Robust Control

How do we deal with model uncertainties?

- Structured uncertainties
  - Good model of dynamics, not sure about values
  - Equations are good, #'s aren't
- Unstructured uncertainties
  - Unmodeled dynamics + disturbances

Need to make a distinction between

- Regulation (stabilization) control = maintaining a desired position ← PID control — provides robust regulati in presence of model uncertainty (Lyapunov guarantees asymptotic stability)
- Tracking control = moving along a trajectory

Integral guarantees $\tilde{\Theta} \xrightarrow{t \to \infty} 0$

But doesn't guarantee perfect robust tracking control

We could settle for $|\tilde{\Theta}| < \varepsilon$ ← tracking precision

O.L. dynamics of robot

$$\tau = \mathbb{H}(\Theta)\ddot{\Theta} + \mathbb{N}(\Theta, \dot{\Theta})$$

Cancel non-linearities as best we can w/ inverse dynamics control (I.D.C.)

$$\tau = \hat{\mathbb{H}}(\Theta)u + \hat{\mathbb{N}}(\Theta, \dot{\Theta})$$

C.L. dynamics

$$\mathbb{H}\ddot{\Theta} + \mathbb{N} = \hat{\mathbb{H}}u + \hat{\mathbb{N}}$$

let $\tilde{\mathbb{N}} = \hat{\mathbb{N}} - \mathbb{N}$ $\qquad$ $\tilde{\mathbb{N}}$ = error in model

$$\therefore \ddot{\Theta} = \mathbb{H}^{-1}(\hat{\mathbb{H}}u + \tilde{\mathbb{N}})$$

$$= \underbrace{\mathbb{H}^{-1}\hat{\mathbb{H}}}_{\text{might have modeling error}} u + \mathbb{H}^{-1}\tilde{\mathbb{N}}$$

$$= u + \underbrace{(\mathbb{H}^{-1}\hat{\mathbb{H}} - \mathbb{I})}_{\mathbb{I} = \text{identity}} u + \mathbb{H}^{-1}\tilde{\mathbb{N}}$$

We do this so we account for modelling error

$$= u + \underbrace{(\mathbb{H}^{-1}\hat{\mathbb{H}} - \mathbb{I})u + \mathbb{H}^{-1}\tilde{\mathbb{N}}}_{\eta}$$

if $\eta = 0 \Rightarrow$ perfect tracking

Can we design u to compensate for $\eta \neq 0$?

Define a "sliding surface"

e.g. 1-DOF

$$z(\theta, t) = (s + \lambda)\tilde{\theta}$$

s = LAPLACE OPERATOR

λ = CONSTANT

$$\underline{z = 0} = \dot{\tilde{\theta}} + \lambda\tilde{\theta}$$

DIFFERENTIAL EQN WHOSE SOLUTION IS $\tilde{\theta} = 0$

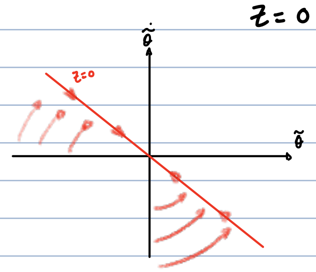
MAKE z^2 A LYAPUNOV FUNCTION V

$$V(z) = z^2$$

DEFINE CONTROL INPUT u TO GUARANTEE z CONVERGES TO ZERO W/ASYMPTOTIC STABILITY

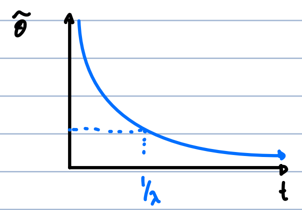
$\left(\frac{d}{dt} z^2\right)$ NEGATIVE DEFINITE

$$z = 0 = \dot{\tilde{\theta}} + \lambda\tilde{\theta}$$



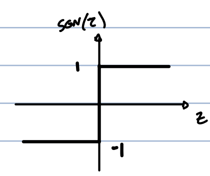
WE WANT TO PUSH SYSTEM'S STATES ONTO THE SURFACE, SO STATES WILL SLIDE TO ZERO

$\rightsquigarrow \tilde{\theta} \to 0$ W/ TIME CONSTANT $1/\lambda$



HOW DO WE MAKE u DO THIS?

$$u = \underbrace{K_p\tilde{\theta} + K_d\dot{\tilde{\theta}} + \ddot{\theta}_d}_{\hat{u}} + \underbrace{\rho \operatorname{sgn}(z)}_{\text{NON-LINEAR SWITCHING ACTION}}$$



WE WANT ρ LARGE ENOUGH SO WE CAN OVERCOME MODELLING ERROR

- SATISFIES LYAPUNOV PROOF IF $\rho \geq \max\|\eta\|$

UNITS OF ACCELERATION

LARGE UNCERTAINTY ⟶ LARGE SWITCHING ACTION

- THIS HITS YOUR SYSTEM AS HARD AS POSSIBLE TO GET BACK TO SLIDING SURFACE
  - YOU'LL NEVER <u>EXACTLY</u> LAND ON THE SLIDING SURFACE (CHATTER)
    - BAD HIGH-FREQUENCY CHATTER
    - SATURATES AMPLIFIER
    - CAN EXCITE UNMODELED DYNAMICS

