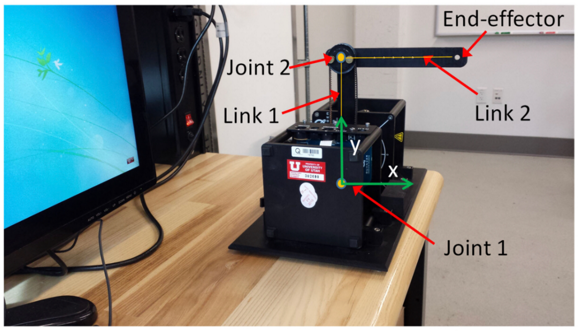
Problem Set 1



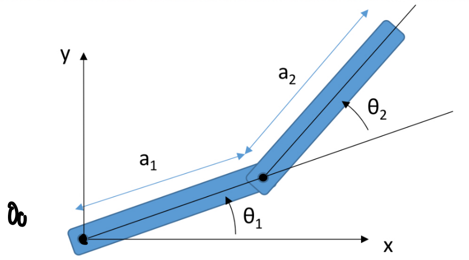


Figure 1. A 2-DOF Serial Manipulator

1. Diagram the robot links, assign 3D coordinate axes, and write out a table of DH parameters (Hollerbach's convention) for this robot. Assume the zero angle configuration is when both links are horizontal.

	a_i	d_i	α_i	θ_i
1	a_1	0	0	*
2	a_2	0	0	*

$$x = a_1 C\theta_1 + C(\theta_1 + \theta_2) a_2$$

$$y = a_1 S\theta_1 + S(\theta_1 + \theta_2) a_2$$

2. Derive the 4x4 homogenous transform matrix ${}^0\mathbf{T}_2$ in terms of the DH parameters. Identify the 3x3 rotation matrix ${}^0\mathbf{R}_2$ and the 3x1 end-effector location ${}^0\boldsymbol{d}_{02}$. Please use vector and subscript conventions consistent with lecture.

$${}^0R_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^1R_2 = \begin{bmatrix} \text{SAME BUT} \\ \theta_1 \text{ INSTEAD OF } \theta_2 \end{bmatrix}$$

$${}^0T_2 = {}^0R_1\,{}^1R_2 \quad {}^0R_1\,{}^0R_2\;{}^0d_{02}$$

$${}^0d_2 = \begin{bmatrix} C\theta_1 a_1 + C\theta_{12} a_2 \\ S\theta_1 a_1 + S\theta_{12} a_2 \\ 0 \end{bmatrix}$$

3. Note that since this robot is planar, we can simplify this end-effector position to 2D coordinates:

$$ {}^{0}\mathbf{d}_{02} = \begin{bmatrix} x \\ y \end{bmatrix} $$

Derive the 2x2 *Manipulator Jacobian*, **J**, by taking derivatives of ${}^{0}\mathbf{d}_{02}$, such that

$$ {}^{0}\dot{\mathbf{d}}_{02} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} $$

Use the determinant of **J** to algebraically solve for the joint angles at which this robot's singularities occur. Draw the robot in a singular configuration and explain what this means about the possible velocities of the robot at this configuration.

$$ x = a_1 C\theta_1 $$

$$ C(\theta_1 + \theta_2) = C_{12} $$

$$ S(\theta_1 + \theta_2) = S_{12} $$

$$ J = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{bmatrix} $$

$$ x = a_1 C\theta_1 + C(\theta_1 + \theta_2) a_2 $$

$$ y = a_1 S\theta_1 + S(\theta_1 + \theta_2) a_2 $$

$$ \det(J) = \frac{\partial x}{\partial \theta_1} \frac{\partial y}{\partial \theta_2} - \frac{\partial y}{\partial \theta_1} \frac{\partial x}{\partial \theta_2} $$

$$ \frac{\partial x}{\partial \theta_1} = -\dot{\theta}_1 a_1 S\theta_1 - \dot{\theta}_1 S_{12} a_2 \qquad \frac{\partial x}{\partial \theta_2} = -\dot{\theta}_2 S_{12} a_2 $$

$$ \frac{\partial y}{\partial \theta_1} = a_1 \dot{\theta}_1 C\theta_1 + \dot{\theta}_1 C_{12} a_2 \qquad \frac{\partial y}{\partial \theta_2} = \dot{\theta}_2 C_{12} a_2 $$

$$ J = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} & -a_2 S_{12} \\ a_1 C_1 + a_2 C_{12} & a_2 C_{12} \end{bmatrix} $$

4. Use the matrix/vector formula to derive the planar 3x2 *Velocity Jacobian* $\mathbf{J}_v$, such that

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \omega_z \end{bmatrix} = \mathbf{J}_v \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}.$$

Compare the first two rows with the manipulator Jacobian. Which Jacobian would you use to compute inverse velocities? Explain using both mathematical and physical reasoning.

CROSS PRODUCT OF 2 VECTORS

$$\dot{d}_{02} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = J_v \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \rightsquigarrow J_v = \begin{bmatrix} \underline{z}_0 \times \underline{d}_{02} & \underline{z}_1 \times \underline{d}_{12} \\ z & \underline{z}_1 \end{bmatrix}$$

$$z_1 \times d_{12}$$

$$= \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_1 C_1 + a_2 C_{12} \\ a_1 S_1 + a_2 S_{12} \\ 0 \end{bmatrix} & {}^0R_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_2 C_{12} \\ a_2 S_{12} \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & {}^0R_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

$$\begin{array}{c} x \\ y \\ z \\ \omega_x \\ \omega_y \\ \omega_z \end{array} = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} & -a_2 S_{12} \\ a_1 C_1 + a_2 C_{12} & a_2 C_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

5. Using the principle of virtual work, derive an equation involving the manipulator Jacobian that relates the joint torques $\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$ to the planar end-effector forces $\begin{bmatrix} F_x \\ F_y \end{bmatrix}$. If the robot is at a singularity, can you find a non-zero set of end-effector forces that result in zero joint torque? Explain using both mathematical and physical reasoning.

6. Suppose we allow for planar torque τ_z on the end-effector, in addition to end-effector forces $\begin{bmatrix} F_x \\ F_y \end{bmatrix}$. Write an equation relating the joint torques $\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$ to the planar wrench $\begin{bmatrix} F_x \\ F_y \\ \tau_z \end{bmatrix}$. Can you invert this relationship and compute the wrench, given the joint torques? Explain using both mathematical and physical reasoning.