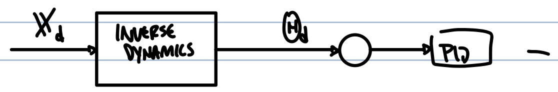
16 Operational Space Control

$\dot{\hat{\alpha}} = \Gamma^{-1} Y^T \sigma$ } Last time stuff

σ: tracking error

All controllers have been designed in joint space

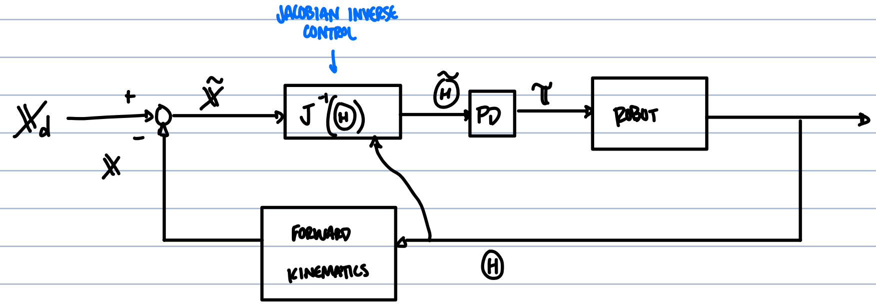
- Desired, error, & actual thetas are in joint space



We could also penalize error in terms of end-effector position
"Operational space" aka "Task Space"

Jacobian Inverse Control

$$X_d = \begin{bmatrix} x \\ y \\ z \\ \theta \\ \phi \\ \psi \end{bmatrix}$$



$J^{-1}(\Theta)$

$$\dot{X} = J(\Theta)\dot{\Theta}$$

small

$$\frac{dX}{dt} = J \frac{d\Theta}{dt}$$

small Δt

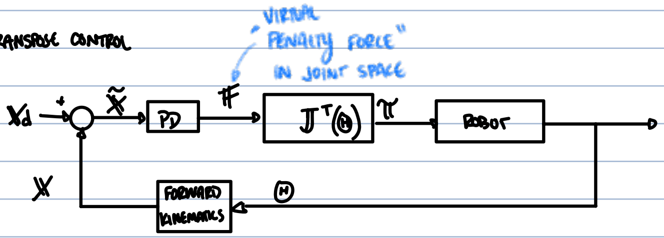
$$\therefore \Delta X = J \Delta \Theta$$

The Jacobian maps between differential motion in joint space & op-space (operational space)

$$\Delta \Theta = J^{-1} \Delta X$$

$$\therefore \tilde{\Theta} \approx J^{-1} \tilde{X}$$

Jacobian Transpose Control



We use a J^T to map between forces & a J^{-1} for mapping between velocities

In both J^{-1} & J^T control, PD gains act like virtual stiffness/damping

J^{-1} control: stiffness damping wrt joint

J^{-1} control: stiffness damping wrt end-effector motion

i.e. K_{p_x} K_{p_y} K_{p_z}

- You can make different stiffness levels

How would we make ${}^{\Theta}K_p = {}^{X}K_p$

If we pick $({}^{\Theta}K_p) J^{-1} = J^T ({}^{X}K_p)$ then J^{-1} & J^T control are equivalent

LEFT-TO-RIGHT IS OPPOSITE TO BLOCK DIAGRAM

For these to be equal...

$${}^{\Theta}K_p = J^T ({}^{X}K_p) J$$

$\therefore$ ${}^{\Theta}K_p$ will have off-diagonal gains even if ${}^{X}K_p$ doesn't have diagonal

· Maps gains from op-space to joint-space

Pros/Cons of op-space control

Pros

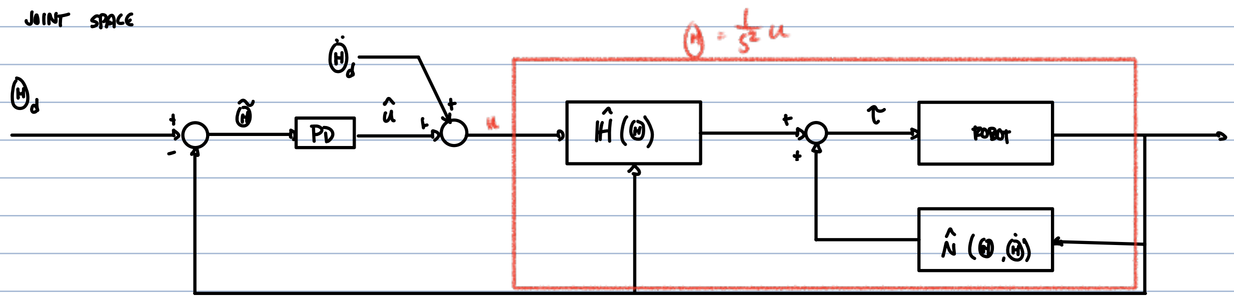
- More intuitive
  - Especially with force control
- Don't need inverse dynamics

Cons

- Assumes small errors for J^{-1} & J^T mapping to be correct
- Singularities are an issue
  - J^{-1} blows up, $J^T \to 0$ $(\tau \to \infty)$
- Redundancies (non-square Jacobian) are difficult to handle (pseudo-inverse)
  - Singularities act as "toggle points"
    - i.e. elbow up vs elbow down
  - Op space has no control over that configuration

