY PLANE WALL W/ CONVECTION SOLUTIONS

$$Bi = \frac{R_{t,cond}}{R_{t,conv}}$$

L_c = CHARACTERISTIC LENGTH

BOX $L_c = \frac{V}{A_s}$ OR $L = \frac{AL}{A}$

CYLINDER $L_c = \frac{r_o}{2}$ r_o = OUTER

SPHERE $L_c = \frac{r_o}{3}$

NON-DIMENSIONAL TO AVOID ODES

$$\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}$$

$$x^* = \frac{x}{L} \qquad \theta = f(Fo, x^*, Bi)$$

$$t^* = \frac{\alpha t}{L^2} = Fo$$

CONDITION: $Fo \geq 0.2$

TRANSIENT CONVECTION ONE TERM APPROXIMATIONS

SYMMETRIC B.C. TABLE 5.1

$$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\xi_n^2 Fo) \cos \xi_n x^*$$

ξ_n VALUES IN APPENDIX B.3 ($\xi_n \tan \xi_n = Bi$)

TABLE 5.1

$$C_n = \frac{4 \sin \xi_n}{2\xi_n + \sin(2\xi_n)}$$

INFINITE MIDPLANE

$Fo > 0.2$ $\theta^* = \theta_0^* \cos(\xi_1 x^*)$

$$\theta_0^* = \frac{T_0 - T_\infty}{T_i - T_\infty} = C_1 \exp(-\xi_1 x^*)$$

T_0 = TEMP WE WANT
θ_0 = MIDPLANE TEMP
T_i =

$Q = \dot{E}_{out}$

$$\frac{Q}{Q_0} = 1 - \frac{\sin \xi_1}{\xi_1} \theta_0^*$$

$Q_0 = Q_{max} = \rho c \forall (T_i - T_\infty)$ ONLY VALID LATE-STAGE & GETS RID OF TRANSIENTS (UNDERPREDICTS)

INFINITE CYLINDER W/CONVECTION

$Fo > 0.2$ $\theta^* = \theta_0^* J_0(\xi_1 r^*)$ $r^* = \frac{r}{r_0}$ J_0 & J_1 TABULATED IN APPENDIX B.4

$$\theta^* = C_1 \exp(-\xi_1^2 Fo)$$

$$\frac{Q}{Q_0} = 1 - \frac{2\theta_0^*}{\xi_1} J_1 \xi_1 \qquad Fo = \frac{\alpha t}{r_0^2}$$

$Fo > 0.2$ SPHERE W/CONVECTION

$$\theta^* = \theta_0^* \frac{1}{\xi_1 r^*} \sin(\xi_1 r^*)$$

θ_0 = CENTER TEMP

$$\theta_0^* = C_1 \exp(-\xi_1^2 Fo)$$

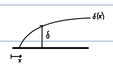
SEMI-INFINITE MEDIA
- ALLOWS US TO TREAT ANY MEDIUM AS SEMI-INFINITE AS LONG AS t IS SHORT ENOUGH

$$\delta_t \approx 2\sqrt{\alpha t}$$

$$\delta_m \approx 2\sqrt{\nu t}$$ ν = KINEMATIC VISCOSITY

$$\tau_{diff} \approx \frac{L^2}{4\alpha}$$

δ_m = LENGTH OF VELOCITY LAYER
δ_t = LENGTH OF THERMAL WAVE



- CONDITIONS:

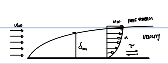
$$Fo = \frac{\alpha t}{L_c} < \frac{1}{4}$$

τ_{diff} = DIFFUSION TIME FOR A GIVEN L

$$\frac{T - T_s}{T_\infty - T_i} = \operatorname{erf} \eta = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$ FIGURE 5.8

