INTRO TO ROBOTICS FORMULA SHEET

HOMOGENEOUS = SAME COORDINATE SPACE

NOTATION

POINTS: $O_j,\ P_j$

SCALAR: $a_j,\ b_j$

VECTOR $\,{}^{i}\underline{V}_j$ $\quad$ i = COORDINATE AXES, j = j^{TH} COORDINATE OF $\vec{V}_j$

$${}^{j}\vec{V} = {}^{j}\nu_1\,{}^{j}\vec{x}_j + {}^{j}\nu_2\,{}^{j}\vec{y}_j = \begin{bmatrix} {}^{j}\nu_1 \\ {}^{i}\nu_2 \end{bmatrix}$$ ← ASSUME ALL VECTORS ARE COLUMNS

$\underline{d}_{12}$ = DISPLACEMENT FROM P_1 TO P_2

ORTHONORMAL BASE

$${}^{j}\underline{x} \cdot {}^{j}\underline{y} = 0$$

$$\|\underline{x}_j\| = \|\underline{y}_j\| = 1$$

VELOCITY

$$\underline{x}' = \dot{\underline{x}}(t) \rightsquigarrow \underbrace{\underline{x}_0'}_{\text{INITIAL VELOCITY}} = \dot{\underline{x}}(0)$$

$$\underline{x}_1'' = \ddot{\underline{x}}(t_1)$$

$${}^{1}\underline{P} = {}^{1}\underset{\sim}{R}_2\,{}^{2}\underline{P}$$

$${}^{j}\underline{V} \cdot {}^{j}\underline{W} = {}^{j}\underline{V}^{T}\,{}^{j}\underline{W} = {}^{j}V_1\,{}^{j}W_1 + {}^{j}V_2\,{}^{j}W_2$$

$${}^{j-1}\underset{\sim}{R} = \underset{\sim}{R}(\theta_j)$$ $\quad$ i & j RELATED BY ${}^{i}\underset{\sim}{R}_j$

${}^{1}\underline{x}_2 = x_2$ IN TERMS OF $\{1\}$

$${}^{1}\underset{\sim}{R}_2 = \begin{bmatrix} {}^{1}x_{22} & {}^{1}y_{21} \\ {}^{1}x_{22} & {}^{1}y_{22} \end{bmatrix} = \begin{bmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{bmatrix}$$

$$\underset{\sim}{Rot}(\theta_j) = \begin{bmatrix} \underset{\sim}{R}(\theta_j) & \underline{0} \\ \underline{0}^{T} & 1 \end{bmatrix}$$

$${}^{0}P = {}^{0}\underset{\sim}{T}_1\,{}^{1}P$$

${}^{0}\underset{\sim}{T}_1$ = 3x3 HOMOGENEOUS TRANSFORMATION

$$\underset{\sim}{Trans}(d) = \begin{bmatrix} \underset{\sim}{I} & \underline{d} \\ \underline{0}^{T} & 1 \end{bmatrix}$$

i.e. $\mathbb{R}^2$ $\quad Trans(\underline{v}) = \begin{bmatrix} 1 & 0 & v_1 \\ 0 & 1 & v_2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \underset{\sim}{I} & \underline{v} \\ \underline{0}^{T} & 1 \end{bmatrix}$

$${}^{i}\underset{\sim}{T}_j = Trans({}^{i}d_{ij})\,Rot(\theta_{ij}) = \begin{bmatrix} \underset{\sim}{R}(\theta_{ij}) & {}^{i}\underline{d}_{ij} \\ \underline{0}^{T} & 1 \end{bmatrix}$$

NOTE: $Rot(\theta_i)\,Trans({}^{0}\underline{d}_{01}) \neq Trans({}^{0}\underline{d}_{01})\,Rot(\theta_i)$