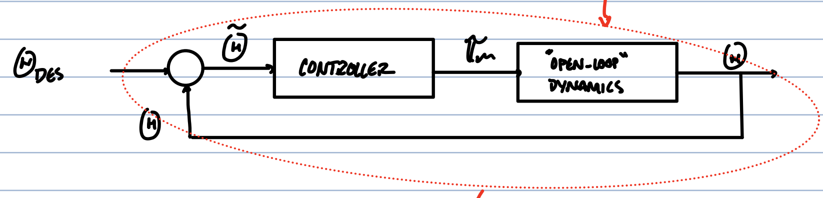
06 CLASSICAL CONTROL REVIEW

FEEDBACK CONTROL

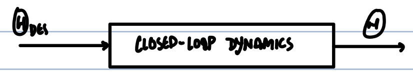
WE WANT OUR DYNAMICS IN THESE TERMS

$$\tau = H(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) + F(\dot{\theta})$$

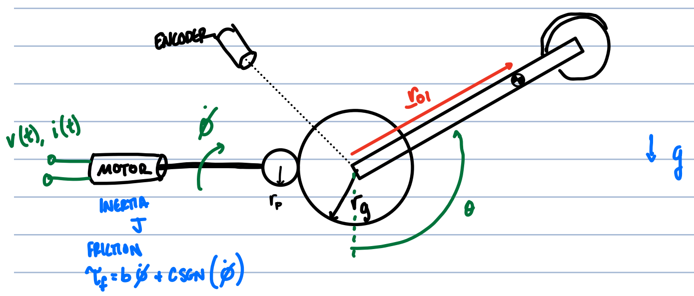
- CONTROL = MODIFYING DYNAMICS IN A DESIRED FASHION



EQUAL TO



1 DOF EXAMPLE (SRV02 MODULE)



$$N = \frac{r_g}{r_p} \quad (N > 1) \qquad \therefore \quad \dot{\phi} = N\dot{\theta}$$

DYNAMICS FOR 1-DOF

$$\tau_{ARM} = I_{LINK}\ddot{\theta} + \ddot{\underline{r}}_{01} \times m_{LINK}\, g$$

$$\tau_m = J\ddot{\phi} + b\dot{\phi} + c\,\mathrm{sgn}(\dot{\phi}) + \frac{1}{N}\tau_{ARM}$$

OR IN TERMS OF θ

$$N\tau_m = NJ(N\ddot{\theta}) + NbN\dot{\theta} + Nc\,\mathrm{sgn}(N\dot{\theta}) + I\ddot{\theta} + mg r_{01} S\theta_1$$

$$N\tau_m = \underbrace{(I + N^2 J)\ddot{\theta}}_{\text{INERTIA}} + \underbrace{mg r_{01} S\theta_1}_{\text{GRAVITY}} + \underbrace{bN^2\dot{\theta} + cN\,\mathrm{sgn}(\dot{\theta})}_{\text{FRICTION}}$$

IF WE CONTROL $i(t)$, THEN $\tau_m = K_T\, i$

$$\underbrace{NK_t}_{A_m = \text{EFFECTIVE GAIN}} i = \underbrace{(I + N^2 J)}_{I_e = \text{EFFECTIVE INERTIA}} \ddot{\theta} + mg\, r_{01} S\theta_1 + \underbrace{bN^2}_{b_e = \text{EFFECTIVE DAMPING}} \dot{\theta} + cN\, \text{sgn}(\dot{\theta})$$

OPEN LOOP DYNAMICS:

$$A_m i = I_e \ddot{\theta} + b_e \dot{\theta} + mg\, r_{01} S\theta_1 + cN\, \text{sgn}(\dot{\theta})$$

TO USE CLASSICAL CONTROL, WE NEED TO LINEARIZE DYNAMICS ABOUT SOME OPERATING POINT

$\theta = 0$ $\dot{\theta} = 0$

TAKE A TAYLOR SERIES:

$$f(x) = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x - x_0) + \left.\frac{\partial^2 f}{\partial x^2}\right|_{x=x_0} \frac{(x-x_0)^2}{2!} + \ldots$$

