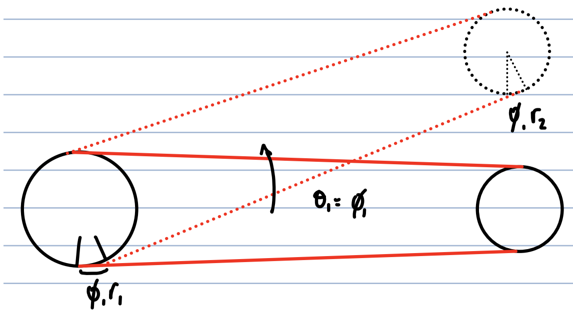
5

LAST TIME



PULLEY #2 ROTATES BY $\frac{-r_1 \phi_1}{r_2}$

$$\rightsquigarrow \theta_2 = \frac{r_1}{r_2}\phi_2 - \frac{r_1}{r_2}\phi_1$$

TRANSMISSION JACOBIAN

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{r_1}{r_2} & \frac{r_1}{r_2} \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix}$$

J_t — MATRIX THAT MAPS BETWEEN MOTOR VELOCITIES + JOINT SPACE VELOCITIES

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & r_2/r_1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

J_t^{-1} — THIS ALSO RELATES FORCES/TORQUES

$$\tau_m = J_t^{-1} \tau_{JOINT}$$

IF $r_2 = r_1$

$$\dot{\phi}_2 = \dot{\theta}_1 + \dot{\theta}_2$$

$\phi_2 = \theta_1 + \theta_2$ ← IF $r_2 = r_1 \Rightarrow \phi_2 =$ ABSOLUTE JOINT $\measuredangle$ WRT GND

↑ IF WE EXPRESS DYNAMICS IN MOTOR SPACE, CORIOLIS TERMS WILL DISAPPEAR

NEW STUFF

TOTAL DYNAMICS w/TRANSMISSION ARE NOW:

$$\tau_{JOINT} = H'(\Theta)\dot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta) + F(\dot{\Theta}) \qquad \text{JOINT SPACE}$$

OR

$$\tau_m = H'(\Phi)\ddot{\Phi} + V(\Phi, \dot{\Phi}) + G(\Phi) + F(\Phi) \qquad \text{MOTOR SPACE}$$

$\dot{\Theta} = J_t \dot{\Phi}$ $\qquad \Phi$ = MOTOR POSITION (USUALLY RAD)

$$H'(\Theta) = \begin{bmatrix} J_1 + \frac{H_{11}}{N_1^2} & \frac{H_{12}}{N_1 N_2} & \cdots & \\ \frac{H_{21}}{N_2 N_1} & J_2 + \frac{H_{22}}{N_2^2} & & \\ \vdots & & & \\ & & & J_n + \frac{H_{nn}}{N_n^2} \end{bmatrix}$$

WHAT IS THE OPTIMAL GEAR RATIO?

- IF WE INCREASE GEAR RATIO, WE INCREASE INERTIA

LARGER GEAR RATIO $\rightsquigarrow N_1^2$

CONSIDER A SINGLE-LINK ROBOT

JOINT TORQUE INERTIA

$$N \tau_m = \left(I_{arm} + N^2 J_{motor\ inertia}\right)\ddot{\theta} + \dots$$

WHAT IS WE WANT TO MAXIMIZE $\ddot{\theta}$ GIVEN A CERTAIN τ_m?
IS THERE AN OPTIMAL GEAR RATIO N?

ASSUME ROBOT ACCELERATES FROM REST $(\dot{\theta}=0)$, AND NEGLECT GRAVITY/FRICTION

$$\ddot{\theta} = \left(\frac{N}{1+N^2J}\right)\tau_m$$

TAKE DERIVATIVE WRT GEAR RATIO, SET TO ZERO

$$\rightsquigarrow \frac{d}{dN}\left(\frac{N}{1+N^2J}\tau_m\right) = \frac{1(1+N^2J) - 2NJ(N)}{(1+N^2J)^2} = 0$$

SET NUMERATOR EQUAL TO ZERO

$$1 + N^2J - 2NJ^2 = 1 - N^2J = 0$$

$$\Rightarrow N = \sqrt{\frac{I}{J}}$$ ← OPTIMAL GEAR RATIO TO MAXIMIZE $\ddot{\theta}$ WITH GEAR RATIO

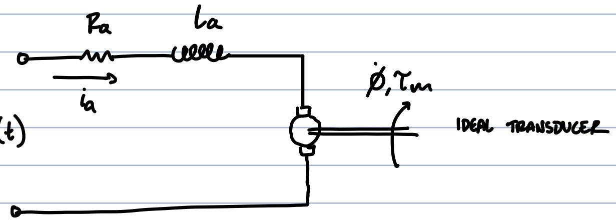
$$\rightsquigarrow I_{eff} = 1 + N^2J = I + I = 2I$$

