ME EN 5230/6230, CS 6330

Intro to Robot Control – Spring 2023

Problem Set #4: 1-DOF Linear Control

Suppose we have a single DOF robot, namely the Quanser SRV-02, which has been turned on its side with a pendulum attached to the output shaft, as shown in Figure 1.


**Figure 1. Single DOF Robot**

Depending on whether the SRV-02 is controlled by a current or voltage amplifier, the linearized dynamics are described by the following differential equations:

A. Voltage amp:

$$\left(\frac{Nk_t}{R_a}\right) V(t) = (I_1 + N^2 J_m)\,\ddot{\theta} \;+ N^2\left(b + \frac{k_t^2}{R_a}\right)\dot{\theta} + m_1 g\, r_{01}\theta$$

B. Current amp:

$$(Nk_t)\, i(t) = (I_1 + N^2 J_m)\,\ddot{\theta} \;+ N^2 b\dot{\theta} + m_1 g\, r_{01}\theta$$

where $\theta$ is the angle of the output shaft, $V(t)$ is the motor voltage, and $i(t)$ is the motor current. Notice that with the voltage amp, there is an extra damping term due to the effect of the back EMF.

Assume that the parameters are known to be:

| | |
|---|---|
| Motor torque constant: $k_t$ | 0.0077 N·m/A |
| Armature resistance: $R_a$ | 2.6 Ω |
| Gear ratio: $N$ | 70 |
| Moment of inertia of link: $I_1$ | $0.83\text{x}10^{-3}$ kg·m$^2$ |
| Motor inertia: $J_m$ | $0.65\text{x}10^{-6}$ kg·m$^2$ |
| Mechanical damping: $b$ | $3.1\text{x}10^{-6}$ N·m·s/rad |
| Gravitational torque constant: $m_1 g\, r_{01}$ | 0.067 N·m/rad |

1. **Impedance Matching:** Imagine that you as the robot designer could have chosen a set of gears with a different gear ratio *N*. For the SRV-02 parameters shown in the table above, use impedance matching to find the ideal gear ratio (to the nearest integer) that would maximize the ability of the robot to accelerate from rest.

2. **Open-Loop Response:** Now assume that the gear ratio is *N*=70 as shown in the table.

   **2.1** For case A (Voltage amp), sketch by hand the open-loop response $\theta(t)$ of the system, assuming that a unit step input $V(t)$ is applied at $t = 0$ and all initial conditions are zero. First, find the open loop poles and decide what kind of a step response to expect (overdamped, underdamped, or critically damped). If the response looks like a first-order response, then compute the time constant and final steady-state value. If the response looks like a second-order response, then compute peak time, 2% settling time, percent overshoot, and final value. Use these to sketch the response of the system.

   **2.2** For case B (Current amp), sketch by hand the open-loop response $\theta(t)$ of the system, assuming that a unit step input $i(t)$ is applied at $t = 0$ and all initial conditions are zero. First, find the open loop poles and decide what kind of a step response to expect (overdamped, underdamped, or critically damped). If the response looks like a first-order response, then compute the time constant and final steady-state value. If the response looks like a second-order response, then compute peak time, 2% settling time, percent overshoot, and final value. Use these to sketch the response of the system.

3. **Closed-Loop Control:** Suppose we have a feedback system as shown below, where $G_P$ is the open loop transfer function of the SRV-02 using the current amp (case B) with *N*=70, and $G_C$ is a controller.

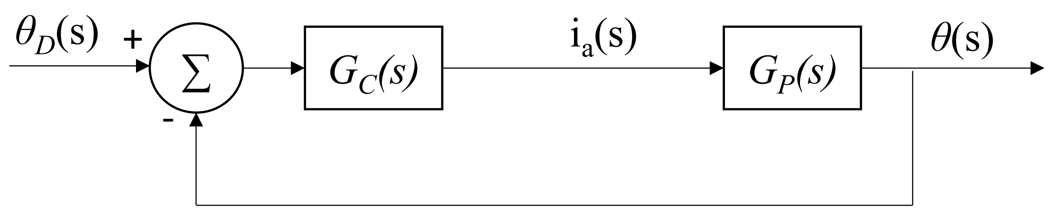


**Figure 2. Block Diagram of the Closed-Loop Controller.**

   **3.1** Use root locus techniques to design a closed-loop PD controller: $G_c(s) = K_P + K_D s$. Design for 10% overshoot and settling time of 0.2 sec.

