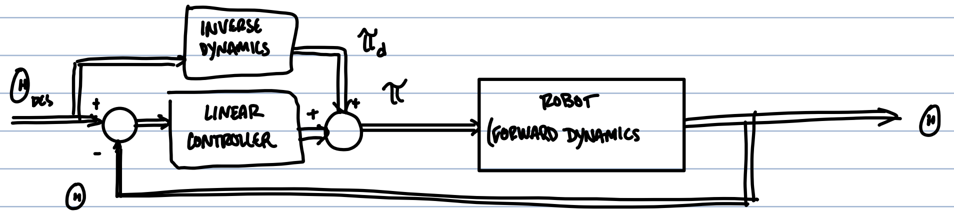
ROBOT CONTROL 4

INVERSE DYNAMICS (RECAP)

$$\tau = H(\Theta)\ddot{\Theta} + V(\Theta,\dot{\Theta}) + G(\Theta)$$

↑ REALLY USEFUL IF WE HAVE A GOOD DYNAMICS MODEL



$$\tau_i = \sum_{j=1}^{n} \underbrace{H_{ij}\,\ddot{\theta}_j}_{} + \sum_{j=1}^{n}\sum_{k=1}^{n} \underbrace{h_{ijk}\,\dot{\theta}_j\,\dot{\theta}_k}_{n^4 \text{ OPERATIONS}}$$

IF n DOF

$n \times n \times n$ COMPUTATIONS $O(n^3)$

IF $n = 6$ ⤳ 70,000 OPERATIONS

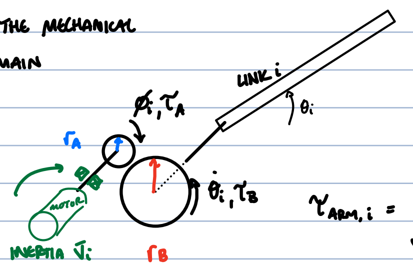
⇒ DOING THIS NUMERICALLY (NEWTON - EULER) IS MUCH BETTER

⤳ > 1000 OPERATIONS

DRIVETRAIN DYNAMICS

TYPICAL ROBOT CONTROL SYSTEM

JUST THE MECHANICAL DOMAIN



$$\tau_{ARM,i} = \sum_{j=1}^{n} H_{ij}(\Theta)\,\ddot{\theta}_j + V_i + G_i$$

ROTATIONAL INERTIA J_i

FRICTION $f_i(\dot{\phi}_i) = \underset{\uparrow}{b_i\,\dot{\phi}_i} + c_i\,\text{SGN}(\dot{\phi})$

VISCOUS FRICTION ↖ COULOMB FRICTION

IF WE THROW A MOTOR ON THIS

- DC MOTOR

MOTOR

$\tau_{m,i} =$

GEAR RATIO $\quad N_i = \dfrac{r_B}{r_A}$

VELOCITY RELATIONSHIP $\quad \dot{\phi}_i = N_i\,\dot{\theta}_i$

TORQUE $\quad \tau_B = N_i\,\tau_A \qquad P_{IN} = P_{OUT}$ FROM GEAR TRAIN

τ_B IS THE TORQUE ON THE ARM

$$N_i\,\tau_A = \tau_B$$
$$= \tau_{ARM,i}$$

$$N_i\left[\tau_{m,i} - J_i\,\ddot{\phi}_i - f_i(\dot{\phi}_i)\right] = \tau_{ARM,i}$$

$$N_i \tau_{m,i} = \tau_{ARM,i} + N_i J_i N \ddot{\theta}_i + N_i f_i(N_i \dot{\theta}_i)$$

$$\rightsquigarrow \underbrace{N_i \tau_{m,i}}_{\tau_{JOINT,i}} = \sum_{j=1}^{n} H_{ij}(\Theta)\ddot{\Theta} + V_i + G_i + N_i^2 J_i \ddot{\theta}_i + \underbrace{N_i f_i(\underbrace{N_i \dot{\theta}_i})}_{\text{FUNCTION (NOT MULTIPLICATION)}}$$

FOR ALL n JOINTS

$$\tau_{JOINT} = \mathbb{H}'(\Theta)\ddot{\Theta} + \mathbb{V}(\Theta, \dot{\Theta}) + \mathbb{G}(\Theta) + \mathbb{F}(\dot{\Theta})$$

IN "JOINT SPACE"

Θ IS JOINT θ'S, ~~NOT~~ MOTOR θ'S

WHERE

$$\mathbb{H}'(\Theta) = \begin{bmatrix} H_{11} + N_1^2 J_1 & H_{12} & \cdots & H_{1n} \\ H_{21} & H_{22} + N_2^2 J_2 & & \\ H_{n1} & & & H_{nn} \end{bmatrix}$$

AUGMENTING INERTIA BY ADDING
MOTOR INERTIA ON DIAGONALS

WE CAN WRITE THESE SAME DYNAMICS IN <u>MOTOR SPACE</u>

BACK TO SCALAR EQN...

$$N_i \tau_{m,i} = \sum_{j=1}^{n} H_{ij}(\Theta)\ddot{\Theta} + V_i + G_i + N_i^2 J_i \ddot{\theta}_i + N_i f_i(N_i \dot{\theta}_i)$$

JOINT SPACE

$$\rightsquigarrow \tau_{m,i} = \frac{1}{N_i}\sum_{j=1}^{n} H_{ij}(\theta)\frac{\ddot{\phi}_j}{N_j} + \frac{1}{N_i}\mathbb{V}(\theta, \dot{\theta}) + \frac{1}{N_i}\mathbb{G}(\theta) + f_i(\dot{\phi}_i) + J_i \ddot{\phi}_i$$

