10 Centralized Control

- Decentralized controllers have not decoupled influence of controller gains on the joints
- We want a controller that linearizes and decouples dynamics

O.L. Dynamics: $\tau = H'(\Theta)\ddot{\Theta} + \underbrace{V(\Theta,\dot{\Theta}) + G(\theta) + F(\dot{\theta})}_{N(\Theta,\dot{\Theta})}$ non-linearities

$$\rightsquigarrow \quad ① \quad \tau = H(\Theta)\ddot{\Theta} + N(\Theta,\dot{\Theta})$$

Consider a new control law:

u is a control effort

$$② \quad \tau = \hat{H}(\Theta)\, u + N(\theta,\dot{\theta})$$

where $u = \underbrace{K_p\tilde{\Theta} + K_D\dot{\tilde{\Theta}} + K_I\int\tilde{\Theta}\,dt}_{\hat{u}} + \ddot{\Theta}_d$

If we set ① = ②

$$H(\Theta)\ddot{\Theta} + N(\Theta,\dot{\Theta}) = \hat{H}(\Theta)\, u + N(\theta,\dot{\theta})$$

If we have a perfect model... $\hat{N} = N \qquad \hat{H} = H$

$$\Rightarrow \quad \ddot{\Theta} = H^{-1} H\, u = u$$

$$= \hat{u} + \ddot{\Theta}_d$$

desired $\ddot{\Theta}$

$$\ddot{\Theta}_d - \ddot{\Theta} = -\hat{u}$$

$$\therefore \quad \boxed{\ddot{\tilde{\Theta}} = -\hat{u}} \quad \text{closed loop dynamics}$$

acceleration of error = $-\hat{u}$

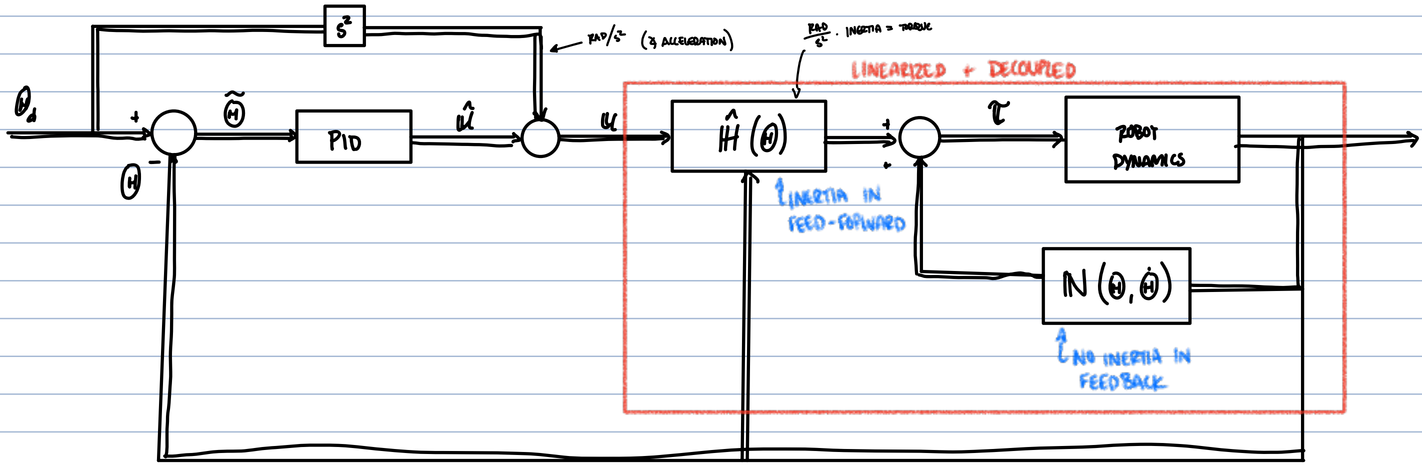
Decoupled:
output of PD on channel 1 reduces $\tilde{\theta}_1$
output of PD on channel 2 reduces $\tilde{\theta}_2$

$$\therefore \begin{bmatrix} \ddot{\tilde{\theta}}_1 \\ \vdots \\ \ddot{\tilde{\theta}}_n \end{bmatrix} = \begin{bmatrix} -\hat{u}_1 \\ \vdots \\ -\hat{u}_n \end{bmatrix} = \begin{bmatrix} -(K_{P_1} + K_{D_1}s + K_I/s)\,\tilde{\theta}_1 \\ \vdots \\ (K_{P_n} + K_{D_n}s + K_{In}/s)\,\tilde{\theta}_n \end{bmatrix} = s^2 \begin{bmatrix} \tilde{\theta}_1 \\ \vdots \\ \tilde{\theta}_n \end{bmatrix}$$

fully decoupled

$\tilde{\tau}_i$ accelerates $\tilde{\theta}_i$, not other joints $(\tilde{\theta}_j)$

