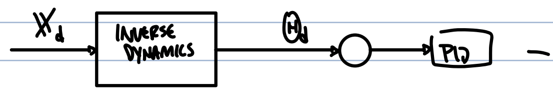
16 Operational Space Control

$\dot{\hat{\alpha}} = \Gamma^{-1} Y^T \sigma$ ← tracking error } last time stuff

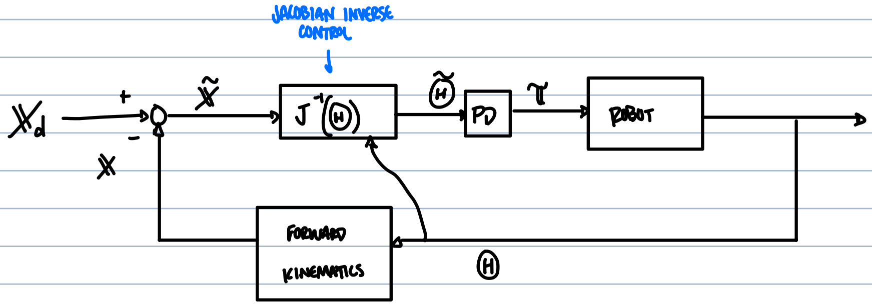
All controllers have been designed in joint space
- Desired, error, & actual thetas are in joint space



We could also penalize error in terms of end-effector position
"Operational space" aka "Task space"

Jacobian Inverse Control

$$X_d = \begin{bmatrix} x \\ y \\ z \\ \theta \\ \phi \\ \psi \end{bmatrix}$$



$J^{-1}(\Theta)$

$\dot{X} = J(\Theta)\dot{\Theta}$
small

$\frac{dX}{dt} = J \frac{d\Theta}{dt}$

small Δt

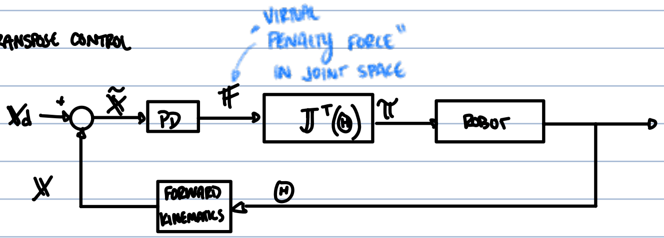
$\therefore \Delta X = J \Delta \Theta$

The Jacobian maps between differential motion in joint space & op-space
↑ operational space

$\Delta\Theta = J^{-1} \Delta X$

$\therefore \tilde{\Theta} \approx J^{-1} \tilde{X}$

Jacobian Transpose Control



We use a J^T to map between forces
& a J^{-1} for mapping between velocities

How does ___ change wrt ___

In both J^{-1} & J^T control, PD gains act like virtual stiffness/damping

J^{-1} control: stiffness damping wrt joint

J^{-1} control: stiffness damping wrt end-effector motion
i.e. K_{p_x} K_{p_y} K_{p_z}

- You can make different stiffness levels

HOW WOULD WE MAKE ${}^{\Theta}K_p = {}^{*}K_p$

IF WE PICK $({}^{\Theta}K_p) J^{-1} = J^T({}^{*}K_p)$ THEN J^{-1} & J^T CONTROL ARE EQUIVALENT

LEFT-TO-RIGHT IS OPPOSITE TO
BLOCK DIAGRAM

FOR THESE TO BE EQUAL...

$${}^{\Theta}K_p = J^T({}^{*}K_p) J$$

$\therefore$ ${}^{\Theta}K_p$ WILL HAVE OFF-DIAGONAL GAINS EVEN
IF ${}^{*}K_p$ DOESN'T HAVE DIAGONAL

