FORMULA SHEET ROBOT CONTROL

$$a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

CHAIN RULE $\frac{d}{dx}\left[f(x)\, g(x)\right] = f'(x)\, g(x) + f(x)\, g'(x)$

PRODUCT RULE $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\, g'(x)$

QUOTIENT RULE $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

DH PARAMETERS

a_i : FROM $\underline{z}_{i-1}$ TO $\underline{z}_i$ ALONG $\underline{x}_i$

d_i : FROM $\underline{x}_{i-1}$ TO $\underline{x}_i$ ALONG $\underline{z}_{i-1}$

α_i : ∡ FROM z_{i-1} TO z_i ABOUT $\underline{x}_i$

θ_i : ∡ FROM $\underline{x}_{i-1}$ TO $\underline{x}_i$ ABOUT $\underline{z}_{i-1}$

IF ROTARY, $\underline{x}_i = \underline{x}_{i-1}$ IN ZERO ∡

JOINT i : CONNECTS LINK i TO LINK $i-1$

z_{i-1} : LOCATED AT JOINT i

θ_i : INTERSECTION OF $\underline{n}_i$ FROM z_i

$\underline{x}_i$: // TO $\underline{n}_i$ IN DIRECTION FROM $\underline{z}_{i-1}$ TO $\underline{z}_i$

$${}^0R_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 C\alpha_1 & S\theta_1 S\alpha_1 \\ S\theta & C\theta_1 C\alpha_1 & -C\theta_1 S\alpha_1 \\ 0 & S\alpha_1 & C\alpha_1 \end{bmatrix}$$

$${}^0T_1 = \begin{bmatrix} {}^0R_1 & -{}^0R_1^T\, {}^0\underline{d}_{01} \\ \underline{0} & 1 \end{bmatrix}$$

$${}^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

JACOBIANS

JACOBIAN: MATRIX REPRESENTING DERIVATIVES OF A VECTOR FUNCTION W.R.T. ITS INPUTS

- JACOBIAN OF f FROM R^n TO R^m IS SIZE $m \times n$

$$J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}$$

$\dot{\Theta}$ = STATE VECTOR (JOINT ∡'S)

${}^0\dot{d}_{on}$ = VELOCITY END EFFECTOR

${}^0\omega_{on}$ = ∡ VELOCITY OF END EFFECTOR

MANIPULATOR

$${}^0\dot{\underline{d}}_{on} = J\dot{\theta}$$

$$J = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \cdots & \frac{\partial x}{\partial \theta_n} \\ \frac{\partial y}{\partial \theta_1} & \cdots & \frac{\partial y}{\partial \theta_n} \\ \frac{\partial z}{\partial \theta_1} & \cdots & \frac{\partial z}{\partial \theta_n} \end{bmatrix}$$

J= MANIPULATOR JACOBIAN

- LOTS OF COMPUTATION, ONLY POSITION

TRANSMISSION:

VELOCITY:

$$\underbrace{\begin{bmatrix} {}^0\dot{\underline{d}}_{on} \\ {}^0\underline{\omega}_{on} \end{bmatrix}}_{\text{TYPICALLY } 6\times 1} = \underbrace{J_v}_{\text{TYPICALLY } 6\times n} \underbrace{\dot{\Theta}}_{n\times 1}$$

J_v = VELOCITY JACOBIAN

$$\tau = J_v^T\, W_{n+1}$$

$$\tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix}$$

$$\underbrace{W}_{\text{WRENCH}} = \begin{bmatrix} f_{n,n+1} \\ n_{n,n+1} \end{bmatrix} \begin{matrix} \text{FORCES} \\ \text{TORQUES} \end{matrix}$$

$$J_v = \begin{bmatrix} {}^0\underline{z}_0 \times {}^0\underline{d}_{on} & {}^0\underline{z}_1 \times {}^0\underline{d}_{1n} & \cdots & {}^0\underline{z}_{n-1} \times {}^0 d_{n-1,n} \\ {}^0\underline{z}_0 & {}^0\underline{z}_1 & \cdots & {}^0\underline{z}_{n-1} \end{bmatrix}$$

