Robot Control 17

Force Control

Reasons to do it

- Safety for:
  - Robot
  - Environment
  - Tool
- Ensure success of task (geometric constraints)
  - Navigating obstacles by touch
  - Avoid jamming/wedging during tasks
- Teleoperation/haptics (force as communication)
  - Use robot to apply forces to human operator so operator can feel environment

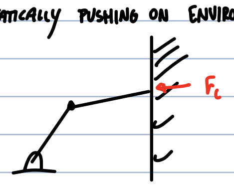
Types of Force Control

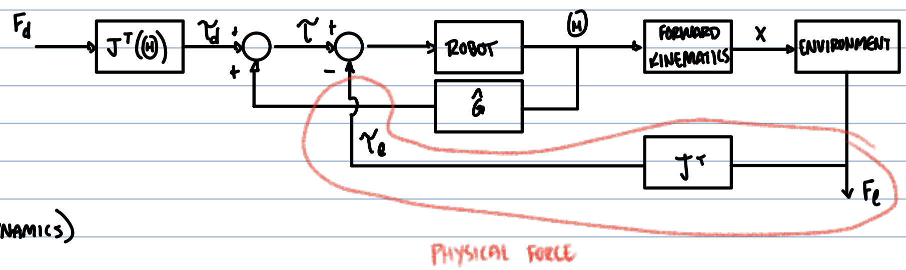
	Force Feedback	No Force Feedback
Direct :	Direct Force Control	Feedforward Force Control
Indirect :	Impedance/Admittance Control	Stiffness Control

6 axis force/torque sensors are expensive ~ \$5K so force feedback is often <u>not</u> affordable

Feedforward Force Control (open-loop control)

e.g. robot quasi-statically pushing on environment





1. $\tau - J^T F_e = H\cancel{\ddot{\Theta}} + \cancel{V} + G$ (open-loop dynamics)

   Neglect due to quasi-static (body moving)

2. Control law: $\tau = J^T F_d + \hat{G}$

Eqn ① = ②

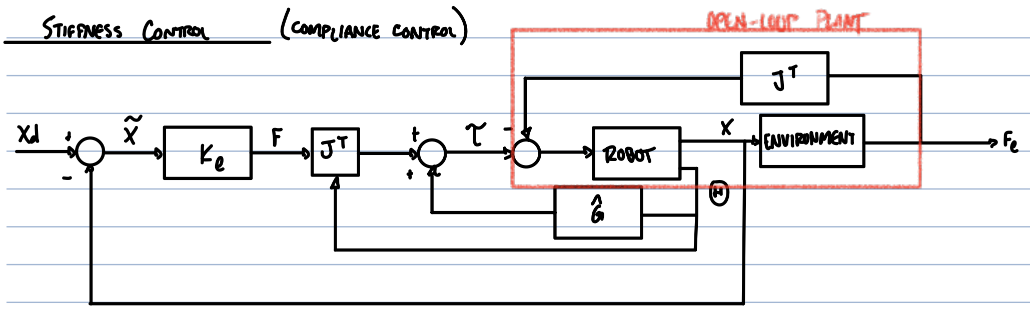
$$J^T F_d + \hat{G} = G + J^T F_e$$

$\therefore F_d = F_e$ if perfect model

If there are unmodeled disturbances, no way to correct for errors in force, since there's no way to penalize force errors

i.e. $\tilde{F} = F_d - F_e$

STIFFNESS CONTROL (COMPLIANCE CONTROL)



1. OPEN-LOOP DYNAMICS $\tau - J^T F_e = \cancel{H\ddot{\theta}} + \cancel{V} + G \rightsquigarrow \tau = J^T F_e + G$

QUASI-STATIC BEHAVIOR

2. CONTROL LAW: $\tau = J^T K_e \tilde{X} + \hat{G}$

(1) = (2)

- CLOSED LOOP DYNAMICS:

$$J^T F_e + G = J^T K_e \tilde{X} + \hat{G}$$

ASSUMING PERFECT MODEL (CANCEL J^T)

$$F_e = K_e \tilde{X}$$

K_e IS END-EFFECTOR (MATRIX)

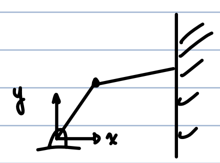
IF ROBOT IS DISPLACED BY $\tilde{X}$, ROBOT RESPONDS W/ F_e

