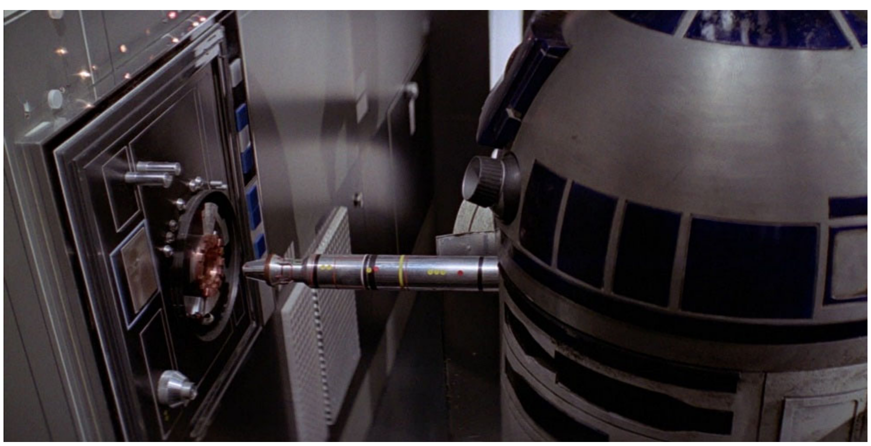
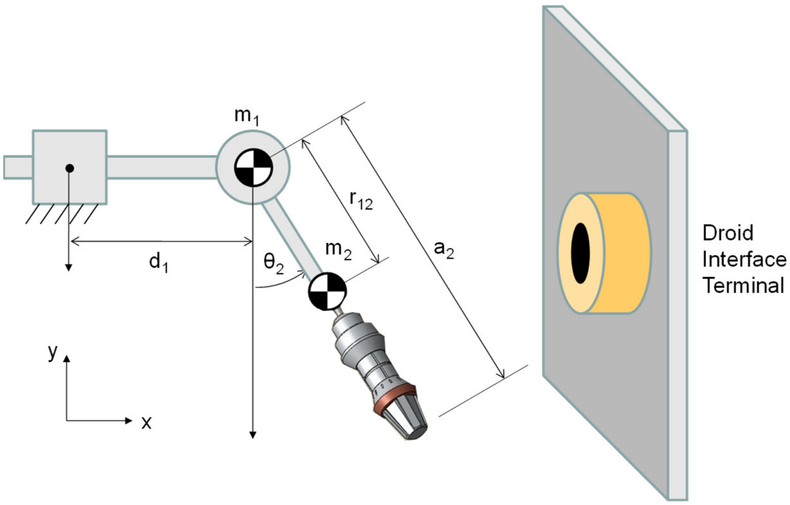
ME EN 6230, CS 6330, ECE 6651
Intro to Robot Control
Practice Final Exam
Spring 2023
120 Minutes
May the 4th be with you

- **closed book**
- **2 sheets of notes allowed**
- **show all work**

Problem 1: Indirect Force Control (Total 45 Points)

R2D2 needs to connect to a droid access panel. Assume his interface arm/tool can be modeled as a 2-DOF planar manipulator with a prismatic joint and rotary joint as shown below. He needs to limit the force of his tool on the socket (x-direction), while maintaining good position tracking in the y-direction to align his tool with the socket. You may assume his arm is moving slow enough to neglect centripetal/Coriolis forces/torques. The forward kinematics are given, as well as the inertia matrix and gravity torques in joint space.





$$x = d_1 + a_2 \sin\theta_2$$
$$y = -a_2 \cos\theta_2$$

$$\mathbf{H}_\Theta = \begin{bmatrix} m_1 + m_2 & r_{12} m_2 \cos\theta_2 \\ r_{12} m_2 \cos\theta_2 & r_{12}^2 m_2 \end{bmatrix}$$

$$\mathbf{G}_\Theta = \begin{bmatrix} 0 \\ r_{12} m_2\, g \sin\theta_2 \end{bmatrix}$$

1.1 (10 pts) Derive the manipulator Jacobian and the derivative of the Jacobian.