- WE STILL HAVE SOME Θ IN HERE
- WE'LL NEED TO INDIVIDUALLY SUBSTITUTE $\theta_i = \dfrac{\phi_i}{N_i}$ $\qquad \dot{\theta}_i = \dfrac{\dot{\phi}_i}{N_i}$

$\dot{\Phi}$ = MOTOR VELOCITIES

$$\rightsquigarrow \tau_m = \mathbb{H}'(\Phi)\ddot{\Phi} + \mathbb{V}(\Phi, \dot{\Phi}) + \mathbb{G}(\Phi) + \mathbb{F}(\dot{\Phi})$$

MOTOR SPACE

WHERE

$$\mathbb{H}' = \begin{bmatrix} J_1 + \frac{H_{11}}{N_1^2} & \frac{H_{12}}{N_1 N_2} & \cdots & \\ \frac{H_{21}}{N_2 N_1} & J_2 + \frac{H_{22}}{N_2^2} & & \\ \vdots & & & \\ & & & J_n + \frac{H_{nn}}{N_n^2} \end{bmatrix}$$

TO FACILITATE THE MAPPING BETWEEN JOINT SPACE + MOTOR SPACE WE CAN DEFINE A TRANSMISSION JACOBIAN J_t

$$\dot{d}_{n} = J\,\dot{\Theta}$$

$$\dot{\Theta} = \underbrace{J_t}_{\text{TYPICALLY CONSTANT (MATRIX GEAR RATIO}}\,\dot{\Phi}$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{N_2} & & \\ \vdots & & \ddots & \\ 0 & & & \frac{1}{N_n} \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \\ \vdots \\ \dot{\phi}_n \end{bmatrix}$$

JUST AS $\tau_{JOINT} = J^T W_{n+1}$

$\tau_m = J_t^T \tau_{JOINT}$

$$\tau_m = J_t^T J^T W_{n+1} = (J J_t)^T W_{n+1}$$

NOW WE CAN USE J_t TO REFLECT DYNAMICS BACK + FORTH BETWEEN MOTOR SPACE + JOINT SPACE

$$\tau_{JOINT} = H'(\Theta)\ddot{\Theta} + V(\Theta,\dot{\Theta}) + G(\Theta) + F(\dot{\Theta})$$

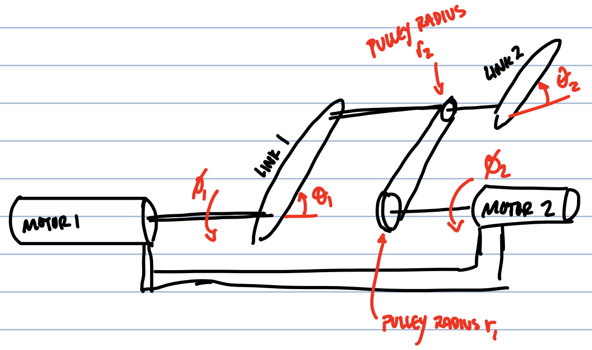
$$\tau_m = J_t^T \tau_{JOINT} = \underbrace{J_t^T H'(\Theta) J_t}_{\text{JACOBIAN SANDWICH}}\ddot{\Phi} + \underbrace{J_t^T V(\Theta,\dot{\Theta}) + J_t^T G(\Theta) + J_t^T F(\dot{\Theta})}_{\text{WE'LL NEED TO SWAP OUT INDIVIDUAL VARIABLES HERE TO GET THE NICE } V(\Phi,\dot{\Phi}),\ G(\Phi),\ F(\dot{\Phi})}$$

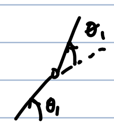
$H'(\Phi)$ MATRIX OPERATION

INVERSING LEADS TO OTHER PROBLEMS

$$\tau_{JOINT} = J_t^{-T}\tau_m = \underbrace{J^{-T} H'(\Phi) J_t^{-1}}_{\text{INVERSE JACOBIAN SANDWICH}}\ddot{\Theta}$$

J_t IS TYPICALLY DIAGONAL IF MOTOR i IS LOCATED ON LINK $i-1$. IF ROBOT IS CABLE/BELT-DRIVEN, THERE MAY BE OFF-DIAGONAL TERMS. THIS IS THE CASE FOR OUR LAB ROBOTS (AND MANY ROBOTS TODAY)



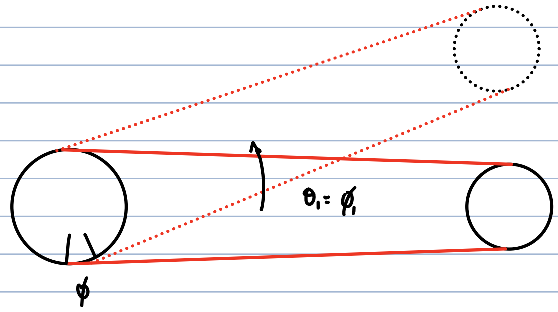


$\theta_1 = \phi_1$

$\theta_2 = \frac{r_1}{r_2}\phi_2 - \frac{r_1}{r_2}\phi_1$

MOTOR 1'S ROTATION WILL AFFECT θ_2

i.e. $\theta_2 = f(\phi_1, \phi_2)$



PULLEY #2 ROTATES BY $\frac{-r_1 \phi_1}{r_2}$