THERMO FORMULA SHEET

PROPERTIES
PROPERTIES = CHARACTERISTICS OF A SYSTEM (P, T, m, VISCOSITY, VELOCITY, ...)

INTENSIVE: INDEPENDENT OF MASS

EXTENSIVE: DEPENDENT ON MASS

DENSITY

$$\rho = \frac{m}{V} \quad \left[\frac{kg}{m^3}\right] \qquad \text{FOR A DIFFERENTIAL} \quad \rho = \frac{\delta m}{\delta V}$$

- A RIGID CONTAINER CAN BE COMPLETELY SPECIFIED BY TWO INDEPENDENT, INTENSIVE PROPERTIES

SPECIFIC VOLUME

$$v = \frac{V}{m} = \frac{1}{\rho}$$

SPECIFIC WEIGHT

$$\gamma_s = \rho g \quad \left[\frac{N}{m^3}\right]$$

SYSTEMS + VOCAB

CLOSED: ENERGY + HEAT CAN TRAVEL ACROSS BOUNDARY

ISOLATED: ONLY HEAT CAN CROSS BOUNDARY

OPEN "CONTROL VOLUME": ENERGY, HEAT, + MASS CAN TRAVEL THROUGH BOUNDARY

SIMPLE COMPRESSIBLE: ABSENCE OF ELECTRIC, MAGNETIC, GRAVITATIONAL, MOTION, AND SURFACE TENSION EFFECTS

INTERPOLATION

$$y = y_1 + \left(\frac{x - x_1}{x_2 - x_1}\right)(y_2 - y_1)$$

ADIABATIC: NO HEAT TRANSFER

ISOTHERMAL: NO CHANGE IN TEMP

REVERSIBLE PROCESS: A PROCESS THAT CAN BE REVERSED WITHOUT LEAVING ANY TRACE ON ITS SURROUNDINGS (AN IDEALIZATION)
- LOST DUE TO FRICTION, ELECTRIC RESISTANCE, & UNRESTRAINED EXPANSION

ISENTROPIC PROCESS: NO CHANGE IN ENTROPY
- IF ADIABATIC + REVERSIBLE => ISENTROPIC
- IF ISENTROPIC ≠> ADIABATIC + REVERSIBLE

ISOBARIC: CONSTANT PRESSURE

UNITS + CONVERSIONS

$$\text{JOULE} = \frac{kg \cdot m^2}{s^2} = N \cdot m \qquad T[K] = T[°C] + 273.15$$

$$\text{NEWTON} = \frac{kg \cdot m}{s^2} \qquad T[R] = T[°F] + 459.67$$

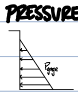
$$1\ kPa \cdot m^3 = 1\ kJ \qquad T[R] = 1.8\ T[K]$$

$$1\ \frac{kJ}{kg} = 1000\ \frac{m^2}{s^2} \qquad 1\ kW = 1000\ \frac{kg\ m^2}{s^3}$$

IMPORTANT UNITS

$$1\ Pa = 1\ \frac{kg}{m s^2} \qquad 1\ kPa = 1000\ \frac{kg}{m s^2}$$

PRESSURE



$$\Delta P = P_2 - P_1 = \rho g \Delta z$$

Δz = PRESSURE HEAD (VERTICAL DISTANCE)

- DON'T FORGET TO ACCOUNT FOR P_{ATM}

$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g\, dz$$

$$\frac{F_2}{F_1} = \frac{A_2}{A_1}$$

UP IS POSITIVE, DOWN IS NEGATIVE

$$P_{DESIRED} = g\left[\ \ \ \right] + P_{ATM}$$

REMEMBER kPa TO Pa CONVERSION

LAWS OF THERMODYNAMICS

ZEROTH LAW: IF TWO BODIES ARE IN THERMAL EQUILIBRIUM W/A THIRD BODY THEY ARE ALSO IN THERMAL EQUILIBRIUM W/EACH OTHER

FIRST LAW: IN A CLOSED SYSTEM W/ADIABATIC PROCESSES, THE NET WORK DONE IS THE SAME REGARDLESS OF THE SYSTEM + PROCESS DETAILS
- THEREFORE ↳ NET WORK ONLY DEPENDS ON STATE 1 & STATE 2
- ↳ CONSERVATION OF ENERGY