   **3.2** Use root locus techniques to design a closed-loop PID controller: $G_c(s) = K_P + K_D s + \frac{K_I}{s}$. Design for 10% overshoot, settling time of 0.2 sec, and no steady-state error.

   **3.3** Simulate the closed-loop step responses of your systems from problems 3.1 and 3.2 using Simulink. Send data to the workspace and use MATLAB plotting commands to plot the step response. Label your plots/axes in MATLAB and indicate the % overshoot, settling time, and steady-state error. Also include a picture of your Simulink model(s). Comment on whether the performance matches your design specifications.

**3.4** Now change your PD and PID controllers to PV and PIV controllers, respectively. As shown in Figures 3 and 4, remove the derivative term from $G_C$ and implement a separate velocity feedback, using the same values for $K_V$ as you used for $K_D$. Show that this results in the same closed-loop pole locations as in problems 1 and 2, but now there is one less closed-loop zero. How does the presence/absence of the closed-loop zero influence the step response? Simulate and use plots to compare the responses. Also include a picture of your Simulink model(s).

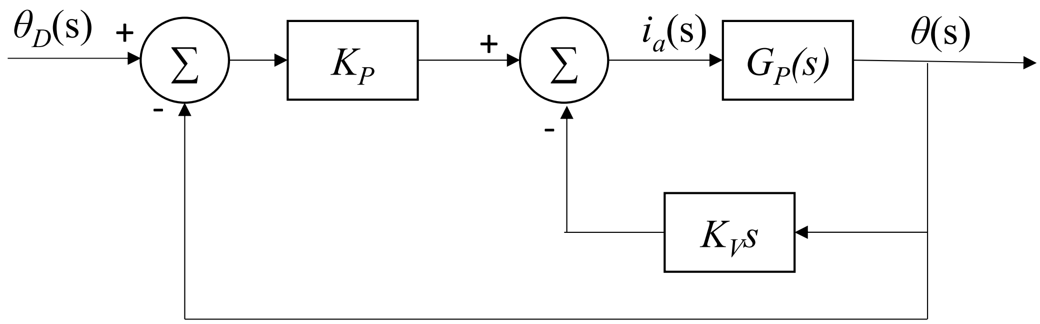


**Figure 3. Block diagram of PV Control**

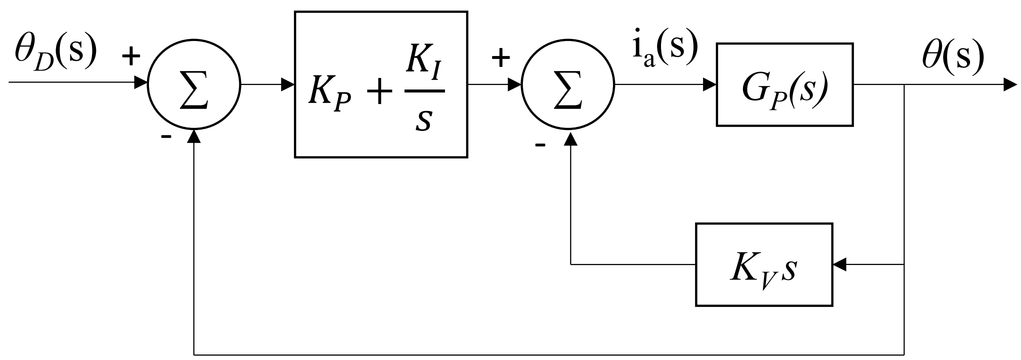


**Figure 4: Block diagram of PIV Control**

**3.5** Now add a constant disturbance *d(s)* right before the plant. Use a large enough disturbance to induce a noticeable steady-state error in your PD or PV controlled system. Simulate and use plots to compare the performance of your PD/PV vs. your PID/PIV controllers in the presence of this disturbance.