15 Adaptive Control

- Last time was a sliding disk

Suppose we have vaguely structured uncertainty in dynamics, and we don't want to use sliding mode control due to large control effort

Adaptive Control

- Adapts parameter values within controller

Ex 2-DOF Quanser Robot w/uncertain mass parameters

1st joint

$$\tau_1 = (N_1^2 J_{m_1} + I_1 + m_2 a_1^2)\ddot{\phi}_1 + a_1 r_{12} m_2 \cos(\phi_2 - \phi_1)\ddot{\phi}_2$$
$$- a_1 r_{12} m_2 \sin(\phi_2 - \phi_1)\dot{\phi}_2^2 + (r_{01} m_1 + a_1 m_2) g \cos(\phi_1) + F_1$$

What is minimum # of parameters we need to know to solve this?

$$\tau_1 = \underbrace{(N_1^2 J_{m_1} + I_1 + m_2 a_1^2)}_{\alpha_1}\ddot{\phi}_1 + a_1 \underbrace{r_{12} m_2}_{\alpha_2} \cos(\phi_2 - \phi_1)\ddot{\phi}_2$$
$$- a_1 \underbrace{r_{12} m_2}_{\alpha_2} \sin(\phi_2 - \phi_1)\dot{\phi}_2^2 + \underbrace{(r_{01} m_1 + a_1 m_2)}_{\alpha_3} g \cos(\phi_1) + F_1$$

We're assuming we know a values, but m & r values are uncertain

There are 3 different parameters we need values for

2nd joint

$$\tau_2 = a_1 \underbrace{r_{12} m_2}_{\alpha_2} \cos(\phi_2 - \phi_1)\ddot{\phi}_1 + \underbrace{(N_2^2 J_{m_2} + I_2)}_{\alpha_4}\ddot{\phi}_2$$
$$+ a_1 \underbrace{r_{12} m_2}_{\alpha_2} \sin(\phi_2 - \phi_1)\dot{\phi}_1^2 + \underbrace{r_{12} m_2}_{\alpha_2} g \cos\phi_2 + F_2$$

4 α's

$\therefore$ We need 4 #'s to solve

- Important that unknown parameters are linearly involved in dynamics
  - No squared terms

$$\rightsquigarrow \quad \alpha = \begin{bmatrix} (N_1^2 J_{m_1} + I_1 + m_2 a_1^2) \\ r_{12} m_2 \\ (r_{01} m_1 + a_1 m_2) \\ (N_2^2 J_{m_2} + I_2) \end{bmatrix}$$

Now we put dynamics in form:

$$\underset{2\times1}{\tau} = \underset{2\times4}{Y(\Phi, \dot{\Phi}, \ddot{\Phi})}\, \underset{4\times1}{\alpha} + \underset{2\times1}{F}$$

If friction params are also unknown, we can incorporate these into Y & α

$$Y(\Phi, \dot{\Phi}, \ddot{\Phi}) = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \end{bmatrix}$$

CORRESPONDS TO τ_1 & a_1

$Y_{11} = \ddot{\phi}_1$ $\qquad$ $Y_{21} = 0$

τ_1 & a_2

$Y_{12} = a_1 \cos(\phi_2 - \phi_1)\ddot{\phi}_2 - a_1 \sin(\phi_2 - \phi_1)\dot{\phi}_2^2$ $\qquad$ $Y_{22} = a_1 \cos(\phi_2 - \phi_1)\ddot{\phi}_1 + a_1 \sin(\phi_2 - \phi_1)\dot{\phi}_1^2 + g\cos\phi_2$

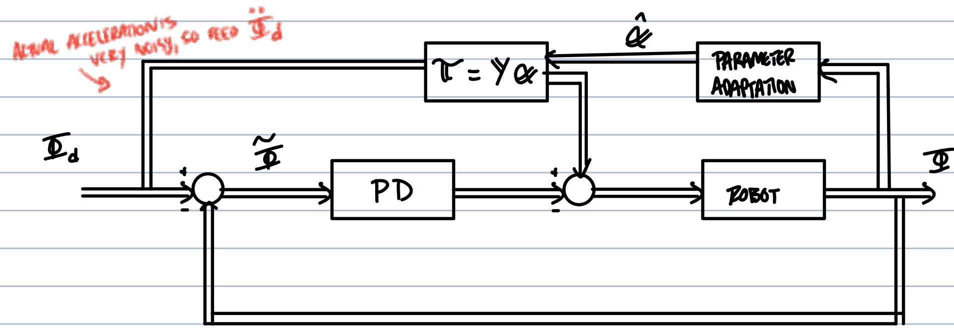
$Y_{13} = g\cos\phi_1$ $\qquad$ $Y_{23} = 0$