SECOND LAW: PROCESSES OCCUR IN A CERTAIN DIRECTION, ENERGY HAS QUALITY, NO

ENERGY

ENERGY ANALYSIS

$$e_{MECH} = \underbrace{\frac{P}{\rho}}_{\text{FLOW ENERGY}} + \frac{V^2}{2} + g(z) \quad \left[\frac{J}{kg}\right]$$

P = PRESSURE
ρ = DENSITY

$$\Delta E = E_{IN} - E_{OUT}$$

$$\Delta E = (Q_{in} - Q_{out}) + (W_{in} - W_{out}) + (E_{mass,in} - E_{mass,out})$$

ENERGY TIPS:
- LOWERCASE MEANS PER UNIT MASS (i.e. $e = E/m$)
- E CAN BE SET TO ZERO AT A CONVENIENT REFERENCE POINT

$$\dot{E} = \dot{m}e \quad \left[\frac{kJ}{s} \text{ or } kW\right]$$

$$\dot{m} = \rho \dot{V} = \rho A_c V_{AVG} \quad \left[\frac{kg}{s}\right]$$

ρ = FLUID DENSITY
$\dot{V}$ = VOLUME FLOW RATE
$\dot{m}$ = MASS FLOW RATE
A_c = CROSS-SECTIONAL AREA
V_{AVG} = AVERAGE FLUID VELOCITY ⊥ TO A_c

$$\dot{V} = \dot{m}/\rho$$

Q = AMOUNT OF HEAT TRANSFERRED

q = HEAT TRANSFER PER UNIT MASS

$$E = U + KE + PE = U + m\frac{V^2}{2} + mgz \quad [J]$$

STATIONARY SYSTEM: VELOCITY + SPEED ARE CONSTANT
$\therefore \Delta KE = \Delta PE = 0$

SIGN CONVENTION
- $+$ { W DONE BY SYSTEM; Q TO THE SYSTEM
- $-$ { W DONE ON SYSTEM; Q FROM SYSTEM

$$E = \dot{P}\Delta t = \dot{E}\Delta t$$

$$\Delta E_{sys} = Q_{NET} - W_{NET}$$

- CONSTANT PRESSURE W_b ⟶ ADD WORK TERM W/CHANGE IN INTERNAL ENERGY TO FIND CHANGE IN ENTHALPY

STATIONARY SYSTEMS: $\Delta U = Q_{NET} - W_{NET}$

ENERGY SAVINGS + EFFICIENCY

$$\text{SIMPLE PAYBACK} = \frac{\text{EXCESS INITIAL COST}}{\text{ANNUAL COST SAVINGS}}$$

$$\text{POWER SAVINGS} = (\text{RATED POWER})(\text{LOAD FACTOR})\left[1/\eta_{st} - 1/\eta_{eff}\right]$$

$$\eta = \frac{\text{OUTPUT}}{\text{INPUT}}$$

$$\eta_{12} = \eta_1 \eta_2$$

HEAT:

$$q = \frac{Q}{m} \quad \left[\frac{kJ}{kg}\right]$$

$$Q = \int_1^2 \dot{Q}\, dt \quad [kJ]$$

WORK:

MECHANICAL:

$$W_{sh} = 2\pi n T \quad [W]$$

$$W_{sh} = 2\pi \dot{n} T \quad [W]$$

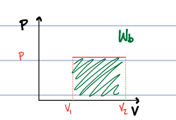
$$W_{spring} = \frac{1}{2}k(x_2^2 - x_1^2)$$

W_{sh} = SHAFT WORK
n = NUMBER OF REVOLUTIONS
$\dot{n}$ = # REVOLUTIONS / UNIT TIME

T = TORQUE = F·r

BOUNDARY:

CONSTANT PRESSURE $W_b = \int_1^2 P\,dV \qquad W_b = mP(v_2 - v_1)$



CONSTANT PRESSURE CYCLE $W_{NET} = \int_1^2 P\,dV - \int_2^1 P\,dV$

DRAW OUT GRAPH EVERY TIME

$W_b < 0$ ⟵ COMPRESSION
$W_b > 0$ ⟵ EXPANSION

CONSTANT TEMP IDEAL GAS (CONFIRM THIS!!)

