Mechanics Formula Sheet

Stress

Method of Joints
- Every member is two-force (only tension/compression)
- Resultant forces are on joints

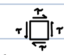
Assume Equilibrium
$$\Sigma M = 0$$
$$\Sigma F = 0$$

Normal:

$$\sigma_{AVG} = \frac{P}{A_0} \qquad \sigma = \frac{N}{A_0} \qquad \int dN = \int_A \sigma \, dA$$

Units [psi or Pa]

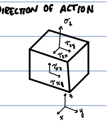
Shear:



$$\tau_{AVG} = \frac{V}{A_{FACE}}$$

Units [psi or Pa]

Conventions:
- Define face by 1. Normal component 2. Direction of action



Method of Sections
- Assumes material is uniform
- Make imaginary cuts between changes in material, geometry, and loads
  - Everywhere within each section has the same load
- At least one force in global FBD must be left out

Assumptions:
- Material is cohesive (all portions are connected)
- Uniform distribution
- Cross-section is constant
  - This assumption is not correct, but we don't have a way to fix it

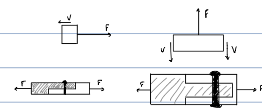
Convention for FBD's
- Within section tension is positive

Torsion

$$\tau_{MAX} = \frac{Tc}{J}$$

Solid $J = \frac{\pi}{2} c^4$ Hollow $J = \frac{\pi}{2} c_o^4 - \frac{\pi}{2} c_i^4$

Single Shear / Double Shear



$V = F$

$V = \frac{F}{2}$

$2F = V$

Statistically Indeterminate
- Use displacement equations and sum constraint forces
  - Remember that compression is negative

Factor of Safety

$$FS = \frac{F_{FAIL}}{F_{ALLOW}} = \frac{\sigma_{FAIL}}{\sigma_{ALLOW}} = \frac{\tau_{FAIL}}{\tau_{ALLOW}}$$

- Creates a way to measure allowable strain
- Some buildings use various load amounts to account for variability

Strain
- Deformations within a body
- Temperature is a type of strain

Normal:

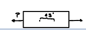
$$\varepsilon_{AVG} = \frac{L - L_0}{L_0} = \frac{\delta}{L_0}$$

- Does not work with two different materials
- ε is dimensionless [in/in]

L_0 = original length

$\varepsilon > 0$ tension
$\varepsilon < 0$ compression

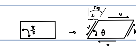
$$\varepsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$



Shear:

$$\gamma_{ENG} = \frac{\pi}{2} - \theta$$

[RAD]



$\gamma > 0$ shear is positive
$\gamma < 0$ shear is negative

θ = [radians]

Double shear vs single shear

Shear vs Strain

Original area → Engineering stress vs True stress ← Instant area

Vocab:

Brittle: Low deformation or it fractures
Ductile: Can absorb large amounts of strain before fracture
- Behaviors shown below

Plastic Region
- Permanent deformation

Yielding
- Permant deformation begins. Additional deformation will occur without any increase in stress

Offset method ← both shear + strain
- Go to $\gamma, \varepsilon = 0.002$, and draw a straight line with slope E, G. Intersection is yield point

Strain Hardening
- Past yielding, additional stress will be supported until ultimate stress. If load is removed, material will have a much higher yield point with same modulus of toughness. It will be less ductile
- Ultimate stress: point where max stress is supported σ_u
  - Necking: cross-sectional area deforms, and deforms until fracture ← Only in ductile

Modulus of Resilience

$$u_r = \frac{1}{2} \sigma_{pl} \varepsilon_{pl}$$
$$u_r = \frac{1}{2} \frac{\sigma_{pl}^2}{E}$$

σ_{pl} = proportional limit stress

Units $\left[\frac{N}{in^2} \cdot \frac{in}{in}\right] = \left[\frac{N \cdot in}{in^3}\right]$

Strain Energy
- Area under σ-ε curve

$$u = \frac{\Delta U}{\Delta V} = \frac{1}{2} \sigma \varepsilon$$

If linear elastic $u = \frac{1}{2} \frac{\sigma^2}{E}$

Modulus of Toughness

- Deformation is not yet permanent
- Ends at proportional limit σ_{pl}

Elastic Region

E = Youngs modulus [psi or Pa]

G = modulus of rigidity [psi or Pa]

Normal	Strain
$\sigma = E\varepsilon$	$\tau = G\gamma$
δ	ϕ
A	Solid $J = \frac{\pi}{2} c^4$ or Hollow $J = \frac{\pi}{2} c_o^4 - \frac{\pi}{2} c_i^4$
P	T
$\delta = \frac{NL}{AE}$	$\phi = \frac{TL}{GJ}$

$P > 0 \Rightarrow$ tension
$P < 0 \Rightarrow$ compression

$T > 0 \Rightarrow$ cross product pointing into bar
$T < 0 \Rightarrow$ cross product pointing away

Conventions:
- $\delta_{AB} > 0 \Rightarrow$ tension
- $\delta_{AB} < 0 \Rightarrow$ compression
- $\delta_{A/B} > 0 \Rightarrow$ moving away
- $\delta_{A/B} < 0 \Rightarrow$ moving closer

