

Robot Control 17

FORCE CONTROL

REASONS TO DO IT

- SAFETY FOR:
 - ROBOT
 - ENVIRONMENT
 - TOOL
- ENSURE SUCCESS OF TASK (GEOMETRIC CONSTRAINTS)
 - NAVIGATING OBSTACLES BY TOUCH
 - AVOID JAMMING / WEDGING DURING TASKS
- TELEOPERATION / HAPTICS (FORCE AS COMMUNICATION)
 - USE ROBOT TO APPLY FORCES TO HUMAN OPERATOR
SO OPERATOR CAN FEEL ENVIRONMENT

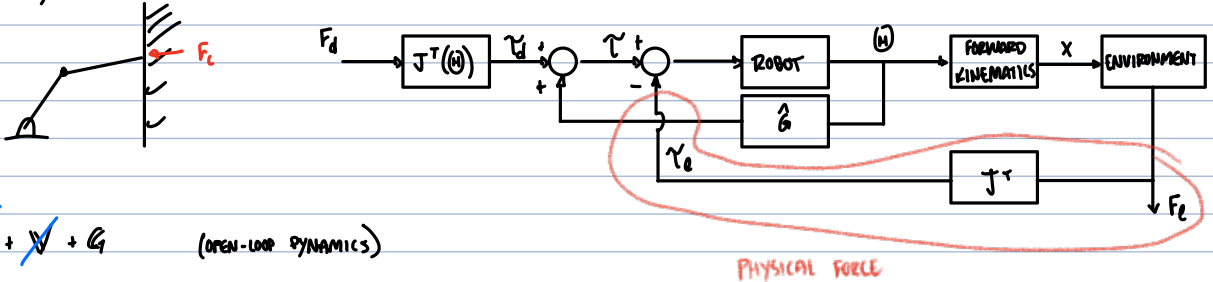
Types of Force Control

	FORCE FEEDBACK	NO FORCE FEEDBACK
DIRECT :	DIRECT FORCE CONTROL	FEEDFORWARD FORCE CONTROL
INDIRECT :	IMPEDANCE / ADMITTANCE CONTROL	STIFFNESS CONTROL

6 AXIS FORCE/TORQUE SENSORS ARE EXPENSIVE ~ \$ 5K SO FORCE FEEDBACK IS OFTEN NOT AFFORDABLE

FEEDFORWARD FORCE CONTROL (OPEN-LOOP CONTROL)

E.G. ROBOT QUASI-STATICALLY PUSHING ON ENVIRONMENT



$$1. \tau - J^T F_e = H \ddot{q} + \dot{V} + G \quad (\text{OPEN-LOOP DYNAMICS})$$

NEGLECT DUE TO QUASI-STATIC (BODY MOVING)

$$2. \text{CONTROL LAW: } \tau = J^T F_d + \hat{G}$$

$$\text{EQN } ① = ②$$

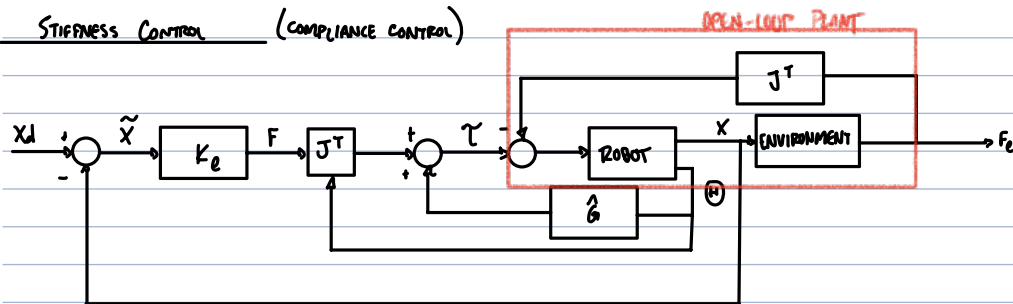
$$J^T F_d + \hat{G} = G + J^T F_e$$

$$\therefore F_d = F_e \quad \text{IF PERFECT MODEL}$$

IF THERE ARE UNMODELLED DISTURBANCES, NO WAY TO CORRECT FOR ERRORS IN FORCE, SINCE THERE'S NO WAY TO PENALIZE FORCE ERRORS

$$i.e. \bar{F} = F_d - F_e$$

STIFFNESS CONTROL (COMPLIANCE CONTROL)



1. OPEN-LOOP DYNAMICS $\tau - J^T F_e = H \ddot{\theta} + \text{viscous} + G \rightsquigarrow \tau = J^T F_e + G$

QUASI-STATIC BEHAVIOR

2. CONTROL LAW: $\tau = J^T k_e \tilde{x} + \hat{G}$

(1) = (2)

