05 Coordinate Transformations

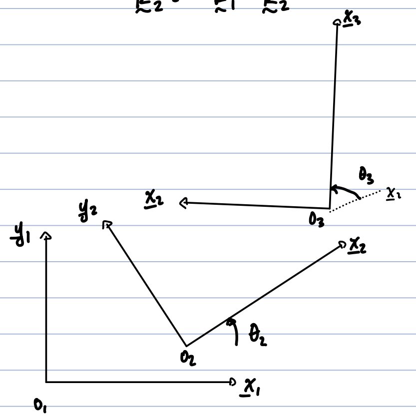
Chaining Rotation Matrices

$^0R_1\,{}^1R_2$

$^0R_2 = {}^0R_1\,{}^1R_2$

- Match subscripts/superscripts

$^0R_2 = R(\theta_2)\,R(\theta_3)$



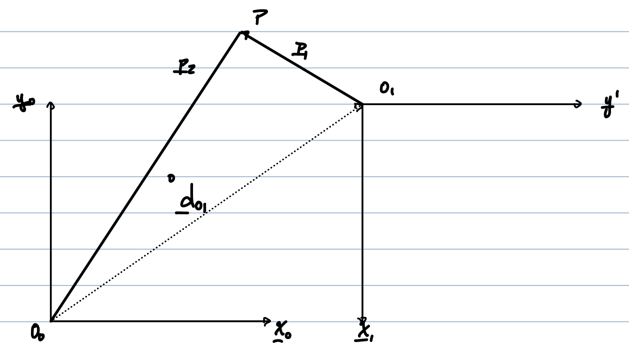
Only in 2D $^0R_2 = R(\theta_1 + \theta_2)$

Not 3D

Homogeneous Transformations

Determinate of all rotation matrices is 1

$\det(R) = 1$



$^0R_1 = R\left(\frac{3\pi}{2}\right) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$= R\left(\frac{-\pi}{2}\right)$

Givens:

$^1d_{01} = O_1 - O_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$^1P_1 = P - O_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

Coordinate transform = combo of coordinate rotations + translations

We want to find $^0P_1$

Distance from O_0 to O_1

Triangle equality: $^0P_1 = {}^0d_{01} + {}^1P_1$

$\therefore = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Ex 2

New given: $^0P_0 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ Find $^1P_0$

Flips d_{10} to d_{10}

$^1P_0 = {}^1P_0 - {}^1d_{01}$

$${}^0\underset{\sim}{R}_1 \, {}^1\underline{p}_1 = {}^0\underline{p}_0 - {}^0\underline{d}_{01}$$

$${}^1\underline{p}_1 = {}^0\underset{\sim}{R}_1^T({}^0\underline{p}_0 - {}^0\underline{d}_{01}) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

EX 3 HOMOGENEOUS TRANSFORMATION

$$\begin{bmatrix} {}^0\underline{p}_0 \\ 1 \end{bmatrix} = \begin{bmatrix} {}^0\underset{\sim}{R}_1 & {}^0\underline{d}_{01} \\ \underline{0}^T & 1 \end{bmatrix}\begin{bmatrix} {}^1\underline{p}_1 \\ 1 \end{bmatrix}$$

3x3 MATRIX

ROW VECTOR OF ZEROS (1x2)

$${}^0\underset{\sim}{T}_1 = \text{TRANSFORMATION FROM 1 TO 0}$$

$$= \begin{bmatrix} {}^0R_1 \, {}^1\underline{p}_1 + {}^1\underline{d}_{01} \\ 1 \end{bmatrix}$$

$$\boxed{{}^0\underset{\sim}{T}_1 = \begin{bmatrix} {}^0\underset{\sim}{R}_1 & {}^0d_{01} \\ \underline{0}^T & 1 \end{bmatrix}}$$

ROTATION AND TRANSLATION

$$\begin{bmatrix} {}^0\underline{p}_0 \\ 1 \end{bmatrix} = {}^0\underset{\sim}{T}_1 \begin{bmatrix} {}^1\underline{p}_1 \\ 0 \end{bmatrix}$$

$${}^0\underline{T}_1 = \begin{bmatrix} {}^0\underline{x}_1 & {}^0\underline{y}_1 & {}^0\underline{d}_{01} \\ 0 & 0 & 1 \end{bmatrix}$$