Conventions:

$$G = \frac{E}{2(1 + \nu)}$$

G = shear modulus of elasticity [psi or Pa]

Thermal Stress

$$\delta_T = \alpha \Delta T L$$

α = coefficient of thermal expansion

ΔT = change in temp L = length δ_T = change in member

Poisson's Ratio

$$\nu = \frac{\varepsilon_{LAT}}{\varepsilon_{LAT}}$$

For most metals: $0.25 < \nu < 0.355$

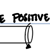
Torsion

$$\tau_{MAX} = \frac{Tc}{J}$$

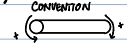
for rods

$$J = \frac{\pi}{2}(c)^4 \quad \text{or} \quad J = \frac{\pi}{2}(c_o^4 - c_i^4)$$

ϕ and T are positive with right hand rule



Convention



Assumptions:
- Closed cross section
- Constant thickness

Thin wall

$$\tau_{AVG} = \frac{T}{2tA_m}$$

T = torque
τ = shear stress
t = thickness
A_m = area enclosed by centerline of thickness

$$q = \tau_{AVG} t$$
$$q = \frac{T}{2A_m}$$

q = shear flow (force per unit length across the cross section)

$$\tau = K \frac{Tc}{J}$$

$$\phi = \frac{TL}{4A_m^2 G} \oint \frac{ds}{t}$$

$\Sigma \phi_i$ if multiple T's

G = shear modulus of elasticity
Integral calculated around tube cross section
ϕ = angle of twist
L = length
ds = change in length around centerline

BENDING

FLEXURE FORMULAS

$$\sigma_{max} = \frac{Mc}{I}$$

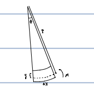
$$\sigma = -\frac{My}{I}$$

C = ⊥ DISTANCE FROM NEUTRAL AXIS TO A POINT FURTHEST AWAY FROM NEUTRAL AXIS (ON TOP OR BOTTOM)
I = MOMENT OF INERTIA ABOUT NEUTRAL AXIS
y = DISTANCE FROM NEUTRAL AXIS

STRAIN

$$\varepsilon_x = \frac{-y}{\rho}$$

y = DISTANCE FROM NEUTRAL AXIS
ρ = RADIUS OF CURVATURE FROM NEUTRAL AXIS



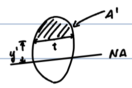
SHEAR IN BENDING

$$\tau = \frac{VQ}{It}$$

Q = MOMENT OF CROSS-SECTIONAL AREA ABOVE OR BELOW THE POINT WHERE SHEAR STRESS IS TO BE DETERMINED (DISTANCE FROM NA)
I = MOMENT OF INERTIA
t = WIDTH OF CROSS SECTION AT SHEAR POINT

$$Q = \sum \bar{y}' A'$$

$\bar{y}'$ = DISTANCE OF AREA CENTROID OF FROM NA
A' = AREA OF TOP OR BOTTOM PORTION OF MEMBER'S CROSS SECTION



NEUTRAL AXIS

- MUST OCCUR IN THE CENTROID OF CROSS-SECTION
- DOES NOT CHANGE IN LENGTH

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

$\bar{y}$ = CENTROID DISTANCE FROM REFERENCE POINT
A_i = INDIVIDUAL SEGMENT'S AREA
y_i = SEGMENT'S CENTROID DISTANCE FROM A REFERENCE

COMMON CENTROIDS

CIRCLE + RECTANGLES: CENTER $\left[\frac{r}{2}, \frac{h}{2}, \frac{b}{2}\right]$

SEMI-CIRCLE: $\bar{y} = \frac{4r}{3\pi}$ ← FROM BASE ALONG RADIAL LINE

MOMENT OF INERTIA

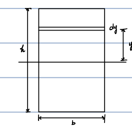
NON-UNIFORM

$$I_y = \int_A y^2 \, dA$$

y = DISTANCE FROM y AXIS TO dA

$$I_x = \int_A x^2 \, dA$$

x = DISTANCE FROM x AXIS TO dA



NEUTRAL AXIS THEOREM → $I = \bar{I} + Ad^2$

COMMON

$$I_{rectangle} = \frac{1}{12} b h^3$$

$$I_{circular} = \frac{\pi r^4}{4}$$

$$I_{semicircle} = \frac{\pi r^4}{8}$$

SHEAR FLOW

- A WAY TO DETERMINE FORCE NECESSARY FOR FASTENERS TO HOLD AGAINST ℓ NOT PSI
- A FORCE PER UNIT LENGTH

$$q = \frac{VQ}{I}$$

V = SHEAR FORCE
q = SHEAR FLOW
Q = $\bar{y}'A'$ WHERE A' IS THE CROSS-SECTIONAL AREA OF THE SEGMENT CONNECTED TO THE BEAM AT JUNCTURE WHERE SHEAR FLOW IS CALCULATED, AND $\bar{y}'$ IS CENTROID

