FORMULA SHEET

SERIES/SEQUENCES

ARITHMETIC SEQUENCE
d = COMMON DIFFERENCE
$a_n = a_1 + (n-1)d$
(LOOKS LIKE LINEAR POLYNOMIAL)

LINEARITY OF CONVERGENT SERIES
IF $\sum_{k=1} a_k$ AND $\sum_{k=1} b_k$ BOTH CONVERGE AND c IS CONSTANT THEN $\sum_{k=1} ca_k$ AND $\sum_{k=1} a_k + b_k$ BOTH CONVERGE

WEIRD SERIES
$\sum_{n=1}^{\infty} (-1)^n$ DIVERGES
$\sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGES

POWER SERIES
$\sum_{n=0}^{\infty} a_n x^n$
ASSUME IT CONVERGES BASED ON x VALUE (ALWAYS USE ART TO TEST)
3 POSSIBILITIES
1. SINGLE POINT AT x=0 $(a_0, a_1 + a_2$
2. A RANGE $[-R,R]$ $(-R,R)$
3. $(-\infty, \infty)$ CONVERGES AT ALL x VALUES

TEST ENDPOINTS

FLOWCHART
1. N^{TH} TERM TEST FOR DIVERGENCE
2. ~~P SERIES~~
2. GEOMETRIC
3. IF $\frac{\text{POWER OF } n}{\text{POWER OF } n}$ LCT
   IF EXPONENTIAL OR FACTORIAL RT
4. INTEGRAL TEST
5. COLLAPSABLE SUM (PARTIAL SUMS)

N^{TH} TERM TEST FOR DIVERGENCE
IF $\sum_{n=1}^{\infty} a_n$, AND $\lim_{n\to\infty} a_n \neq 0$, THEN $\sum_{n=1} a_n$ DIVERGES

GEOMETRIC SERIES
r = COMMON RATIO
$a_n = a_1(r)^{n-1}$
SUM $\frac{a_1}{1-r}$
DIVERGES IF $|r| \geq 1$
CONVERGES IF $|r| < 1$

P-SERIES
$\sum_{n=1}^{\infty} \frac{1}{k^p}$ $p \leq 1$ DIVERGES, $p > 1$ CONVERGES

OPERATIONS ON POWER SERIES
1) $\frac{1}{1-\theta} = \sum_{n=0}^{\infty} \theta^n$ FOR $|\theta| < 1$
2) $e^{\theta} = \sum_{n=0}^{\infty} \frac{\theta^n}{n!}$ $\theta \in \mathbb{R}$

LCT
PICK A b_n THAT CAPTURES "ESSENCE" OF a_n (USUALLY A P-SERIES)
$\lim_{n\to\infty} \frac{a_n}{b_n} = L$
IF $0 < L < \infty$ THEN a_n AND b_n CONVERGE OR DIVERGE TOGETHER
IF $L = 0$ AND b_n CONVERGES, THEN $\sum a_n$ ALSO CONVERGES OTHERWISE THERE IS NO CONCLUSION

ABSOLUTE RATIO TEST
$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = p$
IF $p < 1$ CONVERGES
IF $p > 1$ DIVERGES
IF $p = 1$ NO CONCLUSION

INTEGRAL TEST
IF $f(x)$ IS CONTINUOUS, POSITIVE, NONINCREASING ON $[N, \infty)$ AND $a_n = f(x)$ FOR ALL POSITIVE INTEGERS, k, THEN $\sum_{n=N}^{\infty} a_n$ CONVERGES IF AND ONLY IF $\int_k^{\infty} f(x)\,dx$ CONVERGES

CHECKING FOR CONDITIONS
NEGATIVE
· TAKE DERIVATIVE + LOOK FOR NEGATIVE SLOPE OR SLOPE THAT IS LESS THAN 1

INCREASING:
- IF $k+1 > k$ IT IS INCREASING
- IF YOU HAVE ALL POSITIVE TERMS AND $\lim_{n\to\infty} a_n = 0$, YOU HAVE A DECREASING FUNCTION

TAYLOR SERIES
$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \dots \frac{f^n(a)}{n!}(x-a)^n + R_N(x)$$
$$R_N(x) = \frac{f^{n+1}(c)}{(n+1)!}(x-a)^{n+1}$$
↑ MAXIMIZE REMAINDER/ERROR

MACLAURIN SERIES
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ $\forall x \in (-1,1)$
$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{n}$ $\forall x \in (-1,1)$
$\arctan = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n-1}$ $\forall x \in [-1,1]$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\forall x \in \mathbb{R}$
$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ $\forall x \in \mathbb{R}$
$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2n!}$ $\forall x \in \mathbb{R}$
$\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$ $\forall x \in \mathbb{R}$
$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{2n!}$ $\forall x \in \mathbb{R}$
$e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ $\forall x \in \mathbb{R}$

PARTIAL SUMS
(SERIES $S_p = \sum_{n=0}^{p} a_n$)
· WRITE OUT PARTIAL SERIES
- WRITE TO p AND p-1 TERMS
· CROSS OUT WHAT CANCELS
- TAKE INFINITE LIMIT OF REMAINING TERMS
DETERMINE SUM

ALTERNATING SERIES TEST
CONDITIONS: · ALTERNATING SERIES
· SEQUENCE IS DECREASING
IF $\lim_{n\to\infty} a_n = 0$, IT AT LEAST CONDITIONALLY CONVERGES

SQUEEZE THEOREM

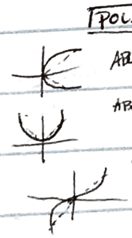
REPEATING SERIES
$.9191919191 = \sum_{k=1}^{\infty} 91\left(\frac{1}{100}\right)^k$

LIMITS
IF $\lim_{x\to a} f(x) = \pm\infty$ AND $\lim_{x\to a} g(x) = \pm\infty$
THEN $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

INDETERMINATE FORMS
$\frac{0}{0}, \frac{\pm\infty}{\pm\infty}, 0\cdot\infty, \infty-\infty, 0^0, \infty^0, 1^{\infty}, 0^0, 0^{\infty}$

DETERMINATE
$\frac{0}{\infty} \to 0, \frac{\infty}{0} \to \infty, \infty+\infty \to \infty, \infty\cdot\infty \to \infty,$
$0^{\infty} \to 0, \infty^{\infty} \to \infty, 1^0 \to 1$

$\boxed{\text{POLAR SYMMETRY}}$



ABOUT x AXIS $(r,\theta) = (r,-\theta)$
ABOUT y-AXIS $(r,\theta) = (-r,-\theta)$
ABOUT ORIGIN $(r,\theta) = (-r,\theta)$

$x = r\cos\theta$ $r^2 = x^2 + y^2$
$y = r\sin\theta$ $\tan\theta = \frac{y}{x}$

RADIAL LINE (GOES THROUGH ORIGIN)
AREA BETWEEN TWO POLAR CURVES
$A = \frac{1}{2}\int_{\alpha}^{\beta} \text{radius}^2\, d\theta$ OR $A = \frac{1}{2}\int_{\alpha}^{\beta} (\text{outer radius}^2 - \text{inner radius}^2)\, d\theta$
$(g(\theta))^2$ $(f(\theta))^2$

DERIVATIVES

CHAIN RULE $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$

PRODUCT RULE $\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x)$

QUOTIENT RULE $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$\frac{d}{dx} x^n = n(x)^{n-1}$

$\frac{d}{dx} c = 0$ (c IS CONSTANT)

$\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

WATCH OUT FOR $x^2 + c$

(DIVIDE BY CONSTANT) $\frac{1}{a}\tan^{-1}\frac{u}{a} = \int \frac{du}{a^2+u^2}$

TRIG / HYPERBOLIC

$\cosh x = \frac{e^x + e^{-x}}{2}$ $\sinh x = \frac{e^x - e^{-x}}{2}$

$\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$\cos\theta = \pm\sqrt{1-\sin^2\theta}$

$\sin^2 x = \frac{1-\cos 2x}{2}$ $\cos^2 x = \frac{1+\cos 2x}{2}$

$\sin^2 x + \cos^2 x = 1$

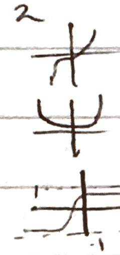
$\sinh x$, $\sin\theta$ ODD

$\cosh x$, $\cos\theta$ EVEN

SINH X

COSH X

TANH X



TRIG

$D_x(\sin x) = \cos x$

$D_x(\cos x) = -\sin x$

$D_x(\tan x) = \sec^2 x$

$D_x(\sec x) = \sec x \tan x$

$D_x(\csc x) = -\csc x \cot x$

$D_x(\cot x) = -\csc^2 x$

$D_x(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$D_x(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

$D_x(\tan^{-1} x) = \frac{1}{1+x^2}$

$D_x(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

$D_x(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$

$D_x(\cot^{-1} x) = \frac{-1}{1+x^2}$

$\csc x = \frac{1}{\sin x}$

$\sec x = \frac{1}{\cos x}$

SOLVE FOR VARIABLE

$\boxed{\text{SQUARE ROOTS}}$

1) $\sqrt[n]{\text{LINEAR POLYNOMIAL}}$

$u = \sqrt[n]{\text{LINEAR POLYNOMIAL}}$

2) TRIG SUBSTITUTION (SOLVE FOR dx ...)

A) $\sqrt{x^2 - a^2}$ $x = a\sec\theta$

B) $\sqrt{x^2 + a^2}$ $x = a\tan\theta$

C) $\sqrt{a^2 - x^2}$ $x = a\sin\theta$

MAKE x^2 POSITIVE

THINGS TO TRY:

- COMPLETE THE SQUARE
- DIVIDE BY POLYNOMIAL (LONG DIVISION)
- TRIG SUBSTITUTION (
- U SUB (u = ANYTHING INSIDE OF $\sqrt{\ }$)
- SIMPLIFY TRIG (SEPARATE (ADD))
- MULTIPLY BY FANCY $1/1$
- INTEGRATE BY PARTS

$\boxed{\text{COMPLETING THE SQUARE}}$

$ax^2 + bx + c \rightarrow a(x+d)^2 + e = 0$ INVERSE TRIG AFTER

$d = \frac{b}{2a}$ $e = c - \frac{b^2}{4a}$

$\boxed{\text{INTEGRATION BY PARTS}}$ SOMETHING WE DON'T KNOW HOW TO INTEGRATE

$\int u\,dv = uv - \int v\,du$

(u dv MUST ACCOUNT FOR ENTIRE INTEGRAND)

(dx GOES W/ dv)

- IN EXPONENTIAL TIMES A SIN OR COS, IT DOESN'T MATTER WHAT YOU PICK AS u AS LONG AS YOU'RE CONSISTENT
- IN POLYNOMIAL TIMES SIN OR COS, LET u BE POLYNOMIAL

$\boxed{\text{PARTIAL FRACTION DECOMPOSITION}}$

- SPLIT DENOMINATOR INTO FACTORS
- SET EQUAL TO OTHER VARIABLES
- LARGER DENOMINATOR (HIGHER DEGREE)

$\frac{1}{x+b} = \frac{A}{x+b}$

$\frac{1}{x^2+b} = \frac{Ax+B}{(x^2+b)}$

$\frac{1}{(x+b)^2} = \frac{A}{(x+b)} + \frac{B}{(x+b)^2}$

$\frac{1}{(x^2+b)^2} = \frac{Ax+B}{x^2+b} + \frac{Cx+D}{(x^2+b)^2}$

$x^3 = (x-0)^3 \leftarrow$ REPEATED FACTOR

TRIG INTEGRALS

1. IF n IS ODD, USE $\sin^2 x + \cos^2 x = 1$
   IF n IS EVEN, HALF ANGLE IDENTITIES
2. IF m OR n IS ODD FACTOR OUT $\sin x$ OR $\cos x$ AND USE $\sin^2 x + \cos^2 x = 1$
   IF m AND n ARE EVEN AND POSITIVE, USE HALF ANGLE IDENTITIES
3. PRODUCT IDENTITIES

1. $\int \sin^n x\,dx$, $\int \cos^n x\,dx$
2. $\int \sin^m x \cos^n x\,dx$
3. $\int \sin(mx)\cos(nx)\,dx$

$\int \csc x = -\ln|\csc x + \cot x| + c$

$\int \sec x = \ln|\tan x + \sec x| + c$

$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{a\to-\infty}\int_a^0 f(x)\,dx + \lim_{b\to\infty}\int_0^b f(x)\,dx$

- IF ANYTHING DIVERGES / GOES TO INFINITY, IT DIVERGES

ALGEBRA

$y = \log_a x \iff a^y = x$

$(x^n)^m = x^{nm}$

$x^n \cdot x^m = x^{n+m}$

$\lim_{h\to 0}(1+h)^{\frac{1}{h}} = e$

$\lim_{n\to\infty}\left(1 - \frac{1}{n}\right)^n = \frac{1}{e}$

$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

$e^{\ln x} = x$

$\ln ab = \ln a + \ln b$

$\ln\frac{a}{b} = \ln a - \ln b$

$\ln(a)^r = r\ln a$

$\sin(\cos^{-1}x) = \sqrt{1-x^2} = \cos(\sin^{-1}x)$

$\sec(\tan^{-1}x) = \sqrt{1+x^2}$

$\tan(\sec^{-1}x) = \begin{cases}\sqrt{x^2-1}, & x \geq 1 \\ -\sqrt{x^2-1}, & x \leq -1\end{cases}$

(THE e^{n^2} CANCELS)

$\frac{n+1}{e^{(n+1)^2}} \cdot \frac{e^{n^2}}{n} = \frac{(n+1)}{n e^{2n+1}}$

INVERSE TRIG

$x = \sin^{-1} y \iff y = \sin x$ $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x = \cos^{-1} y \iff y = \cos x$ $x \in [0, \pi]$

$x = \tan^{-1} y \iff y = \tan x$ $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x = \sec^{-1} y \iff y = \sec x$ $x \in [0, \pi]$ $x \neq \pi/2$

PRODUCT IDENTITIES / HALF ANGLE

$\sin^2 x = \frac{1-\cos 2x}{2}$ $\cos^2 x = \frac{1+\cos 2x}{2}$

$\sin(mx)\cos(nx) = \frac{1}{2}[\sin((m+n)x) + \sin((m-n)x)]$

$\sin(mx)\sin(nx) = \frac{1}{2}[\cos((m+n)x) - \cos((m-n)x)]$

$\cos(mx)\cos(nx) = \frac{1}{2}[\cos((m+n)x) + \cos((m-n)x)]$

INVERTIBLE FUNCTION: "REVERSES" ANOTHER FUNCTION

- NEEDS TO PASS THE HORIZONTAL LINE TEST

IF f IS DIFFERENTIABLE AND MONOTONIC ON INTERVAL I AND $f'(x) \neq 0$, THEN $f^{-1}(x)$ IS DIFFERENTIABLE AT $y = f(x)$ AND $(f^{-1})'(y) = \frac{1}{f'(x)}$

$\frac{dx}{dy} = 1 / \frac{dy}{dx}$

HALF LIFE $A = A_0 \left(\frac{1}{2}\right)^{t/h}$

A = AMOUNT

h = HALF LIFE

t = TIME

A_0 = INITIAL AMOUNT

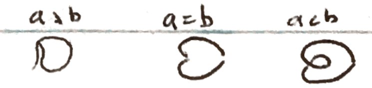
POLAR GRAPHS

LIMACON $r = a \pm b\cos\theta$ $r = a \pm b\sin\theta$

COS VERTICALLY ORIENTED

SIN HORIZONTALLY ORIENTED

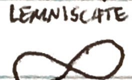
$a > b$ $a = b$ $a < b$



CARDIOID

LEMNISCATE

LEMNISCATE $r^2 = \pm a\cos(2\theta)$ $r^2 = \pm a\sin(2\theta)$



ROSE $r = a\cos(n\theta)$ $r = a\sin(n\theta)$

(n LEAVES IF ODD)
($2n$ LEAVES IF EVEN)

LONG DIVISION (LOWER OR EQUAL TERM)

DIVIDE BY LEADING TERM