CLOSED LOOP DYNAMICS:

$$J^T F_e + G = J^T k_e \tilde{x} + \hat{G}$$

ASSUMING PERFECT MODEL (CANCEL J^T)

$$F_e = k_e \tilde{x}$$

k_e IS END-EFFECTOR (MATRIX)

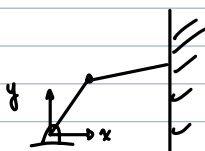
IF ROBOT IS DISPLACED BY \tilde{x} , ROBOT RESPONDS w/ F_e

$$K_p = J^T k_e J \quad \text{OR} \quad k_e = J^{-T} K_p J^{-1}$$

EQUIVALENT JOINT SPACE STIFFNESS

IF k_e IS DIAGONAL, K_p WILL HAVE OFF-DIAGONAL ELEMENTS, CONFIGURATION DEPENDENT

e.g. 2-DOF ROBOT



CHOOSE $k_e = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$

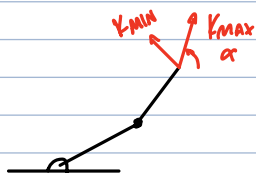
CHOOSE SMALL \rightarrow (pointing to k_x)

CHOOSE LARGE \rightarrow (pointing to k_y)

SCENARIO: DRAWING ON BOARD w/ ROBOT

IN GENERAL, $k_e = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{xy} & k_{yy} \end{bmatrix}$ \leftarrow SYMMETRIC

WE CAN CONTROL 3 ASPECTS OF OP-SPACE STIFFNESS



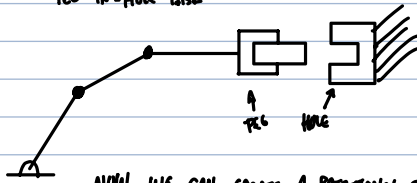
K_{min} , K_{max} ARE PRINCIPLE STIFFNESS

EIGENVALS OF STIFFNESS MATRIX

α = ORIENTATION OF EIGENVECTOR FOR K_{max}

IF WE ADD A 3RD DOF TO PLANAR ROBOT

PEG-IN-HOLE TASK



NOW WE CAN CREATE A ROTATIONAL STIFFNESS w/ TRANSLATIONAL STIFFNESS TOO

$$\text{WRENCH} = \begin{bmatrix} F_x \\ F_y \\ \tau \end{bmatrix} = K_c \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\theta} \end{bmatrix} \quad \tilde{\theta} = \text{ORIENTATION ERROR}$$

(3x3)

$$K_c = \begin{bmatrix} K_{xx} & K_{xy} & K_{x\theta} \\ K_{xy} & K_{yy} & K_{y\theta} \\ K_{x\theta} & K_{y\theta} & K_{\theta\theta} \end{bmatrix} \quad 6 \text{ INDEPENDENT PARAMETERS TO CHOOSE}$$

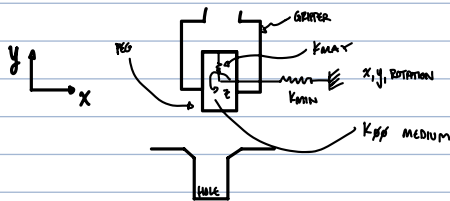
$K_{\theta\theta}$ = ROTATIONAL STIFFNESS

r_{cc} = LOCATION OF COMPLIANCE CENTER

COMPLIANCE CENTER (POINT ABOUT WHICH THE END EFFECTOR WILL ROTATE)

A FORCE APPLIED AT COMPLIANCE CENTER WILL CAUSE DISPLACEMENT BUT NOT ROTATION

CONTROL (G)	
K_{max}	$K_{\theta\theta}$
K_{min}	r_{cc}
α	



WHITNEY FOUND THAT CHOOSING $L = d/2$ TO BEST AVOID JAMMING/WEDGING

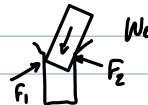
BETTER TO HAVE COMPLIANCE CENTER

TOWARDS THE END OF PEG AS IF YOU'RE PULLING PEG INTO HOLE RATHER THAN PUSHING IT

JAMMING = WRENCH FROM ROBOT IS IN WRONG PROPORTION FOR INSERTION TO PROCEED

WEDGING = ELASTICITY AT PEG

CAUSES REACTION FORCES F_1 , F_2 TO EXERT WRONG FORCE ON PEG



TO ROTATE OR TRANSLATE THE COMPLIANCE CENTER TO A DESIRED LOCATION/ORIENTATION

$$\text{WANT: } W_{cc} = K_{cc} \tilde{x}_{cc}$$

↑
DIAGONAL MATRIX

MAP BETWEEN CC FRAME & END-EFFECTOR FRAME

$$\tilde{x}_{cc} = J_{cc} \tilde{x}_e$$

↑ ↑
COMPLIANCE CENTER DISPLACEMENT END-EFFECTOR DISPLACEMENT

$$K_{cc} = \begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{\theta\theta} \end{bmatrix}$$