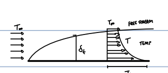
BOUNDARY LAYER

VELOCITY



- VISCOUS EFFECTS AND GRADIENT LARGEST @WALL

THERMAL



- NOTE: WITH INTERNAL FLOW, BOUNDARY LAYERS WILL INTERSECT

$$C_f = \frac{\tau_s}{\rho u_\infty^2/2}$$

C_f = LOCAL FRICTION COEFFICIENT (UNITLESS FRICTION)

$$h = \frac{-k_f\, \partial T/\partial y|_{y=0}}{T_s - T_\infty}$$

CONSERVATION OF MASS

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h\, dA_s$$

BOUNDARY LAYER

$$\delta_m = 2\sqrt{\nu t}$$ ν = KINEMATIC VISCOCITY

$$\delta_t = 2\sqrt{\alpha t}$$

SIMILARITY PROBLEMS

YOU CAN ALSO USE $Re = \frac{\rho u_\infty x}{\mu}$

1. COMPARE Re W/ GIVEN L's $Re_1 = \frac{L_1 u_\infty}{\nu}$ $Re_2 = \frac{L_2 u_\infty}{\nu}$
2. $Nu = \frac{\bar{h} L}{k_f} \rightsquigarrow \frac{h_1 L_1}{k_f} = \frac{h_2 L_2}{k_f} \rightsquigarrow h_2 = h_1 \frac{L_1}{L_2}$
3. FIND $\bar{h}$

GENERAL FORM OF SOLUTION FOR BOUNDARY LAYERS

$$\overline{Nu} = C\, Re_x^m Pr^n$$

$$\overline{Nu} = \frac{\bar{h} x}{k_f}$$

$$q''_{conv} = \bar{h}(\Delta T_{conv})$$

$$\alpha = \frac{k\rho}{c}$$ - THERMAL DIFFUSIVITY

$$Bi = \frac{hL}{k}$$
- INTERNAL THERMAL RESISTANCE OF SOLID TO BOUNDARY LAYER THERMAL RESISTANCE
  - CONVECTION/CONDUCTION

$$C_f = \frac{\tau_s}{\rho V^2/2}$$
- UNITLESS SHEAR STRESS
- $V = u_\infty$

FLAT PLANE LAMINAR
- STEADY, INCOMPRESSIBLE FLOW,
- CONSTANT FLUID PROPERTIES
- NEGLIGIBLE VISCOUS DISSIPATION

$$C_{f,x} = 0.664\, Re_x^{-1/2} = \frac{\tau_{s,x}}{\rho u_\infty^2/2}$$

$\tau_{s,x}$ = LOCAL WALL SHEAR STRESS

$$\frac{\delta}{\delta_t} = Pr^{1/3}$$

$$\delta = \frac{5x}{\sqrt{Re_x}}$$ ← BLASIUS SOLUTION

TO GET AN AVERAGE VALUE, MULTIPLY NON-DIMENSIONAL

FOR BOUNDARY LAYERS, APPROXIMATE THERMOPHYSICAL PROPERTIES WITH $T_f = \frac{T_s + T_\infty}{2}$
- A-4 FOR GASES

μ = ABSOLUTE VISCOCITY = DYNAMIC VISCOCITY

$$Fo = \frac{\alpha t}{L^2}$$
- RATIO OF HEAT CONDUCTION RATE TO RATE OF THERMAL ENERGY STORAGE IN A SOLID
- DIMENSIONLESS TIME

$$Nu = \frac{hL}{k_f}$$
- CONVECTION TO PURE CONDUCTION RATIO

$$Re_x = \frac{\rho u_\infty x}{\mu}$$
- ρ, μ COME FROM FLUID FILM THERMODYNAMIC TABLES ($T_f = \frac{T_\infty + T_s}{2}$)
- DETERMINES TRANSITION FROM LAMINAR → TURBULENT
  - CRITICAL $Re = Re_{x,c} = 5 \cdot 10^5$ ← FOR FLAT PLATE

$$Nu_x = \frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3}$$ AS LONG AS: $Pr \geq 0.6$

$$T^* = \frac{T - T_s}{T_\infty - T_s} \qquad Nu_x = 0.564\, Re^{1/2}\, Pr^{1/2} \quad \text{IF } Pr \ll 1$$
$$T^* = T^*(\eta)$$

