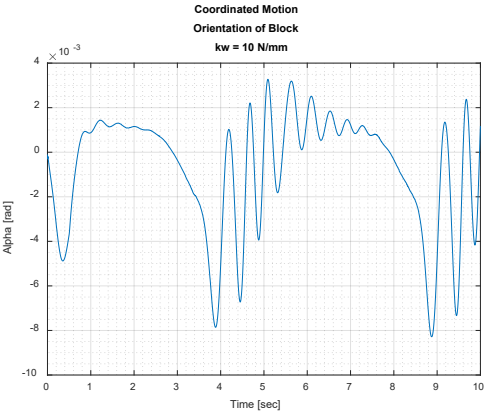
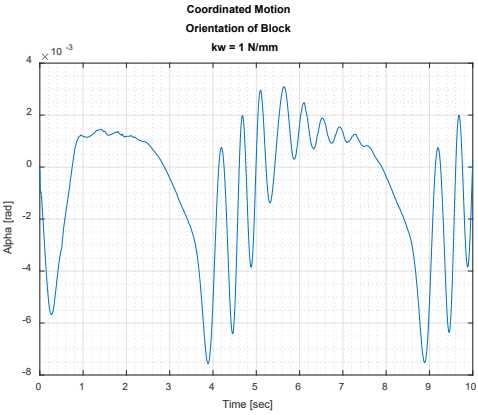
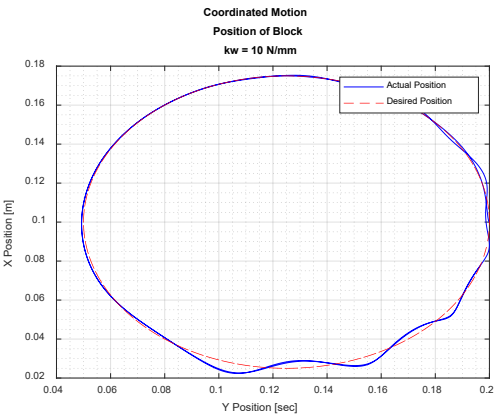
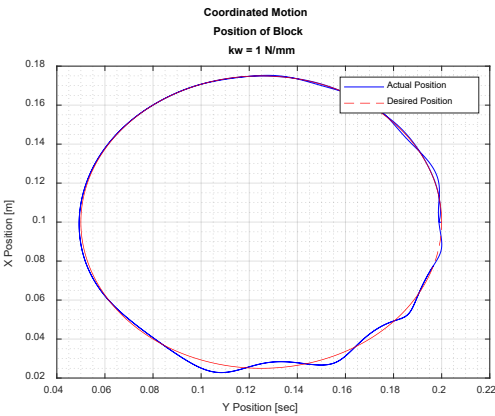
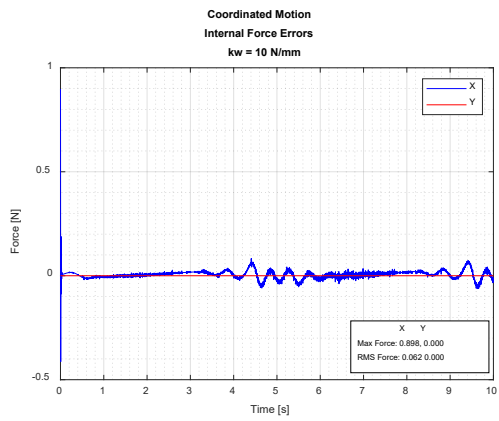
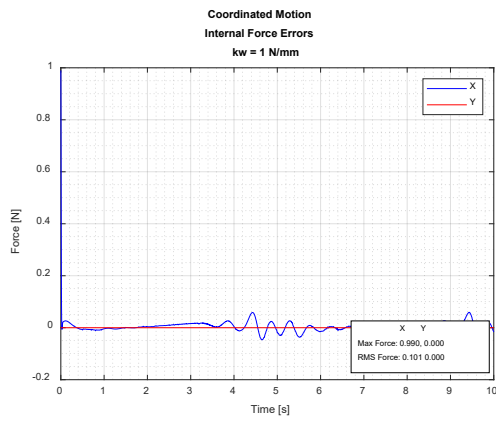
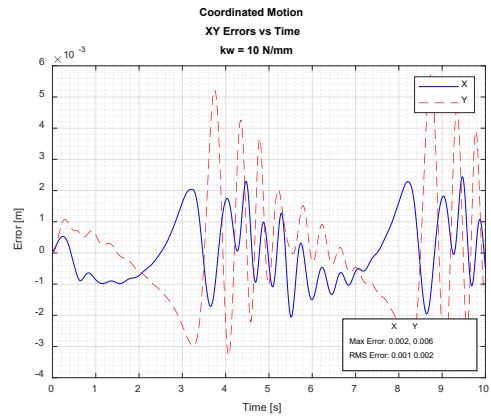
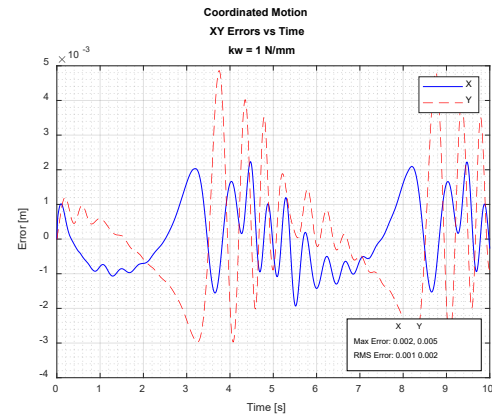