$K_p = J^T K_e J$ OR $K_e = J^{-T} K_p J^{-1}$

EQUIVALENT JOINT SPACE STIFFNESS

IF K_e IS DIAGONAL, K_p WILL HAVE OFF-DIAGONAL ELEMENTS, CONFIGURATION DEPENDENT

e.g. <u>2-DOF ROBOT</u>



CHOOSE $K_e = \begin{bmatrix} K_x & 0 \\ 0 & K_y \end{bmatrix}$

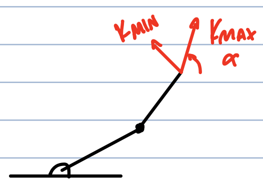
CHOOSE SMALL

CHOOSE LARGE

SCENARIO: DRAWING ON BOARD W/ROBOT

IN GENERAL, $K_e = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{xy} & K_{yy} \end{bmatrix}$ SYMMETRIC

WE CAN CONTROL 3 ASPECTS OF OP-SPACE STIFFNESS



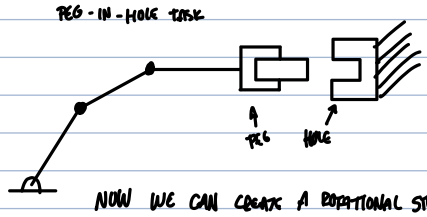
K_{MIN}, K_{MAX} ARE PRINCIPLE STIFFNESS

EIGENVALS OF STIFFNESS MATRIX

α = ORIENTATION OF EIGENVECTOR FOR K_{MAX}

IF WE ADD A 3RD DOF TO PLANAR ROBOT

PEG-IN-HOLE TASK



NOW WE CAN CREATE A ROTATIONAL STIFFNESS W/TRANSLATIONAL STIFFNESS TOO

$$\text{WRENCH} = \begin{bmatrix} F_x \\ F_y \\ \tau \end{bmatrix} = \underset{(3\times3)}{K_e} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\phi} \end{bmatrix}$$

$\tilde{\phi}$ = ORIENTATION ERROR

$$K_e = \begin{bmatrix} K_{xx} & K_{xy} & K_{x\phi} \\ K_{xy} & K_{yy} & K_{y\phi} \\ K_{x\phi} & K_{y\phi} & K_{\phi\phi} \end{bmatrix}$$

6 INDEPENDENT PARAMETERS TO CHOOSE

$K_{\phi\phi}$ = ROTATIONAL STIFFNESS

r_{cc} = LOCATION OF COMPLIANCE CENTER

CONTROL (6)

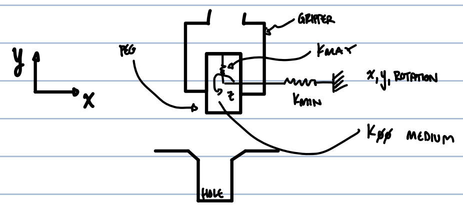
K_{max}, $K_{\phi\phi}$

K_{min}, r_{cc}

α

COMPLIANCE CENTER (POINT ABOUT WHICH THE END EFFECTOR WILL ROTATE

A FORCE APPLIED AT COMPLIANCE CENTER WILL CAUSE DISPLACEMENT BUT NOT ROTATION

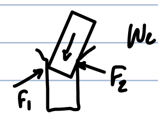


WHITNEY FOUND THAT CHOOSING $L = d/2$ TO BEST AVOID JAMMING/WEDGING

BETTER TO HAVE COMPLIANCE CENTER TOWARDS THE END OF PEG AS IF YOU'RE PULLING PEG INTO HOLE RATHER THAN PUSHING IT

JAMMING = WRENCH FROM ROBOT IS IN WRONG PROPORTION FOR INSERTION TO PROCEED

WEDGING = ELASTICITY AT PEG CAUSES REACTION FORCES F_1 F_2 TO EXERT WRONG FORCE ON PEG



TO ROTATE OR TRANSLATE THE COMPLIANCE CENTER TO A DESIRED LOCATION/ORIENTATION

WANT: $W_{cc} = K_{ee} \; \tilde{X}_{cc}$

DIAGONAL MATRIX

MAP BETWEEN CC FRAME & END-EFFECTOR FRAME

$$K_{cc} = \begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{bmatrix}$$

$\tilde{X}_{cc} = J_{ce} \tilde{X}_e$

END-EFFECTOR DISPLACEMENT

COMPLIANCE CENTER DISPLACEMENT

$\therefore\ W_e = J_{ce}^T W_{cc} = J_{ce}^T K_{cc} \tilde{X}_{cc}$

$$W_e = \underbrace{\left(J_{ce}^T K_{cc} J_{cc}\right)}_{K_e} \tilde{X}_e$$

OP-SPACE P GAINS NEEDED TO LOCATE / ORIENT COMPLIANCE CENTER REMOTELY

OR JUST DEFINE END EFFECTOR FRAME TO COINCIDE WITH OTHER DESIRED COMPLIANCE CENTER THEN: $K_e = K_{cc}$

STIFFNESS CONTROL IS THE SAME AS J^T CONTROL (PD CONTROL IN OP-SPACE) BUT W/ SELECTIVE P GAINS

- SAME PRINCIPLE CAN BE APPLIED TO D GAINS TO CONTROL DAMPING IN DIFFERENT DIRECTIONS

WE WOULD LIKE TO REPROGRAM THE INERTIA OF THE ROBOT

PROGRAM ROBOT TO BEHAVE LIKE A MASS-SPRING-DAMPER SYSTEM WHERE WE GET TO CHOOSE K_e, B_e, M_e

DESIRED CLOSED-LOOP DYNAMICS: $F_e = K_e \tilde{X} + B_e \dot{\tilde{X}} + M_e \ddot{\tilde{X}}$

OR $\tau = J^T \left(K_e \tilde{X} + B_e \dot{\tilde{X}} + M_e \ddot{\tilde{X}}\right)$

OPEN-LOOP DYNAMICS: $\tau = H \ddot{\Theta} + V + G + \tau_e$

JOINT TORQUES DUE TO ENV CONTACT

CONTROL LAW: $\tau = \hat{H}\ddot{\Theta} + \hat{V} + \hat{G} + J^T\left(K_e \tilde{X} + B_e \dot{\tilde{X}} + M_e \ddot{\tilde{X}}\right)$

RECALL $\ddot{\Theta} = J^{-1}\left(\ddot{x} - \dot{J}\dot{\Theta}\right)$

$$\tau = \hat{H} J^{-1}\left(\ddot{x} - \dot{J}\dot{\Theta}\right) + \hat{V} + \hat{G} + J^T\left(K_e \tilde{X} + B_e \dot{\tilde{X}} + M_e \ddot{\tilde{X}}\right)$$

$$= J^T \left({}^{x}\hat{H}\right)\left(\ddot{X}_d - \ddot{\tilde{X}} - \dot{J}\dot{\Theta}\right) + \hat{V} + \hat{G} + J^T\left(K_e \tilde{X} + B_e \dot{\tilde{X}} + M_e \ddot{\tilde{X}}\right)$$

${}^{x}\hat{H} = J^{-T}\hat{H}J^{-1}$ OP-SPACE TRANSFORM

$$= \hat{V} + \hat{G} + J^T\left[{}^{x}\hat{H}\ddot{X}_d + K_e \tilde{X} + B_e \dot{\tilde{X}} + \left(M_e - {}^{x}\hat{H}\right)\ddot{\tilde{X}} - {}^{x}\hat{H}\dot{J}\dot{\Theta}\right]$$

PROBLEM: WE CAN'T MEASURE $\ddot{\tilde{X}}$ (OR WE SHOULDN'T)

TRICK TO GET AROUND THIS

REWRITE C.L. DYNAMICS

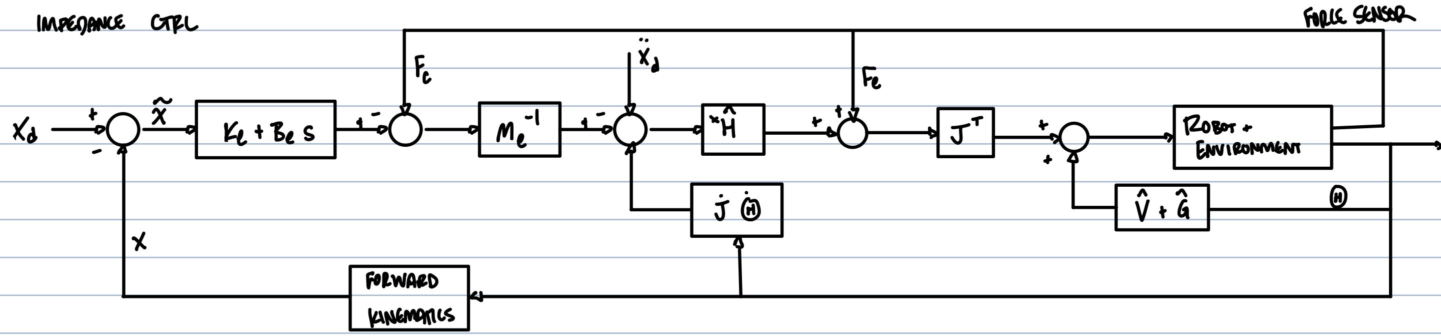
$$\ddot{\tilde{X}} = M_e^{-1}\left(F_e - K_e \tilde{X} - B_e \dot{\tilde{X}}\right)$$

SUB INTO CONTROL LAW

$$\tau = \hat{V} + \hat{G} + J^T\left[{}^{x}\hat{H}\ddot{x}_d + K_e \tilde{X} + B_e \dot{\tilde{X}} + \left(M_e - {}^{x}\hat{H}\right)M_e^{-1}\left(F_e - K_e \tilde{X} - B_e \dot{\tilde{X}}\right) - {}^{x}\hat{H}\dot{J}\dot{\Theta}\right]$$

NEW CONTROL LAW: $\tau = \hat{V} + \hat{G} + J^T\,{}^{x}\hat{H}\left[\ddot{X}_d - \dot{J}\dot{\Theta} + M_e^{-1}\left(K_e \tilde{X} + B_e \dot{\tilde{X}} - F_e\right)\right] + J^T F_e$

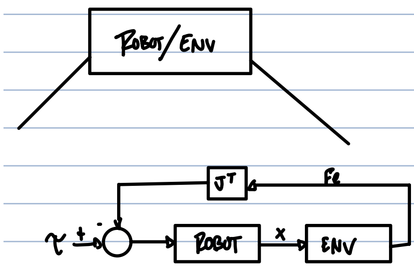
IMPEDANCE CTRL



Impedance Control

LOOKS LIKE IDC IN OP-SPACE BUT W/EXTRA FEEDBACK FROM FORCE SENSOR IN 2 SPOTS TO:

1. SUBSTITUTE FOR MEASURING ACCELERATION
2. CANCEL PHYSICAL FORCE FEEDBACK IN ROBOT/ENV/PLANT



$\hat{H}$ IN CONTROLLER INVERTS/DECOUPLES DYNAMICS SO THAT THE ROBOT BEHAVES LIKE $\frac{1}{s^2}$

$H M_e^{-1}$ = RATIO OF ACTUAL INERTIA TO DESIRED INERTIA
EFFECTIVELY REPLACES ACTUAL INERTIA OF ROBOT W/DESIRED INERTIA

OR... Alternative

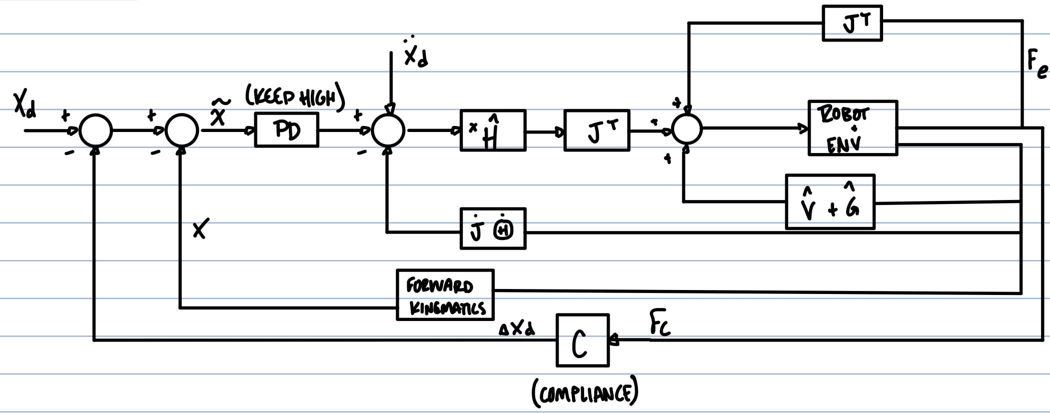
Admittance Control

STILL GOING TO MAKE ROBOT BEHAVE LIKE A MASS-SPRING-DAMPER BUT CAUSALITY IS REVERSED.

=> ROBOT WILL RESPOND W/FORCE RATHER THAN DISPLACEMENT

FORCE SENSOR IS REQUIRED

Compliance Control



CAUSALITY : A FORCE F_e CAUSES A DEFLECTION IN DESIRED TRAJECTORY ΔX_d. ROBOT COMPLIES W/ FORCE F_e BY BACKING OFF DESIRED POSITION