IF BY 2, i.e. $\overline{Nu}_x = \frac{1}{2}\overline{Nu}_x$, $C_{f,x} = \frac{1}{2}\overline{C}_{f,x}$

$$\nu = \frac{\mu}{\rho}$$

· INERTIA VS VISCOUS FORCES

FOR CYLINDERS: $Re = \frac{VD}{\nu}$ V = VELOCITY (UPSTREAM), ν = KIN VISCOSITY, $Re_D = 2 \cdot 10^5$

$$Pr = \frac{c\mu}{k} = \frac{\nu}{\alpha}$$

- RATIO OF MOMENTUM + THERMAL DIFFUSIVITIES
- ν = KINEMATIC VISCOSITY = μ/ρ
- $Pr \ll 1 \Rightarrow$ ENERGY X-FER BY DIFFUSION DOMINANT (LIQUID METAL)
- $Pr \gg 1 \Rightarrow$ MOMENTUM X-FER BY DIFFUSION DOMINANT (OILS)

$$\frac{\delta_m}{\delta_t} = \sqrt{\frac{\nu}{\alpha}} = \sqrt{Pr}$$

ALL Pr #'s

$$Nu_x = \frac{0.3387\, Re_x^{1/2}\, Pr^{1/3}}{\left[1 + (0.0468/Pr)^{2/3}\right]^{1/4}}$$ AS LONG AS $Pe_x = Re_x Pr \geq 100$

$$Pe = \frac{u_\infty x}{\alpha}$$

<u>TURBULENT FLOW OVER A FLAT PLATE</u>

$$\delta_{m,turb} = 0.37\, x\, Re_x^{-1/5}$$

3 LAYERS OF TURBULENT B.L.
- VISCOUS
- BUFFER
- TURBULENT

<u>AVERAGE $\bar{h}_L$ ACROSS LAMINAR + TURBULENT</u>

$$\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} \frac{k_f}{x}\, 0.332\, Re_x^{1/2}\, Pr^{1/3}\, dx + \int_{x_c}^{L} \frac{k_f}{x}\, 0.0296\, Re^{4/5}\, Pr^{1/3}\, dx\right)$$

$$\frac{\delta_{m,turb}}{\delta_{t,turb}} \approx Pr_{turb}$$

Pr_{turb} = NOT A THERMOPHYSICAL PROPERTY, USUALLY ASSUMED TO BE 1

δ_m = VELOCITY B.L. δ_t = THERMAL B.L.

x_c = DISTANCE TO TURBULENCE

$$\overline{Nu}_L = \left(0.037\, Re_L^{4/5} - A\right) Pr^{1/3}$$ $0.6 \leq Pr \leq 60$ AND $Re_{x,c} \leq Re_L \leq 10^8$

$$A = 0.037\, Re_{x,c}^{4/5} - 0.664\, Re_{x,c}^{1/2}$$ ← A BECOMES CONSTANT $Re_{x,c} = 5 \cdot 10^5$

Re_L = REYNOLDS # @ $x = L$ $\qquad Re_{x,c}$ = CRITICAL REYNOLDS #

IF $Pr_{turb} = 1$, $\delta_m = \delta_t$

$$C_{f,x} = 0.0592\, Re^{-1/5}$$ AS LONG AS: $Re_{x,c} < Re < 10^8$ ($Re_{x,c}$: CRITICAL Re)

- SLOWER GROWTH OF VISCOUS SUBLAYER
- GRADUAL DECAY

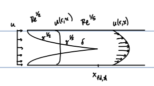
$$Nu_x = 0.0296\, Re^{4/5}\, Pr^{1/3}$$ $0.6 < Pr < 60$

<u>CYLINDER IN CROSS-FLOW</u> — WEEK 11

$$Re_D = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$ V = BULK VELOCITY, D = DIAMETER

$$\overline{Nu}_D = \frac{\bar{h} D}{k} = C\, Re_D^m\, Pr^{1/3}$$ ← TABLE 7.2

INTERNAL FLOW



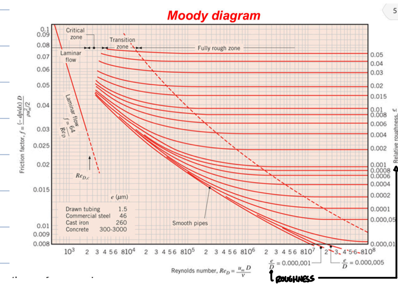
· NO SLIP BOUNDARY CONDITION

$$\nu = \frac{\mu}{\rho} \qquad V = \frac{\dot{m}}{\rho A_c}$$

DEVELOPING FLOW $u = f(r,x)$

DEVELOPED FLOW $u = f(r)$

ASSUMPTIONS:
- LOW SPEED
- FORCED CONVECTION
- NO PHASE CHANGE



VELOCITY PROFILE IN FULLY DEVELOPED FLOW:

$$u(r) = -\frac{1}{4\mu}\frac{dP}{dx} r_0^2 \left[1 - \left(\frac{r}{r_0}\right)^2\right]$$ ASSUMES: $\frac{dP}{dx} < 0$ (REDUCED PRESSURE W/FLOW)

$$u_m = \frac{-r_0^2}{8\mu}\frac{dP}{dx}$$ ASSUMES $dP/dx < 0$, $u_m > 0$

$$u_{max}(r) = 2 u_m \quad @\ r = 0$$

PRESSURE GRADIENT AND FRICTION FACTOR

IF FULLY DEVELOPED LAMINAR:

$$f = \frac{64}{Re_D}$$ f = FRICTION FACTOR

$$C_f = \frac{f}{4}$$ C_f = COEFFICIENT OF FRICTION

$$C_f = \frac{\tau_s}{\rho u_m^2/2}$$ $C_f \neq f$ $\quad u_m \neq u_\infty$ $\quad D_h = D$ FOR DIFFERENT PIPE GEOMETRIES

TURBULENT

$$f = \frac{(-dP/dx)D}{\rho u_m^2/2}$$ f = FRICTION FACTOR MOODY (DARCY)

<u>REYNOLD'S # IN A TUBE</u> ($Re_{D,c} \approx 2300$ ← CRITICAL Re)

$$Re_D = \underbrace{\frac{\rho u_m D}{\mu}}_{\text{USE THIS ONE}} = \frac{u_m D}{\nu} = \rho D\left(\frac{\dot{m}}{\rho A_c}\right)$$

ν = KINEMATIC VISCOSITY, u_m = MEAN VELOCITY

$A_c = \frac{\pi D^2}{4}$ ← IN CIRCULAR PIPE

<u>NON-CIRCULAR TUBE</u>

$$D_h = \frac{4A_c}{Per}$$ A_c = X-SECTIONAL AREA, Per = PERIMETER

IF CIRCULAR... $$Re_D = \frac{4\dot{m}}{\pi D \mu}$$

LAMINAR
$$u_m = \frac{-r_0^2}{8\mu}\left(\frac{dP}{dx}\right)$$
$$C_f = f/4$$
$$\tau = -\mu \left.\frac{\partial u}{\partial r}\right|_{r=r_0}$$

<u>HYDRODYNAMIC ENTRY LENGTH</u> VS <u>THERMAL ENTRY LENGTH</u>

LAMINAR: $\left(\frac{x_{fd,h}}{D}\right)_{lam} \approx 0.05\, Re_D$ $\qquad$ LAMINAR: $\frac{x_{fd,t}}{D} \approx 0.05\, Re_D Pr$

TURBULENT: $\left(\frac{x_{fd,h}}{D}\right)_{turb} \approx 10$ $\qquad$ TURBULENT: $\frac{x_{fd,t}}{D} \approx 10$

NOTES ABOUT $x_{fd,h}$ & $x_{fd,t}$
- YOU CAN HAVE $x_{fd,t} > x_{fd,h}$ OR $x_{fd,t} < x_{fd,h}$
- IF $Pr \gg 1$ FAVORS MOMENTUM X-FER ($x_{fd,h} \ll x_{fd,t}$)
- IF $Pr \ll 1$ FAVORS THERMAL X-FER ($x_{fd,h} \gg x_{fd,t}$)

<u>BOUNDARY CONDITIONS</u>

1. CONSTANT SURFACE HEAT FLUX $\qquad q_{conv} = q''(P \cdot L)$

$$T_m(x) = T_{m,i} + \frac{P}{\dot{m} c_p} q_s'' x$$ ← FOR T_{out} $x = L$

2. CONSTANT SURFACE TEMP

$$\frac{\Delta T(x)}{\Delta T_i} = \frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left[-\frac{Px}{\dot{m} c_p}\bar{h}\right]$$

$$\dot{m} c_p = \frac{-PL\bar{h}}{\ln(\Delta T_o/\Delta T_i)}$$ $\bar{h}$ = HEAT X-FER COEFFICIENT

$$q_{conv} = \bar{h} A_s \Delta T_{lm}$$ A_s = AREA OF SURFACE, ΔT_{lm} = LOG MEAN TEMP DIFF · SEE HEAT EXCHANGERS

