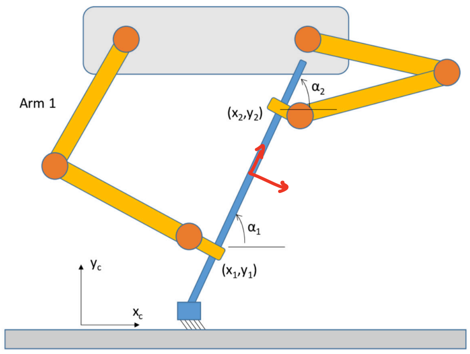
2. A pair of 3-DOF planar robot arms are attached to an overhead gantry and given the task of sweeping the floor with a broom. Each robot has a firm grip on the broomstick and can independently control the planar position (x, y) and orientation (α) of its end-effector, imparting both force (F_x, F_y) and torque (T_z) to the broomstick. So the robots have a combined total of 6-DOF in operational space $(\dot{x}_1, \dot{y}_1, \dot{\alpha}_1, \dot{x}_2, \dot{y}_2, \dot{\alpha}_2)$.



2.1 Formulate an appropriate task space using a combination of absolute and relative velocities, and show how to map between this task space and joint space. (i.e. derive an expression for J_{rel} in terms of J_1 and J_2).

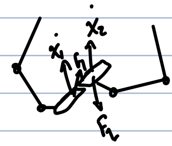
2.2 Setup a table of natural and artificial constraints with respect to your task space (consider the interactions of the robot with the broomstick as well as the interaction of the broomstick with the floor). Derive the corresponding selection matrix.

2.3 Show how you would implement a hybrid position/force controller for this task. Draw a block diagram. Show what you would use as the desired positions and forces.

2.1)
$\dot{x}_c$ = VELOCITY OF BROOM BRISTLES IN X
$\dot{y}_c$ = " IN y
$\dot{\alpha}_1$ = ∡ OF BROOM
$\dot{x}_{REL}$ = VELOCITY OF BROOM'S TOP IN X
$\dot{y}_{REL}$ = " IN y
$\dot{\alpha}_2$ = RELATIVE ∡ OF α_1

$\dot{x}_{REL} = \dot{x}_c + C\dot{\alpha}_1 \frac{L}{2}$ L = LENGTH OF BROOM
$\dot{y}_{REL} = \dot{y}_c + S\alpha_1 \frac{L}{2}$

$K_L \tau_b = \alpha_1 - \alpha_2$



$X_{REL} = \dot{X}_2 - \dot{X}_1 = J_2\dot{\Theta}_2 - J_1\dot{\Theta}_1$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_{REL} \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ -J_1 & J_2 \end{bmatrix} \begin{bmatrix} \dot{\Theta}_1 \\ \dot{\Theta}_2 \end{bmatrix}$$

2)

$V_{x,REL}$

	KINETIC	STATIC
NATURAL	$V_{y_c} = 0$ $\omega_2 = 0$ $V_{x,REL} = 0$ $V_{y,REL} = 0$	$\tau_1 = 0$ $f_{x,c} = 0$
ARTIFICIAL	$V_{xc} = V_{xc,d}$ $\omega_1 = \omega_{1,d}$	$f_{yc} = f_{yc,d} = 0$ $\tau_2 = \tau_{2,d} = 0$ $f_{x,REL} = f_{x,REL} = 0$ $f_{y,REL} = f_{y,REL} = 0$

3)
$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_{REL} \end{bmatrix} = \underbrace{\begin{bmatrix} J_1 & 0 \\ -J_1 & J_2 \end{bmatrix}}_{J_{REL}} \begin{bmatrix} \dot{\Theta}_1 \\ \dot{\Theta}_2 \end{bmatrix}$$

$S = \mathrm{diag}([1\ 0\ 1\ 0\ 0\ 0])$