1.2 (10 pts) Draw a block diagram and write the control law for **inverse dynamics control** in **operational space**. Compute the numerical values for each block in your controller when $\theta_2 = \pi/2$ and $\dot{\theta}_2 = 1$, including $\mathbf{J}$, $\dot{\mathbf{J}}$, and $\mathbf{H}_\mathbf{x}$ (the inertia in operational space). Let $m_1 = m_2 = 2.0$, $r_{12} = a_2 = 0.5$. Use **P** gains of 1000 and **D** gains of 100 in both the x and y directions.

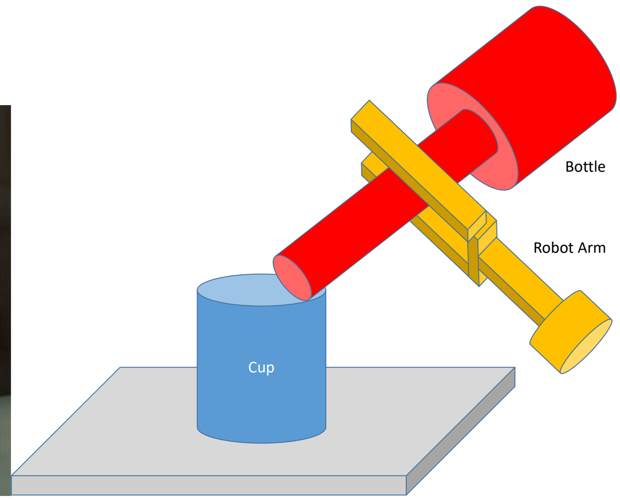
1.3 (10 pts) How would you augment your controller to implement **impedance control** in **operational space**? Redraw your block diagram and rewrite the control law. Suppose you want the robot to exhibit an impedance, characterized by a stiffness of 100 N/m, damping of 10 N·s/m, and inertia of 2 kg in the x-direction. How would you implement this?

1.4 (10 pts) How would you augment your controller to implement **admittance control** in **operational space**? Redraw your block diagram and rewrite your control law. Suppose you want the robot to exhibit an admittance, characterized by a stiffness of 100 N/m, damping of 10 N·s/m, and inertia of 2 kg in the x-direction. How would you implement this? Would you use the same PD gains as in problem **1.3**? Explain.

1.5 (5 pts) Discuss the advantages and disadvantages of impedance vs. admittance control. In what situations would you use impedance control? In what situations would you use admittance control? What would you do if you didn't have a force sensor?

Problem 2: Hybrid Control (25 Points)

I have no idea what this bartender droid's name is. Let's assume he can position/orient his hand with 6 degrees of freedom. Let's also assume that he has a firm grip on the bottle, and that the bottle is in contact with the rim of the cup as he pours the drink. Design a hybrid controller for the droid to pour the drink.



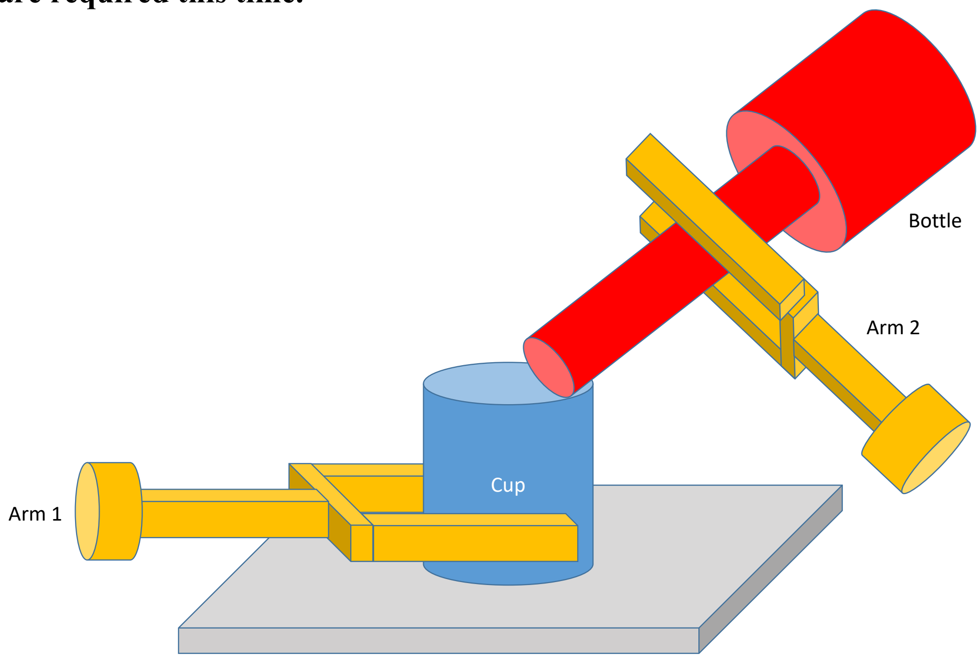
2.1 (10 pts) Set up a table of natural and artificial constraints to show how to best perform this task using a combination of position and force control. Clearly state any assumptions you make, and clearly specify/illustrate the coordinate system you would use to define your constraints. Present your resulting selection matrix.