3. KNOWN EXTERNAL TEMPERATURE

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{UA}{\dot{m} c_p}\right)$$ U = OVERALL HEAT TRANSFER COEFFICIENT

<u>CONVECTION CORRELATIONS INTERNAL FLOW</u>

FULLY DEVELOPED LAMINAR FLOW (CIRCULAR):

CONSTANT SURFACE HEAT FLUX: $$Nu_D = \frac{hD}{k} = 4.36$$

CONSTANT SURFACE TEMP: $$Nu_D = \frac{hD}{k} = 3.66$$

PROPERTIES @ $T_m = \frac{T_{m,i} + T_{m,o}}{2}$

THESE NEGLECT ENTRY LENGTH (8.4.2 HAS ENTRY STUFF)

BULK MEAN TEMP

$$T_m = \frac{2}{u_m r_0^2}\int_0^{r_0} u T r\, dr$$

- AVERAGE TEMP OVER CROSS SECTION

NON-CIRCULAR

$$D_h = \frac{4A_c}{P}$$

IF TURBULENT
$$\Rightarrow Re_D = \frac{\rho u_m D}{\mu}$$
$$\Rightarrow Nu_D = \frac{h D_h}{k}$$

IF LAMINAR ⇒ TABLE 8.1

FULLY DEVELOPED TURBULENT FLOW:

- BOTH CONSTANT SURFACE TEMP + CONSTANT HEAT FLUX $$Nu_D = \frac{hD}{k} = 0.023\, Re_D^{4/5}\, Pr^n$$ IF $T_s > T_m$, $n = 0.4$ (HEATING); IF $T_s < T_m$, $n = 0.3$ (COOLING)
- PROPERTIES AT T_m (SEE ABOVE) $\quad 0.6 \leq Pr \leq 160 \quad Re_D \geq 10{,}000 \quad \frac{L}{D} \geq 10$

REMEMBER TO CHECK IF ASSUMING FULLY DEVELOPED WAS GOOD!
- CHECK BY LOOKING @ THERMAL ENTRY LENGTHS

$$\bar{h} = \frac{\overline{Nu}_D k}{D}$$

Heat Exchangers

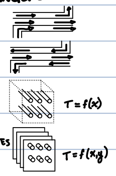
Concentric

Counterflow

Cross flow $T = f(x)$

Plates $T = f(x,y)$

Shell + tube



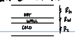
$$R_{TOT} = \frac{1}{UA}$$

U = overall heat exchanger coefficient [W/m²K]
UA = capacitance of heat exchanger
A = area [m²]
R_{TOT} = total thermal resistance [K/W]

$$R_{TOT} = \frac{1}{(hA)_c} + R_w + \frac{1}{(hA)_h}$$

↑ R_w typically ignored as k is high in walls

Heat exchanger wall



Energy balance

LMTD method

Valid regardless of flow configuration — equal:

$$q = \dot{m}_h c_{p,h} (T_{h,i} - T_{h,o}) = C_h (T_{h,i} - T_{h,o})$$
$$q = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i}) = C_c (T_{c,o} - T_{c,i})$$

Assumes: $T_{c,o} > T_{c,i}$, $T_{h,i} > T_{h,o}$
- Constant specific heats
- No phase change

c_p = heat capacity
C = heat capacity rate

$c_{p,h}$ or $c_{p,c}$ = heat

$$q = UA\,\Delta T_{LM}$$

↑ heat transfer between two fluids separated by external surface

$$C_h = \dot{m}_h c_{p,h} \quad \text{(same w/cold)}$$

$$C_r = \frac{C_{min}}{C_{max}}$$

C_r = capacitance ratio

$$Q_{max} = C_{min}(T_{h,i} - T_{c,i})$$

$$\varepsilon \cong \frac{q}{q_{max}} \quad \text{or} \quad \varepsilon = \frac{q}{C_{min}(T_{h,i} - T_{c,i})}$$

$$NTU = \frac{UA}{C_{min}}$$

$$q = \varepsilon\, C_{min}(T_{h,i} - T_{c,i})$$

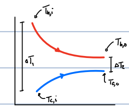
$$q = UA \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}$$

Assume:
- No heat x-fer by HX & surroundings
- No axial conduction in fluid
- Constant specific heat + overall HTC
  - calculate @ $\frac{T_i + T_o}{2}$