$$W_b = P_1V_1 \ln\frac{V_2}{V_1} = P_2V_2 \ln\frac{V_2}{V_1}$$

$$W_b = P_1V_1 \ln\frac{P_2}{P_1} = P_2V_2 \ln\frac{P_2}{V_1}$$

$$W_b = mRT_0 \ln\frac{V_2}{V_1} = mRT_0 \ln\frac{P_2}{P_1}$$

ALL BECAUSE $PV = mRT_0 = C$ OR... $P = \frac{C}{V}$

- IF PRESSURE DROPS, THEN ITS LIKELY HIT THE STOPS

PROPERTY TABLES + PURE SUBSTANCES

GIVEN	SUPERHEATED VAPOR	SUBCOOLED LIQUID
T	$P < P_{sat}$	$P > P_{sat}$
P	$T > T_{sat}$	$T < T_{sat}$
P or T	$v > v_g$	$v < v_g$
P or T	$u > u_g$	$u < u_g$
P or T	$h > h_g$	$h < h_g$

- IF SATURATION IS A CONDITION, THEN THE CORRESPONDING PRESSURE OR TEMPERATURE FROM TABLE MUST BE OCCURING
- IF TEMPERATURE IS ABOVE T_{sat} FOR A GIVEN PRESSURE, GO TO THE SUPERHEATED VAPOR TABLE

CRITICAL POINT: POINT WHERE SATURATED LIQUID + SATURATED VAPOR STATES ARE THE SAME

COMPRESSED LIQUID: A LIQUID UNDER ANY PRESSURE

SATURATED LIQUID: A LIQUID ABOUT TO VAPORIZE

SATURATED VAPOR: A VAPOR ABOUT TO CONDENSE

SUPERHEATED VAPOR: A VAPOR NOT ABOUT TO CONDENSE
- PRESSURE + TEMP NO LONGER DEPENDENT
- TYPICALLY ABOVE CRITICAL TEMP, BUT NOT REQUIRED

- DURING A PHASE CHANGE, PRESSURE + TEMPERATURE ARE DEPENDENT

SUBSCRIPTS
- f → SATURATED LIQUID
- g → SATURATED VAPOR
- fg → DIFFERENCE BETWEEN LIQUID + GAS

(i.e. $u_{fg} = u_g - u_f$)

ENTHALPY ← SUM OF INTERNAL ENERGY

$$h = u + Pv \quad [kJ/kg]$$

v = SPECIFIC VOLUME
P = PRESSURE
u = INTERNAL ENERGY

- IF THE ONLY WORK DONE IS A CHANGE OF VOLUME AT A CONSTANT PRESSURE, THE ENTHALPY CHANGE IS EXACTLY EQUAL TO THE HEAT TRANSFERRED TO THE SYSTEM

CONSTANT PRESSURE ENTHALPY

h_{fg} = ENTHALPY OF VAPORIZATION
- AMOUNT OF ENERGY REQUIRED TO VAPORIZE AT A GIVEN PRESSURE OR TEMPERATURE
- BECOMES ZERO AT THE CRITICAL POINT

$$\Delta H = \Delta U + W_b$$

ONLY TRUE FOR CONSTANT PRESSURE QUASI-EQUILIBRIUM PROCESS

IF $W_b = 0$ (RIGID)
$\therefore \Delta H = \Delta U$

USUALLY $\Delta H = \Delta E_{sys}$ IF $W_b = 0$

QUALITY ← ONLY HAS SIGNIFICANCE FOR LIQUID-VAPOR MIXTURES

$$x = \frac{m_{vapor}}{m_{total}} \qquad h_{avg} = h_f + x h_{fg}$$

$$x = \frac{v_{avg} - v_f}{v_{fg}} \qquad u_{avg} = u_f + x u_{fg}$$

$$v_{avg} = v_f + x v_{fg}$$

COMPRESSED LIQUID

- TABULATED DATA NOT WIDELY AVAILABLE
- APPROXIMATE COMPRESSED LIQUID AS A SATURATED LIQUID AT THE SAME TEMPERATURE

$$y \cong y_{f@T}$$

FOR BETTER ACCURACY...

