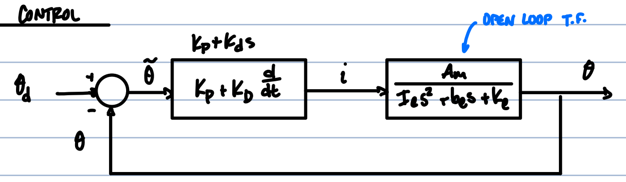
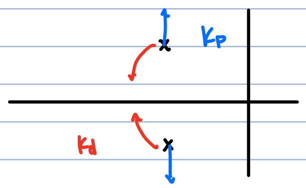
CLASSICAL CONTROL REVIEW CONTINUED

PD CONTROL



$$\frac{\theta(s)}{\theta_d(s)} = \frac{(K_p + K_d s)A_m}{I_e s^2 + \underbrace{(b_e + K_d A_m)}_{\text{DAMPING}}s + \underbrace{(K_e + K_p A_m)}_{\text{STIFFNESS}}}$$



- WE CAN LOCATE CLOSED LOOP POLES ANYWHERE IN COMPLEX PLANE TO ACHIEVE GOOD COMBO OF %OS & T_s
- THERE IS STILL S.S. ERROR

$$e_{ss,PD} = \frac{K_e}{K_e + K_p A_m}$$

- THIS WILL LET US USE LARGER K_p WITHOUT MUCH OSCILLATION

USING ROOT LOCUS FOR DESIGN

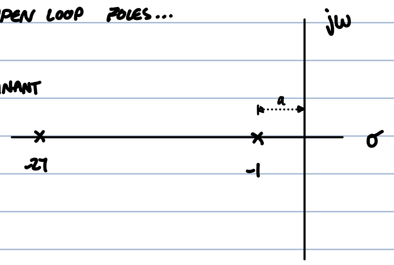
1 DOF ROBOT W/ VOLTAGE CONTROL (WITH NUMBERS :))

OPEN-LOOP T.F. $G_p(s) = \frac{\theta(s)}{V(s)} = \frac{54}{s^2 + 28s + 27} = \frac{54}{(s+1)(s+27)}$

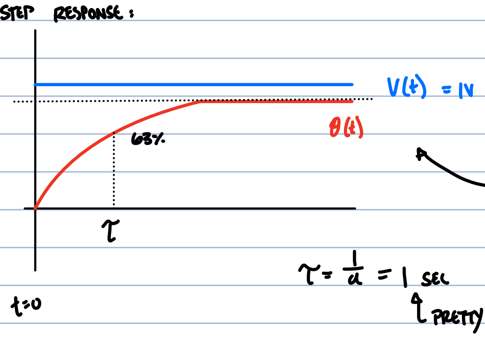
P IS FOR PLANT

OPEN LOOP POLES...

POLE @ $s = 1$ IS DOMINANT



STEP RESPONSE:

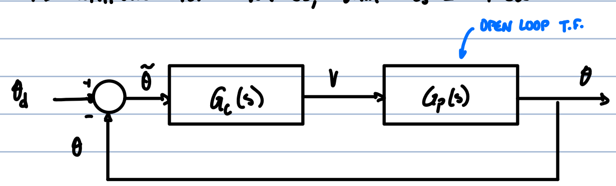


$\tau = \frac{1}{a} = 1$ SEC

PRETTY DANG SLOW...

$$\theta(t \to \infty) = \lim_{s \to 0} s \cdot \frac{1}{s} G_p(s) = \frac{54}{s^2 + 28s + 27}\bigg|_{s=0} = \frac{54}{27} = 2 \text{ RAD} \quad \therefore \theta_{ss}(t) = 2 \text{ RAD}$$

GOAL: DESIGN PD CONTROLLER FOR 10% OS, WITH $t_s \le 0.1$ SEC



FOR PD CONTROLLER

$$G_c(s) = K_p + K_d s$$

LOOP GAIN

OPEN-LOOP ZERO

$$\therefore \; G_c G_p = K_p + K_d s \cdot \frac{54}{s^2+28s+27} = \frac{\overbrace{54 K_p}^{\text{LOOP GAIN}}\left(s + \frac{K_p}{K_d}\right)}{(s+1)(s+27)}$$

← READY FOR R.L.