IF WE ASSUME DEVIATION FROM O.P. IS SMALL, ERROR GET SMALLER + SMALLER

IF $\|x - x_0\| \ll 1$

$$f(x) \approx f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=0} (x - x_0)$$

$$S\theta \approx \sin(\theta)\Big|_{\theta=0} + \cos(\theta)\Big|_{\theta=0} (\theta - \theta_0)$$

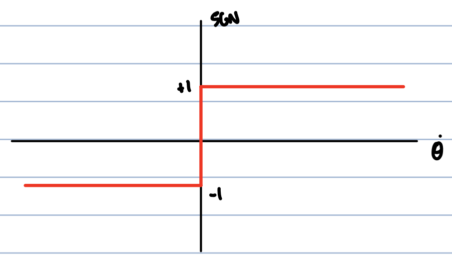
$$= \theta - \theta_0$$

$\uparrow$ ASSUMED TO BE SMALL

$$S\theta \approx \theta$$

$$\therefore A_m i = I_e \ddot{\theta} + b_e \dot{\theta} + \underbrace{mg r_{01} \theta}_{\text{NOW LINEAR :)}} + \underbrace{c\, \text{sgn}(\dot{\theta})}_{\text{CAN WE LINEARIZE THIS?}}$$

SGN FUNCTION



THE DERIVATIVE MUST EXIST AT OUR OPERATING POINT IF WE WANT TO USE A T.S.

• DISCONTINUITY AT $\dot{\theta} = 0$

$\frac{\partial f}{\partial x}$ DOESN'T EXIST $\Rightarrow$ NO LINEARIZING ABOUT OUR OPERATING POINT

$\therefore$ LET'S ASSUME WE CAN NEGLECT COULOMB FRICTION

$$A_m i \approx I_e \ddot{\theta} + b_e \dot{\theta} + \underbrace{mg r_{01}}_{K_e = \text{EFFECTIVE STIFFNESS}} \theta$$

OPEN-LOOP DYNAMICS FOR CONTROL

$$\Rightarrow \boxed{A_m i = I_e \ddot{\theta} + b_e \dot{\theta} + K_e \theta}$$

$\uparrow$ LOOKS LIKE A LINEAR MASS-SPRING DAMPER SYSTEM

ANALYSIS

$$\mathcal{L}\{A_m i\} = \mathcal{L}\left[I_e \ddot{\theta} + b_e \dot{\theta} + k_e \theta \right]$$

$$A_m\, i(s) = I_e\, s^2 \theta(s) + b_e\, s\, \theta(s) + k_e\, \theta(s)$$

$$A_m\, i(s) = \left[I_e s^2 + b_e s + k_e \right] \theta(s)$$

SOLVE FOR $\theta(s)$

OPEN-LOOP TRANSFER FUNCTION

$$\frac{\theta(s)}{i(s)} = \frac{I_e s^2 + b_e s + k_e}{A_m} = \frac{A_m / I_e}{s^2 + (b_e/I_e) s + (k_e/I_e)}$$

$$= \frac{A_m / I_e}{s^2 + 2\xi \omega_n s + \omega_n^2} = \frac{}{(s+p_1)(s+p_2)}$$

ξ = DAMPING RATIO

ω_n = NATURAL FREQUENCY

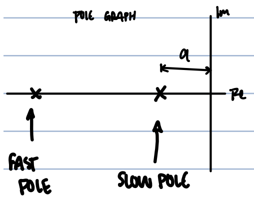
p_1, p_2 = POLES/ROOTS OF OPEN LOOP SYSTEM