$$h \cong h_{f@T} + v_{f@T}(P - P_{sat@T})$$

SPECIFIC HEAT

$$\Delta u = u_2 - u_1 = c_{v,avg}(T_2 - T_1)$$

$$\Delta h = h_2 - h_1 = c_{p,avg}(T_2 - T_1)$$

$\left.\begin{array}{l} mc_p(\Delta T) \\ mc_v(\Delta T) \end{array}\right\}$ VERY USEFUL (TYPICALLY EQUAL TO ΔE_{sys})

EQUATIONS OF STATE

- BEST USED @ HIGH TEMP + LOW PRESSURE
- APPLICABLE TO GAS AT TEMPERATURES ABOVE CRITICAL POINT
- YOU HAVE TO USE ABSOLUTE PRESSURE + KELVINS

IDEAL GAS:

$$PV = mRT$$
OR
$$Pv = RT$$

P = ABSOLUTE PRESSURE
v = VOLUME
R = GAS CONSTANT [kJ/kg·K]
T = TEMP [K]

$$R = \frac{R_u}{M}$$

R_u = UNIVERSAL GAS CONSTANT
M = MOLAR MASS

IDEAL GAS ASSUMPTIONS:
- NEGLECT MOLECULAR INTERACTIONS
- NEGLECT VOLUME OCCUPIED BY MOLECULES
- APPLICABLE AT LOW DENSITY
- LOW PRESSURE, HIGH TEMP ← IMPORTANT

$$\frac{P}{P_{cr}} \ll 1 \qquad \frac{T}{T_{cr}} \gg 2$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

COMPRESSIBILITY FACTOR

$$PV = mZRT \qquad Z = \frac{v_{actual}}{v_{ideal}}$$

POLYTROPIC PROCESS: A PROCESS WHEN PRESSURE + VOLUME ARE RELATED BY $PV^n = C$, WHERE n & C ARE CONSTANT

MASS CONSERVATION

MASS FLOW RATE

$$\dot{m} = \int_{A_c} \rho V_n \, dA_c \quad [kg/s]$$

$$\dot{m} = \rho V_{avg} A_c \quad [kg/s]$$

ASSUMING ρ IS CONSTANT ACROSS CROSS SECTION

VOLUME FLOW RATE

$$\dot{V} = \int_{A_c} V_n \, dA$$

$$\dot{V} = V_{avg} A_c$$

MASS BALANCE EQNS

$$\Delta m_{cv} = m_{in} - m_{out}$$

$$\frac{dm_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

ONE INLET, ONE OUTLET → $\dot{m}_{in} = \dot{m}_{out}$

TOTAL ENERGY OF A FLOWING FLUID: θ

$$\theta = Pv + (u + ke + pe) \qquad h = Pv + u$$

$$\theta = h + \frac{V^2}{2} + gz \qquad h = \text{ENTHALPY}$$

$$\dot{E}_{mass} = \dot{m}\theta$$

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v} \qquad v = \text{SPECIFIC VOLUME}$$

IDEAL GAS NOTE: $\dot{m} = \frac{V_{avg}A_c}{v}$ → $Pv = RT$

LIQUIDS CAN BE CONSIDERED INCOMPRESSIBLE

ONE INLET, ONE OUTLET → $\dot{V}_{in} = \dot{V}_{out}$ ← NOT TRUE FOR GASES

ENERGY OF A FLOWING FLUID

$$W_{flow} = PAL = PV$$

P = PRESSURE
A = AREA
L = LENGTH
V = VOLUME

VERY USEFUL FOR STEADY FLOW PROBLEMS

$$w_{flow} = Pv$$

STEADY FLOW

REMEMBER:

$h_2 - h_1 = c_{p,avg}(T_2 - T_1) = \Delta h$
↑ CONSTANT

IF NOT CONSTANT, USE TABLES?

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

1 kJ/kg = 1000 m²/s²

$$\dot{Q} - \dot{W} = \sum_{in} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

$W_b = 0$
$\sum_{in} \dot{m} = \sum_{out} \dot{m}$
$\dot{E}_{in} = \dot{E}_{out}$
Q & W CONSTANT WRT TIME

h
- IDEAL GAS, VARIABLE SPECIFIC HEAT (A17 → A25)
- IDEAL GAS, CONSTANT c_p $\quad h_2 - h_1 = c_p(T_2 - T_1)$
- H_2O (A4 → A7) $\quad$ R-134a (A11 → A13)

ke, pe
- CONVERT TO kJ/kg $\quad$ 1 kJ/kg = 1000 m²/s²

STEADY FLOW DEVICES

NOZZLE/DIFFUSER
- $W = 0$
- $Q \approx 0$
- $\Delta PE \approx 0$

COMPRESSOR/PUMP ← INCREASES PRESSURE
- $Q \approx 0$
- $\Delta PE = 0$
- $\Delta KE = 0$

