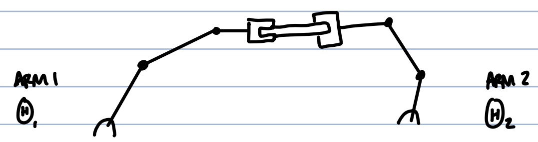
23 Multi-Arm Coordination

Multi-fingered grasping

Multi-legged walking

Two arms gripping a shared object



Θ_1, Θ_2 are joint $\dot{x}$'s

If both robots try to do motion control (with high PD gains) they may fight each other, resulting in large forces on object or each other

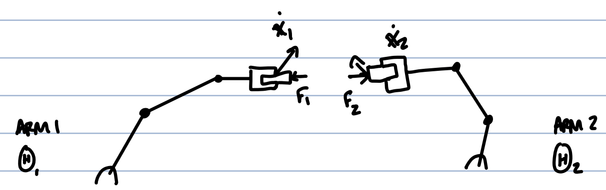
$\Theta_1 \in \mathbb{R}^{n_1}$ $\quad n_1 = n_2 = 3$

$\Theta_2 \in \mathbb{R}^{n_2}$ $\quad m = 3$

$\dot{X} \in \mathbb{R}^m$

master-slave control : one controls motion/position, the other concerns force control

A more balanced approach is to share responsibility for motion & force control



relative velocity

$$\dot{X}_{rel} = \dot{X}_2 - \dot{X}_1 = J_2 \dot{\Theta}_2 - J_1 \dot{\Theta}_1$$

define new task space

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_{rel} \end{bmatrix} = \underbrace{\begin{bmatrix} J_1 & 0 \\ -J_1 & J_2 \end{bmatrix}}_{J_{rel}} \begin{bmatrix} \dot{\Theta}_1 \\ \dot{\Theta}_2 \end{bmatrix}$$

J_{rel} = Jacobian for newly defined task space

This Jacobian will <u>also</u> map forces & torques (duality)

$$\underbrace{\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}}_{\text{joint torques}} = \begin{bmatrix} J_1^T & -J_1^T \\ 0 & J_2^T \end{bmatrix} \underbrace{\begin{bmatrix} F_{EXT} \\ F_{INT} \end{bmatrix}}_{\text{forces in new task space}}$$

$$\text{Power} = \underbrace{(F_{EXT}^T - F_{INT}^T)\dot{X}_1}_{\text{Arm 1 Power}} + \underbrace{F_{INT}^T \dot{X}_2}_{\text{Arm 2 Power}} \qquad (\dot{X}_2 = \dot{X}_1 + \dot{X}_{rel})$$

DUALITY PROOF

$$\begin{bmatrix} F_{EXT} \\ F_{INT} \end{bmatrix}^T \begin{bmatrix} \dot X_1 \\ \dot X_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}^T \begin{bmatrix} \dot \Theta_1 \\ \dot \Theta_2 \end{bmatrix}$$

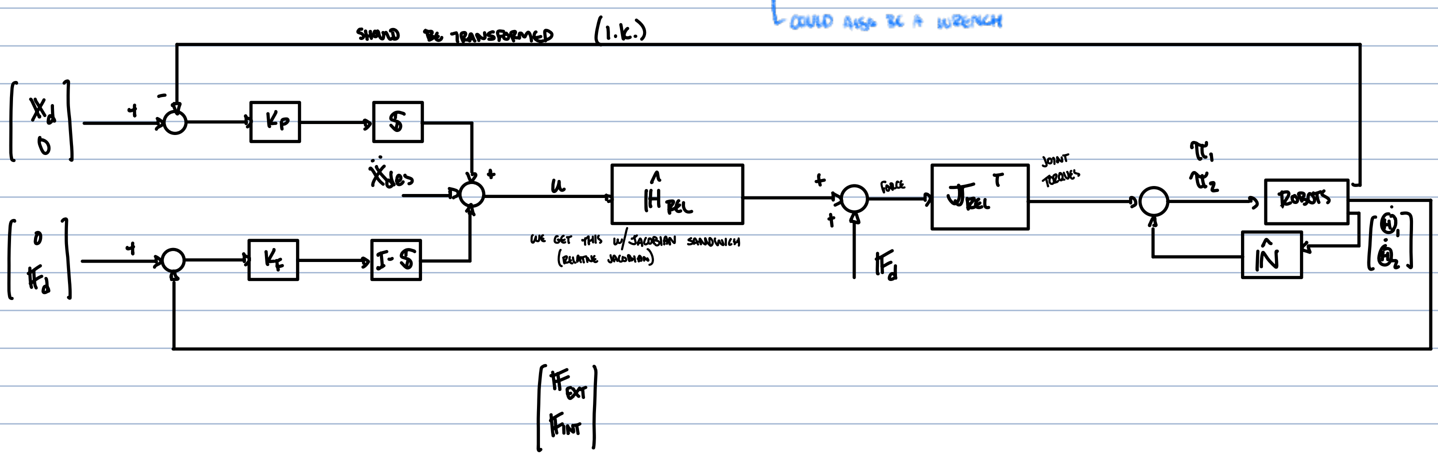
$$\begin{bmatrix} F_{EXT} \\ F_{INT} \end{bmatrix}^T J_{REL} \begin{bmatrix} \dot \Theta_1 \\ \dot \Theta_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}^T \begin{bmatrix} \dot \Theta_1 \\ \dot \Theta_2 \end{bmatrix}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = J_{REL}^{\;T} \begin{bmatrix} F_{EXT} \\ F_{INT} \end{bmatrix}$$