# 1.1 BLOCK STIFFNESS

## 1.1.1 Plots



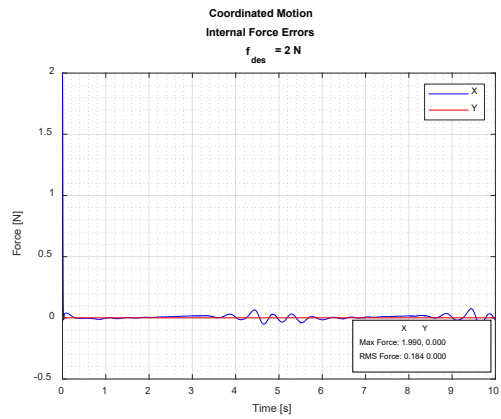
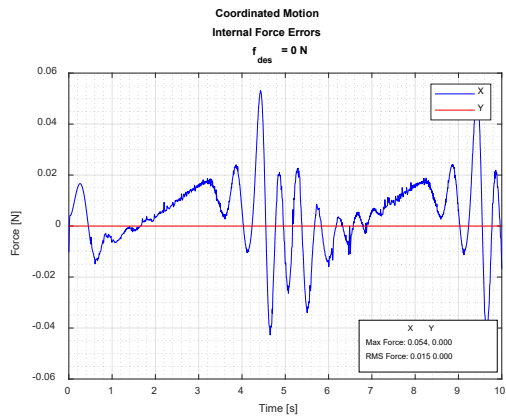
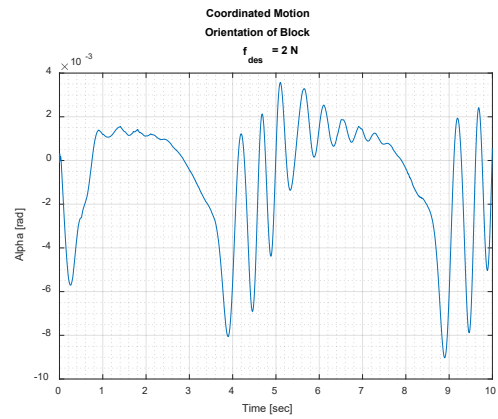
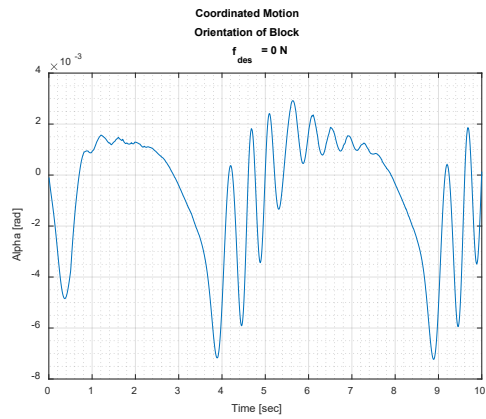
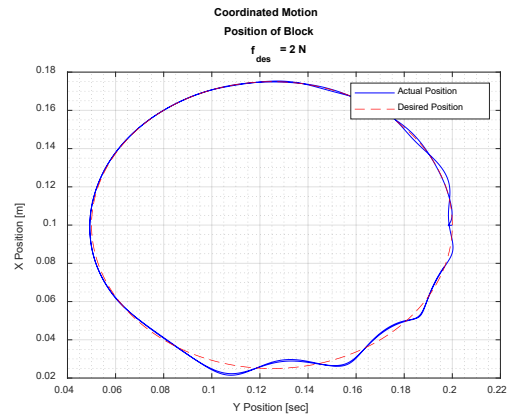
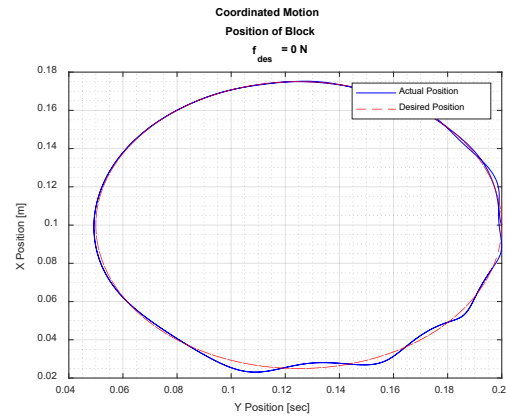