2.2 (10 pts) Sketch a complete block diagram and write the control law for a hybrid position/force controller for this task. Make sure to use inverse dynamics control within your hybrid framework to decouple and linearize your dynamics. Show what you would use for your desired inputs. Be sure to account for any rotations between frames of reference.

2.3 (5 pts) Assuming that the inverse dynamics control is functioning properly, your position control and force control should behave as completely independent controllers in their respective degrees of freedom. Discuss the stability of the position control vs. the stability of the force control. Is there any guarantee of stability for the position control? What about the force control? Sketch a root locus for the force control and discuss what factors influence the stability.

Problem 3: Multi-Arm Coordination (Total 10 pts)

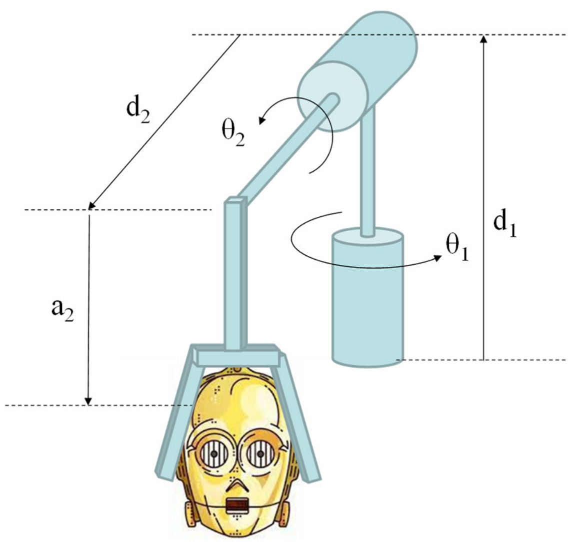
Consider the same bartender droid from Problem 2. However let's now assume that his other arm (also 6-DOF) is grasping the cup. Assume everything else is the same about the task (the cup is still resting on the table, and the bottle is still resting on the rim of the cup). How can you set up a hybrid controller to have the arms cooperate to do this task? Define your task space and make a table of constraints. Be sure to state any assumptions you make. **No Jacobians or block diagrams are required this time.**



Problem 4: Adaptive Control (Total 20 Points)

"Oh my goodness! Shut me down. Machines building machines. How perverse!"

As plant manager at the Geonosis droid factory, you have just installed a 2-DOF rotary-jointed robot for rapidly assembly of battle droids. However you don't have an accurate measurement of the robot parameters. Assume d_1, d_2, a_2, m_1, and m_2 are all unknown. Design an adaptive controller. Draw a complete block diagram of the control system, and write the control law and adaptation law. Be sure to specify the elements of $\boldsymbol{\alpha}$ and Y based on the given dynamic equations.



The dynamics are given by:

$$\tau_1 = (m_2 d_2^2 + a_2^2 m_2 \cos\theta_2)\ddot{\theta}_1 - a_2 d_2 m_2 sin\theta_2 \ddot{\theta}_2 - 2a_2^2 m_2 sin\theta_2 \cos\theta_2 \dot{\theta}_1 \dot{\theta}_2 - a_2 d_2 m_2 \cos\theta_2 \dot{\theta}_2^2 + m_2 g(d_2 \cos\theta_1 - a_2 \cos\theta_2 sin\theta_1)$$

$$\tau_2 = -a_2 d_2 m_2 sin\theta_2 \ddot{\theta}_1 + a_2^2 m_2 \ddot{\theta}_2 + a_2^2 m_2 sin\theta_2 \cos\theta_2 \dot{\theta}_1^2 - a_2 m_2 g sin\theta_2$$