03 Review Kinematics + Dynamics

Jacobian

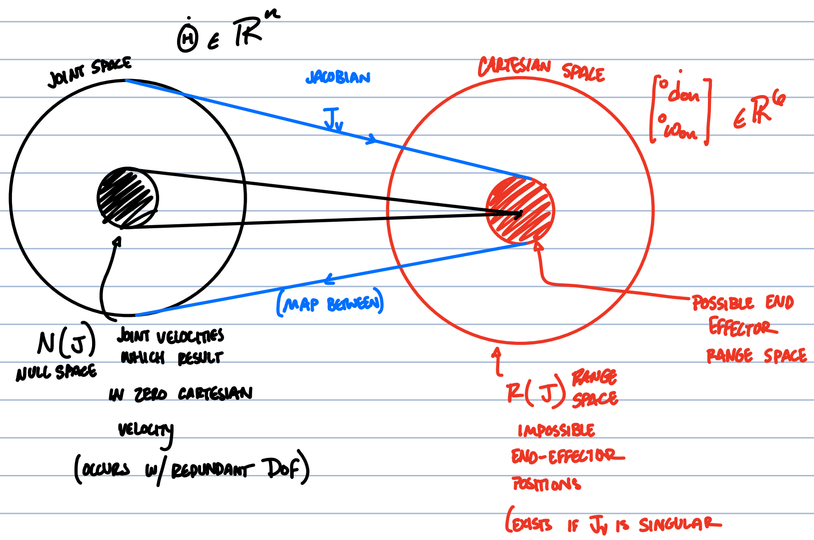
- Maps end effector velocity, forces, torques, into joint velocity, force, torque

Duality between Kinematics + Statics

$$\underbrace{\begin{bmatrix} {}^{o}\dot{d}_{on} \\ {}^{o}\omega_{on} \end{bmatrix}}_{6\times 1} = \underbrace{J_v}_{6\times n}\ \underbrace{\dot{\Theta}}_{n\times 1 \text{ vector}}$$

← changes w/ configuration of robot

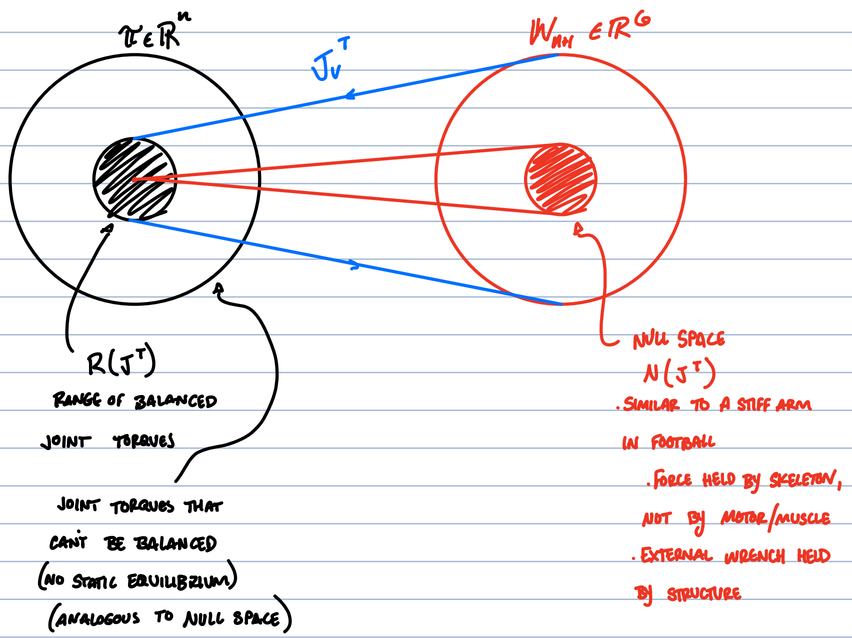
Instantaneous kinematics @ particular set of $\dot{\Theta}$



Statics

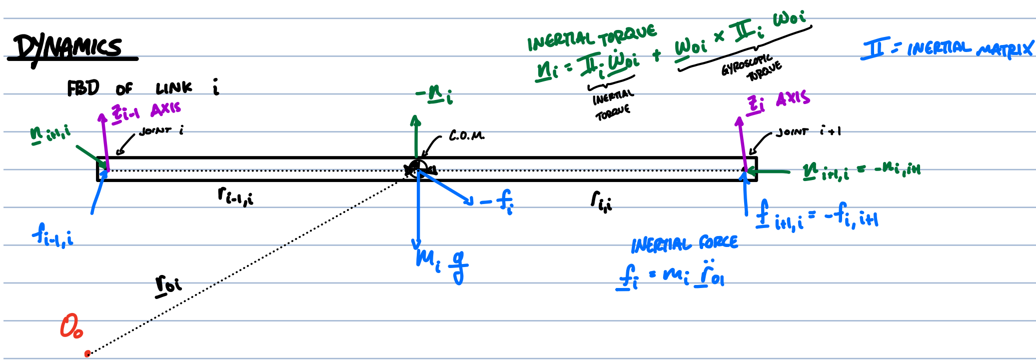
$$\underbrace{\tau}_{\substack{\text{torques} \\ (n\times 1)}} = \underbrace{J_v^T}_{(n\times 6)}\ \underbrace{W_{n\times 1}}_{\substack{\text{wrench (forces + torques)} \\ (6\times 1)}}$$

