1. (30 pts) Suppose $^1\mathbf{x}_2 = \frac{1}{2}\begin{bmatrix} -\sqrt{3} \\ -1 \end{bmatrix}$.

   (a) (10pts) What is $^1\mathbf{y}_2$? Draw a diagram showing the orientation of axes 2 relative to axes 1.

   (b) (10pts) What is the matrix $^1\mathbf{R}_2$?

   (c) (10pts) What is the rotation angle θ such that $^1\mathbf{R}_2 = \mathbf{R}(\theta)$?

2. (30pts) Suppose $^1\mathbf{x}_2 = \frac{1}{5}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$.

   (a) (6pts) What is $^1\mathbf{y}_2$?

   (b) (6pts) What is $^1\mathbf{R}_2$?

   (c) (6pts) What is $^2\mathbf{R}_1$?

   (d) (6pts) Given $^2\mathbf{p} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$, what is the numerical value of $^1\mathbf{p}$?

   (e) (6pts) Given $^1\mathbf{q} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, what is the numerical value of $^2\mathbf{q}$?

3. (30 pts) Consider the coordinate system 2 in relation to coordinate system 1 below. Suppose O_2 is located at (-1,2) relative to coordinate system 1, and point P is located at (1,2) relative to coordinate system 2. Let $\mathbf{p}_i = P - O_i$. What are the following vectors (i.e., their x and y components)?

   (a) (6 pts) $^1\mathbf{x}_2$ and $^1\mathbf{y}_2$.

   (b) (8 pts) $^2\mathbf{p}_2$ and $^1\mathbf{p}_2$.

   (c) (8 pts) $^1\mathbf{p}_1$ and $^2\mathbf{p}_1$.

   (d) (8 pts) $^1\mathbf{d}_{12}$ and $^2\mathbf{d}_{12}$.
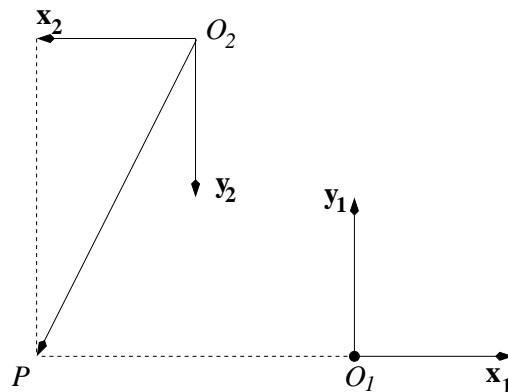
4. (40 pts) Consider the combination of robot, table, block, and camera in the figure below, with associated coordinate systems as shown.
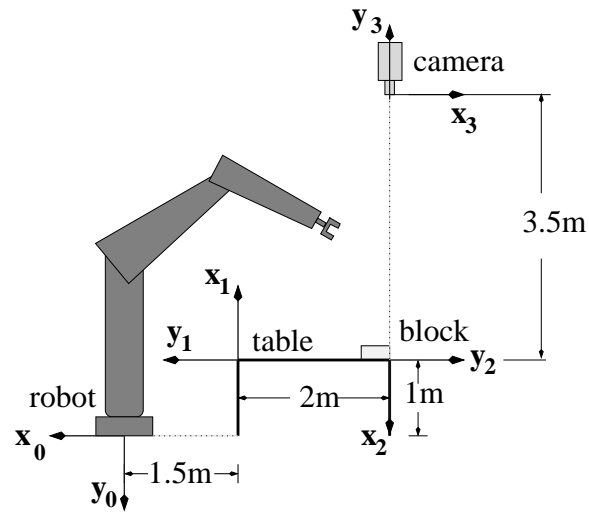
(a) (28pts) Find $^0\mathbf{R}_1$, $^1\mathbf{R}_2$, $^2\mathbf{R}_3$ and $^0\mathbf{R}_3$ by inspection.

(b) (12pts) Suppose $^0\mathbf{p} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. By inspection, find $^1\mathbf{p}$, $^2\mathbf{p}$, and $^3\mathbf{p}$.