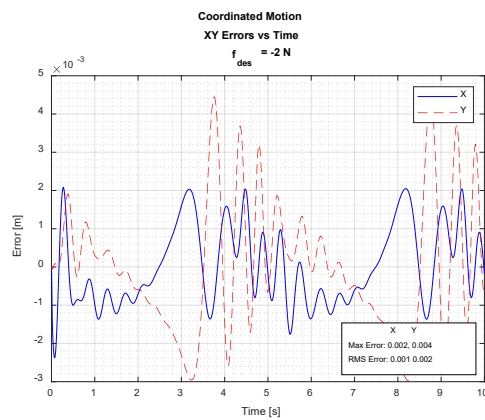
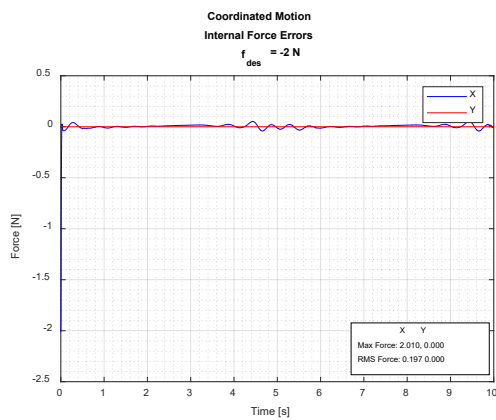
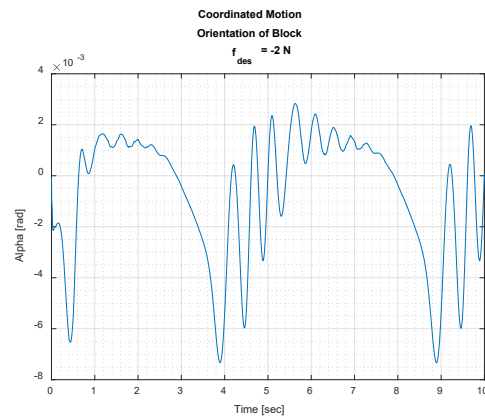
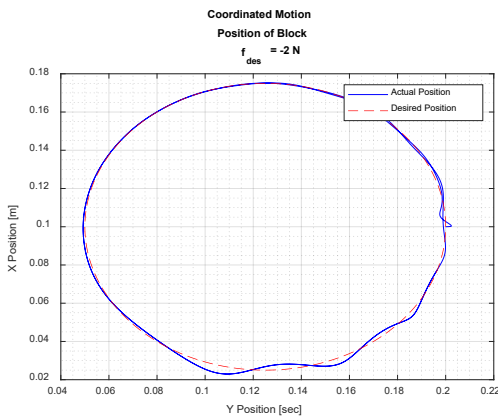
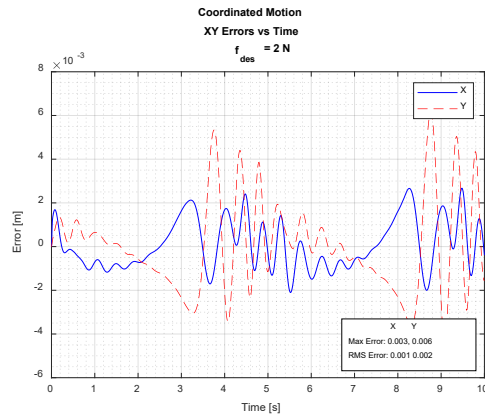
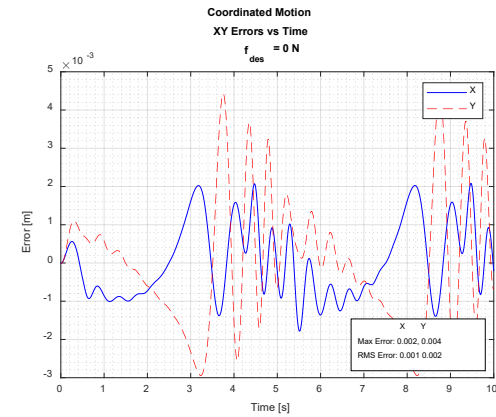
### 1.1.2 Analysis