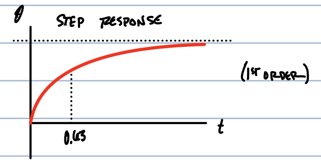
POSSIBLE BEHAVIORS

OVERDAMPED

$\xi > 1$

REAL POLES





τ_c = TIME CONSTANT = TIME TO 63% OF SS VALUE $= \frac{1}{a}$

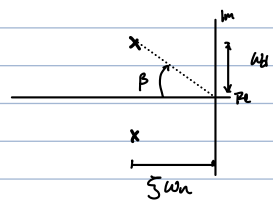
$\frac{1}{a}$ = TIME CONSTANT

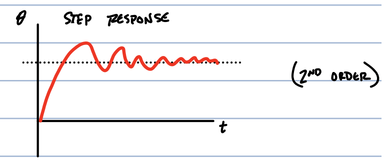
.WE NEGLECT FAST POLE IF 5 X LARGER THAN SLOW POLE

UNDERDAMPED

$\xi < 1$

COMPLEX CONJUGATE POLES





OS

$\cos\beta = \xi$

$\%OS = e^{-\pi\xi/\sqrt{1-\xi^2}}$

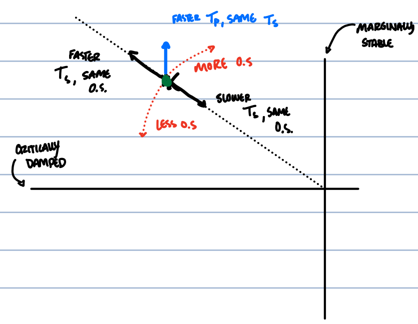
T_p = TIME TO PEAK $= \frac{\pi}{\omega_d}$

T_s = TIME TO SETTLE (98% SS) $= 4/\xi\omega_n$

$\xi\omega_n$ = TIME TO S.S.

ω_d = DAMPED NATURAL FREQUENCY

MODIFYING POLES

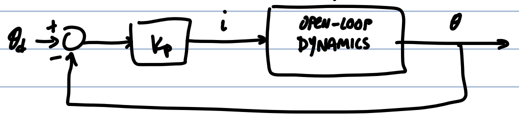


CONTROL TECHNIQUES

PROPORTIONAL CONTROL

K_p

G_p = OPEN LOOP T.F. $= \dfrac{\theta(s)}{i_s}$



TO GET CLOSED LOOP TRANSFER FUNCTION... $\dfrac{\theta(s)}{\theta_d(s)}$

$$\theta = G_p i$$

$$i = K_p(\theta_d - \theta)$$

$$\therefore \theta = G_p K_p(\theta_d - \theta)$$

$$\theta(1 + K_p G_p) = K_p G_p \theta_d$$

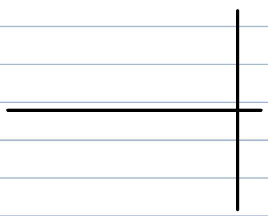
$$\Rightarrow \boxed{\frac{\theta}{\theta_d} = \frac{K_p G_p}{1 + K_p G_p}}$$ FEEDBACK FORMULA

$$\frac{\theta(s)}{\theta_d(s)} = K_p \frac{G_p}{1 + K_p G_p} = \frac{K_p A_m}{I_e s^2 + b_e s + \underbrace{(k_e + K_p A_m)}}$$

STIFFNESS WE CAN MODIFY (CHANGING NATURAL FREQUENCY)

ROOT LOCUS

- GRAPH OF ALL CLOSED LOOP POLE LOCATIONS



- AS WE INCREASE K_p, $\omega_d \uparrow$, T_s STAYS THE SAME

FOR A STEP INPUT IN θ_d

$$\theta\Big|_{t\to\infty} = \lim_{s\to 0} s\,\theta_d(s)\frac{\theta(s)}{\theta_d(s)} = \lim_{s\to 0} \overbrace{\frac{1}{s}}^{\mathcal{L}\text{ of step}} s \frac{\theta(s)}{\theta_d(s)}$$

$$= \lim_{s\to 0} \frac{\theta(s)}{\theta_d(s)} = \frac{K_p A_m}{k_e + K_p A_m}$$

$$\%\, e = \frac{\theta_d - \theta}{\theta_d} =$$

WE WANT TO INCREASE S.S. ERROR

- IF YOU HAVE ZERO ERROR, CURRENT WILL NO LONGER FLOW $\Rightarrow$ NEVER CORRECT