(we need to flip dimensions to make it work)



Dynamics

FBD of link i



Force Balance

Force:

$$\sum \underline{f} = 0 \rightsquigarrow \underline{f}_i = \underline{f}_{i-1,i} + m_i \underline{g} - \underline{f}_{i,i+1}$$

Torque:

About C.O.M.

$$\sum \underline{n} = 0 \qquad \underline{n}_i = \underline{n}_{i-1,i} - \underline{n}_{i,i+1} - \underline{r}_{i-1,i} \times \underline{f}_{i-1,i} + \underline{r}_{i,i} \times \underline{f}_{i,i+1}$$

We pick C.O.M. as point

r is going towards C.O.M. rather than away from it (definition of a moment)

How does this affect motor torques?

We're controlling these

Motor torque/force is scalar projection onto z axis

- Rotary joint

$$\tau_i = \underline{z}_{i-1} \cdot \underline{n}_{i-1,i}$$

scalars

- Prismatic

$$f_i = \underline{z}_{i-1} \cdot \underline{f}_{i-1,i}$$

The end effector forces can be included in dynamic formulation OR separately mapped to joint-space using J_v^T, and added at the end

Can be done for all n links

Two different methods for deriving dynamics

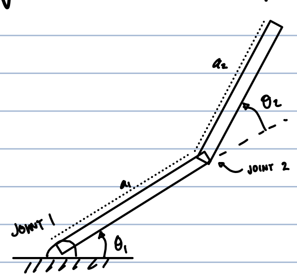
1. Newton/Euler formulation (Hollerbach Ch 10)
   - Recursive, computationally efficient
2. Lagrangian formulation (Siciliano Ch 7)
   - Easier to get closed-form algebraic eqns w/ comprehensible terms

In general, solution for dynamics of a robot manipulator can be expressed as closed form

$$\underset{\text{joint torques}}{\tau} = \underset{\text{inertia matrix}}{\mathbb{H}(\Theta)}\, \underset{\text{joint acceleration}}{\ddot{\Theta}} + \underset{\text{velocity terms}}{\mathbb{V}(\Theta, \dot{\Theta})} + \underset{\text{gravity force}}{G(\Theta)}$$

velocity terms (coriolis terms) can also be C

e.g. 2 DoF Planar Robot



Link 1: m_1, I_1 (about joint 1)

Link 2: m_2, I_2 (about joint 2)

Problem set 2 answer, but ours doesn't have C.O.M. assumption (look @ Hollerbach Ch 10 stuff)

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & -2h & -h \\ h & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_2^2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$$

Assuming C.O.M. of each link is exactly in middle of each link

$$\therefore H_{11} = I_1 + I_2 + m_2 a_1 (a_1 + a_2 C\theta_2)$$

$$H_{22} = I_2$$

$$H_{21} = H_{12} = I_2 + \tfrac{1}{2} m_2 a_1 a_2 C\theta_2$$

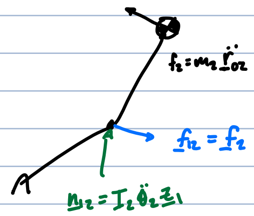
Non-linear + configuration dependent

$$h = \tfrac{1}{2} m_2 a_1 a_2 S\theta_2$$

$$G_1 = \tfrac{1}{2} a_1 m_1 g \, C\theta_1 + m_2 g \left(a_1 C\theta_1 + \tfrac{1}{2} a_2 \cos(\theta_1 + \theta_2)\right)$$

$$G_2 = \tfrac{1}{2} a_2 m_2 g \cos(\theta_2 + \theta_1)$$

Intuition of Dynamic Coupling



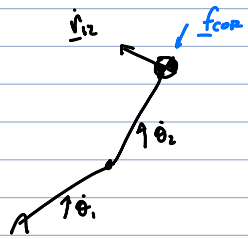
Assume 1^{st} joint $\ddot{\theta}_1 = 0$

$\ddot{\theta}_2 \neq 0$

$\underline{f}_{12}$ needs to be there to balance out other force

$$\tau_1 = \underline{z}_0 \cdot \left(\underline{n}_{12} + \underline{d}_{01} \times \underline{f}_{12}\right)$$

$$= H_{12} \ddot{\theta}_2$$



If we have link 2 accelerating, joint 1 will feel a torque

$$\underline{f}_{cor2} = 2 m_2 \underline{\omega}_{01} \times \dot{\underline{r}}_{12}$$

only causes torque on joint 1

artifact of expressing things in an old coordinate frame

$$\tau_1 = \underline{z}_0 \left(d_{01} \times \underline{f}_{cor}\right) = -2h \dot{\theta}_1 \dot{\theta}_2$$

Coriolis force

- is not a real force
- is an artifact of using relative γ velocities in our dynamic formulation
- We could define absolute joint γ's : $\{\phi_1, \phi_2\}$ then the Coriolis terms would go away

$$\phi_1 = \theta_1$$

$$\phi_2 = \theta_1 + \theta_2$$