Parallel flow HX LMTD

$\Delta T_1 = T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i}$
$\Delta T_2 = T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i}$ ← $T_{h,o} > T_{c,i}$ (if not true, => HX is CF)

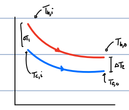
Notes:
- As L↑, ΔT goes down
- If UA fixed, $q_{PF} < q_{CF}$



Counterflow HX LMTD

$\Delta T_1 = T_{h,i} - T_{c,o}$
$\Delta T_2 = T_{h,o} - T_{c,i}$

Notes: As L↑, ΔT constant
- If fixed UA, counterflow heat x-fer larger
- $T_{c,o}$ can exceed $T_{h,o}$ in CF



- If U, ΔT_{LM} equal, counterflow HX smaller in area

Special conditions:

If $C_h >> C_c$ <=> condensing vapor

If $C_h << C_c$ <=> evaporating liquid

If $C_h = C_c$ => counterflow heat exchanger w/equal c_p

Heat exchanger ε-NTU solutions:

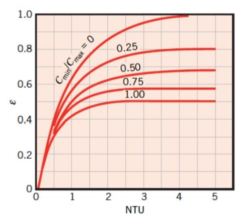


FIGURE 11.10 Effectiveness of a parallel-flow heat exchanger (Equation 11.28).

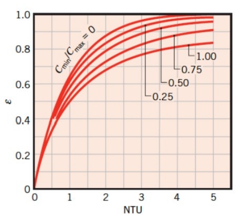


FIGURE 11.11 Effectiveness of a counterflow heat exchanger (Equation 11.29).

$$\frac{\delta_t}{} \approx 0.05\, Re_D Pr$$

$$\frac{\delta_h}{} \approx 0.05\, Re_D$$

Pr << 1

(liquid metals)

RADIATION

$$\lambda = \frac{c}{v} = \frac{c/n}{v}$$ λ = WAVELENGTH

$T_{sun} = 5800$ [K]

AS $T \uparrow$ $\lambda \downarrow$ (TYPICAL FOR EMISSIVE BODIES)

SPECTRAL $\Rightarrow f(\lambda)$

$$q = \varepsilon \sigma T_s^4 - \alpha G$$

ε = EMISSIVITY

σ = STEPHAN BOLTZMANN CONST

E $\left[\frac{W}{m^2}\right]$ EMISSIVE POWER (RADIATION OUT OF SURFACE)

G $\left[\frac{W}{m^2}\right]$ IRRADIATION (INCIDENT RADIATION ON SURFACE)

J $\left[\frac{W}{m^2}\right]$ RADIOSITY (RATE OF RADIATION LEAVING SURFACE)

OVER ALL WAVELENGTHS + DIRECTIONS

ρ REFLECTIVITY (FRACTION OF IRRADIATION REFLECTED)

α ABSORPTIVITY (FRACTION OF IRRADIATION ABSORBED)

τ TRANSMISSIVITY (FRACTION OF IRRADIATION TRANSMITTED)

ε EMISSIVITY (RATIO OF RADIATION EMITTED BY A SURFACE TO RADIATION EMITTED BY A BLACKBODY AT THE SAME TEMPERATURE)

$$\rho + \alpha + \tau = 1$$

BLACKBODY RADIATION

- EMITS MAX AMOUNT OF RADIATION (IDEALIZED)
  - ABSORBS ALL INCOMING RADIATION

$$E_{b,\lambda}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]}$$

$C_1 = 3.74177 \cdot 10^8$

$C_2 = 1.43878 \cdot 10^4$

λ NEEDS TO BE IN μm

$$\lambda_{max} T = 2898 \ [\mu m K]$$

$$E_b = \sigma T^4$$ $\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$

EMISSION

$$F_{(0\to\lambda)} = \frac{\int_0^\lambda E_{\lambda,b}\, d\lambda}{\sigma T^4}$$

TABLE 12.2 ← NEEDS λT

σT^4 ASSUMES IDEAL BLACKBODY (THIS COURSE)

$$\int_{\lambda_1}^{\lambda_2} E_{\lambda,b}\, d\lambda = \sigma T^4 \left[F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)}\right]$$

HEMISPHERICAL EMISSIVITY

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}\, d\lambda}{E_b(T)}$$ $\varepsilon = f(\lambda)$

