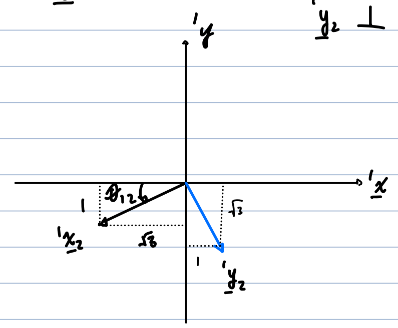
# ROBOTICS HW 1

1. ${}^1\underline{x}_2 = \frac{1}{2}\begin{bmatrix}-\sqrt{3}\\-1\end{bmatrix}$

A) WHAT IS ${}^1\underline{y}_2$ ?

$${}^1\underline{y}_2 \perp {}^1\underline{x}_2 \Rightarrow \boxed{{}^1\underline{y}_2 = \frac{1}{2}\begin{bmatrix}1\\-\sqrt{3}\end{bmatrix}}$$




$$\begin{bmatrix}{}^2\underline{x} & {}^2\underline{y}\end{bmatrix} = \underset{\sim}{R} \quad \begin{bmatrix}{}^1\underline{x}\\ {}^1\underline{y}\end{bmatrix}$$

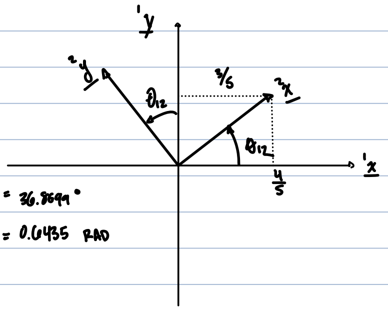
B) $${}^1\underset{\sim}{R}_2 = Z(\underbrace{\text{ATAN2}(-1/2, -\sqrt{3}/2)}_{=\,-2.618\ \text{RAD}\ =\ \frac{-5\pi}{6}\ =\ \frac{7\pi}{6}})$$

$$\therefore\ {}^1\underset{\sim}{R}_2 = \begin{bmatrix}C_\theta & -S_\theta\\ S_\theta & C_\theta\end{bmatrix} = \begin{bmatrix}-0.866 & 0.5\\ -0.5 & -0.866\end{bmatrix} = \begin{bmatrix}\frac{-\sqrt{3}}{2} & \frac{1}{2}\\ \frac{-1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

C) $\theta = -150° = 210°$

OR... $\frac{-5\pi}{6} = \frac{7\pi}{6}$

2) ${}^1\underset{\sim}{x}_2 = \frac{1}{5}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$




$\theta_{12} = \tan^{-1}\left(\frac{3/5}{4/5}\right) = 36.8699°$

$= 0.6435$ RAD

A) ${}^1\underset{\sim}{y}_2 = \frac{1}{5}\begin{bmatrix} -3 \\ 4 \end{bmatrix}$

B) ${}^1\underset{\sim}{R}_2 = \begin{bmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{bmatrix} = \begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$ $\quad \therefore \boxed{{}^1\underset{\sim}{R}_2 = \begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}}$

C) ${}^2\underset{\sim}{R}_1 = \underbrace{({}^1\underset{\sim}{R}_2)^T = ({}^1\underset{\sim}{R}_2)^{-1}}_{\text{PROPERTY OF ROTATION MATRICES}} = \begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}$ $\quad \therefore \boxed{{}^2\underset{\sim}{R}_1 = \begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}}$

D) ${}^2\underset{\sim}{p} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ WHAT IS ${}^1\underset{\sim}{p}$

${}^1\underset{\sim}{p} = {}^1\underset{\sim}{R}_2 \, {}^2\underset{\sim}{p} \quad \leadsto \quad {}^1\underset{\sim}{p} = \underset{2\times 2}{\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} -1 \\ -1 \end{bmatrix}} = \begin{bmatrix} -0.8 + 0.6 \\ -0.6 - 0.8 \end{bmatrix} = \begin{bmatrix} -0.2 \\ -1.4 \end{bmatrix}$

$\therefore \boxed{{}^1\underset{\sim}{p} = \begin{bmatrix} -0.2 \\ -1.2 \end{bmatrix}}$

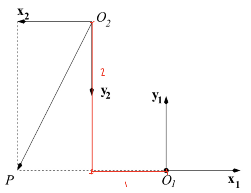
E) ${}^1\underset{\sim}{q} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ FIND ${}^2\underset{\sim}{q}$

${}^2\underset{\sim}{q} = {}^2\underset{\sim}{R}_1 \, {}^1\underset{\sim}{q} = \begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 0.8 - 1.6 \\ -0.6 - 2.4 \end{bmatrix} = \begin{bmatrix} -0.8 \\ -3 \end{bmatrix}$

$\boxed{{}^2\underset{\sim}{q} = \begin{bmatrix} -0.8 \\ -3 \end{bmatrix}}$

3. (30 pts) Consider the coordinate system 2 in relation to coordinate system 1 below. Suppose $O_2$ is located at (-1,2) relative to coordinate system 1, and point $P$ is located at (1,2) relative to coordinate system 2. Let $\mathbf{p}_i = P - O_i$. What are the following vectors (i.e., their x and y components)?