Op-space version of inverse dynamics control

Joint space



How do we relate this to op-space

$$\dot{X} = J\dot{\Theta}$$

$$\ddot{X} = J\ddot{\Theta} + \dot{J}\dot{\Theta}$$

$$\ddot{\Theta} = J^{-1}(\ddot{X} - \dot{J}\dot{\Theta})$$

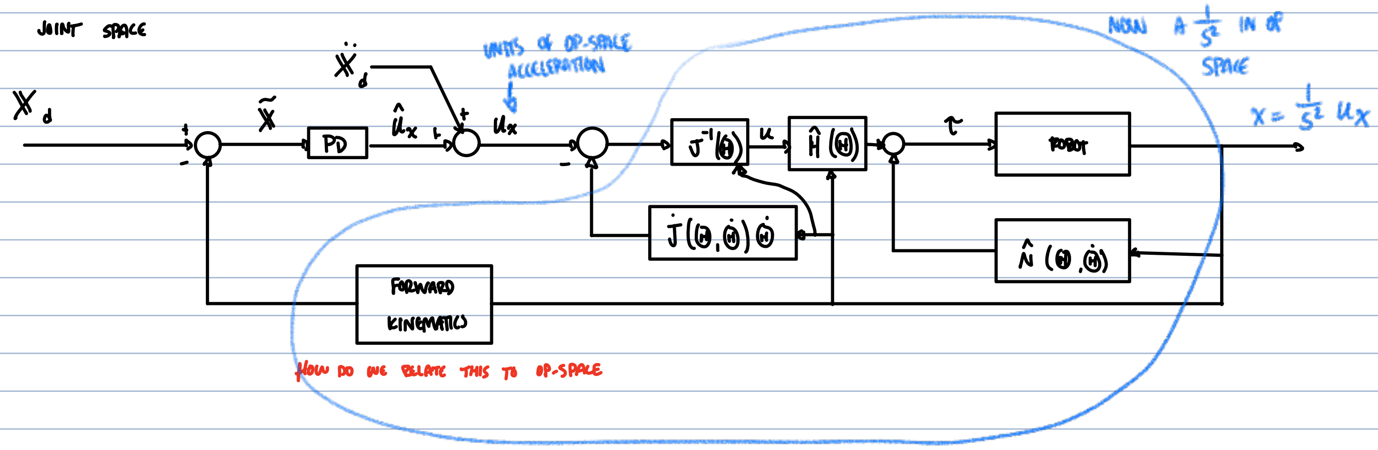
u is in units of acceleration

Suggests to choose control law where $u = J^{-1}(u_x - \dot{J}\dot{\Theta})$

u in joint space; $u_x = u$ in op-space

$$u_x = \ddot{X}_d + \hat{u}_x$$

JOINT SPACE



LYAPUNOV SAYS THIS IS STABLE

STILL HAS J^{-1} INSIDE OF IT

· IS THIS J^{-1} CONTROL OR J^T CONTROL ?

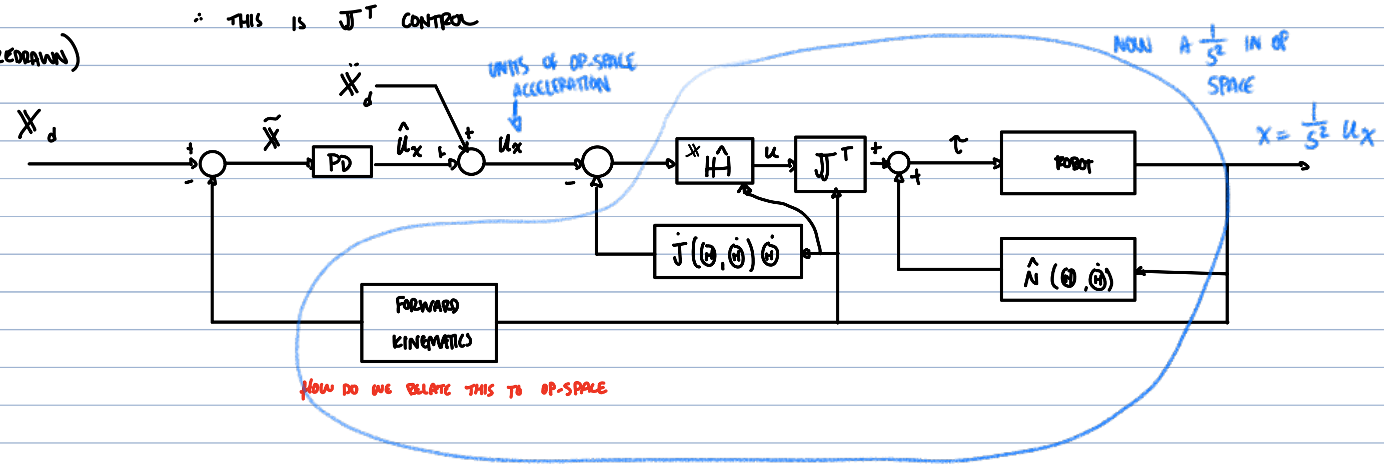
$$\hat{H}(\Theta)\, J^{-1}(\Theta) = J^T\big(\underbrace{J^{-T}\hat{H}(\Theta)\, J^{-1}}_{\text{JACOBIAN SANDWICH}}\big)$$

MAPS THINGS

$$= {}^{x}\hat{H}$$ = EQUIVALENT INERTIA IN OP-SPACE

∴ THIS IS J^T CONTROL

(REDRAWN)



STILL HAS J^{-1} INSIDE OF IT

BEHAVES LIKE 3 INDEPENDENT, DECOUPLED CONTROLLERS IN X Y Z CONTROLLERS
(OR 6 IN $xyz\ \theta\ \phi\ \psi$)