Block stiffness didn't seem to change the results much. There was a slightly smaller maximum force, but other parameters were identical, or within a few thousandths of a unit.

## 1.2 MAGNITUDE OF INTERNAL FORCE PLOTS

### 1.2.1 Plots



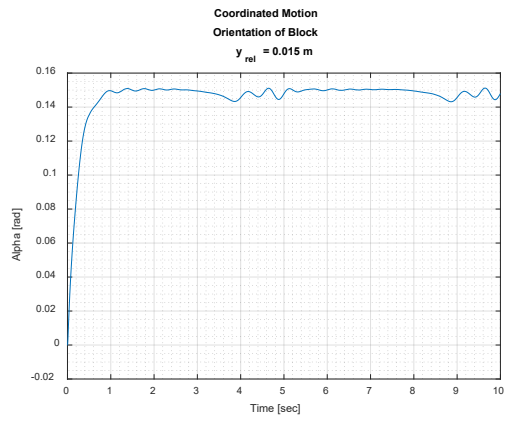
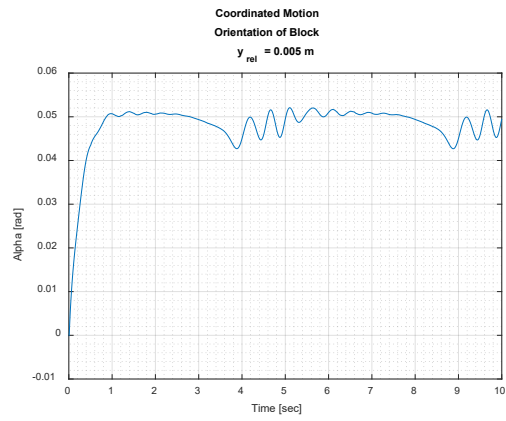
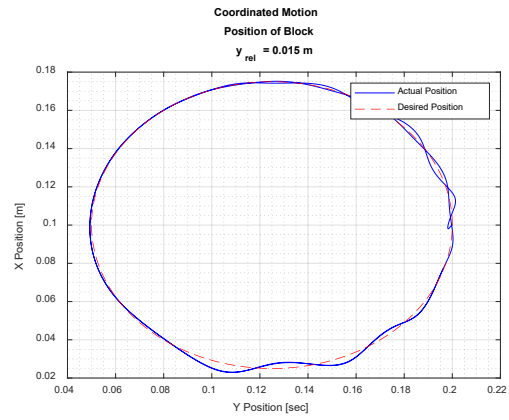
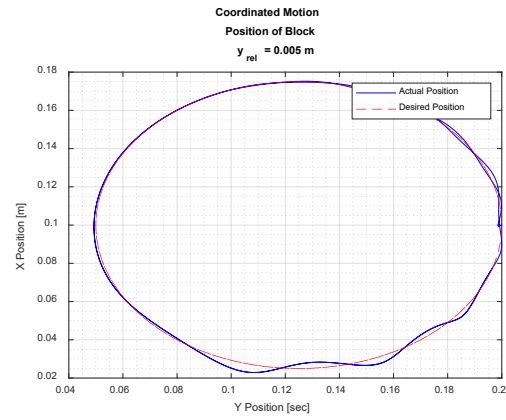