OPEN LOOP POLES

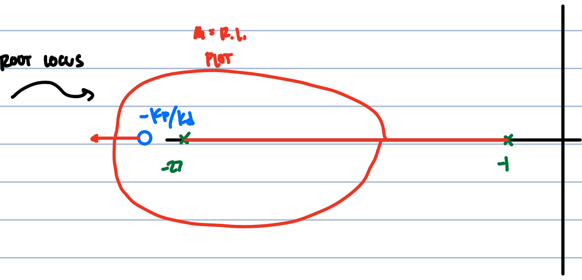
DESIRED

$$\zeta_{des} = \frac{-\ln(\%OS)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.6$$

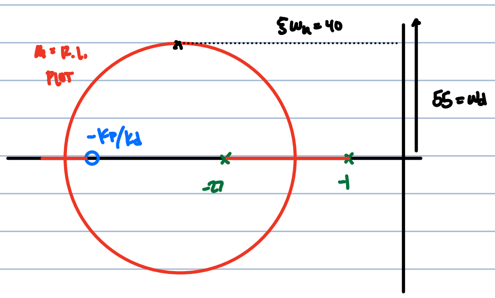
$$T_s = \frac{4}{\zeta \omega_n} \qquad \omega_n = \frac{4}{T_s \zeta} = \frac{4}{(0.1)(0.6)} = 67$$

$$\therefore \; \omega_d = \omega_n \sqrt{1-\zeta^2} = 55$$

ROOT LOCUS →

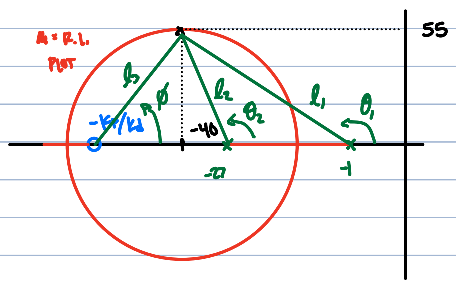


- ROOT LOCUS TELLS US ALL POSSIBLE CLOSED LOOP POLE LOCATIONS AS YOU VARY LOOP GAIN
- WE'RE ABLE TO MOVE LOCATION OF ZEROS/POLES WITH DERIVATIVE AND/OR INTEGRAL CONTROL



WE WANT POLES @ $s = -40 \pm 55j$

WE CAN USE THE ∡ CONDITION TO ENSURE THAT ROOT LOCUS GOES THROUGH DESIRED CLOSED LOOP LOCATIONS



IN ORDER FOR THAT POINT TO EXIST, WE NEED ∡'S FROM ALL OTHER POLES + ZEROS

$$\phi = \pm 180^\circ + \tan^{-1}\left(\frac{55}{-39}\right) + \tan^{-1}\left(\frac{55}{-13}\right)$$

$$\phi = \pm 180 + 125^\circ + 103^\circ = 48^\circ$$

$$\tan(48) = \frac{55}{-40-z} \qquad \therefore \; z = -90$$

$\therefore$ ZERO MUST BE AT -90

$$\frac{K_p}{K_d} = 90$$

USE MAGNITUDE CONDITION TO FIND THE LOOP GAIN REQUIRED TO PLACE CLOSED LOOP POLES AT DESIRED POINT ON ROOT LOCUS

$$54 K_d = \frac{l_1 l_2}{l_3} = \frac{\sqrt{39^2+54^2}\sqrt{13^2+55^2}}{\sqrt{50^2+55^2}} = 51$$

$$\therefore \; K_D = 0.94$$

$$K_p = 90 K_D = 85$$

WHEN WE SIMULATE, WE DON'T GET 10% O.S.

- WE DIDN'T CONSIDER THE CLOSED LOOP ZERO

CLOSED LOOP T.F.

$$\frac{\theta}{\theta_d} = \frac{(K_p + K_d s) A_m}{I_e s^2 + (b_e + K_d A_m) s + (K_e + K_p A_m)}$$

C.L. ZERO @ $s = -90$ IS <u>NOT</u> 5× LEFT OF CLOSED LOOP POLES @ -40, WHICH WILL INTERFERE W/SYSTEM RESPONSE

PV CONTROL (VELOCITY)

- SAME AS PD CONTROL, BUT DOESN'T DIFFERENTIATE STEP SAME C.L. POLES BUT W/NO C.L. ZERO

PID CONTROL

$$G_c = K_p + K_d s + K_I \frac{1}{s}$$

$$\rightsquigarrow G_c G_p = \frac{54 K_d (s + z_1)(s + z_2)}{s(s+1)(s+27)}$$

1. PLACE 2 ZEROS USING THE ∡ CONDITION