$\Rightarrow$ OPTIMAL GEAR RATIO MAKES ARM INERTIA SIMILAR OR EQUAL TO MOTOR INERTIA } THIS IS IMPEDANCE MATCHING

DC MOTOR DYNAMICS

RESISTANCE

INDUCTANCE



KVL

$$V_a - R_a i_a - L_a \frac{di}{dt} - V_b = 0$$

V_b = BACK EMF

FARADAY'S LAW

$$V_b = K_b \dot{\phi}$$

K_b UNITS $\left[\frac{V}{RAD/s}\right]$

LORENTZ

$$\tau_m = K_t i_a$$

$\left[\frac{Nm}{A}\right]$

$K_b = K_t$ (IN SI UNITS)

WE KNOW THIS FROM CONSERVATION OF POWER

ELECTRIC POWER$_{IN}$ = MECHANICAL POWER$_{OUT}$

$$V_b i_a = \tau_m \dot{\phi}$$

$$K_b \dot{\phi} i_a = K_t i_a \dot{\phi}$$

$$\Rightarrow K_b = K_t$$

MOTOR INDUCTANCE IS OFTEN NEGLECTED

IF $L_a \ll R_a$, ITS OKAY TO NEGLECT IT

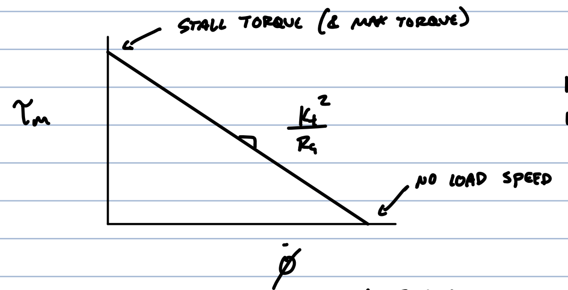
BECOMES MUCH MORE IMPORTANT AT HIGH FREQUENCY

$$\therefore V_a = R_a i_a + K_t \dot{\phi}$$

$$i_a = \frac{V_a - K_t}{R_a} \dot{\phi}$$

$$K_t i_a = \tau_m$$

$$\therefore \tau_m = \frac{K_t V_a}{R_a} - \frac{K_t^2}{R_a} \dot{\phi}$$ ← TIES MOTOR TORQUE TO MOTOR VOLTAGE



IF WE INCREASE V_a, IT SHIFTS MOTOR CURVE TO BE LARGER AT ALL $\dot{\phi}$

IN PRACTICE:

- CHOOSE BETWEEN VOLTAGE OR CURRENT AMPLIFIER
  - WE HAVE THESE IN LAB
  - WILL KEEP MOTORS FROM BURNING OUT (HEAT $\propto i_a$)

CONSIDERATIONS

$\Rightarrow$ WE CAN CONTROL τ_m WITH V_a, BUT V_b (BACK EMF) COMES INTO PLAY

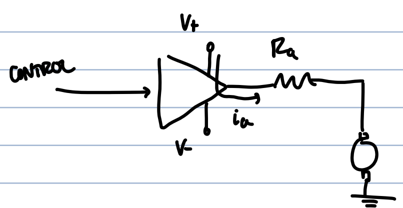
DEPENDS ON $\dot{\phi}$, WHICH DEPENDS ON DYNAMICS

WE COULD CONTROL CURRENT RATHER THAN VOLTAGE SO WE CAN AVOID BACK E.M.F.

$$\tau_m = K_t i_a$$

POWER CONSUMPTION

EX VOLTAGE AMPLIFIER



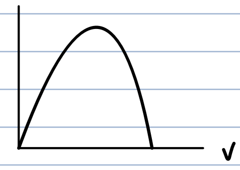
POWER INTO AMP = $V_0 i_a$

POWER OUT OF AMP = $V_a i_a$

POWER LOSS WITHIN OP AMP

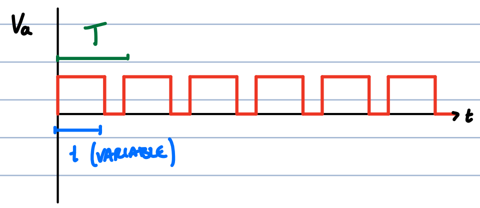
$\Rightarrow$ POWER LOSS = $(V_0 - V_a) i_a$