CAN GENERALIZE TO "ADMITTANCE CONTROL" BY REPLACING C W/ $A(s)$

DESIRED CL DYNAMICS

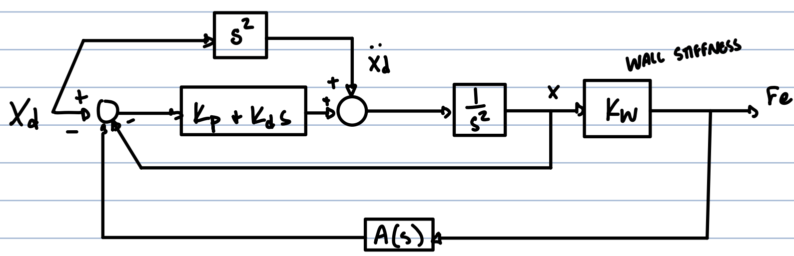
$$M_e \Delta \ddot{X}_d + B_e \Delta \dot{X}_d + K_e \Delta X_d = F_e$$

$$\frac{\Delta X_d}{F_e} = \frac{1}{M_e s^2 + B_e s + K_e} = A(s)$$

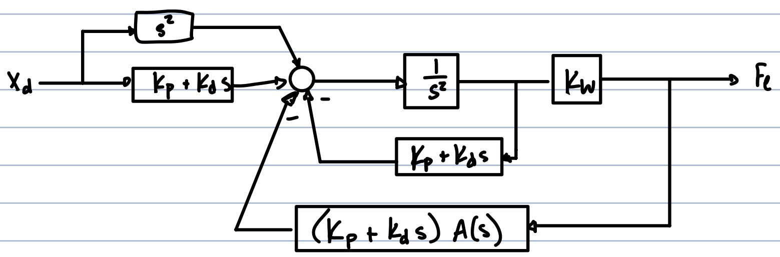
CAN STILL HAVE HIGH PD GAINS IN INNER LOOP TO GUARANTEE GOOD TRACKING & DISTURBANCE REJECTION
BUT OUTER LOOP CAN BE SENSITIVE TO EXTERNAL FORCE AT FORCE SENSOR

IS ADMITTANCE CONTROL STABLE?

ASSUMING INVERSE DYNAMICS CONTROL REDUCES INNER LOOP DYNAMICS TO $\frac{1}{s^2}$



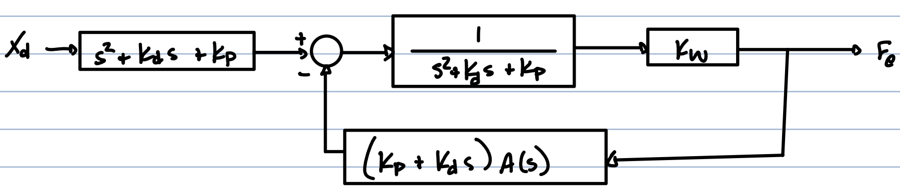
$\Downarrow$

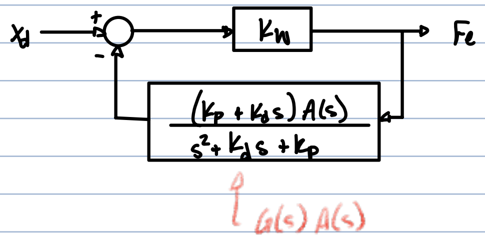


FEEDBACK RULE

$$\frac{\frac{1}{s^2}}{1 + \frac{1}{s^2}(K_p + K_d s)} = \frac{1}{s^2 + K_d s + K_p}$$

$\Downarrow$

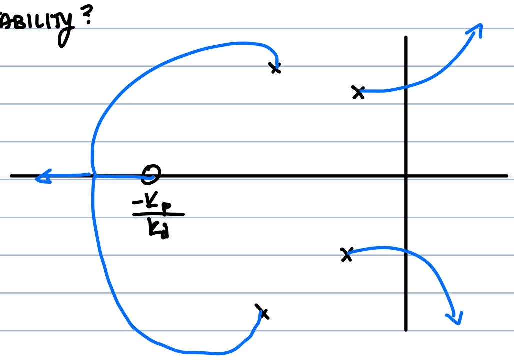




$$A(s) = \frac{1}{M_e s^2 + B_e s + K_e}$$

$$G(s) = \frac{K_p + K_d s}{s^2 + K_d s + K_p}$$

STABILITY?



AT LOW GAIN, SYSTEM IS STABLE, BUT AS LOOP INCREASES, IT WILL GO UNSTABLE

WALL STIFFNESS IS PART OF LOOP GAIN, ADMITTANCE GOES UNSTABLE FOR STIFF ENVIRONMENTS.

OLTF

$$\frac{K_w (K_p + K_d s)}{(s^2 + K_d s + K_p)(M_e s^2 + B_e s + K_e)}$$