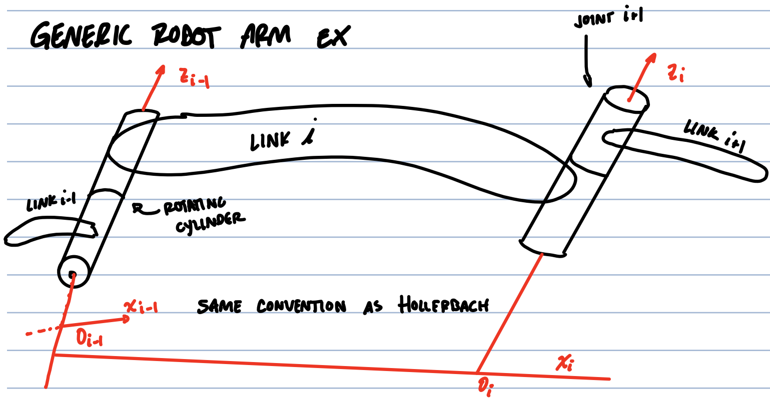
Lecture 2 Kinematics Review

Generic Robot Arm Ex



DH Params

α_i : "SKEW ∡" OR "LINK TWIST"
FROM z_{i-1} TO z_i ABOUT x_i

d_i : "LINK OFFSET DISTANCE"
FROM x_{i-1} TO x_i ALONG z_{i-1}

a_i : "LINK LENGTH" DISTANCE
FROM z_{i-1} TO z_i ALONG x_i

θ_i : "JOINT ∡"
FROM x_{i-1} TO x_i ABOUT z_{i-1}

JOINT	α_i	d_i	a_i	θ_i
1				
2				
3				
4				
5				

Homogeneous Transform

$${}^{i-1}T_i = \begin{bmatrix} {}^{i-1}R_i & {}^{i-1}\underline{d}_{i-1,i} \\ \underline{0} & 1 \end{bmatrix}$$

FROM $i-1$ FRAME TO i

$${}^{i-1}R_i = R_z(\theta_i)\,R_x(\alpha_i)$$

$$= \begin{bmatrix} C\theta_i & -S\theta_i & 0 \\ S\theta_i & C\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i \\ 0 & S\alpha_i & C\alpha_i \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_i & -S\theta_i\,C\alpha_i & S\theta_i\,S\alpha_i \\ S\theta_i & C\theta_i\,C\alpha_i & -C\theta_i\,S\alpha_i \\ 0 & S\alpha_i & C\alpha_i \end{bmatrix}$$

$${}^{i-1}\underline{d}_{i-1,i} = d_i\,{}^{i-1}\underline{z}_{i-1} + a_i\,{}^{i-1}\underline{x}_i$$

$$= d_i\,{}^{i-1}\underline{z}_{i-1} + a_i\,{}^{i-1}R_i\,{}^{i}\underline{x}_i$$

$$= d_i\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + a_i\begin{bmatrix} C\theta_i \\ S\theta_i \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_i\,C\theta_i \\ a_i\,S\theta_i \\ d_i \end{bmatrix}$$

$${}^{0}T_n = {}^{0}T_1\,{}^{1}T_2 \cdots {}^{n-1}T_n$$

${}^{0}R_n$ = ROTATION BETWEEN END-EFFECTOR FRAME AND BASE FRAME (3x3)

${}^{0}d_{0n}$ = DISPLACEMENT OF END-EFFECTOR FROM BASE FRAME

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

FORWARD KINEMATICS

GIVEN JOINT θ's, COMPUTE END-EFFECTOR POSITION/ORIENTATION

INVERSE KINEMATICS

GIVEN END EFFECTOR POS/ORIENTATION $\rightarrow$ COMPUTE JOINT θ's

NO GENERAL SOLUTION

VELOCITY KINEMATICS

$$
{}^0\dot{d}_{0n} = \frac{d}{dt}\left({}^0 d_{0n}\right) = \frac{\partial d_{0n}}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial d_{0n}}{\partial \theta_2}\dot{\theta}_2 + \dots
$$

$$
= \underbrace{\begin{bmatrix} \partial x/\partial\theta_1 & \partial x/\partial\theta_2 & \cdots & \partial x/\partial\theta_n \\ \partial y/\partial\theta_1 & \partial y/\partial\theta_2 & \cdots & \partial y/\partial\theta_n \\ \partial z/\partial\theta_1 & \partial z/\partial\theta_2 & \cdots & \partial z/\partial\theta_n \end{bmatrix}}_{\text{MANIPULATOR JACOBIAN}} \begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}
$$