IF $\dot{\phi} = 0$ (STALL) $\rightsquigarrow$ $V_a = R_a i_a \Rightarrow P_{loss} = (V_0 - V_a)\frac{V_a}{R_a}$



THE ONLY POINTS OF NO POWER LOSS (WHEN $V_a = V_0$, & WHEN MOTOR IS OFF) ← MOTOR IS ALL ON

PWM IS A WAY TO AVOID WASTING POWER



$$V_{AVG} = \frac{V_0 t}{T} \qquad \frac{t}{T} = \text{DUTY CYCLE}$$

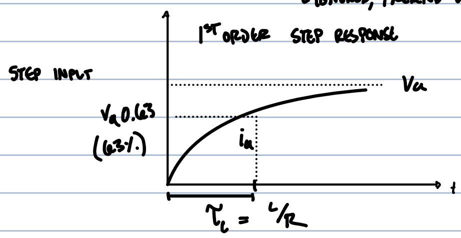
- WE CHOOSE DUTY CYCLE TO GET DESIRED AVERAGE VOLTAGE

$$V_{AVG} = \frac{V_0 t + 0(T-t)}{T}$$

- WE WANT MOTOR TO BEHAVE LIKE A LOW-PASS FILTER
  - WITH INDUCTANCE:

$$V_a = R_a i_a + L\frac{di_a}{dt} + V_b$$

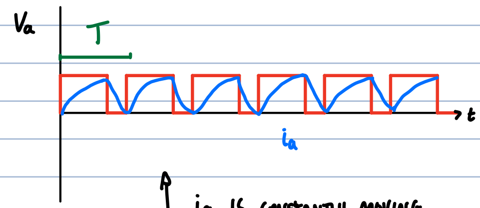
↑ IGNORED, PRETEND WE'RE STALLED



$$\tau_c = \frac{L}{R}$$

IF $T > T_c$

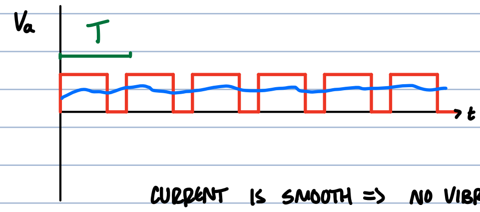
CHOPPY CURRENT



↑ i_a IS CONSTANTLY MOVING
∴ TORQUE RIPPLE

$T < T_c$

SMOOTH CURRENT



CURRENT IS SMOOTH ⇒ NO VIBRATIONS

WE WANT A VERY SMALL T, SO WE HAVE A LARGE FREQUENCY ⇒ $f = \frac{1}{T}$ IS LARGE (MHz LEVEL)

PWM RECAP:

PROS	CONS
. HIGH EFFICIENCY	. LOTS OF HIGH FREQUENCY VOLTAGE SWITCHING GENERATES ELECTRICAL NOISE WHICH CAN MESS W/ SENSORS
. SOME WILL HAVE BUILT-IN FEEDBACK TO CONTROL TORQUE TO	. MORE EXPENSIVE

SUMMARY : AMPLIFIERS

- CAN EITHER CONTROL VOLTAGE OR METER TO THE CURRENT

VOLTAGE CONTROL

$$\frac{K_t V_a}{R_a} = \tau_m + \underbrace{\frac{K_t^2}{R_a}\dot{\phi}}_{\text{BACK EMF}}$$

(ACTS LIKE FRICTION, CAN BE LUMPED W/ MECHANICAL FRICTION

$$f = b\dot{\phi} + C\,\mathrm{SGN}(\dot{\phi})$$

$$\therefore f' = \left(b + \frac{K_t^2}{R_a}\right)\dot{\phi} + C\,\mathrm{SGN}(\dot{\phi})$$

TOTAL EFFECTIVE VISCOUS DAMPING

- WITH MANIPULATOR DYNAMICS

TOTAL DYNAMICS
$$\begin{bmatrix} \frac{K_t}{R_a} V_1 \\ \frac{K_t}{R_a} V_2 \end{bmatrix} = \mathbb{H}'\ddot{\Phi} + \mathbb{V} + \mathbb{G} + \mathbb{F}'(\dot{\Phi})$$

INCLUDE MECHANICAL FRICTION AND BACK EMF

CURRENT CONTROL

$$\tau_m = K_t i_a$$

TOTAL DYNAMICS
$$\begin{bmatrix} K_t i_1 \\ K_t i_2 \end{bmatrix} = \mathbb{H}'\ddot{\Phi} + \mathbb{V} + \mathbb{G} + \mathbb{F}$$

ONLY MECHANICAL FRICTION

ASSUMES K_t IS SAME ON BOTH MOTORS