ABSORPTIVITY

$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda) E_{\lambda,b}(\lambda,T)\, d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda,T)\, d\lambda}$$

KIRCHOFF'S LAWS

$$\alpha(T) = \varepsilon(T)$$

$$\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}$$ ← ALWAYS VALID IF:

1. DIFFUSE IRRADIATION INDEPENDENT OF DIRECTION
2. DIFFUSE SURFACE INDEPENDENT OF DIRECTION

$\varepsilon_{\lambda,\theta}$ = SPECTRAL DIRECTIONAL EMISSIVITY

$\alpha_{\lambda,\theta}$ = SPECTRAL DIRECTIONAL ABSORPTIVITY

θ = DIRECTION

$\varepsilon_\lambda \neq f(\theta)$ DIFFUSE SURFACE: $\varepsilon_\lambda = \alpha_\lambda$

$\varepsilon_\lambda \neq f(\lambda)$ AND $\varepsilon_\lambda \neq f(\theta)$ GRAY SURFACE: $\varepsilon = \alpha$

BE CAREFUL

ENERGY BALANCE

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\underset{\text{ABSORBED RADIATION}}{\sum \alpha_i G_i} + \dot{Q}_{in} = \underset{\text{EMISSION}}{\sum \varepsilon_j E_j} + \dot{Q}_{out}$$

$Q_{in} = q'' A$ $\quad Q_{out} = q'' A$

DIFFUSE SURFACE

$$\varepsilon_\lambda = \alpha_\lambda$$

RADIATION ENERGY BALANCE

- DRAW CONTROL SURFACE

$$\dot{E}_{in} = \dot{E}_{out}$$

DONT FORGET EMISSIVE RADIATION OUT

TABLE 12.2 DIFFUSE & GRAY Blackbody Radiation Functions

$\varepsilon = \kappa$

λT (μm · K)	$F_{(0 \to \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ (μm · K · sr)$^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000

TABLE 12.2 *Continued*

λT (μm · K)	$F_{(0 \to \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ (μm · K · sr)$^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.696720
4,600	0.579280	0.467343	0.647004
4,800	0.607559	0.433109	0.599610
5,000	0.633747	0.400813	0.554898
5,200	0.658970	0.370580×10^{-4}	0.513043
5,400	0.680360	0.342445	0.474092
5,600	0.701046	0.316376	0.438002
5,800	0.720158	0.292301	0.404671
6,000	0.737818	0.270121	0.373965
6,200	0.754140	0.249723×10^{-4}	0.345724
6,400	0.769234	0.230985	0.319783
6,600	0.783199	0.213786	0.295973
6,800	0.796129	0.198008	0.274128
7,000	0.808109	0.183534	0.254090
7,200	0.819217	0.170256×10^{-4}	0.235708
7,400	0.829527	0.158073	0.218842
7,600	0.839102	0.146891	0.203360
7,800	0.848005	0.136621	0.189143
8,000	0.856288	0.127185	0.176079

TABLE 12.2 $\varepsilon \sigma T_s^4$ *Continued*

λT (μm · K)	$F_{(0 \to \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ (μm · K · sr)$^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
8,000	0.856288	0.127185	0.176079
8,500	0.874608	0.106772×10^{-4}	0.147819
9,000	0.890029	0.901463×10^{-5}	0.124801
9,500	0.903085	0.765338	0.105956
10,000	0.914199	0.653279×10^{-5}	0.090442
10,500	0.923710	0.560522	0.077600
11,000	0.931890	0.483321	0.066913
11,500	0.939959	0.418725	0.057970
12,000	0.945098	0.364394×10^{-5}	0.050448
13,000	0.955139	0.279457	0.038689
14,000	0.962898	0.217641	0.030131
15,000	0.969981	0.171866×10^{-5}	0.023794
16,000	0.973814	0.137429	0.019026
18,000	0.980860	0.908240×10^{-6}	0.012574
20,000	0.985602	0.623310	0.008629
25,000	0.992215	0.276474	0.003828
30,000	0.995340	0.140469×10^{-6}	0.001945
40,000	0.997967	0.473891×10^{-7}	0.000656
50,000	0.998953	0.201605	0.000279
75,000	0.999713	0.418597×10^{-8}	0.000058
100,000	0.999905	0.135752	0.000019