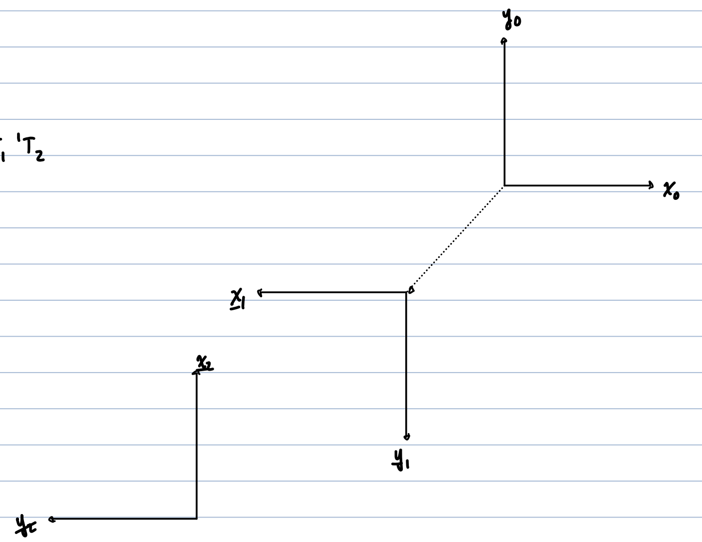
06 Operators

$\underset{\sim}{{}^0T_1} = \text{Trans}\begin{bmatrix}-1\\-1\end{bmatrix}\text{Rot}(\pi)$

$\underset{\sim}{{}^1T_2} = \text{Trans}\begin{bmatrix}2\\1\end{bmatrix}\text{Rot}\left(\frac{-\pi}{2}\right)$

$\underset{\sim}{{}^0T_2} = {}^0T_1\,{}^1T_2$



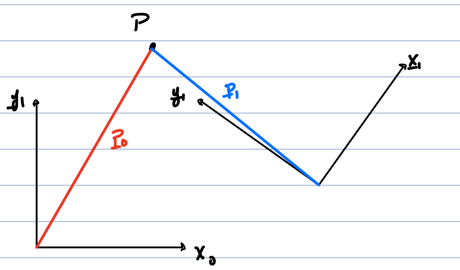
COORDINATE SYSTEM TRANSFORMATION EQN:

$\underset{\sim}{{}^0p_0} = {}^0\underline{d}_{01} + \underset{\sim}{{}^0R_1}\,{}^1p_1$

NOW...

$\underset{\sim}{{}^0p_0} = {}^0\underline{d} + \underset{\sim}{R}\,{}^0p_1$

SAYS THE SAME AS ABOVE



HOMOGENEOUS COORDINATE TRANSFORMATION

$$\begin{bmatrix}{}^0p_0\\1\end{bmatrix} = \begin{bmatrix}{}^0R_1 & {}^0d_{01}\\ \underline{0}^T & 1\end{bmatrix}\begin{bmatrix}{}^1p_1\\1\end{bmatrix}$$

$$= \begin{bmatrix}R & {}^0\underline{d}\\ \underline{0}^T & 1\end{bmatrix}\begin{bmatrix}{}^0p_1\\1\end{bmatrix}$$

$$= \underset{\sim}{{}^0D_1}\begin{bmatrix}{}^0p_1\\1\end{bmatrix}$$

OPERATOR WORKING IN FRAME 1

(NOT A COORDINATE TRANSFORMATION)

$\underset{\sim}{{}^1D_1} = \text{Trans}\begin{bmatrix}2\\1\end{bmatrix}\text{Rot}\left(\frac{-\pi}{2}\right) = \begin{bmatrix}0 & 1 & 2\\-1 & 0 & 1\\0 & 0 & 1\end{bmatrix}$ ${}^1D_1$

1ST OPERATOR OPERATING IN FRAME 1

. DIFFERENT

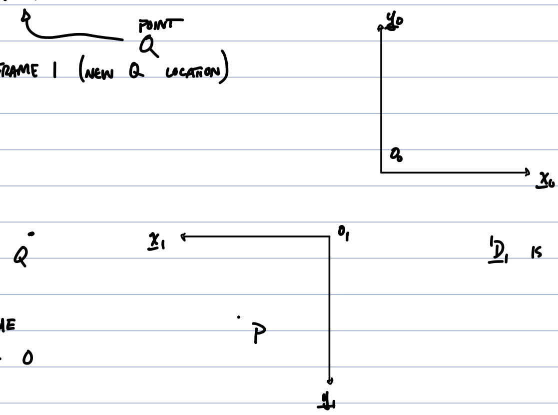
EXAMPLE

MOVE $\begin{bmatrix}1\\1\\1\end{bmatrix}$ IN FRAME 1 W/ OPERATOR ${}^1\tilde{D}_1$

$${}^1\tilde{D}_1 \begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}3\\0\\1\end{bmatrix}$$ WITHIN FRAME 1

POINT Q

MOVED $\underset{P}{\begin{bmatrix}1\\1\\1\end{bmatrix}}$ WITHIN FRAME 1 (NEW Q LOCATION)



${}^1\tilde{D}_1$ IS WITHIN FRAME 1

REFERRING OPERATORS TO ANOTHER FRAME
- FIND POINT Q IN FRAME OF 0

$$\underbrace{{}^0\tilde{D}_0 = {}^0\tilde{T}_1\, {}^1\tilde{D}_1\, {}^0\tilde{T}_1^{-1}}$$

TENSOR TRANSFORM
- ROTATING/TRANSFORMING A MATRIX TO ANOTHER COORDINATE SYSTEM OR FRAME

POINT Q IN FRAME 0

EX:

$${}^0\tilde{D}_0 \begin{bmatrix}-2\\-2\\1\end{bmatrix} = {}^0\tilde{T}_1\, {}^1\tilde{D}_1\, {}^0\tilde{T}_1^{-1} \begin{bmatrix}-2\\-2\\1\end{bmatrix}$$

TRANSFORM TO FRAME 0

DISPLACE

TRANSFORM <u>BACK</u>

$$\begin{array}{cccc} & {}^0T_1 & {}^1\tilde{D}_1 & {}^1\tilde{T}_0 = ({}^0T_1)^{-1} \\ {}^0D_0 = & \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

TEST BY APPLYING OPERATOR TO SEE IF THE POINTS END @ THE SAME POINT

Q WRT FRAME 0

$${}^0\tilde{D}_0 \begin{bmatrix}-2\\-2\\1\end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix}-2\\-2\\1\end{bmatrix} = \begin{bmatrix}-4\\-1\\1\end{bmatrix}$$

POLE OF A PLANAR TRANSFORMATION

- <u>EVERY</u> OPERATOR HAS A POINT THAT DOESN'T MOVE (INVARIANCE)

POLE

$${}^1\tilde{p} = \tilde{R}\,{}^1\tilde{p} + {}^1\tilde{d}$$

$${}^1\tilde{p} - \tilde{R}\,{}^1\tilde{p} = {}^1\tilde{d}$$

$$(\tilde{I} - \tilde{R})\,{}^1\tilde{p} = {}^1\tilde{d}$$

$${}^1\tilde{p} = (\tilde{I} - \tilde{R})^{-1}\,{}^1\tilde{d}$$

HOW TO FIND THE POLE

(IN PURE TRANSLATION, EVERYTHING WILL APPEAR TO BE A POLE)

IF YOU REFER YOUR COORDINATE SYSTEM TO A POLE, <u>EVERYTHING</u> CAN BE TREATED AS A ROTATION