· MAPS GAINS FROM OP-SPACE TO JOINT-SPACE

PROS/CONS OF OP-SPACE CONTROL

PROS

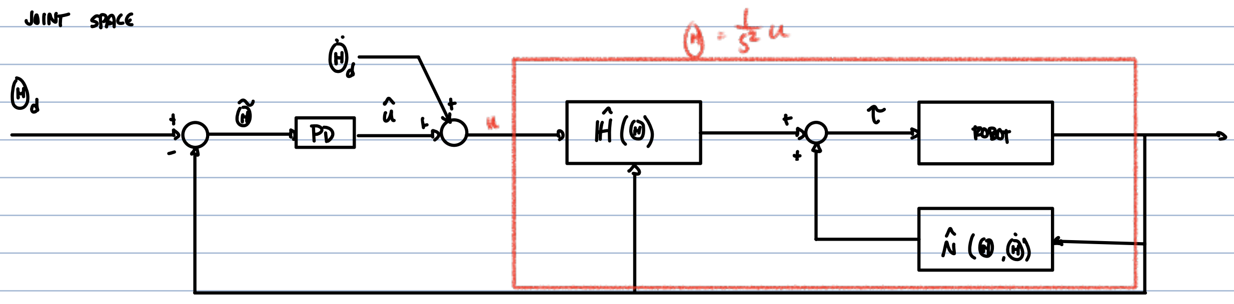
- MORE INTUITIVE
  - ESPECIALLY WITH FORCE CONTROL
- DON'T NEED INVERSE DYNAMICS

CONS

- ASSUMES SMALL ERRORS FOR J^{-1} & J^T MAPPING TO BE CORRECT
- SINGULARITIES ARE AN ISSUE
  - J^{-1} BLOWS UP, $J^T \to 0$ $(\tau \to \infty)$
- REDUNDANCIES (NON-SQUARE JACOBIAN) ARE DIFFICULT TO HANDLE (PSEUDO-INVERSE)
  - SINGULARITIES ACT AS "TOGGLE POINTS"
    i.e. ELBOW UP VS ELBOW DOWN
  - OP SPACE HAS NO CONTROL OVER THAT CONFIGURATION

OP-SPACE VERSION OF INVERSE DYNAMICS CONTROL

JOINT SPACE



HOW DO WE RELATE THIS TO OP-SPACE

$$\dot{X} = J\dot{\Theta}$$

$$\ddot{X} = J\ddot{\Theta} + \dot{J}\dot{\Theta}$$

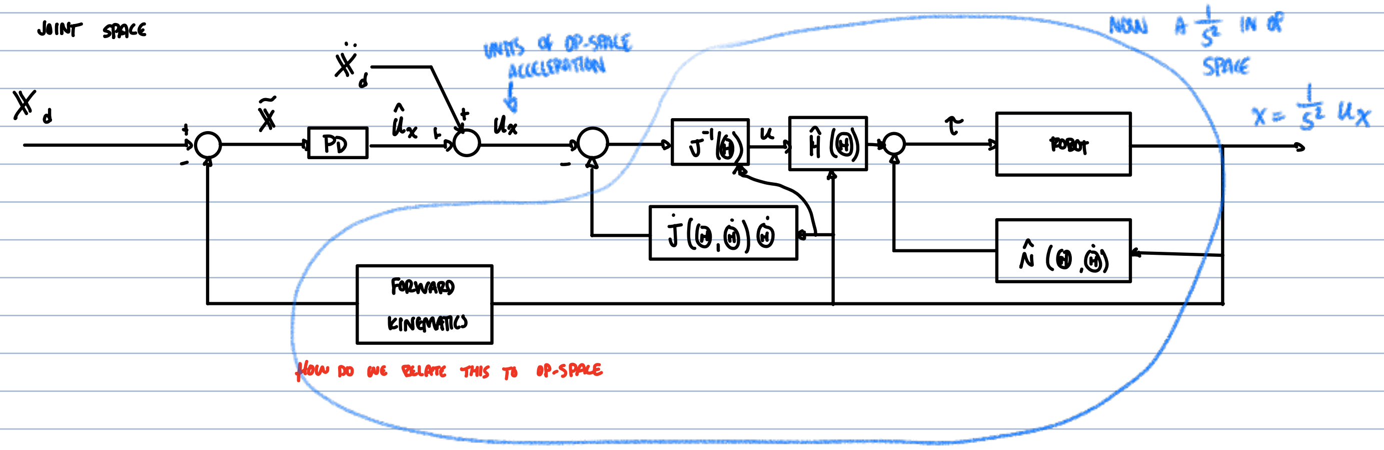
$$\ddot{\Theta} = J^{-1}(\ddot{X} - \dot{J}\dot{\Theta})$$