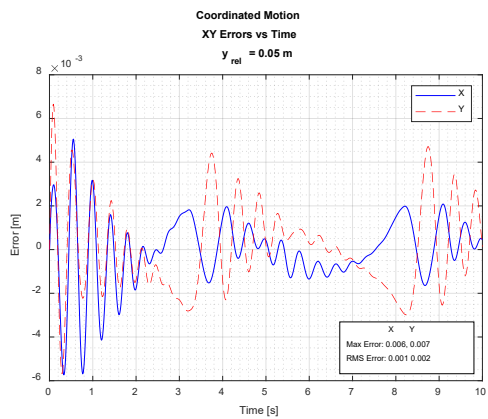
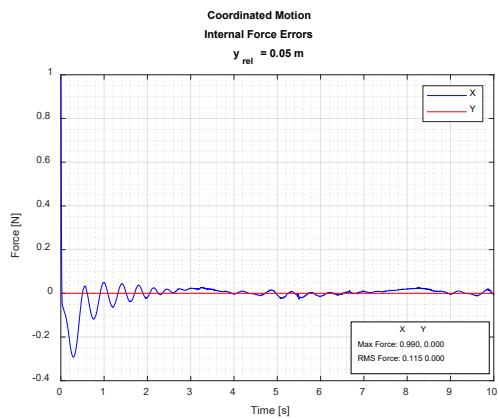
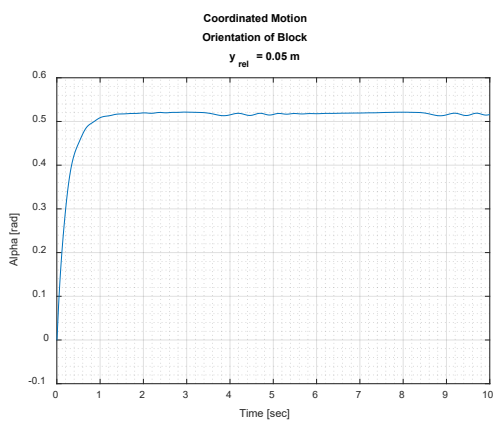
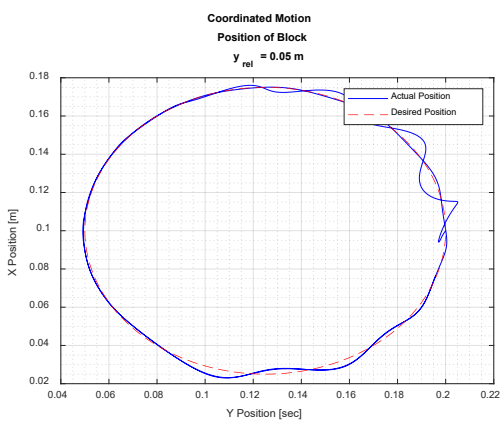
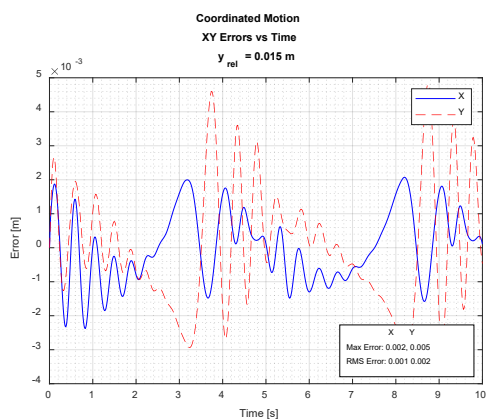
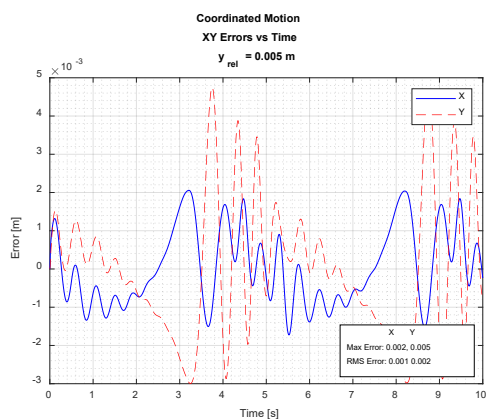
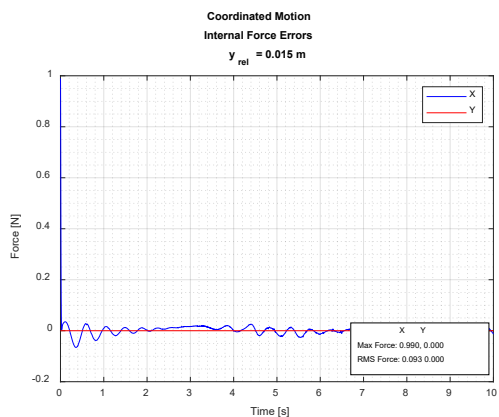
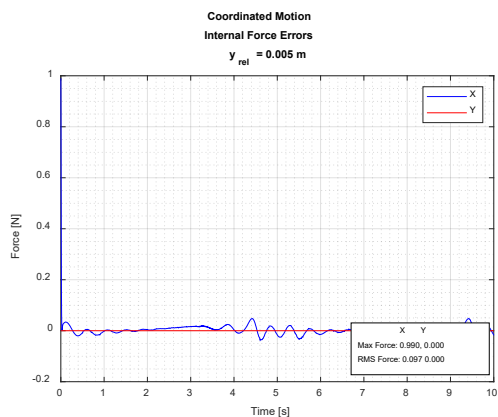
### 1.2.2 Analysis

The desired force metric led to much different plot results. The sign of the desired force led to a large initial spike of that force's error but was eventually able to settle out. With this large desired force, there was decent trajectory, but it had a few circles and unexpected disturbances. Alpha was similar across the desired forces, and max xy errors were interestingly largest when internal forces were in tension. RMS x errors were similar across all desired internal forces.

