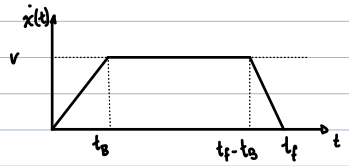
08 Trajectories

LSPB TRAJECTORIES

$\dot{x}(0) = 0$

$\dot{x}(t_B) = v$



x_1 x_2 x_3

$x_1(t) = a_0 + a_1 t + a_2 t^2 \qquad 0 \le t \le t_B$

$x_2(t) = b_0 + b_1 (t_f - t_B) \qquad t_B \le t \le t_f - t_B$

$x_3(t) = c_0 + c_1 (t - (t_f - t_B)) + c_2 (t - (t_f - t_B))^2 \qquad t_f - t_B \le t \le t_f$

9 UNKNOWNS $\Rightarrow$ 9 CONSTRAINTS

$x(0) = 0 \qquad x_1(0) = 0$

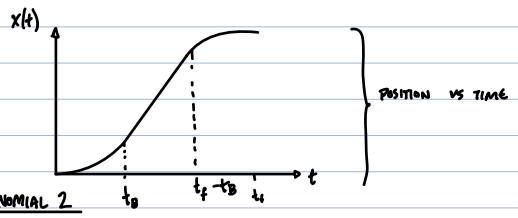
$\dot{x}(0) = 0$

$\dot{x}_1(t_B) = v$

$\dot{x}_2(t_f - t_B) = v \qquad x_1(t_B) = x_2(t_B)$

$\dot{x}_3(t_f - t_B) = v \qquad x_2\left(\frac{t_f}{2}\right) = \frac{x_1 + x_0}{2}$ ⟵ SYMMETRY CONSTRAINT

$\dot{x}_3(t_f) = 0$ (COULD BE CONTINUITY, BUT THIS IS TRADITIONAL)



POLYNOMIAL 1

$\Rightarrow x_1(0) = a_0 = x_0$

$\dot{x}_1(0) = a_1 = 0$

$\dot{x}_2(t_B) = 2a_2 t_B = v$

$\rightsquigarrow a_2 = \frac{v}{2t_B}$

POLYNOMIAL 2

$x_2\left(\frac{t_f}{2}\right) = \frac{x_0 + x_1}{2} = b_0 + v\left(\frac{t_f}{2} - t_B\right)$

$\Rightarrow b_0 = \frac{x_0 + x_1}{2} - v\left(\frac{t_f}{2} - t_B\right)$

POLYNOMIAL 3

$\dot{x}_3(t_f - t_B) = v = c_1$

$\dot{x}_3(t_f) = v + 2c_2\left(t_f - (t_f - t_B)\right) = 0$

$\rightsquigarrow c_2 = \frac{-v}{2t_B}$

$x_3(t_f) = c_0$

MULTIPLE DIMENSION POLYNOMIALS

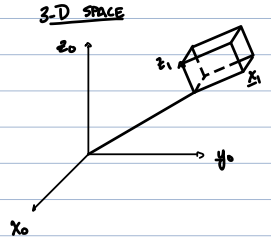
$\underline{x}(t) = \underline{a}_0 + \sum_{i=1}^{n} \underline{a}_i t^i$

$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_m(t) \end{bmatrix} = \begin{bmatrix} a_{01} \\ a_{02} \\ \vdots \\ a_{0m} \end{bmatrix} + \begin{bmatrix} a_{10} \\ a_{11} \\ & \\ a_{1m} \end{bmatrix}$

PLANE

$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ y_1(t) \\ \theta_1(t) \end{bmatrix}$

3-D SPACE



$$x(t) = \begin{bmatrix} x_1(t) \\ y_1(t) \\ z_1(t) \\ \phi_1(t) \\ \theta_1(t) \\ \psi_1(t) \end{bmatrix}$$ ← STATE SPACE REPRESENTATION

PLANNING TRAJECTORY OF ROBOT IN JOINT SPACE

$$\underset{\text{MULTIDIMENSIONAL POLYNOMIAL}}{\underline{x}(t)} = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \vdots \\ \theta_n(t) \end{bmatrix}$$

LINEAR AXIS DESCRIBED BY d

ROLL AXIS

PITCH AXIS