- LESS COMPUTATION, PLUS IT GIVES YOU ∡ VELOCITY

JACOBIAN'S USES

INVERSE VELOCITY:
- YOU CAN INVERT J_v TO GIVE VELOCITIES
- ONLY WORKS IF J_v IS SQUARE, AND THERE AREN'T SINGULARITIES $(\det(J_v) = 0)$
- SINGULARITIES OCCUR AT EDGE OF ROBOT'S REACH, OR OUTSIDE OF ITS CAPABILITIES
- IF YOU HAVE REDUNDANT DOF, YOU USE A PSEUDO-INVERSE TO CANCEL REDUNDANCIES

TRANSMISSION

- LETS YOU REWRITE FROM JOINT SPACE INTO MOTOR SPACE

$$\dot{\Theta} = J_t\, \dot{\Phi}$$

$\dot{\Phi}$ = MOTOR SPEEDS

$$\tau_M = J_t^{-1}\, \tau_{JOINT}$$

← DOESN'T WORK IF J_t NOT SQUARE

$$J_t = \begin{bmatrix} \frac{1}{N_1} & 0 & \cdots \\ 0 & \frac{1}{N_2} & \\ \vdots & & \ddots \\ & & & \frac{1}{N_n} \end{bmatrix}$$

← NOT A HARD/FAST RULE

WRITE EQNS OF JOINT VELOCITIES, & HOW THEY RELATE TO MOTOR VELOCITIES. USE THAT TO BUILD J_t

EXAMPLE FULL DYNAMICS

$$\tau = H(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta)$$

H = INERTIAL MATRIX

V = VELOCITY TERMS (A.K.A CORIOLIS)

G = GRAVITY

FULL DYNAMICS W/ TRANSMISSION

$$\tau_{JOINT} = H'(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta)$$

$$J_t^T \tau_{JOINT} = J_t^T H'(\Theta) J_t \ddot{\Phi} + J_t^T V(\Theta, \dot{\Theta}) + J_t^T G(\Theta)$$

$$\tau_m = H'(\Phi)\ddot{\Phi} + V(\Phi, \dot{\Phi}) + G(\Phi)$$

τ_m = MOTOR TORQUES

WHERE
$$H' = \begin{bmatrix} J_1 + \frac{H_{11}}{N_1^2} & \frac{H_{12}}{N_1 N_2} & \cdots \\ \frac{H_{21}}{N_1 N_2} & J_2 + \frac{H_{22}}{N_2^2} & \\ & & \ddots \\ & & & \boxed{J_n + \frac{H_{nn}}{N_n^2}} \end{bmatrix}$$

NOT SURE IF THAT'S RIGHT (MAYBE $J_n + \frac{H_{nn}}{H_n^2}$)

DRIVETRAIN DYNAMICS

$$N_i = \frac{r_B}{r_A}$$

N_i = GEAR RATIO

$$\dot{\phi}_i = N\, \dot{\theta}_i$$

$\dot{\theta}_i$ = JOINT VELOCITY

$\dot{\phi}_i$ = MOTOR VELOCITY

$$\tau_B = N_i \tau_A$$

DC MOTOR DYNAMICS

$$V_b = K_b \dot{\phi}$$

ϕ = MOTOR SPEED

$$\tau_m = K_t\, i_a$$

K_b = BACK EMF

IF IN S.I. UNITS : $K_t = K_b$

$$V_a = R_a i_a + K_t \dot{\phi}$$

V_a = ARMATURE VOLTAGE

R_a = ARMATURE RESISTANCE

i_a = ARMATURE CURRENT

$$\tau_m = \frac{K_t V_a}{R_a} - \frac{K_t}{R_a}\dot{\phi}$$

$$\Omega_{zyz} = \dot{\phi}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \dot{\theta}\begin{bmatrix} -S\phi \\ C\phi \\ 0 \end{bmatrix} + \dot{\psi}\begin{bmatrix} C\phi S\theta \\ S\phi S\theta \\ C\theta \end{bmatrix}$$

VELOCITY + ACCELERATION

$\dot{R} = \omega R$ $\quad\quad$ ${}^{0}\dot{\underline{p}}_{1} = {}^{0}\dot{R}_{1}\,{}^{1}\underline{p}_{1} = S(\underline{\omega})\,{}^{0}\dot{R}_{1}\,{}^{1}\underline{p}_{1} = {}^{0}\underline{\omega}_{01} \times {}^{0}\underline{p}_{1}$

CONTROL

P:

PD:

PID: