ME EN 5230/6230, CS 6330

Intro to Robot Control – Spring 2023

Problem Set #1: Kinematics and Statics

For this problem, we consider the kinematics of the planar 2-DOF Serial Manipulator that we will be using in lab, shown in Figure 1. We will only be concerned with the algebraic solution at this time, so don't worry about numerical values. Feel free to use shorthand notation for the trigonometric terms (e.g. sθ = sinθ, cθ = cosθ), but do SHOW ALL WORK.

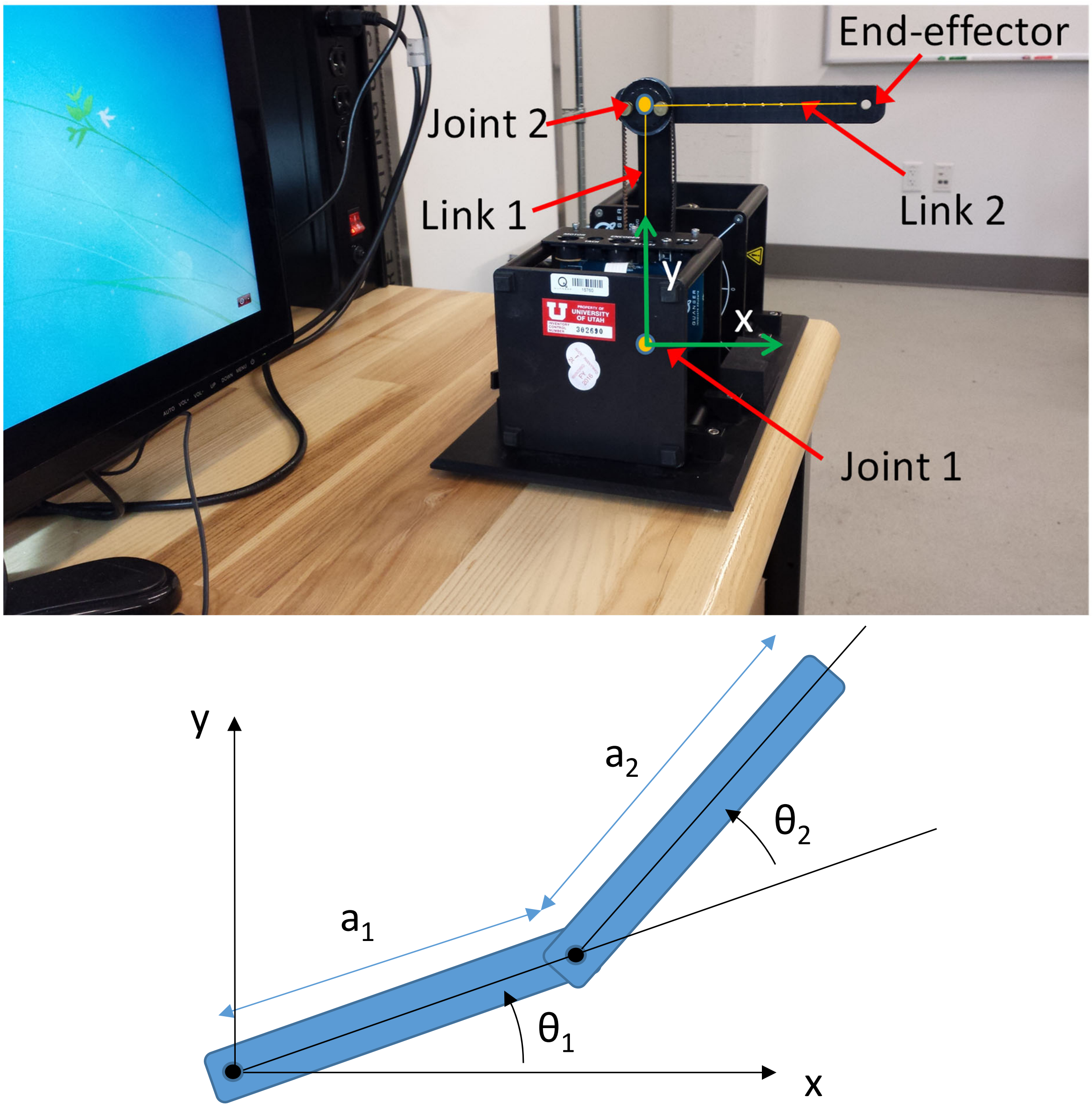

Figure 1. A 2-DOF Serial Manipulator

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1. Diagram the robot links, assign 3D coordinate axes, and write out a table of DH parameters (Hollerbach's convention) for this robot. Assume the zero angle configuration is when both links are horizontal.

2. Derive the 4x4 homogenous transform matrix ${}^{0}\mathbf{T}_2$ in terms of the DH parameters. Identify the 3x3 rotation matrix ${}^{0}\mathbf{R}_2$ and the 3x1 end-effector location ${}^{0}\boldsymbol{d}_{02}$. Please use vector and subscript conventions consistent with lecture.

3. Note that since this robot is planar, we can simplify this end-effector position to 2D coordinates:
$$ {}^{0}\boldsymbol{d}_{02} = \begin{bmatrix} x \\ y \end{bmatrix} $$
Derive the 2x2 *Manipulator Jacobian*, **J**, by taking derivatives of ${}^{0}\boldsymbol{d}_{02}$, such that
$$ {}^{0}\dot{\boldsymbol{d}}_{02} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} $$
Use the determinant of **J** to algebraically solve for the joint angles at which this robot's singularities occur. Draw the robot in a singular configuration and explain what this means about the possible velocities of the robot at this configuration.

4. Use the matrix/vector formula to derive the planar 3x2 *Velocity Jacobian* $\mathbf{J}_v$, such that
$$ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \omega_z \end{bmatrix} = \mathbf{J}_v \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}. $$
Compare the first two rows with the manipulator Jacobian. Which Jacobian would you use to compute inverse velocities? Explain using both mathematical and physical reasoning.

5. Using the principle of virtual work, derive an equation involving the manipulator Jacobian that relates the joint torques $\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$ to the planar end-effector forces $\begin{bmatrix} F_x \\ F_y \end{bmatrix}$. If the robot is at a singularity, can you find a non-zero set of end-effector forces that result in zero joint torque? Explain using both mathematical and physical reasoning.

6. Suppose we allow for planar torque τ_z on the end-effector, in addition to end-effector forces $\begin{bmatrix} F_x \\ F_y \end{bmatrix}$. Write an equation relating the joint torques $\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$ to the planar wrench $\begin{bmatrix} F_x \\ F_y \\ \tau_z \end{bmatrix}$. Can you invert this relationship and compute the wrench, given the joint torques? Explain using both mathematical and physical reasoning.