FAN
- $Q \approx 0$
- $\Delta PE = 0$

TURBINE
- WORK PRODUCING DEVICE

HEAT EXCHANGER
- HEAT X-FER
  - $Q_{cold} = Q_{hot}$
- ENTIRE SYSTEM
  - $Q = 0$
- EACH FLUID STREAM
  - $W = 0$
  - $\Delta KE = 0$
  - $\Delta PE = 0$

$\sum_{in} \dot{m} = \sum_{out} \dot{m}$

THROTTLING VALVE
- $Q = 0$
- $W = 0$
- $\Delta PE = 0$
- $\Delta KE = 0$

$\Rightarrow$ $h_1 = h_2$

MIXING CHAMBER
- $Q \approx 0$
- $W = 0$
- $\Delta KE = 0$
- $\Delta PE = 0$

$\sum_{in} \dot{m}h = \sum_{out} \dot{m}h$
$\sum_{in} \dot{m} = \sum_{out} \dot{m}$

UNSTEADY FLOW

$$m_{in} - m_{out} = \Delta m$$

FILLING A RIGID CONTAINER

$$m_i h_i = m_2 u_2 - m_1 u_1$$

$$Q - W = \sum_{out} mh - \sum_{in} mh + (m_2u_2 - m_1u_1)$$

SAME GOALS

HEAT ENGINE

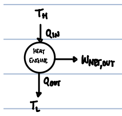
- NET WORK OUTPUT
- IT IS NOT POSSIBLE TO MAKE A HEAT ENGINE THAT DOES NOT REJECT ANY HEAT
  - TO OPERATE A CYCLE IT MUST RETURN TO ITS ORIGINAL STATE

$$W_{NET,OUT} = W_{OUT} - W_{IN}$$

$$W_{NET,OUT} = Q_{IN} - Q_{OUT}$$

$$W_{NET,OUT} = Q_H - Q_L$$

$$\eta_{Th} = \frac{W_{NET,OUT}}{Q_{TOTAL,IN}} = 1 - \frac{Q_{OUT}}{Q_{IN}} = 1 - \frac{Q_L}{Q_H}$$



(CARNOT)
REVERSIBLE HEAT ENGINES

$$\eta_{TH,REV} = 1 - \frac{T_L}{T_H}$$ MUST BE IN KELVIN

$$\left(\frac{Q_H}{Q_L}\right)_{REV} = \frac{T_H}{T_L}$$

REFRIGERATOR

$$W_{NET,IN} + Q_L = Q_H$$

$$COP_R = \frac{Q_L}{W_{NET,IN}} = \frac{1}{Q_H/Q_L - 1}$$

- Q_L IS THE AMOUNT OF HEAT YOU WANT TO LOSE

HEAT PUMP

HEAT PUMP: DOES THE OPPOSITE OF A REFRIGERATOR, BUT RUNS ON THE SAME CYCLE

$$COP_{HP} = \frac{Q_H}{W_{NET,IN}}$$

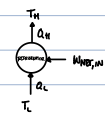
$$COP_{HP} = COP_R + 1$$

CARNOT REVERSIBLE REFRIGERATOR

$$COP_{R,REV} = \frac{1}{T_H/T_L - 1}$$

CARNOT REVERSIBLE HEAT PUMP

$$COP_{HP,REV} = \frac{1}{1 - T_L/T_H}$$



ENTROPY

- INCREASES WITH IRREVERSIBILITIES
- NO CHANGE IN ENTROPY WILL PRODUCE MOST EFFICIENT PROCESSES
- ENTROPY IS A MEASURE OF THE NET RANDOMNESS OF A SUBSTANCE