$Y_{14} = 0$ $\qquad$ $Y_{24} = \ddot{\phi}_2$

τ_1, a_4 AREN'T MULTIPLIED

IF WE TAKE A LOT OF DATA W/ $\Phi, \dot{\Phi}, \ddot{\Phi}, \tau$ WE CAN USE LINEAR REGRESSION TO FIND α (SYSTEM ID)

WE COULD USE A PD CONTROLLER W/ A TRAJECTORY, WE CAN USE A LINEAR REGRESSION TO SOLVE FOR α



SIMILAR TO FEEDFORWARD COMPUTED TORQUE CONTROL
BUT W/ PARAMETER ADAPTATION

LETS AUGMENT OUR ADAPTIVE CONTROL TO USE ACTUAL + DESIRED TRAJECTORY (CHANGES HOW WE CALCULATE Y IN ABOVE DIAGRAM)

DEFINE A REFERENCE VELOCITY

$$\dot{\Phi}_r = \dot{\Phi}_d + \Lambda\tilde{\Phi} \qquad \Lambda = \frac{K_P}{K_D}$$

ERROR CORRECTION TERM

$$\ddot{\Phi}_r = \ddot{\Phi}_d + \Lambda\dot{\tilde{\Phi}}$$

USE CONTROL LAW:

$$\tau = \hat{\tau} + \mathbb{K}_d\left(\dot{\tilde{\Phi}} + \Lambda\tilde{\Phi}\right)$$

PD CONTROL

$K_p = K_P\Lambda$

WHERE $\hat{\tau} = Y(\Phi, \dot{\Phi}, \dot{\Phi}_r, \ddot{\Phi}_r)\hat{\alpha}$

NEED TO PUT DYNAMICS IN FORM (WE DO IT TO SATISFY LYAPUNOV PROOF, WE DON'T HAVE TIME TO GO OVER IT)

$$\hat{\tau} = \hat{H}(\Phi)\ddot{\Phi}_r + \underbrace{\hat{C}(\Phi, \dot{\Phi})\dot{\Phi}_r}_{\dot{V}} + \hat{G}(\Phi) + B\dot{\Phi}_r$$

WE NEED TO MODIFY Y

$$Y_{11} = \ddot{\phi}_{1r}$$

$$Y_{12} = a_1 C(\phi_2 - \phi_1)\ddot{\phi}_{2r} - a_1 S(\phi_2 - \phi_1)\dot{\phi}_2 \dot{\phi}_{2r}$$

BIG DEAL SO ITS STABLE W/LYAPUNOV PROOF

$$Y_{13} = g\, C(\phi_1)$$

$$Y_{22} = a_1 C(\phi_2 - \phi_1)\ddot{\phi}_r + a_1 S(\phi_2 - \phi_1)\dot{\phi}_1 \dot{\phi}_{1r} + g\, C(\phi_2)$$

$$Y_{24} = \ddot{\phi}_{2r}$$

ADAPTATION LAW :

$$\dot{\hat{\alpha}} = \underbrace{\Gamma^{-1}}_{\text{ADAPTATION GAINS}} \underbrace{Y^T(\Phi, \dot{\Phi}, \dot{\Phi}_r, \ddot{\Phi}_r)}_{\text{ROBOT DYNAMICS}}\, \sigma$$

$$\sigma = \dot{\tilde{\Phi}} + \Lambda \tilde{\Phi}$$

Γ^{-1} IS INVERSE, BECAUSE IT APPEARS IN THE LYAPUNOV PROOF

$\tilde{\alpha} = \hat{\alpha} - \alpha$

ESTIMATE TRUE VALUES

IS IT STABLE? $\tilde{\Phi} \to 0$?

$\tilde{\alpha} \to 0$?

LYAPUNOV CANDIDATE : $V(\sigma, \tilde{\Phi}, \hat{\alpha}) = \frac{1}{2}\sigma^T H \sigma + \tilde{\Phi}^T \Lambda k_D \tilde{\Phi} + \frac{1}{2}\tilde{\alpha}^T \Gamma \tilde{\alpha}$

IF WE DO THAT, IT IS ASYMPTOTICALLY STABLE $\tilde{\Phi} = 0$ (GOES TO ZERO)

BUT PARAMETER ERROR $\tilde{\alpha}$ IS ONLY STABLE (BOUNDED, BUT WON'T GO TO ZERO)

- DEPENDS ON TRAJECTORY
  - IF TRAJECTORY SUFFICIENTLY EXCITES THE DYNAMICS, THEN $\tilde{\alpha} \to 0$

ADAPTIVE CONTROL GUARANTEES THAT THE $\hat{\alpha}$ DRIVES TRACKING ERROR TO ZERO, EVEN THOUGH WE MAY NOT GET TRUE α VALUE

$\Phi_r = \Phi_d$ w/ ERROR CORRECTION