CS 5310/6310 & ME 5220/6220 **Homework 2** **Due September 14, 2022**

1. (24 pts) Consider the homogeneous transformations

$$ {}^0\mathbf{T}_1 = \mathbf{Trans}([1 \ \ -1]^T)\ \mathbf{Rot}(\pi) \qquad {}^1\mathbf{T}_2 = \mathbf{Trans}([-1 \ \ -2]^T)\ \mathbf{Rot}(-\pi/2) $$

   (a) (8pts) Draw a diagram to scale showing the relative locations of frames 0, 1, and 2.

   (b) (4pts) Suppose point P is fixed in coordinate system 2, and is located by $\mathbf{p}_2 = P - O_2$. Suppose ${}^2\mathbf{p}_2 = [1\ 1]^T$; draw $\mathbf{p}_2$ in your diagram. Find ${}^1\mathbf{p}_2$ and ${}^0\mathbf{p}_2$.

   (c) (6pts) Suppose $\mathbf{p}_1 = P - O_1$. Draw $\mathbf{p}_1$, and find ${}^2\mathbf{p}_1$, ${}^1\mathbf{p}_1$, and ${}^0\mathbf{p}_1$.

   (d) (6pts) Suppose $\mathbf{p}_0 = P - O_0$. Draw $\mathbf{p}_0$, and find ${}^2\mathbf{p}_0$, ${}^1\mathbf{p}_0$, and ${}^0\mathbf{p}_0$.

2. (20pts) Again consider the homogeneous transformations

$$ {}^0\mathbf{T}_1 = \mathbf{Trans}([1 \ \ -1]^T)\ \mathbf{Rot}(\pi) \qquad {}^1\mathbf{T}_2 = \mathbf{Trans}([-1 \ \ -2]^T)\ \mathbf{Rot}(-\pi/2) $$

   (a) (10pts) Show a figure giving an interpretation of the product ${}^0\mathbf{T}_1\ {}^1\mathbf{T}_2$ as a coordinate transformation.

   (b) (10pts) Show a figure giving an interpretation of the product ${}^0\mathbf{T}_1\ {}^1\mathbf{T}_2$ as an operator.

3. (10pts) Again consider the transformation ${}^0\mathbf{T}_1$ from above:

$$ {}^0\mathbf{T}_1 = \mathbf{Trans}([1 \ \ -1]^T)\ \mathbf{Rot}(\pi) $$

   but now consider that

$$ \mathbf{D}_1 = \mathbf{Trans}([-1 \ \ -2]^T)\ \mathbf{Rot}(-\pi/2) $$

   is an operator in frame 1. Find the equivalent operator $\mathbf{D}_0$ in frame 0.

4. (42pts) This question concerns the concatenation of two polynomial trajectories $x_1(t)$ and $x_2(t)$.

   - Polynomial $x_1(t)$ is valid in the time range $0 \le t \le t_1$. At $t = 0$, the position and velocity are zero.
   - Polynomial $x_2(t)$ is valid in the time range $t_1 \le t \le t_2$. At $t = t_2$, the position is $x2$, and the velocity is zero.
   - At $t = t_1$, the two polynomials have the same position, velocity and acceleration.

   (a) (8pts) Identify the constraints at time 0, t_1 and t_2.

   (b) (6pts) What are the minimal degrees of the two polynomials? Write their equations, and first and second derivatives.

(c) (5pts) Substitute the unary constraints into the equations above. Identify which lead immediately to coefficient solutions.

(d) (3pts) Now substitute the above polynomial equations into the binary constraints.

(e) (16pts) Find the coefficients of polynomial 2.

(f) (4pts) Find the coefficients of polynomial 1.