APPLY HYBRID CONTROL TO RELATIVE TASK SPACE

CONSTRAINTS	KINEMATIC	STATIC
NATURAL	$\dot X_{REL} = 0$	$F_{EXT} = 0$
ARTIFICIAL	$\dot X_1 = \dot X_{des}$	$F_{INT} = F_{des}$

COULD ALSO BE A WRENCH

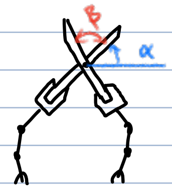


$$\hat H_{REL} = J_{REL}^{-T} \begin{bmatrix} \hat H_1(\Theta_1) & 0 \\ 0 & \hat H_2(\Theta_2) \end{bmatrix} J_{REL}^{\;T}$$

FOR QUASI-STATIC OR SLOW TRAJECTORIES, DON'T NEED TO USE I.D.C. JUST USE J^T

$$S = \begin{bmatrix} \underbrace{\begin{matrix} 1 & & \\ & 1 & \\ & & 1 \end{matrix}}_{\substack{x\ y\ \theta \\ \text{ABS}}} & \\ & \underbrace{\begin{matrix} 0 & & \\ & 0 & \\ & & 0 \end{matrix}}_{x,y,\theta\ \text{REL}} \end{bmatrix}$$

ANOTHER EXAMPLE: TWO ROBOTS (FINGERS) OPERATING A PAIR OF SCISSORS



PLACE END-EFFECTOR FRAMES OF BOTH ROBOTS AT PIVOT x, y

α = ABSOLUTE ∡

β = RELATIVE ∡

DEFINE RELATIVE TASK SPACE:

COMBINED 6-DOF

TASK SPACE

$$\left.\begin{array}{l} 1.\ V_x \\ 2.\ V_y \\ 3.\ \dot{\alpha} \end{array}\right\} \text{3 ABSOLUTE DOF}$$

$$\left.\begin{array}{l} 4.\ V_{x,REL} \\ 5.\ V_{y,REL} \\ 6.\ \beta \end{array}\right\} \text{3 RELATIVE DOF}$$

LET'S ASSUME THE SCISSORS ARE FRICTIONLESS

	KIN	STATIC
NATURAL	$V_{x,REL} = V_{y,REL} = 0$ (IF $\neq 0$ WHEN SCISSORS BREAK)	$f_x = 0$ $\quad f_y = 0$ $\tau_\alpha = 0$ $\quad \tau_\beta = 0$
ARTIFICIAL	$V_x = V_{x,d}$ $V_y = V_{y,d}$ $\dot{\alpha} = \dot{\alpha}_d$ $\dot{\beta} = \dot{\beta}_d$	$f_{x,REL} = f_{x,d}$ $f_{y,REL} = f_{y,d}$

$$\therefore \quad S = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 1 \end{bmatrix}$$

$$J_{REL} = \begin{bmatrix} J_1 & 0 \\ -J_1 & J_2 \end{bmatrix}$$