(a) (6 pts) ${}^1\mathbf{x}_2$ and ${}^1\mathbf{y}_2$.
(b) (8 pts) ${}^2\mathbf{p}_2$ and ${}^1\mathbf{p}_2$.
(c) (8 pts) ${}^1\mathbf{p}_1$ and ${}^2\mathbf{p}_1$.
(d) (8 pts) ${}^1\mathbf{d}_{12}$ and ${}^2\mathbf{d}_{12}$.




$p_i = P - O_i$

A) $\boxed{{}^1\underline{x}_2 = \begin{bmatrix}-1\\0\end{bmatrix} \qquad {}^1\underline{y}_2 = \begin{bmatrix}0\\-1\end{bmatrix}}$

B) ${}^2\underline{p}_2 = P - O_2 = \begin{bmatrix}1\\2\end{bmatrix}$

${}^1\underline{p}_2 = {}^1\underset{\sim}{R}_2\,{}^2\underline{p}_2 \qquad \theta = \pi \rightsquigarrow {}^1\underset{\sim}{R}_2 = \begin{bmatrix}\cos(\pi) & -\sin(\pi)\\ \sin(\pi) & \cos(\pi)\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

${}^1\underline{p}_2 = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}-1\\-2\end{bmatrix}$

$\boxed{{}^2\underline{p}_2 = \begin{bmatrix}1\\2\end{bmatrix} \qquad {}^1\underline{p}_2 = \begin{bmatrix}-1\\-2\end{bmatrix}}$

c) ${}^1\underline{p}_1 = P - O_1 = \begin{bmatrix}-2\\0\end{bmatrix} \qquad {}^2\underline{p}_1 = {}^2\underset{\sim}{R}_1\,{}^1\underline{p}_1$

${}^2\underset{\sim}{R}_1 = ({}^1R_2)^T = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

$\Rightarrow {}^2\underline{p}_1 = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}-2\\0\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}$

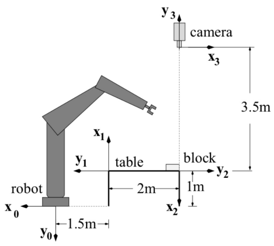
$\boxed{{}^1\underline{p}_1 = \begin{bmatrix}-2\\0\end{bmatrix} \qquad {}^2\underline{p}_1 = \begin{bmatrix}2\\0\end{bmatrix}}$

D) ${}^1\underline{d}_{12} = O_2 - O_1 = \begin{bmatrix}-1\\2\end{bmatrix}$

${}^2\underline{d}_{12} = {}^2\underset{\sim}{R}_1\,{}^1\underline{d}_{12} = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}1\\-2\end{bmatrix}$

$\boxed{{}^1\underline{d}_{12} = \begin{bmatrix}-1\\2\end{bmatrix} \qquad {}^2\underline{d}_{12} = \begin{bmatrix}1\\-2\end{bmatrix}}$

4. (40 pts) Consider the combination of robot, table, block, and camera in the figure below, with associated coordinate systems as shown.




(a) (28pts) Find ${}^0\mathbf{R}_1$, ${}^1\mathbf{R}_2$, ${}^2\mathbf{R}_3$ and ${}^0\mathbf{R}_3$ by inspection.

(b) (12pts) Suppose ${}^0\mathbf{p} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. By inspection, find ${}^1\mathbf{p}$, ${}^2\mathbf{p}$, and ${}^3\mathbf{p}$.

A) ${}^0\underset{\sim}{R}_1 = R(\frac{\pi}{2}) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ ${}^1\underset{\sim}{R}_2 = R(\pi) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

${}^2\underset{\sim}{R}_3 = R(\frac{3\pi}{2}) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$$\therefore\ {}^0\underset{\sim}{R}_3 = {}^0\underset{\sim}{R}_1\, {}^1\underset{\sim}{R}_2\, {}^2\underset{\sim}{R}_3 = \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}_{\begin{bmatrix} 0+0 & 0+1 \\ -1+0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\therefore\ \boxed{{}^0\underset{\sim}{R}_3 = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}_{R(\pi)}}$$

B) $\boxed{{}^0\underset{\sim}{p} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad {}^1\underset{\sim}{p} = \begin{bmatrix} -2 \\ -1 \end{bmatrix} \quad {}^2\underset{\sim}{p} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad {}^3\underset{\sim}{p} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}}$