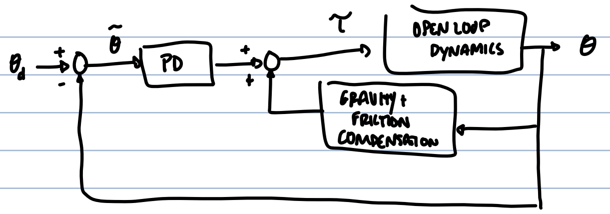


1.1 (THIS WITHOUT GRAVITY/FEEDBACK)
1.2 (THIS)

OPEN LOOP DYNAMICS

$$I\ddot{\theta} + f(\theta,\dot{\theta}) = \tau$$

NONLINEAR GRAVITY/FRICTION

$\hat{f}$ = MODEL OF f's REAL EFFECT (WE'D MODEL FRICTION)

CONTROL LAW: $\tau = K_p\tilde{\theta} + K_d\dot{\tilde{\theta}} + \underbrace{\hat{f}(\theta,\dot{\theta})}_{\text{MODEL OF GRAVITY/FRICTION}}$

WHAT IF $\|\hat{f}(\theta,\dot{\theta}) - f(\theta,\dot{\theta})\| = \|\tilde{f}(\theta,\dot{\theta})\|$

MODEL ERROR

WE CAN'T GUARANTEE ASYMPTOTIC STABILITY $\tilde{\theta} = 0$
USING PD CONTROL

IF WE WANT $\tilde{\theta} \to 0$, USE PID CONTROL

$\therefore$ CONTROL LAW: $\tau = K_p\tilde{\theta} + K_d\dot{\tilde{\theta}} + K_i\xi + \hat{f}(\theta,\dot{\theta})$

$$\dot{\xi} = \tilde{\theta} \quad \therefore \xi = \int\tilde{\theta}$$

LYAPUNOV CANDIDATE

$$V(\tilde{\theta}, \dot{\theta}, \xi)$$

NEW 3RD STATE
(3RD ORDER SYSTEM)
- WE WANT ALL ERROR TO GO TO ZERO

LYAPUNOV RESULTS
- GIVEN CERTAIN CONDITIONS + BOUNDS ON THE GAINS, WE CAN SHOW THE SYSTEM IS ASYMPTOTICALLY STABLE BY LYAPUNOV/LASALLE

MORE GENERALLY, WE CAN SHOW PID CONTROL IS ASYMPTOTICALLY STABLE FOR A MULTI-DOF ROBOT

PROOF DONE BY:
[ARIMOTO + MIYAZAKI, 1983]
PROOF TO GUARANTEE PID CONTROL ON A MULTI-DOF ROBOT
(IT'S A MESS, IT'S DONE ☺)

GUARANTEED ERROR GOING TO ZERO.. GIVEN INFINITE TIME

NO GUARANTEE HOW FAST $\tilde{\theta} \to 0$

CAN WE COMPENSATE FOR ALL NONLINEARITIES IN A WAY THAT IMPROVES TRACKING PERFORMANCE?

SPLIT OPEN-LOOP DYNAMICS INTO LINEAR + NON-LINEAR DYNAMICS

1. $$\tau = [I + \underbrace{H^*(\Theta)}_{\text{NON-LINEAR}}]\ddot{\Theta} + V(\Theta, \dot{\Theta}) + \underbrace{B\dot{\Theta}}_{\text{LINEAR DAMPING}} + \underbrace{F^*(\dot{\Theta})}_{\text{NON-LINEAR DAMPING}} + G\Theta$$

2. CONTROL LAW: $$\tau = \underbrace{\left(K_P + K_d s + \frac{K_I}{s}\right)}_{\text{MATRICES}}\tilde{\Theta} + \hat{H}(\Theta)\ddot{\Theta} + \tilde{V}(\Theta, \dot{\Theta}) + G(\Theta) + \underbrace{F^*(\dot{\Theta})}_{\text{MODEL OF NON-LINEAR DAMPING}}$$

JOINT ERRORS

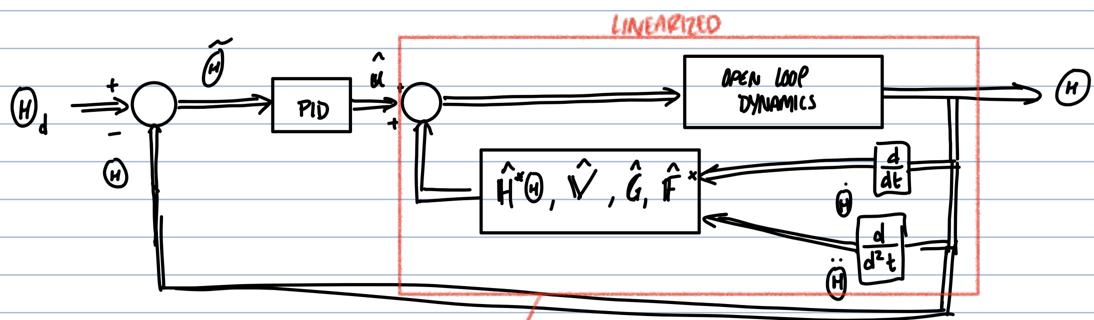
MODEL OF NON-LINEAR

IF WE WANT C.L.