INVERSE DYNAMICS CONTROL a.k.a. FEEDBACK LINEARIZATION

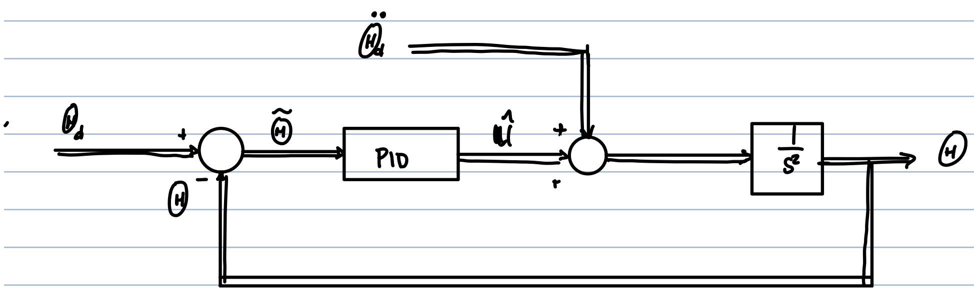


SPREADS OUT ACTION OF PD ON ALL JOINTS

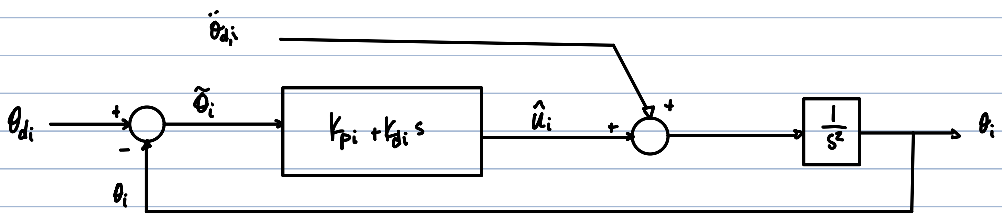
- u RESULTS IN A TORQUE ON ALL JOINTS
- ERROR IS DISTRIBUTED TO ALL JOINTS TO DECOUPLE

$s^2 \ddot{\tilde{\Theta}} = u$

$\rightsquigarrow \frac{\tilde{\Theta}}{u} = s^2 \rightsquigarrow$ EVERYTHING IN RED BLOCK BEHAVES LIKE A TRANSFER FUNCTION $\frac{1}{s^2}$

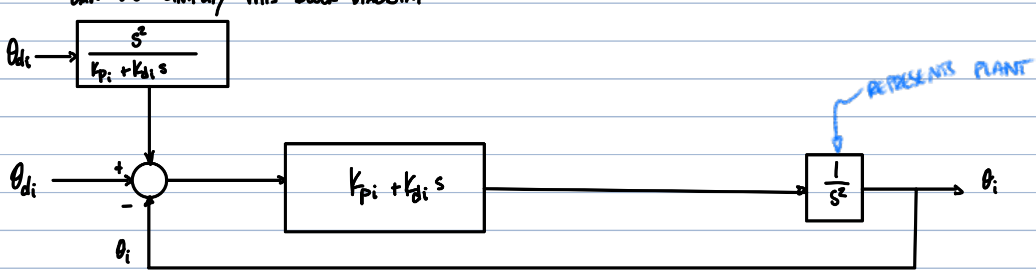


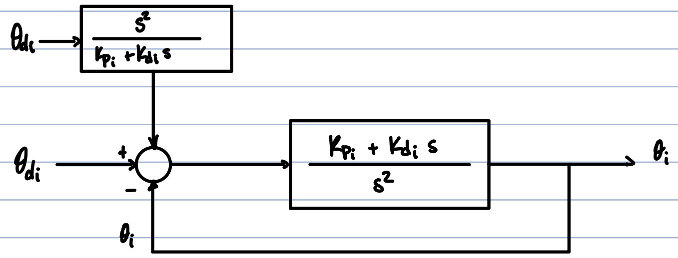
EQUIVALENT TO n DECOUPLED 1-DOF CONTROL SYSTEMS

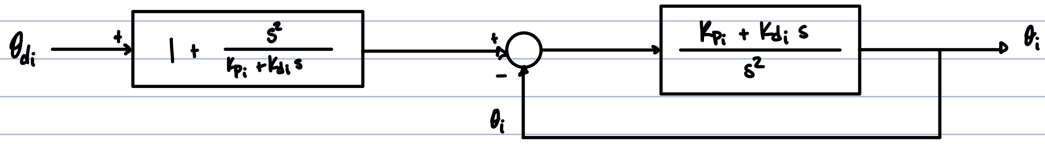


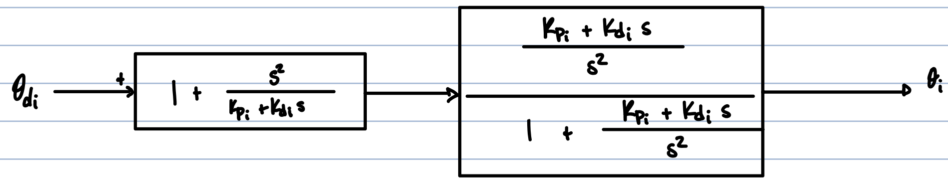
INSTEAD OF DOING A TORQUE OUTPUT, WE CAN THINK OF THIS AS OUTPUTTING THE CORRECT ACCELERATION W/ DECOUPLING + DYNAMICS DONE BY SYSTEM

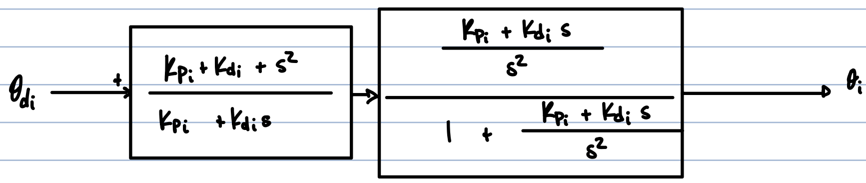
CAN WE SIMPLIFY THIS BLOCK DIAGRAM?

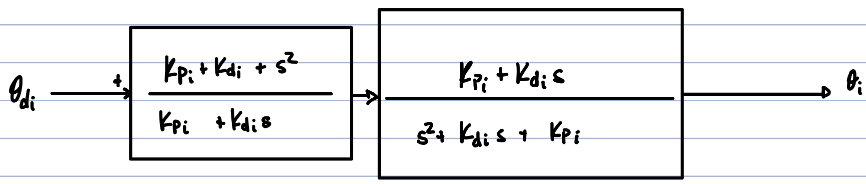


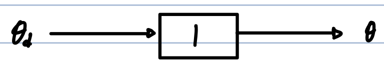








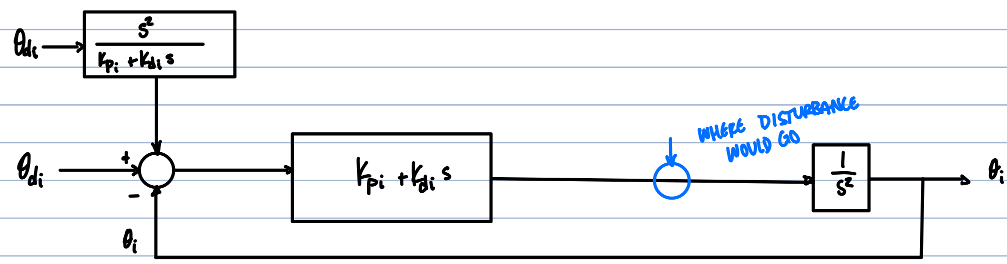




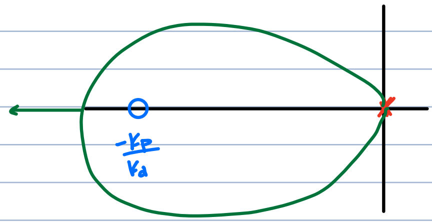
WE HAVE PERFECT TRACKING, REGARDLESS OF PD GAINS

- WE'LL NEVER HAVE A PERFECT MODEL OF EVERYTHING

→ IN PRACTICE, WE'LL STILL NEED A GOOD SET OF PD GAINS FOR DISTURBANCE REJECTION



$$\rightsquigarrow \quad \frac{\theta(s)}{d(s)} = \frac{1}{s^2 + K_{d_i} s + K_{p_i}}$$



Poles from $\frac{1}{s^2}$ plant

= Root locus plot

- We can choose $\frac{-K_p}{K_d}$ to put C.L. poles wherever I want to set a good disturbance rejection

We can use the same PD (or PID) on all n joints

- We can even use same PD on different robots

Simulating Forward Dynamics

Open loop eqn

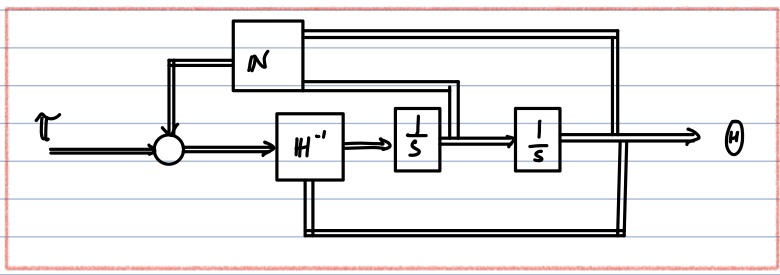
$$\tau = H(\Theta)\ddot{\Theta} + N(\Theta, \dot{\Theta})$$

We want to solve for Θ as the output

$$H(\Theta)\ddot{\Theta} = \tau - N(\Theta, \dot{\Theta})$$

$$s^2\Theta(s) = \ddot{\Theta} = H^{-1}(\Theta)\left[\tau - N(\theta, \dot{\theta})\right]$$

$$\Theta(s) = \frac{1}{s^2} H^{-1}\left[\tau - N\right]$$



Forward dynamics
(Simulating the robot)

If we insert this into controller block diagram, then inverse dynamics cancels forward dynamics

$$\Theta_d \rightarrow \boxed{1} \rightarrow \Theta$$