Q IS THE AREA THE FASTENER NEEDS TO HOLD ONTO

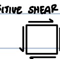
SHEAR FLOW IS USED TO MEASURE HOW MUCH SHEAR FORCE A FASTENER MUST RESIST

$$F = qs$$

F = FORCE TO PREVENT SLIPPING
S = SPACING

PLANE TRANSFORMATION

POSITIVE SHEAR CONVENTION



POSITIVE ACTION ON POSITIVE SIDE

COMBINED LOADING + PRESSURE VESSELS

HOOP/CIRCUMFERENTIAL

$$\sigma = \frac{pr}{t}$$

ℓ AROUND SURFACE

LONGITUDINAL

$$\sigma = \frac{pr}{2t}$$

ℓ ALONG

← SPHERICAL VESSELS ONLY HAVE LONGITUDINAL STRESSES

COMBINED LOADING

$$\sigma = \frac{P}{A}$$

$$\tau = \frac{T\rho}{J}$$

AXIAL	BENDING	TORSION	THERMAL	PRESSURE
$\delta = \frac{PL}{AE}$	$\sigma = \frac{-My}{I}$			
	$\tau = \frac{VQ}{It}$			

STRESS/STRAIN TRANSFORMATION

- PLANE STRESS DOES NOT IMPLY PLANE STRAIN

CONDITIONS FOR PLANE STRESS:
- COMPONENTS IN ONE DIRECTION CAN BE IGNORED
  - USUALLY THE z DIRECTION

PRINCIPAL, AVERAGE, & MAX

NORMAL

ORIENTATION OF PRINCIPAL STRESS

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

NO SHEAR STRESS ACTS ON THE PRINCIPAL PLANES OF STRESS

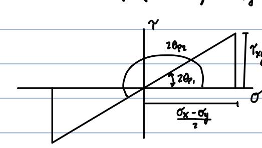
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

SHEAR

ORIENTATION OF MAX SHEAR

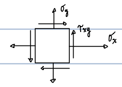
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tau_{max\ in\ plane} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right) + \tau_{xy}^2}$$



PLANE STRESS

- WHEN LOADINGS CAN BE ANALYZED WITHIN A SINGLE PLANE



SIGN CONVENTIONS
- σ_x, σ_y POSITIVE WHEN TENSION
- τ_{xy} POSITIVE WHEN POSITIVE SIDE ACTS IN POSITIVE DIRECTION
- θ IS POSITIVE COUNTER CLOCKWISE
- x' IS DIRECTED POSITIVE OUTWARD

PLANE TRANSFORMATION EQUATIONS

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

PRINCIPAL STRESSES
- σ_x, σ_y & τ_{xy} ARE CONSTANT, BUT θ CAN CHANGE $\sigma_{x'}$ $\tau_{x'y'}$
- MAXIMUM + MINIMUM NORMAL STRESSES
  - ACT ON PRINCIPAL PLANES OF STRESS
  - NO SHEAR STRESS ACTS ON THE PRINCIPAL PLANES

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$ ← WILL HAVE TWO ROOTS θ_{p1}, θ_{p2} (90° APART)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

$\sigma_{1,2}$ = PRINCIPAL STRESSES
$\sigma_1 > \sigma_2$

$$\sigma_{AVG} = \frac{\sigma_x + \sigma_y}{2}$$

SHEAR

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

θ_{s1}, θ_{s2} = MAX SHEAR STRESS ORIENTATION
θ_s & θ_p ARE 45° APART

$$\tau_{\substack{MAX \\ IN\text{-}PLANE}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

MOHR'S CIRCLE

STRAIN

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

$C = (\sigma_{AVG}, 0)$
$A = (\sigma_x, \tau_{xy})$

C = CENTER OF CIRCLE
A = REFERENCE POINT

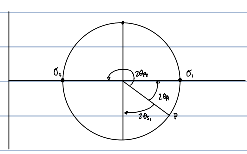
SIGN CONVENTIONS:
- σ → +
- τ ↓ +
- θ_p ↻ +
- θ_s ↺ +

STEPS FOR DRAWING
1. COORDINATE SYSTEM
2. PLOT C
3. PLOT A
4. DRAW CA (CA = R)
5. SKETCH CIRCLE

STRESS

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$C = (\varepsilon_1, 0)$
$A = \left(\varepsilon_1, \frac{\gamma_{xy}}{2}\right)$



ABSOLUTE MAXIMUM

IF $\sigma_1 > \sigma_2 > \sigma_3$ THEN:

$$\tau_{\substack{ABS \\ MAX}} = \frac{\sigma_{MAX} - \sigma_{MIN}}{2}$$

$$\gamma_{\substack{ABS \\ MAX}} = \varepsilon_{MAX} - \varepsilon_{MIN}$$

PLANE STRAIN

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\varepsilon_{AVG} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

$$\tan 2\theta = -\left(\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right)$$

$$\frac{\gamma_{\substack{MAX\ IN \\ PLANE}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

ABSOLUTE MAX

$\gamma^{ABS}_{MAX} = \varepsilon_1 - \varepsilon_2$ <u>IF</u> OPPOSITE SIGNS