1 = 2 IF WE HAVE PERFECT MODELS, THEY'LL CANCEL W/ NON-LINEAR TERMS

$$\therefore I\ddot{\Theta} + B\dot{\Theta} = \left(K_P + K_D s + \frac{K_I}{s}\right)\tilde{\Theta}$$

RESULTING CLOSED LOOP DYNAMICS ARE LINEAR

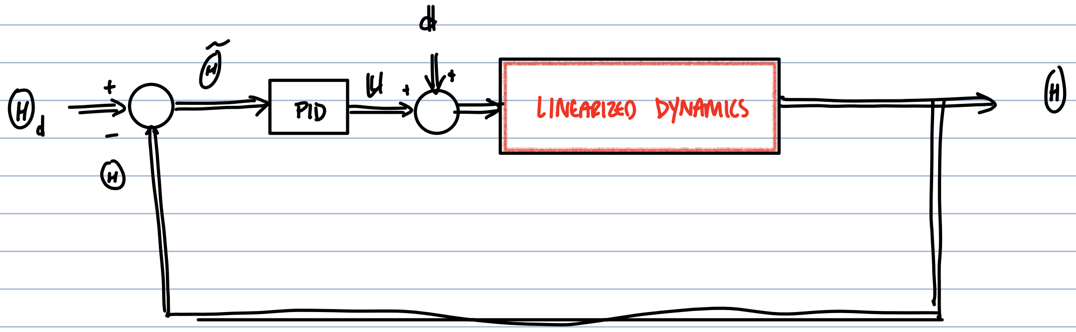


NOW PID CONTROLLER THINKS EVERYTHING IS LINEAR

DYNAMICS LINEARIZED WRT PID CONTROLLER

IF WE HAVE AN IMPERFECT MODEL, WE CAN VIEW LEFTOVER NON-LINEARITIES AS A DISTURBANCE ON OUR SYSTEM

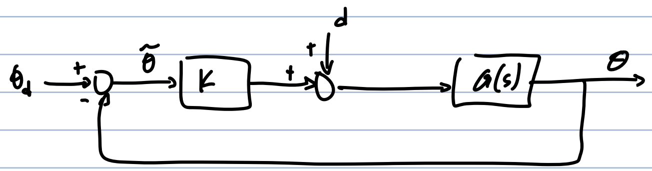
$$I\ddot{\Theta} + B\dot{\Theta} = u + d(\Theta, \dot{\Theta}, \ddot{\Theta})$$



NOW WE WANT TO INCREASE GAINS TO BETTER MINIMIZE EFFECTS OF DISTURBANCE

⇒ LARGER PID GAINS :)

EG. 1-DOF EXAMPLE



$$\theta(s) = G(s)\left[d(s) + \underbrace{K\tilde{\theta}(s)}_{G(s)} \right]$$

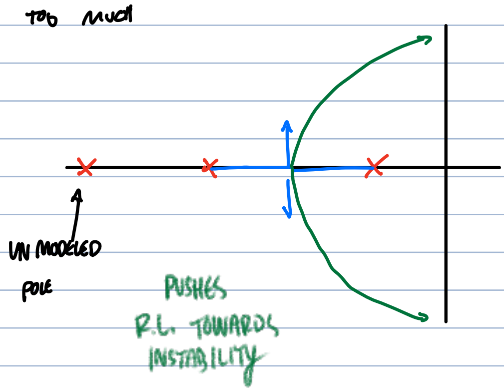
IF $\theta_d = \theta$

$$\theta(s) = \left[\frac{G(s)}{1 + KG(s)} \right] d(s)$$

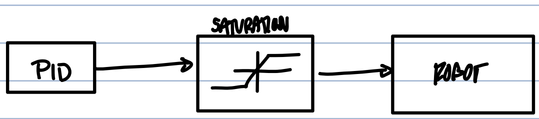
AS $K \uparrow$, SMALLER EFFECT OF DISTURBANCE ON A TRAJECTORY

<u>BUT</u> IF WE INCREASE K TOO MUCH

1. UNMODELED POLES CREATE INSTABILITY



2. CAN SATURATE AMPLIFIER

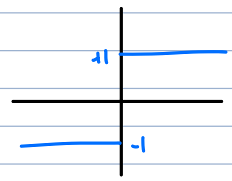


SATURATION IS A NON-LINEARITY THAT CREATES INSTABILITY (LIMIT CYCLING)

OTHER PROBLEMS:

- DIFFERENTIATING Θ TO GET $\dot{\Theta}$ & $\ddot{\Theta}$ AMPLIFIES SENSOR NOISE
- MEASURING $\dot{\Theta}$ & $\ddot{\Theta}$ IS ALSO NOISY
- NONLINEARITIES CAN AMPLIFY NOISE EVEN FURTHER

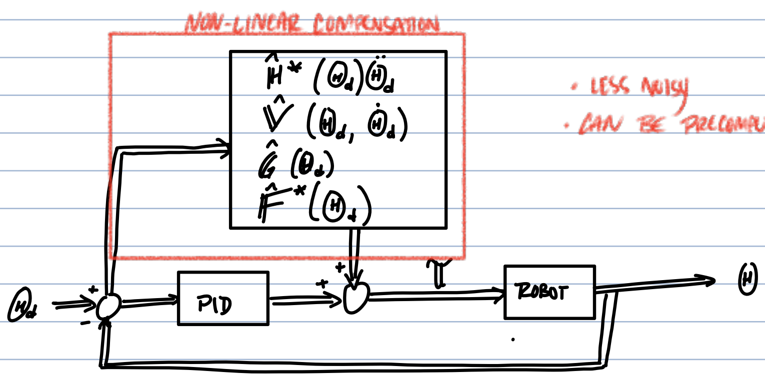
E.G. $\hat{F}(\dot{\theta}) = c\ \mathrm{sgn}(\dot{\theta})$



IF $\dot{\theta}$ HOVERS AROUND 0,
THEN $\hat{F}$ WILL BOUNCE BACK
& FORTH (CHATTER)

ALTERNATIVE: FEEDFORWARD COMPUTED TORQUE CONTROL

- USE DESIRED TRAJECTORY RATHER THAN ACTUAL TRAJECTORY TO COMPUTE NON-LINEAR PARTS OF SYSTEM



- LESS NOISY
- CAN BE PRECOMPUTED

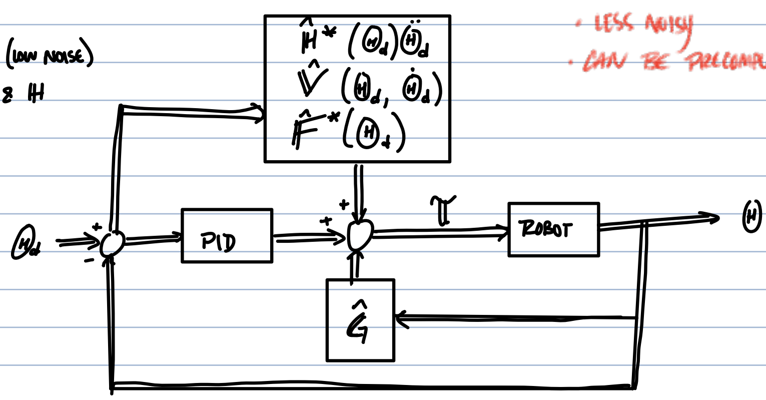
IF θ_d IS KNOWN AHEAD OF TIME, WE CAN PRE-COMPUTE ALL NON-LINEAR COMPENSATION

IF A DISTURBANCE KNOCKS YOU OFF, OR MODEL IS NOT VERY GOOD, THEN Θ_d DEVIATES FROM Θ, MAKING COMPENSATION INACCURATE

ALTERNATIVE: MIX OF FEED FORWARD & FEEDBACK COMPENSATION

EXAMPLE:

- FEED BACK GRAVITY (LOW NOISE)
- FEED FORWARD F & H



- LESS NOISY
- CAN BE PRECOMPUTED

THIS ALL MIGHT BE UNNECESSARY

- MAYBE $\dot{\theta}$ IS SMALL SO WE CAN LEAVE OUT $\hat{V}$ COMPENSATION
- MAYBE ROBOT IS LIGHTWEIGHT OR ACCELERATING SLOWLY $\hat{H}$ COMPENSATION OUT

IF $\ddot{\Theta} \ll 1$ LEAVE $\hat{H}$ OUT

IF $\dot{\Theta} \ll 1$ LEAVE V OUT

IF $N \gg 1$ $\therefore$ J_m DOMINATES $\Rightarrow$ NEGLECT I_{ARM}, NEGLECT $\hat{H}$ COMPENSATION

GEAR RATIO

REGARDLESS OF FEEDBACK VS FEEDFORWARD, PID CONTROL IS STILL DECENTRALIZED

ALL OF THESE CONTROLLERS ARE DECENTRALIZED

CONTROL EFFORT u_i (AND THUS τ_i) DEPEND ONLY ON $\tilde{\theta}_i$

OUR DYNAMICS ARE COUPLED, THIS IS AN ISSUE

WE NEED A CENTRALIZED CONTROLLER THAT LINEARIZES AND DECOUPLES THE PID CONTROL ACTION