J $\leftarrow$ ONLY POSITION OF END EFFECTOR

ALTERNATIVELY

$$
\underset{6\times1}{\begin{bmatrix} {}^0\dot{d}_{0n} \\ {}^0\omega_{0n} \end{bmatrix}} = \underset{\text{VELOCITY JACOBIAN } 6\times n}{J_v} \underset{n\times1}{\begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}}
$$

$$
J_v = \begin{bmatrix} {}^0z_0 \times {}^0d_{0n} & {}^0z_1 \times {}^1d_{1n} & \cdots & {}^{0}z_{n-1} \times {}^{n-1}d_{n-1,n} \\ {}^0z_0 & {}^0z_1 & \cdots & {}^0z_{n-1} \end{bmatrix}
$$

EASY NUMERICAL SOLUTION

INVERSE VELOCITIES

- WE CAN

$$
\underset{n\times1}{\begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}} = \underset{n\times6}{J_v^{-1}} \underset{6\times1}{\begin{bmatrix} {}^0\dot{d}_{0n} \\ {}^0\omega_{0n} \end{bmatrix}}
$$

ONLY WORKS IF WE HAVE FULL VELOCITY JACOBIAN

- NON-SQUARE MATRICES
- SINGULARITIES

  IF $\det(J_v) = 0 \Rightarrow$ SINGULAR

HAPPENS WHEN ROBOT IS AT THE EDGE OF ITS REACH, OR OUTSIDE OF ITS CAPABILITIES

ON THE HUMAN BODY, EXAMPLE SINGULARITIES ARE:

- ELBOW LOCKED

  NO HORIZONTAL VELOCITY
- WRIST DIRECTLY OVER SHOULDER

  CAN'T HAVE VELOCITY OUT OF PLANE

WHAT IF J_v IS NOT SQUARE?

- IF $n < 6$ ... YOU CAN'T INDEPENDENTLY SPECIFY BOTH $\underline{\dot{d}_{0n}}$ & $\underline{\omega_{0n}}$

→ SPECIFY FEWER THINGS

3 DOF ROBOT

SPECIFIES $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$ NOT $\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$

→ USE A SUBSET OF VELOCITY JACOBIAN

EX $n=3$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \underset{3\times3}{J_v^{-1}} \, \dot{d}_{0n}$$

$n=2$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \underset{2\times2}{J_v^{-1}} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

IF $n > 6$ ⇒ MULTIPLE SOLUTIONS ⇒ REDUNDANT DOF

CAN USE PSEUDO-INVERSE TO FIND SOLUTIONS

↑ MULTIPLE (NOT SINGLE)

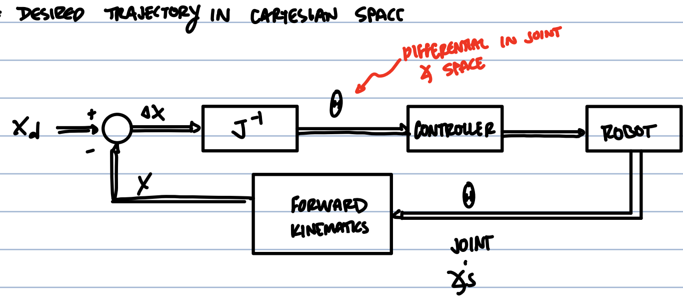
- YOU CAN USE DIFFERENT PSEUDO-INVERSES TO OPTIMIZE CERTAIN PARAMETERS

UTILITY OF THE JACOBIAN

INVERSE JACOBIAN IS USEFUL FOR "OPERATIONAL SPACE CONTROL"

- INVERSE KINEMATICS ARE HARD, BUT INVERSE JACOBIAN IS JUST INVERSE OF A MATRIX

X_d = DESIRED TRAJECTORY IN CARTESIAN SPACE



THE JACOBIAN IS ALSO USEFUL FOR STATICS

- RELATES STATIC FORCES + TORQUES TO JOINT TORQUES

$$\tau = J^T W_{n+1}$$

↑ END EFFECTOR WRENCH

$$\begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix} = J^T \begin{bmatrix} f_{n,n+1} \\ \\ n_{n,n+1} \end{bmatrix} \begin{matrix} \text{FORCES} \\ \\ \text{TORQUES} \end{matrix}$$

↑ FORCES + TORQUES STACKED

PROOF PRINCIPLE OF VIRTUAL WORK/POWER

(VIRTUAL) POWER OUT OF MOTORS = POWER OUT OF END EFFECTOR

$$\tau_1 \dot{\theta}_1 + \tau_2 \dot{\theta}_2 + \ldots = \underline{f}_{n,n+1}^T \, {}^0\dot{\underline{d}}_{on} \quad \underline{n}_{n,n+1}^T \, \underline{\omega}_{on}$$

$$\tau^T \dot{\Theta} = W_{n+1}^T \begin{bmatrix} {}^0\dot{\underline{d}}_{on} \\ \underline{\omega}_{on} \end{bmatrix} = W_{n+1}^T J \dot{\Theta}$$

$$\rightsquigarrow \quad \tau^T = W_{n+1}^T J$$

$$\rightsquigarrow \quad \tau = J^T W_{n+1}$$

JOINT TORQUES = JACOBIAN^T WRENCH