## 1.3 Y RELATIVE

### 1.3.1 Plots



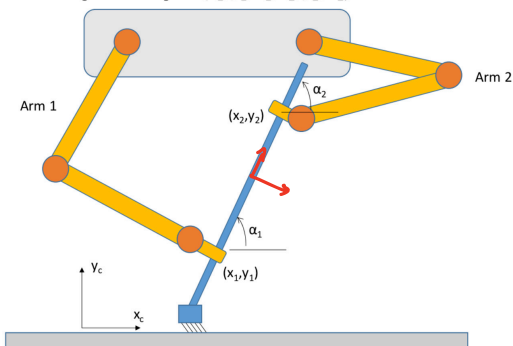


### **1.3.2 Analysis**

The y relative offset doesn't do much below a certain threshold, but then created large errors due to singularities. The y offset made the controller much more unstable once it hit 0.5 m.



2. A pair of 3-DOF planar robot arms are attached to an overhead gantry and given the task of sweeping the floor with a broom. Each robot has a firm grip on the broomstick and can independently control the planar position  $(x, y)$  and orientation  $(\alpha)$  of its end-effector, imparting both force  $(F_x, F_y)$  and torque  $(T_z)$  to the broomstick. So the robots have a combined total of 6-DOF in operational space  $(\dot{x}_1, \dot{y}_1, \dot{\alpha}_1, \dot{x}_2, \dot{y}_2, \dot{\alpha}_2)$ .

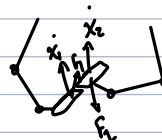


- 2.1 Formulate an appropriate task space using a combination of absolute and relative velocities, and show how to map between this task space and joint space. (i.e. derive an expression for  $J_{rel}$  in terms of  $J_1$  and  $J_2$ ).
- 2.2 Setup a table of natural and artificial constraints with respect to your task space (consider the interactions of the robot with the broomstick as well as the interaction of the broomstick with the floor). Derive the corresponding selection matrix.
- 2.3 Show how you would implement a hybrid position/force controller for this task. Draw a block diagram. Show what you would use as the desired positions and forces.

2.1)  $\dot{x}_c$  = VELOCITY of BROOM BRISTLES IN X  
 $\dot{y}_c$  = " " IN Y  
 $\alpha_1$  =  $\angle$  of BROOM  
 $\dot{x}_{rel}$  = VELOCITY of BROOM'S TOP IN X  
 $\dot{y}_{rel}$  = " " IN Y  
 $\alpha_2$  = RELATIVE  $\angle$  of  $\alpha_1$

$\dot{x}_{rel} = \dot{x}_c + L \dot{\alpha}_1 \frac{1}{2}$   $L$  = LENGTH of BROOM  
 $\dot{y}_{rel} = \dot{y}_c + L \dot{\alpha}_1 \frac{1}{2}$

$L \alpha_2 = \alpha_1 - \alpha_2$



$\dot{x}_{rel} = \dot{x}_2 - \dot{x}_1 = J_2 \dot{\theta}_2 - J_1 \dot{\theta}_1$

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_{rel} \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ -J_1 & J_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$

	KINETIC	STATIC
2) $v_{x,rel}$		
<u>NATURAL</u>	$v_{yc} = 0$ $\omega_2 = 0$ $v_{x,rel} = 0$ $v_{y,rel} = 0$	$\tau_1 = 0$ $f_{x,c} = 0$
<u>ARTIFICIAL</u>	$v_{xc} = v_{xc,d}$ $\omega_1 = \omega_{1,d}$	$f_{yc} = f_{yc,d} = 0$ $\tau_2 = \tau_{2,d} = 0$ $f_{x,rel} = f_{x,rel} = 0$ $f_{y,rel} = f_{y,rel} = 0$

5)  $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_{rel} \end{bmatrix} = \underbrace{\begin{bmatrix} J_1 & 0 \\ -J_1 & J_2 \end{bmatrix}}_{J_{rel}} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$

$S = \text{diag}([1 \ 0 \ 1 \ 0 \ 0 \ 0])$