★ · DON'T FORGET 1ST LAW
$\Delta E_{SYS} = E_{IN} - E_{OUT}$

ΔE_{SYS} CAN BE EQUAL TO $m(u_2 - u_1)$ IN SAME CASES

$$\Delta S = S_2 - S_1$$

$$\Delta S = \frac{Q}{T_0}$$

IF Q IS LEAVING SYSTEM, ΔS IS NEGATIVE

- ISENTROPIC $\Rightarrow \Delta S = 0 \Rightarrow S_1 = S_2$

INTERNALLY REVERSIBLE

$$Q_{INT\ REV} = \int_1^2 T\,dS$$

ENTROPY CHANGE IN LIQUIDS + SOLIDS

$$\Delta S = S_2 - S_1 = \int_1^2 c(T)\frac{dT}{T} \approx c_{AVG} \ln\frac{T_2}{T_1}$$ ABSOLUTE TEMP

IF ISENTROPIC $\Rightarrow T_2 = T_1$

ENTROPY CHANGE IN IDEAL GASES WITH VARIABLE SPECIFIC HEAT

$$\Delta S = S_2 - S_1 = \int c_v(T)\frac{dT}{T} + R\ln\frac{V_2}{V_1}$$

$$\Delta S = S_2^0 - S_1^0 - R\ln\frac{P_2}{P_1}$$

$$\Delta S = S_2 - S_1 = \int c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1}$$

ENTROPY CHANGE IN IDEAL GASES W/CONSTANT SPECIFIC HEAT

$$\Delta S = C_{V,AVG} \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}$$

$$\Delta S = C_{P,AVG} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

IDEAL GAS ISENTROPIC RELATIONSHIPS

W/CONSTANT SPECIFIC HEATS

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{k}$$

$$k = \frac{C_P}{C_V}$$

USE ABSOLUTE TEMPERATURES

USE AVERAGE VALS FOR C_P & C_V

W/VARIABLE SPECIFIC HEATS

$$\frac{P_2}{P_1} = \frac{P_{r_2}}{P_{r_1}}$$

P_r = RELATIVE PRESSURE = $f(T)$

$$\frac{V_2}{V_1} = \frac{V_{r_2}}{V_{r_1}}$$

V_r = RELATIVE SPECIFIC VOLUME = $f(T)$

ISENTROPIC EFFICIENCIES

TURBINE:

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

h_{2a} = ACTUAL ENTHALPY @ STATE 2
h_{2s} = ISENTROPIC ENTHALPY @ STATE 2

POWER CYCLES

$$Q = \int T\,ds \qquad W_b = \int_1^2 P\,dV$$

ASSUMPTIONS:

4 SIMPLIFICATIONS

1. WORKING FLUID IS AIR
   - IDEAL GAS
2. ALL PARTS INTERNALLY REVERSIBLE
   - QUASI-EQUILIBRIUM
3. COMBUSTION PROCESS REPLACED BY HEAT ADDITION FROM EXTERNAL SOURCE
4. EXHAUST REPLACED BY A HEAT-REJECTION PROCESS

COLD-AIR ASSUMPTIONS
- CONSTANT C_P C_V W/ROOM TEMP VALUES

CARNOT CYCLE
- SETS EFFICIENCY LIMITS
- IMPRACTICAL IN APPLICATION

$$\eta_{Th} = \frac{W_{OUT}}{Q_{IN}} = 1 - \frac{T_L}{T_H}$$

$$W_{OUT} = Q_{IN} - Q_{OUT}$$

ENGINE TERMINOLOGY

TOP DEAD CENTER (TDC): PISTON POSITION WHEN V IS MINIMUM

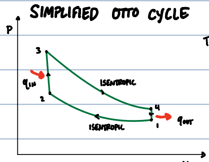
BOTTOM DEAD CENTER (BDC): PISTON POSITION WHEN V IS MAXIMUM

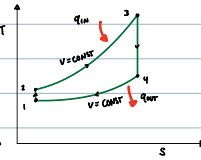
COMPRESSION RATIO (r): $r = \frac{V_{MAX}}{V_{MIN}} = \frac{V_{BDC}}{V_{TDC}}$

MEAN EFFECTIVE PRESSURE (MEP): FAKE PRESSURE ASSIGNED TO ENTIRE POWER STROKE. PRODUCES SAME AMOUNT OF W_{NET} AS DURING ACTUAL CYCLE

$$MEP = \frac{W_{NET}}{V_{MAX} - V_{MIN}} = \frac{W_{NET}}{V_{MAX} - V_{MIN}}$$

SIMPLIFIED OTTO CYCLE





$r = \frac{V_{MAX}}{V_{MIN}} = \frac{V_1}{V_2}$ ← COMPRESSION RATIO

ANY PROCESS/STEP:

$q_{IN} - q_{OUT} + W_{IN} - W_{OUT} = \Delta u$

1→2

$S_2 = S_3 \therefore \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k-1}$

2→3

$q_{IN} = u_3 - u_2$ $\quad \frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2}$

$q_{IN} = c_v (T_3 - T_2)$

$$MEP = \frac{W_{NET,OUT}}{V_1 - V_2}$$

3→4

$S_3 = S_4$

USE ISENTROPIC RELATIONSHIPS FOR 1→2 & 3→4

OVERALL CYCLE

$W_{NET} = W_{OUT} - W_{IN} = q_{IN} - q_{OUT}$

$\eta_{TH,OTTO} = 1 - \left(\frac{V_2}{V_1}\right)^{k-1} = 1 - \frac{1}{r^{k-1}}$

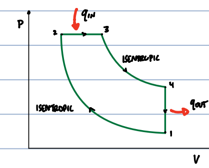
4→1

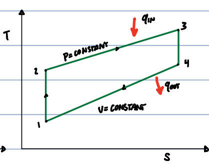
$q_{OUT} = u_4 - u_1$ $\quad \frac{P_4 V_4}{T_4} = \frac{P_1 V_1}{T_1}$

$q_{OUT} = c_v (T_4 - T_1)$

DIESEL CYCLE

$12 < r < 24$





$r_c = \frac{V_3}{V_2} = \frac{V_3}{V_2}$ ← BEFORE + AFTER COMBUSTION

1→2, 3→4 ISENTROPIC RELATIONSHIPS

2→3 IDEAL GAS CONSTANT P $\quad \frac{P_3 V_3}{T_3} = \frac{P_4 V_4}{T_3} \quad P_3 = P_4$

4→1 IDEAL GAS CONSTANT V

$$MEP = \frac{W_{NET,OUT}}{V_1 - V_2} \quad \text{USE } \frac{RT_1}{P_1} = v_1$$

$\eta_{TH,DIESEL} = \frac{W_{NET,OUT}}{q_{IN}}$

IS CONSTANT SPECIFIC HEAT

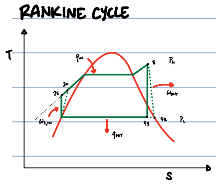
$\eta_{TH,DIESEL} = 1 - \frac{1}{r^{k-1}} \left[\frac{r_c^k - 1}{k(r_c - 1)}\right]$

$W_{NET} = q_{IN} - q_{OUT}$

$q_{IN} = h_3 - h_2$ USE $c_p (T_3 - T_2)$ IF c_p CONSTANT

$q_{OUT} = u_4 - u_1$ USE $c_v (T_4 - T_1)$ IF c_v CONSTANT

RANKINE CYCLE



ENHANCEMENTS

$\}$ HEAT IN

$W_t = W_{t,1} + W_{t,2}$

PUMP 1→2: $W_{P,IN} = h_2 - h_1$

$W_{P,IN} = v_1 (P_2 - P_1)$

IMPORTANT: $\left\{ \begin{array}{l} h_1 = h_{f@P_1} \\ v_1 = v_{f@P_1} \end{array} \right.$

TIPS:
- USE $S_4 = S_3$ TO FIND QUALITY AT 4

BOILER 2→3: $q_{IN} = h_3 - h_2$

TURBINE 3→4: $W_{t,OUT} = h_3 - h_4$ $\quad \eta_{th} = \frac{W_{OUT}}{q_{IN}} = 1 - \frac{q_{OUT}}{q_{IN}}$

CONDENSER 4→1: $q_{OUT} = h_4 - h_1$ $\quad W_{NET} = q_{IN} - q_{OUT}$

ACTUAL:

$\eta_P = \frac{W_s}{W_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$

$\eta_t = \frac{W_a}{W_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$

REHEATING
- REMEMBER POINT 4 COULD BE A MIXTURE AT $P = P_4$

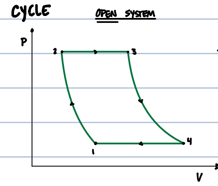
BRAYTON CYCLE

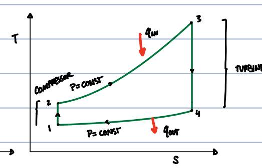
OPEN SYSTEM

$r = \frac{P_2}{P_1} = \frac{P_3}{P_4}$

$P_2 = P_3$

$P_4 = P_1$





1→2 ISENTROPIC COMPRESSION (COMPRESSOR)

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$

2→3 ISOBARIC HEAT ADDITION

$q_{IN} = h_3 - h_2$

$= c_p (T_3 - T_2)$ ← CONSTANT PROPERTIES

3→4 ISENTROPIC EXPANSION (TURBINE)

$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{(k-1)/k}$

4→1 ISOBARIC HEAT REJECTION

$q = h_4 - h_1$

$= c_p (T_4 - T_1)$

BACK WORK RATIO $= \frac{W_{COMP}}{W_{TURB}}$

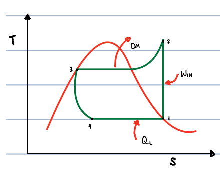
$q_{OUT} = h_4 - h_1$

$r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4}$ $\quad r_p$ = PRESSURE RATIO

$\eta_{TH,BRAYTON} = 1 - \frac{T_1 (T_4/T_1 - 1)}{T_2 (T_3/T_2 - 1)}$

$\eta_{TH,BRAYTON} = \frac{W_{OUT}}{q_{IN}} = 1 - \frac{q_{OUT}}{q_{IN}}$

VAPOR COMPRESSION REFRIGERATION/HEAT PUMP



NOTE:
- IDEAL CASE REQUIRES STATE 1 TO BE A SATURATED VAPOR
- EQUATIONS BELOW WORK FOR IDEAL CASE

STATE 1:

$T_1 = T_{sat@P_1}$ $\quad v_1 = v_{g@T_1}$

$h_1 = h_{g@P_1}$ $\quad S_2 = S_1$

1→2: ISOTHERMAL HEAT ADDITION (COMPRESSOR)

$W_{IN} = h_2 - h_1$

2→3: ISENTROPIC COMPRESSION (CONDENSER)

$q_{OUT} = h_2 - h_3$

STATE 3:

$P_3 = P_2$ $\quad T_3 = T_{sat@P_3}$

$h_3 = h_{f@P_3}$ $\quad v_3 = v_{f@T_3}$

3→4: ISOTHERMAL HEAT REJECTION (EXPANSION VALVE)

$h_3 = h_4$

STATE 4: ← REMEMBER X

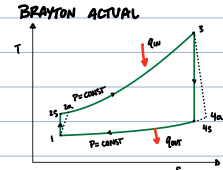
$P_4 = P_1$

4→1: ISENTROPIC EXPANSION (EVAPORATOR)

$q_{IN} = h_1 - h_4$

$COP_R = \frac{q_L}{W_{NET,IN}} = \frac{h_1 - h_4}{h_2 - h_1}$ $\qquad COP_{HP} = \frac{q_H}{W_{NET,IN}} = \frac{h_2 - h_3}{h_2 - h_1}$

BRAYTON ACTUAL



$W_{NET} = W_t - W_c = \eta_t (h_3 - h_{4s}) - \frac{h_{2s} - h_1}{\eta_c}$

$\eta_c = \frac{W_s}{W_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$

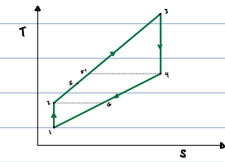
$\eta_t = \frac{W_a}{W_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$

a IS ACTUAL
s IS ISENTROPIC
↳ FIND ISENTROPIC, USE η TO FIND ACTUAL

BRAYTON ENHANCEMENT

$T_{5'} = T_4$

$T_2 = T_6$



$$q_{REGEN,ACTUAL} = h_5 - h_2$$

$$q_{REGEN,MAX} = h_{5'} - h_2 = h_4 - h_2$$

EFFECTIVENESS

$$\longrightarrow \mathcal{E} = \frac{h_5 - h_2}{h_4 - h_2}$$

·HOW CLOSELY IT MODELS AN IDEAL REGENERATOR

IF $\mathcal{E} = 1$ $\quad \eta_{th} = 1 - \frac{T_1}{T_3} r_p^{(k-1)/k}$