INTRO TO ROBOTICS FORMULA SHEET

NOTATION

POINTS: O_j , P_j → REPRESENTATION : ${}^{j}P_i = \begin{bmatrix} {}^{j}p_i \\ 1 \end{bmatrix}$

${}^{j}P_i$ = POINT
${}^{j}\underline{p}_i$ = VECTOR

j = COORDINATE SPACE

SCALAR: a_j , b_j

VECTOR ${}^{i}\underline{V}_j$

i = COORDINATE AXES
j = jTH COORDINATE OF $\vec{V}_j$

$${}^{j}\vec{V} = {}^{j}v_1 \, {}^{j}\vec{x}_j + {}^{j}v_2 \, {}^{j}\vec{y}_j = \begin{bmatrix} {}^{j}v_1 \\ {}^{j}v_2 \end{bmatrix}$$

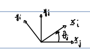
← ASSUME ALL VECTORS ARE COLUMNS

SKEW MATRIX $\underset{\sim}{S}(\underline{k})\underline{v} = \underline{k} \times \underline{v}$

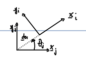
$$\underset{\sim}{S}(\underset{\sim}{k}) = \begin{bmatrix} 0 & -k_3 & k_2 \\ k_3 & 0 & -k_1 \\ -k_2 & k_1 & 0 \end{bmatrix}$$

ROTATION/TRANSLATION

$${}^{i}\underset{\sim}{R}_j = R(\theta_j)$$ = ROTATION FROM AXES $\{j\}$ TO $\{i\}$



$${}^{i}\underline{T}_j = \text{Trans}({}^{i}\underline{d}_{ij})\,\text{Rot}(\theta_{ij})$$ = TRANFORMATION FROM $\{j\}$ TO $\{i\}$



${}^{i}\underline{d}_{ij}$ = DISPLACEMENT FROM O_i TO O_j IN $\{i\}$

${}^{1}\underline{d}_{01} = {}^{1}\underline{d}_{10}$

VELOCITY $\underline{x}' = \dot{\underline{x}}(t) \rightsquigarrow \underline{x}_0' = \dot{\underline{x}}(0)$ (INITIAL VELOCITY)

$\underline{x}_1'' = \ddot{\underline{x}}(t_1)$

$${}^{1}\underline{P} = {}^{1}\hat{R}_2 \, {}^{2}\underline{P}$$

TRANSFORMATIONS ← STATIC DESCRIPTION OF LOCATION BETWEEN LOCATION OF TWO COORDINATE SYSTEMS

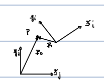
- DESCRIBE A COORDINATE TRANSFORMATION

POINTS: ${}^{1}P$ =

${}^{0}P$ = POINT IN $\{0\}$
${}^{1}P$ IS POINT P IN $\{1\}$

VECTORS ${}^{0}\underline{P}_1 = {}^{0}\underset{\sim}{R}_1 \, {}^{1}\underline{P}_1$

${}^{0}\underline{P}_1$ = HOW DISTANCE FROM O_1 TO P LOOKS IN $\{0\}$
${}^{0}\underline{P}_0$ = HOW DISTANCE FROM O_0 TO P LOOKS IN $\{0\}$



$$\begin{bmatrix} {}^{0}\underline{P}_0 \\ 1 \end{bmatrix} = {}^{0}\underset{\sim}{T}_1 \begin{bmatrix} {}^{1}\underline{P}_1 \\ 1 \end{bmatrix}$$

$${}^{0}\underline{P}_0 = {}^{0}\underline{d}_{01} + {}^{0}\underset{\sim}{R}_1 \, {}^{1}\underline{P}_1$$

OPERATORS

ANGLE - AXIS FORMULA (ROTATION OF BASIS ABOUT ARBITRARY VECTOR $\underline{k}$ BY θ)

$$\underline{R}_k = \underset{\sim}{I}\, C_\theta + \underset{\sim}{S}(\underline{k}) S_\theta + \underline{k}\,\underline{k}^T (1 - C_\theta)$$

$\underset{\sim}{S}(\underline{k})$ = SKEW MATRIX OF $\underline{k}$

HOMOGENEOUS = SAME COORDINATE SPACE

ORTHONORMAL BASE VECTORS

$${}^{j}\underline{x} \cdot {}^{j}\underline{y} = 0$$

$$\|\underline{x}_j\| = \|\underline{y}_j\| = 1$$

$${}^{j}\underline{V} \cdot {}^{j}\underline{W} = {}^{j}\underline{V}^T \, {}^{j}\underline{W} = {}^{j}V_1 \, {}^{j}W_1 + {}^{j}V_2 \, {}^{j}W_2$$

TRANSLATION + ROTATION DEFINITIONS

$${}^{1}\underset{\sim}{R}_2 = \begin{bmatrix} {}^{1}x_{22} & {}^{1}y_{21} \\ {}^{1}x_{22} & {}^{1}y_{22} \end{bmatrix} = \begin{bmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{bmatrix}$$

$$\underset{\sim}{Rot}(\theta_j) = \begin{bmatrix} \underset{\sim}{R}(\theta_j) & \underline{0} \\ \underline{0}^T & 1 \end{bmatrix}$$

TRANSFORMING COORDINATE AXES

$${}^{0}P = {}^{0}\underset{\sim}{T}_1 \, {}^{1}P$$

${}^{0}\underset{\sim}{T}_1$ = 3×3 HOMOGENEOUS TRANSFORMATION

$$\underset{\sim}{Trans}(d) = \begin{bmatrix} \underset{\sim}{I} & \underline{d} \\ \underline{0}^T & 1 \end{bmatrix}$$

I.E. $\mathbb{R}^2$ $\text{Trans}(\underline{v}) = \begin{bmatrix} 1 & 0 & v_1 \\ 0 & 1 & v_2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \underset{\sim}{I} & \underline{v} \\ \underline{0}^T & 1 \end{bmatrix}$

TRANSFORMATIONS

- DESCRIBE THINGS IN OTHER COORDINATE FRAMES

$${}^{i}\underset{\sim}{T}_j = \text{Trans}({}^{i}d_{ij})\,\text{Rot}(\theta_{ij}) = \begin{bmatrix} \underset{\sim}{R}(\theta_{ij}) & {}^{i}\underline{d}_{ij} \\ \underline{0}^T & 1 \end{bmatrix}$$

NOTE: $\text{Rot}(\theta_i)\,\text{Trans}({}^{0}\underline{d}_{01}) \neq \text{Trans}({}^{0}\underline{d}_{01})\,\text{Rot}(\theta_i)$

INVERSE TRANSFORMATION RULES

$$({}^{i}\underset{\sim}{T}_j)^{-1} = {}^{j}\underset{\sim}{T}_i$$

$$({}^{i}\underset{\sim}{T}_j)^{T} \neq {}^{i}\underset{\sim}{T}_j^{-1}$$

OPERATORS ← DISPLACEMENTS RELATIVE TO A COORDINATE FRAME

- THESE MOVE THINGS
- A

$${}^{0}\underset{\sim}{D}_0 = {}^{0}\underset{\sim}{T}_1 \, {}^{1}\underset{\sim}{D}_1 ({}^{0}\underset{\sim}{T}_1)^{-1}$$