ME EN 5230/6230, CS 6330
Intro to Robot Control
Spring 2022

Recursive Newton-Euler Algorithm for a Serial Manipulator

FOR i = 1 to n, compute velocities and accelerations:

Compute angular velocity of link i:

$$\boxed{1} \quad \boldsymbol{\omega}_{0i} = \boldsymbol{\omega}_{0,i-1} + \begin{cases} \dot{\theta}_i \boldsymbol{z}_{i-1} & \text{joint } i \text{ rotary} \\ \mathbf{0} & \text{joint } i \text{ prismatic} \end{cases}$$

Compute angular acceleration of link i:

$$\boxed{2} \quad \dot{\boldsymbol{\omega}}_{0i} = \dot{\boldsymbol{\omega}}_{0,i-1} + \begin{cases} \ddot{\theta}_i \boldsymbol{z}_{i-1} + \dot{\theta}_i \boldsymbol{\omega}_{0,i-1} \times \boldsymbol{z}_{i-1} & \text{joint } i \text{ rotary} \\ \mathbf{0} & \text{joint } i \text{ prismatic} \end{cases}$$

Compute translational acceleration of i^{th} coordinate frame:

$$\boxed{3} \quad \ddot{\boldsymbol{d}}_{0i} = \ddot{\boldsymbol{d}}_{0,i-1} + \dot{\boldsymbol{\omega}}_{0i} \times \boldsymbol{d}_{i-1,i} + \boldsymbol{\omega}_{0i} \times (\boldsymbol{\omega}_{0i} \times \boldsymbol{d}_{i-1,i}) + \begin{cases} \mathbf{0} & \text{joint } i \text{ rotary} \\ 2\dot{d}_i \boldsymbol{\omega}_{0,i-1} \times \boldsymbol{z}_{i-1} + \ddot{d}_i \boldsymbol{z}_{i-1} & \text{joint } i \text{ prismatic} \end{cases}$$

Compute translational acceleration of center of mass of link i:

$$\boxed{4} \quad \ddot{\boldsymbol{r}}_{0i} = \ddot{\boldsymbol{d}}_{0,i-1} + \dot{\boldsymbol{\omega}}_{0i} \times \boldsymbol{r}_{i-1,i} + \boldsymbol{\omega}_{0i} \times (\boldsymbol{\omega}_{0i} \times \boldsymbol{r}_{i-1,i}) + \begin{cases} \mathbf{0} & \text{joint } i \text{ rotary} \\ 2\dot{d}_i \boldsymbol{\omega}_{0,i-1} \times \boldsymbol{z}_{i-1} + \ddot{d}_i \boldsymbol{z}_{i-1} & \text{joint } i \text{ prismatic} \end{cases}$$

END

FOR i = n to 1, compute joint torques/forces:

Apply Newton/Euler equations to compute inertial force and torque on i^{th} link:

$$\boxed{A} \quad \boldsymbol{f}_i = m_i \ddot{\boldsymbol{r}}_{0i}$$

$$\boxed{B} \quad \boldsymbol{n}_i = \mathbf{I}_i \dot{\boldsymbol{\omega}}_{0i} + \boldsymbol{\omega}_{0i} \times \mathbf{I}_i \boldsymbol{\omega}_{0i}$$

Apply dynamic force and torque balance equations about i^{th} link:

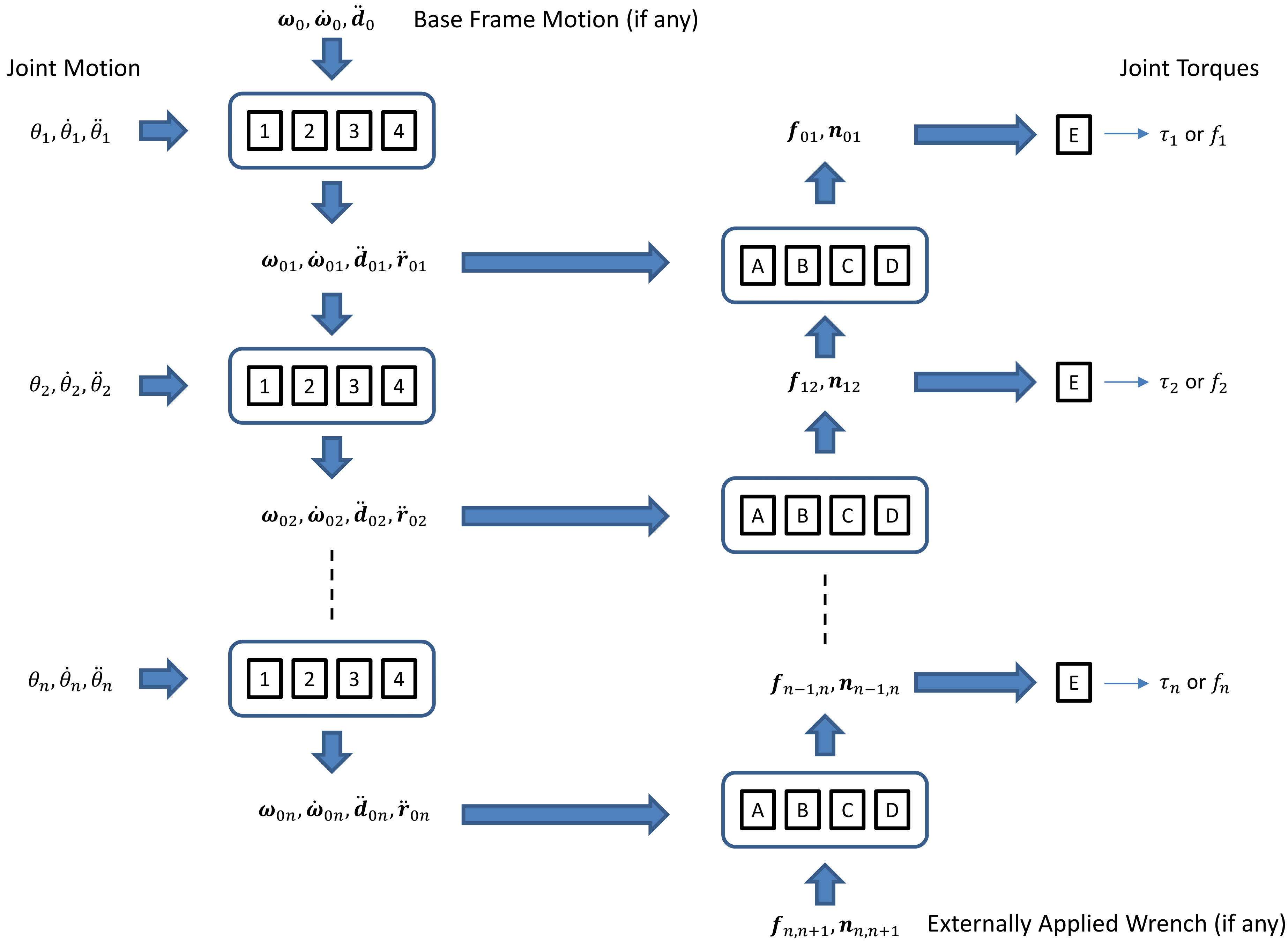
$$\boxed{C} \quad \boldsymbol{f}_{i-1,i} = \boldsymbol{f}_i + \boldsymbol{f}_{i,i+1} - m_i \boldsymbol{g}$$

$$\boxed{D} \quad \boldsymbol{n}_{i-1,i} = \boldsymbol{n}_i + \boldsymbol{n}_{i,i+1} + \boldsymbol{r}_{i-1,i} \times \boldsymbol{f}_{i-1,i} - \boldsymbol{r}_{ii} \times \boldsymbol{f}_{i,i+1}$$

Compute actuation torque/force at the i^{th} joint:

$$\boxed{E} \quad \begin{array}{ll} \tau_i = \boldsymbol{z}_{i-1} \cdot \boldsymbol{n}_{i-1,i} & \text{joint } i \text{ rotary} \\ f_i = \boldsymbol{z}_{i-1} \cdot \boldsymbol{f}_{i-1,i} & \text{joint } i \text{ prismatic} \end{array}$$

END



$\boldsymbol{\omega}_0, \dot{\boldsymbol{\omega}}_0, \ddot{\boldsymbol{d}}_0$
Base Frame Motion (if any)
Joint Motion
Joint Torques
$\theta_1, \dot{\theta}_1, \ddot{\theta}_1$
1
2
3
4
$\boldsymbol{f}_{01}, \boldsymbol{n}_{01}$
E
τ_1 or f_1
$\boldsymbol{\omega}_{01}, \dot{\boldsymbol{\omega}}_{01}, \ddot{\boldsymbol{d}}_{01}, \ddot{\boldsymbol{r}}_{01}$
A
B
C
D
$\theta_2, \dot{\theta}_2, \ddot{\theta}_2$
$\boldsymbol{f}_{12}, \boldsymbol{n}_{12}$
τ_2 or f_2
$\boldsymbol{\omega}_{02}, \dot{\boldsymbol{\omega}}_{02}, \ddot{\boldsymbol{d}}_{02}, \ddot{\boldsymbol{r}}_{02}$
$\theta_n, \dot{\theta}_n, \ddot{\theta}_n$
$\boldsymbol{f}_{n-1,n}, \boldsymbol{n}_{n-1,n}$
τ_n or f_n
$\boldsymbol{\omega}_{0n}, \dot{\boldsymbol{\omega}}_{0n}, \ddot{\boldsymbol{d}}_{0n}, \ddot{\boldsymbol{r}}_{0n}$
$\boldsymbol{f}_{n,n+1}, \boldsymbol{n}_{n,n+1}$
Externally Applied Wrench (if any)