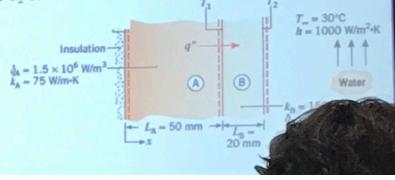


EXAMPLE 3.7

A plane wall is a composite of two materials, A and B. The wall of material A has uniform heat generation $q = 1.5 \times 10^6 \text{ W/m}^3$, $k_A = 75 \text{ W/m}\cdot\text{K}$, and thickness $L_A = 50 \text{ mm}$. The wall material B has no generation with $k_B = 150 \text{ W/m}\cdot\text{K}$ and thickness $L_B = 20 \text{ mm}$. The inner surface of material A is well insulated, while the outer surface of material B is cooled by a water stream with $T_\infty = 30^\circ\text{C}$ and $h = 1000 \text{ W/m}^2\cdot\text{K}$.

1. Sketch the temperature distribution that exists in the composite under steady-state conditions.
2. Determine the temperature T_0 of the insulated surface and the temperature T_2 of the cooled surface.



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WHY HEAT GEN → PARABOLIC?

ASSUMPTIONS:

1. 1D HEAT X-PER- IN X

2. STEADY STATE

3. NEGIGIBLE CONTACT RESISTANCE

4. ADIABATIC WALL ON LEFT

5. CONSTANT K

FEATURES WE EXPECT

WALL A: WE DON'T EXPECT LINEAR DISTRIBUTION

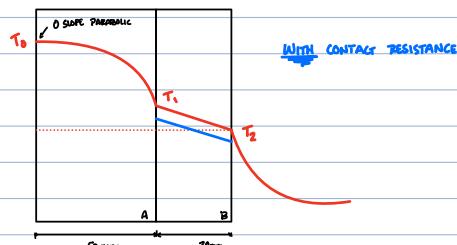
BASED ON SOLUTION: x^2 (PARABOLIC)

INSULATED \Rightarrow 0 SLOPE @ INSULATION INTERFACE

WALL B: LINEAR T PROFILE

$k_B \approx k_A$

CONNECTION AT OUTSIDE WALL B



T_1 :

$$\dot{q}(L_A A) = \dot{q}_{COND} A$$

$$\dot{q}_{COND}'' = \dot{q}_{COND}$$

\dot{q} = VOLUMETRIC HEAT GEN

$$\dot{q}_{COND} A = \dot{q}_{COND} A$$

$$\dot{q} L_A = h(T_2 - T_\infty)$$

$$\Rightarrow T_2 = T_\infty + \frac{\dot{q} L_A}{h} = 105^\circ\text{C}$$

$$T_2 = 105^\circ\text{C}$$

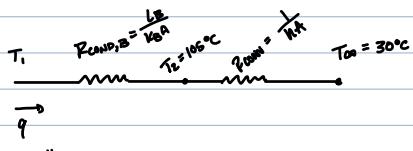
$$T_0 = ?$$

Analytical Solution

$$T_0 = \frac{q l_A^2}{2k_A} + T_1$$

WE DON'T HAVE T_1

BE CAREFUL w/ AREAS



$$q = q'' \cos \theta A = q_{\text{COND}}$$

$$\frac{T_1 - T_\infty}{\left(\frac{l_B}{k_B A} + \frac{1}{k_A} \right)} = q A$$

$$\Rightarrow T_1 = T_\infty + \left(\frac{l_B}{k_B} + \frac{1}{k_A} \right) q l_A$$

$$\therefore T_1 = 115^\circ\text{C}$$

$$\Rightarrow T_0 = 140^\circ\text{C}$$

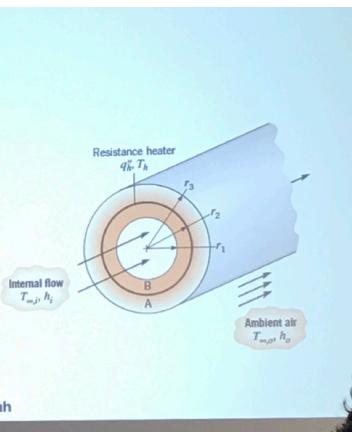


Exam review - Q2

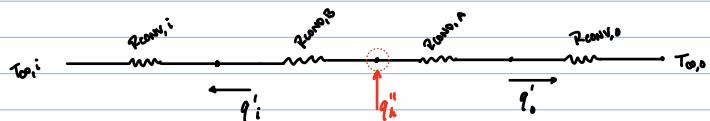
A composite cylindrical wall is composed of two materials of thermal conductivity k_A and k_B , which are separated by a very thin, electric resistance heater for which interfacial contact resistances are negligible.