$$\therefore W_e = J_{ce}^T W_{cc} = J_{ce}^T K_{cc} \tilde{X}_{cc}$$

$$W_e = \underbrace{(J_{ce}^T K_{cc} J_{ce})}_{K_e} \tilde{X}_e$$

K_e

↑ OP-SPACE P GAINS NEEDED TO LOCATE / ORIENT COMPLIANCE CENTER PROPERLY

OR JUST DEFINE END EFFECTOR FRAME TO COINCIDE WITH OTHER DESIRED COMPLIANCE CENTER THEN: $K_e = K_{cc}$

STIFFNESS CONTROL IS THE SAME AS J^T CONTROL (PD CONTROL IN OP-SPACE) BUT W/ SELECTING P GAINS

• SAME PRINCIPLE CAN BE APPLIED TO D GAINS TO CONTROL DAMPING IN DIFFERENT DIRECTIONS

WE WOULD LIKE TO REPROGRAM THE KINEMATICS OF THE ROBOT

PROGRAM ROBOT TO BEHAVE LIKE A MASS-SPRING-DAMPER SYSTEM WHERE WE GET TO CHOOSE K_e, B_e, M_e

DESIRED CLOSED-LOOP DYNAMICS: $F_e = K_e \tilde{X} + B_e \dot{\tilde{X}} + M_e \ddot{\tilde{X}}$

OR $\tau = J^T (K_e \tilde{X} + B_e \dot{\tilde{X}} + M_e \ddot{\tilde{X}})$

OPEN-LOOP DYNAMICS: $\tau = H(\dot{\Theta}) + V + G + \tau_e$
 ↑ JOINT TORQUES DUE TO ENV CONTACT

CONTROL LAW: $\tau = \hat{H} \ddot{\Theta} + \hat{V} + \hat{G} + J^T (K_e \tilde{X} + B_e \dot{\tilde{X}} + M_e \ddot{\tilde{X}})$

RECALL $\ddot{\Theta} = J^{-1}(\ddot{X} - \dot{J} \dot{\Theta})$

$\tau = \hat{H} J^{-1}(\ddot{X} - \dot{J} \dot{\Theta}) + \hat{V} + \hat{G} + J^T (K_e \tilde{X} + B_e \dot{\tilde{X}} + M_e \ddot{\tilde{X}})$

$= J^T \begin{pmatrix} \ddot{X} \\ \hat{H} \end{pmatrix} (\ddot{X}_d - \ddot{\tilde{X}} - \dot{J} \dot{\Theta}) + \hat{V} + \hat{G} + J^T (K_e \tilde{X} + B_e \dot{\tilde{X}} + M_e \ddot{\tilde{X}})$

$J^T \hat{H} J^{-1}$ OP-SPACE TRANSFORM

$= \hat{V} + \hat{G} + J^T \left[{}^x \hat{H} \ddot{X}_d + K_e \tilde{X} + B_e \dot{\tilde{X}} + (M_e - {}^x \hat{H}) \ddot{\tilde{X}} - {}^x \hat{H} \dot{J} \dot{\Theta} \right]$

PROBLEM: WE CAN'T MEASURE $\ddot{\tilde{X}}$ (OR WE SHOULDN'T)

TRICK TO GET AROUND THIS

REWRITE C.L. DYNAMICS

$\ddot{\tilde{X}} = M_e^{-1} (F_e - K_e \tilde{X} - B_e \dot{\tilde{X}})$ SUB INTO CONTROL LAW

$\tau = \hat{V} + \hat{G} + J^T \left[{}^x \hat{H} \ddot{X}_d + K_e \tilde{X} + B_e \dot{\tilde{X}} + (M_e - {}^x \hat{H}) M_e^{-1} (F_e - K_e \tilde{X} - B_e \dot{\tilde{X}}) - {}^x \hat{H} \dot{J} \dot{\Theta} \right]$

NEW CONTROL LAW: $\tau = \hat{V} + \hat{G} + J^T {}^x \hat{H} [\ddot{X}_d - \dot{J} \dot{\Theta} + M_e^{-1} (K_e \tilde{X} + B_e \dot{\tilde{X}} - F_e)] + J^T F_e$