THIS ISN'T A TERRIBLE STRATEGY FOR TEMPERATURE CONTROL (BANG-BANG CONTROL)

- YOU CAN BUILD IN A DEAD-ZONE INTO SGN FUNCTION

GUARANTEES PERFECT TRACKING IN CONTINUOUS DOMAIN

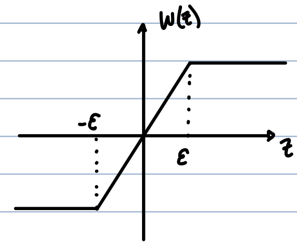
- IN DISCRE TIME DOMAIN, CONTROL LAW CANNOT SWITCH IN INFINITESIMAL TIME STEPS

INSTEAD, CREATE A BOUNDARY LAYER ε AROUND THE SLIDING SURFACE

$$W(z)$$

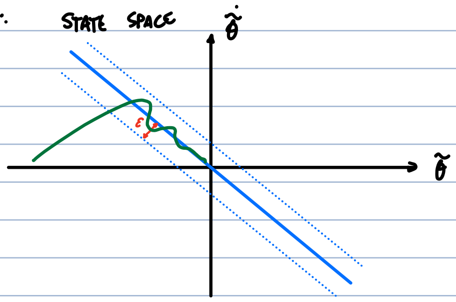
$$u = \hat{u} + \underbrace{\rho \, \mathrm{sat}\left(\frac{z}{\varepsilon}\right)}_{W(z)}$$

SAT IS SIMILAR TO SGN, BUT DOESN'T HAVE DISCONTINUITY

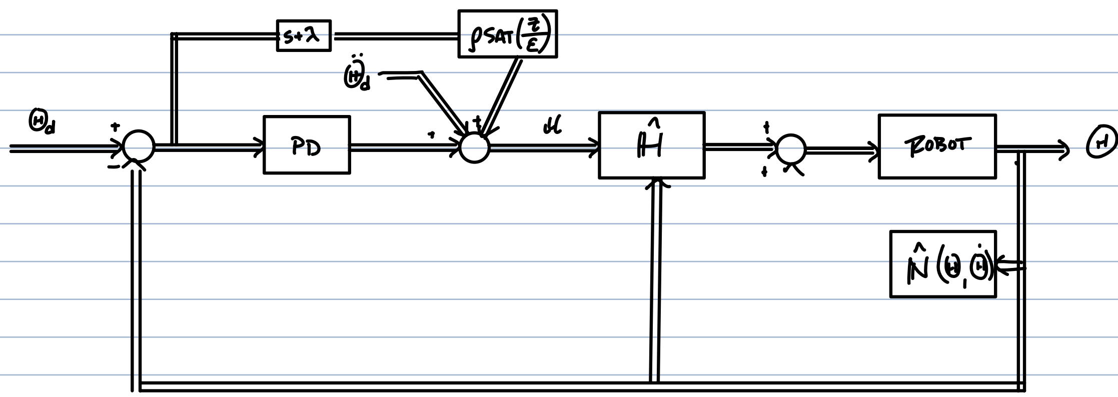


$$\mathrm{sat}\left(\frac{z}{\varepsilon}\right) = \begin{cases} \mathrm{sgn}\left(\frac{z}{\varepsilon}\right) & \text{for } \frac{\|z\|}{\varepsilon} \geq 1 \\ \frac{z}{\varepsilon} & \text{for } \frac{\|z\|}{\varepsilon} < 1 \end{cases}$$

$\therefore$ STATE SPACE



ONCE IN BOUNDARY LAYER, STAY IN BOUNDARY LAYER, SMOOTH TRACKING CONTROL ACTION WITHIN BOUNDARY LAYER



IN MULTI-DOF SPACE

$$\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix} \qquad \frac{1}{\lambda_i} = \text{TIME CONSTANT FOR } i^{\text{TH}} \text{ JOINT}$$

$$W(z) = \begin{bmatrix} \rho_1 \, \mathrm{sat}\left(\frac{z_1}{\varepsilon_1}\right) \\ \vdots \\ \rho_n \, \mathrm{sat}\left(\frac{z_n}{\varepsilon_n}\right) \end{bmatrix}$$

WE NOW NEED TO PICK ALL λ_i's ρ_i's ε_i's

STRATEGY

PICK ρ FIRST

$$\rho \geq \|\eta\|_{MAX} \quad \text{FOR MODEL ERROR / UNCERTAINTY}$$

WE OFTEN DON'T KNOW η_{MAX}

$\therefore$ PICK ρ TO BE AS BIG AS MOTOR/AMP WILL ALLOW :)

PICK λ

- MAKE λ TO BE AT LEAST 5x FASTER THAN CLOSED LOOP POLES FROM PD CONTROL

PICK ε

- TRADEOFF BETWEEN CHATTER & TRACKING ACCURACY
- BOUNDARY LAYER THICKNESS
- COMBO OF POSITION + VELOCITY TRACKING

$\dot{\tilde{\theta}} + \lambda \tilde{\theta} < \varepsilon$ IN GENERAL: $\tilde{\theta} < \frac{\varepsilon}{\lambda}$ TRACKING PRECISION

$\frac{\varepsilon}{\lambda}$ TRACKING PRECISION