Liquid pumped through the tube is at a temperature $T_{in,i}$ and provides a convection coefficient h_i at the inner surface of the composite. The outer surface is exposed to ambient air, which is at $T_{out,o}$ and provides a convection coefficient of h_o . Under steady-state conditions, a uniform heat flux of q''_h is dissipated by the heater.

- Sketch the equivalent thermal circuit of the system and express all resistances in terms of relevant variables.
- Obtain an expression that may be used to determine the heater temperature, T_h .
- Obtain an expression for the ratio of heat flows to the outer and inner fluids, q'_o/q'_i . How might the variables of the problem be adjusted to minimize this ratio?



ASSUME : STEADY STATE

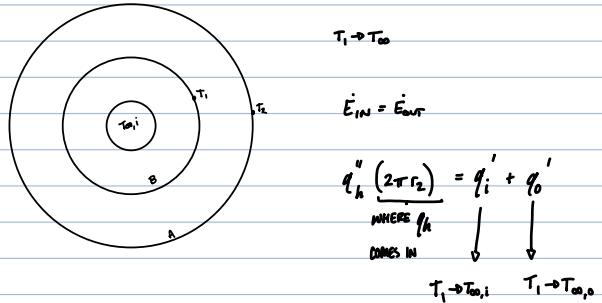


$$R_{conv,h} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L} \quad \text{WE CAN DO THIS ON A PER UNIT LENGTH BASIS}$$

$$R_{conv,i} = \frac{1}{h_A} \quad R_{conv,o} = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_B}$$

$$R_{conv,i} = \frac{1}{h_i 2\pi r_1} \quad R_{conv,o} = \frac{\ln\left(\frac{r_3}{r_1}\right)}{2\pi k_A}$$

$$R_{conv,o} = \frac{1}{h_o 2\pi r_3}$$



$$q''_h \frac{(2\pi r_2)}{h} = q'_i + q'_o$$

WHERE q''_h
COMES IN

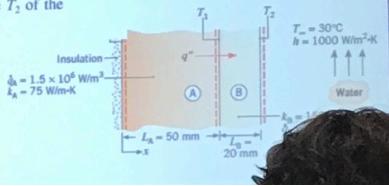
$T_i \rightarrow T_{in,i}$ $T_i \rightarrow T_{out,o}$

TRY 1

EXAMPLE 3.7

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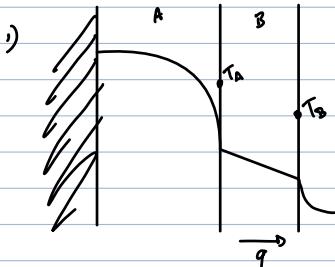
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$$q = 1.5 \times 10^6 \frac{\text{W}}{\text{m}^3}$$

$$k_A = 75 \frac{\text{W}}{\text{m}\cdot\text{K}} \quad l_A = 50 \text{ mm}$$

$$k_B = 150 \frac{\text{W}}{\text{m}\cdot\text{K}} \quad l_B = 20 \text{ mm}$$

$$T_0 = 30^\circ\text{C} \quad h = 1000 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$$



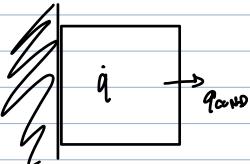
$$T(x) = -\frac{q}{2k}x + C_1 + C_2$$

in A:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_s$$

$$\dot{E}_g = \dot{E}_{out}$$

ASSUME $L=1$



$$\text{in B: } h(T_B - T_{\infty}) = q'' A$$

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_s$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$(T_B - T_{\infty}) h = k \frac{(T_A - T_B)}{l_B}$$

$$q l_A A = q_{conv}$$

$$q l_A = q_{conv}$$

$$q l_A = h (T_B - T_{\infty})$$

$$\Rightarrow T_B = \frac{q l_A}{h} + T_{\infty} = 105^\circ\text{C}$$

$$q l_A = k \left(\frac{T_A - T_B}{l_B} \right) \quad \text{Solve for } T_A$$

$$\frac{l_A}{k} q l_A = T_A - T_B \Rightarrow \frac{l_B}{k_B} q l_A + T_B = T_A = 115^\circ\text{C}$$

$$\frac{(T_A - T_B)}{l_A} k_A = k_B \frac{T_A - T_B}{l_B}$$

Exam review - Q2

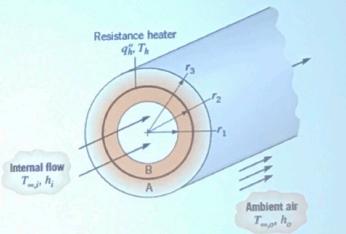
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(b) Obtain an expression that may be used to determine the heater temperature, T_h .

(c) Obtain an expression for the ratio of heat flows to the outer and inner fluids, q'_o/q'_i . How might the variables of the problem be adjusted to minimize this ratio?



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$$\begin{aligned} T_{\infty,i} & \quad T_{\infty,o} \\ h_i & \\ q''_h & \end{aligned}$$