u IN JOINT SPACE $\quad u_x = u$ IN OP-SPACE $\qquad$ u IS IN UNITS OF ACCELERATION

SUGGESTS TO CHOOSE CONTROL LAW WHERE $u = J^{-1}(u_x - \dot{J}\dot{\Theta})$

$$u_x = \ddot{X}_d + \hat{u}_x$$

JOINT SPACE



LYAPUNOV SAYS THIS IS STABLE

STILL HAS J^{-1} INSIDE OF IT

. IS THIS J^{-1} CONTROL OR J^T CONTROL?

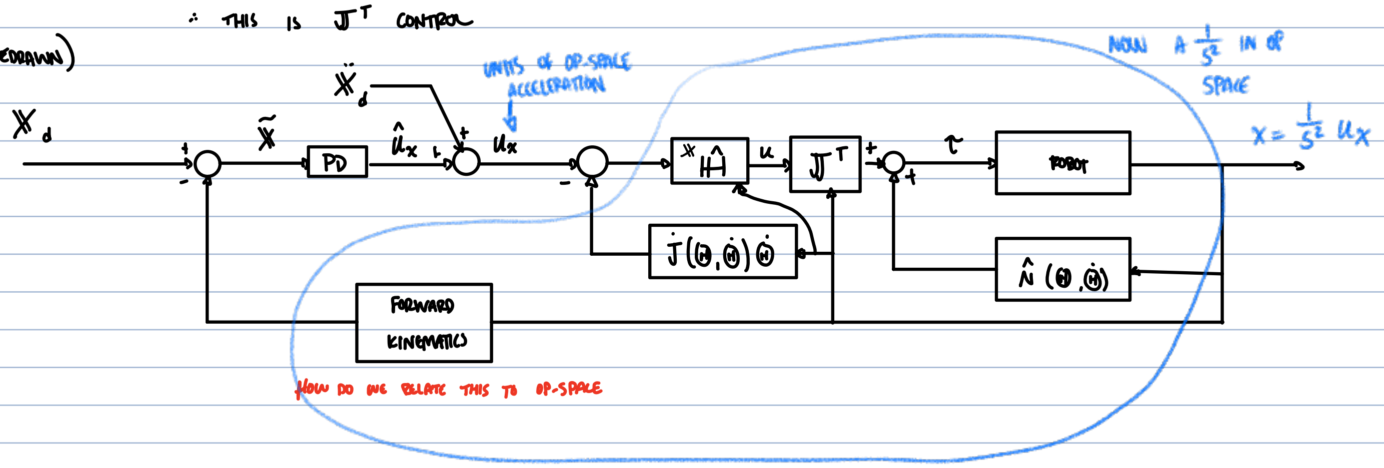
$$\hat{H}(\Theta)\, J^{-1}(\Theta) = J^T \big(\underbrace{J^{-T} \hat{H}(\Theta)\, J^{-1}}_{\text{JACOBIAN SANDWICH}} \big)$$

MAPS THINGS

$${}^{x}\hat{H} = \text{EQUIVALENT INERTIA IN OP-SPACE}$$

∴ THIS IS J^T CONTROL

(REDRAWN)



STILL HAS J^{-1} INSIDE OF IT

BEHAVES LIKE 3 INDEPENDENT, DECOUPLED CONTROLLERS IN x y z CONTROLLERS
(OR 6 IN $xyz\ \theta\ \phi\ \psi$)