The diagram illustrates a control system for a robot arm, likely a 2-DOF system, with the following components and signal flow:

- Inputs:**
 - x_d : Desired position input.
 - \ddot{x}_d : Desired acceleration input.
 - F_e : External force input (applied at two points).
- Control Loop 1 (Position/Velocity):**
 - The desired position x_d is compared with the current position x (feedback from the **FORWARD KINEMATICS** block) at a summing junction.
 - The resulting error signal \tilde{x} is processed by a controller block $K_L + B_L s$.
 - The output of this block is compared with the desired velocity \dot{x}_d (feedback from the **FORWARD KINEMATICS** block) at another summing junction.
 - The resulting error signal is processed by an integrator block M_e^{-1} .
- Control Loop 2 (Acceleration):**
 - The output of the M_e^{-1} block is compared with the desired acceleration \ddot{x}_d at a third summing junction.
 - The resulting error signal is processed by a feedforward block \hat{H} .
- Force Compensation:**
 - The external force F_e is compared with the output of the \hat{H} block at a fourth summing junction.
 - The resulting signal is processed by the Jacobian transpose block J^T .
- Robot Dynamics and Feedback:**
 - The output of J^T is compared with the estimated velocity $\hat{V} + \hat{G}$ (feedback from the **FORWARD KINEMATICS** block) at a fifth summing junction.
 - The resulting signal is the input to the **ROBOT + ENVIRONMENT** block.
 - The **ROBOT + ENVIRONMENT** block outputs the current position x and velocity \dot{x} back to the **FORWARD KINEMATICS** block.
 - The **FORWARD KINEMATICS** block also receives the joint angles θ from the **ROBOT + ENVIRONMENT** block.

LOOKS LIKE IDC IN OP-SPACE BUT N/EXTRA FEEDBACK FROM FORCE SENSOR IN 2 SPOTS TO:

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- The diagram illustrates a Reinforcement Learning system. At the top is a box labeled "Robot/Env". Below it, a feedback loop is shown. A signal x goes from a "Robot" block to an "Env" block. The "Env" block outputs a signal $r + \gamma V^*$ to a summation node (a circle with a plus sign). The summation node also receives a signal $-Q$ from a "Q" block. The output of the summation node goes to the "Robot" block. The "Robot" block also outputs a signal x to the "Env" block. The "Env" block outputs a signal x to a "Q" block. The "Q" block outputs a signal $-Q$ to the summation node. The "Q" block also receives a signal Q from the "Robot/Env" block.

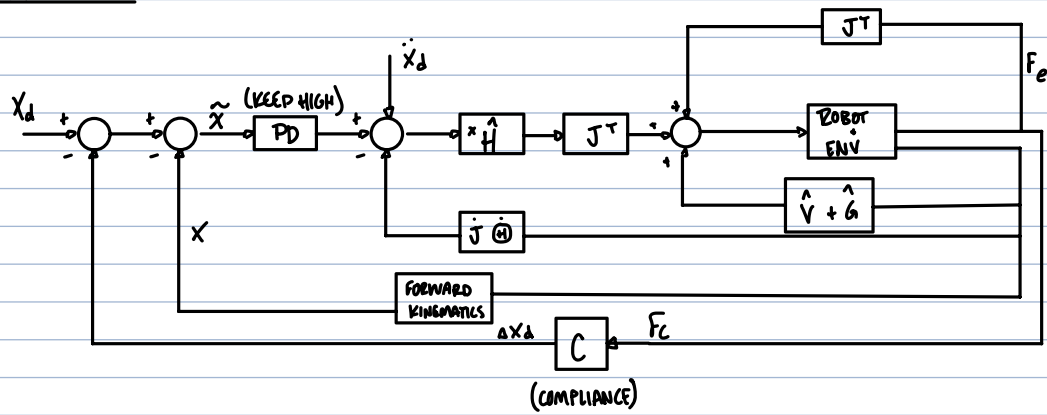
$$^x H M_e^{-1} = \text{RATIO OF ACTUAL INERTIA TO DESIRED INERTIA}$$

EFFECTIVELY REPLACES ACTUAL INERTIA OF ROBOT W/ DESIRED INERTIA

ADMITTANCE CONTROL

FORCE SENSOR IS REQUIRED

COMPLIANCE CONTROL



CAUSALITY : A FORCE F_e CAUSES A DEFLECTION IN DESIRED TRAJECTORY Δx_d . ROBOT COMPLIES w/FORCE F_e BY BACKING OFF DESIRED POSITION

CAN GENERALIZE TO "ADMITTANCE CONTROL" BY REPLACING C w/ $A(s)$

DESIRED CL DYNAMICS

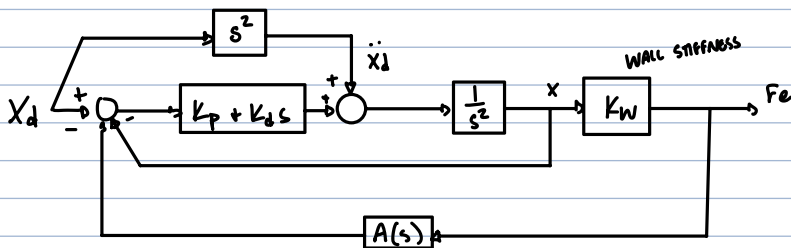
$$M_e \Delta \ddot{x}_d + B_e \Delta \dot{x}_d + K_e \Delta x_d = F_e$$

$$\frac{\Delta x_d}{F_e} = \frac{1}{M_e s^2 + B_e s + K_e} = A(s)$$

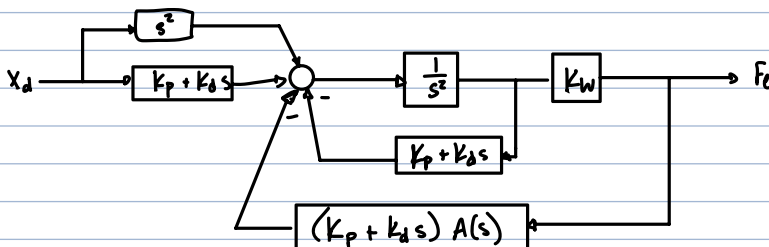
CAN STILL HAVE HIGH PD GAINS IN INNER LOOP TO GUARANTEE GOOD TRACKING & DISTURBANCE REJECTION
BUT OUTER LOOP CAN BE SENSITIVE TO EXTERNAL FORCE AT FORCE SENSOR

IS ADMITTANCE CONTROL STABLE?

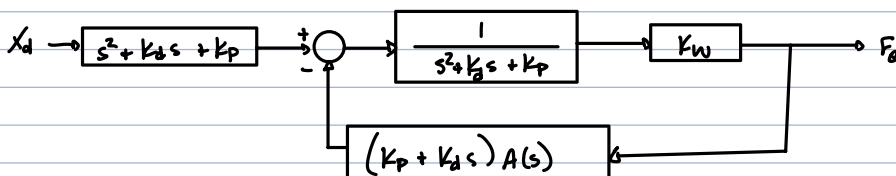
ASSUMING INVERSE DYNAMICS CONTROL REDUCES INNER LOOP DYNAMICS TO $\frac{1}{s^2}$



⇓

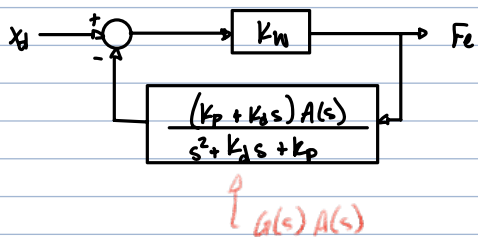


⇓



FEEDBACK RULE

$$\frac{\frac{1}{s^2}}{1 + \frac{1}{s^2}(K_p + K_d s)} = \frac{1}{s^2 + K_d s + K_p}$$

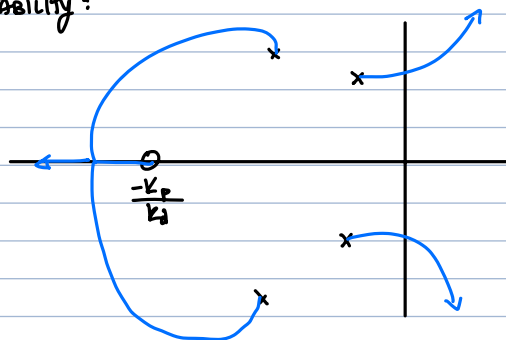


$$A(s) = \frac{1}{M_e s^2 + B_e s + K_e}$$

$$G(s) = \frac{K_p + K_d s}{s^2 + K_d s + K_p}$$

At low gain, system is stable, but as loop increases, it will go unstable

STABILITY?



With stiffness is part of loop gain, admittance goes unstable for stiff environments.

OLTF

$$\frac{K_w (K_p + K_d s)}{(s^2 + K_d s + K_p)(M_e s^2 + B_e s + K_e)}$$