INVERSE OF HOMOGENEOUS TRANSFORM

$$({}^0\underset{\sim}{T}_1)^{-1} \neq ({}^0T_1)^T$$

$$\begin{bmatrix} {}^1\underline{p}_1 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} {}^0\underset{\sim}{R}_1^T & -{}^0\underset{\sim}{R}_1^T \, {}^0\underline{d}_{01} \\ \underline{0}^T & 1 \end{bmatrix}}_{\text{TRANSFORMATION FROM 0 TO 1}}\begin{bmatrix} {}^0\underline{p}_0 \\ 1 \end{bmatrix}$$

(INVERSE OF ABOVE)

$$\Rightarrow \quad {}^0\underset{\sim}{T}_1^{-1} = {}^1\underset{\sim}{T}_0$$

TRANS + ROT TRANSFORMATIONS

$\text{Trans}({}^0\underline{d}_{01})$ = PURE TRANSLATION OF $\underline{d}_{01}$ IN 0 COORDINATES

$$\text{Trans}({}^0\underline{d}_{01}) = \begin{bmatrix} I & {}^0\underline{d}_{01} \\ \underline{0}^T & 1 \end{bmatrix}$$

$\text{Rot}({}^0\underset{\sim}{R}_1)$ = PURE ROTATION

$$R({}^0\underset{\sim}{R}_1) = \begin{bmatrix} {}^0\underset{\sim}{R}_1 & 0 \\ \underline{0}^T & 1 \end{bmatrix}$$

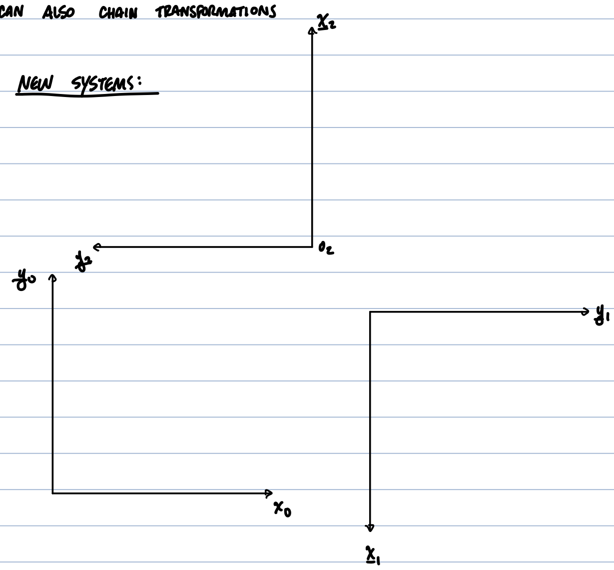
$$\text{Trans}({}^0\underset{\sim}{d}_{01})\,\text{Rot}({}^0\underset{\sim}{R}_1) \neq \text{Rot}({}^0\underset{\sim}{R}_1)\,\text{Trans}({}^0\underset{\sim}{d}_{01})$$

YOU <u>MUST</u> TRANSLATE FIRST

$${}^0\underset{\sim}{T}_1 = \text{Trans}({}^0\underset{\sim}{d}_{01})\,\text{Rot}({}^0\underset{\sim}{R}_1)$$

YOU CAN ALSO CHAIN TRANSFORMATIONS

<u>NEW SYSTEMS:</u>



$${}^1\underset{\sim}{T}_2 = \begin{bmatrix} {}^1x_2 & {}^1y_2 & {}^1\underset{\sim}{d}_{12} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^0\underset{\sim}{T}_1 = \begin{bmatrix} {}^0x_1 & {}^0y_1 & {}^0\underset{\sim}{d}_{01} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

BY INSPECTION...

$${}^0\underset{\sim}{T}_2 = \begin{bmatrix} {}^0x_2 & {}^0y_2 & {}^0\underset{\sim}{d}_{02} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^0\underset{\sim}{T}_2 = {}^0\underset{\sim}{T}_1\,{}^1\underset{\sim}{T}_2 = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

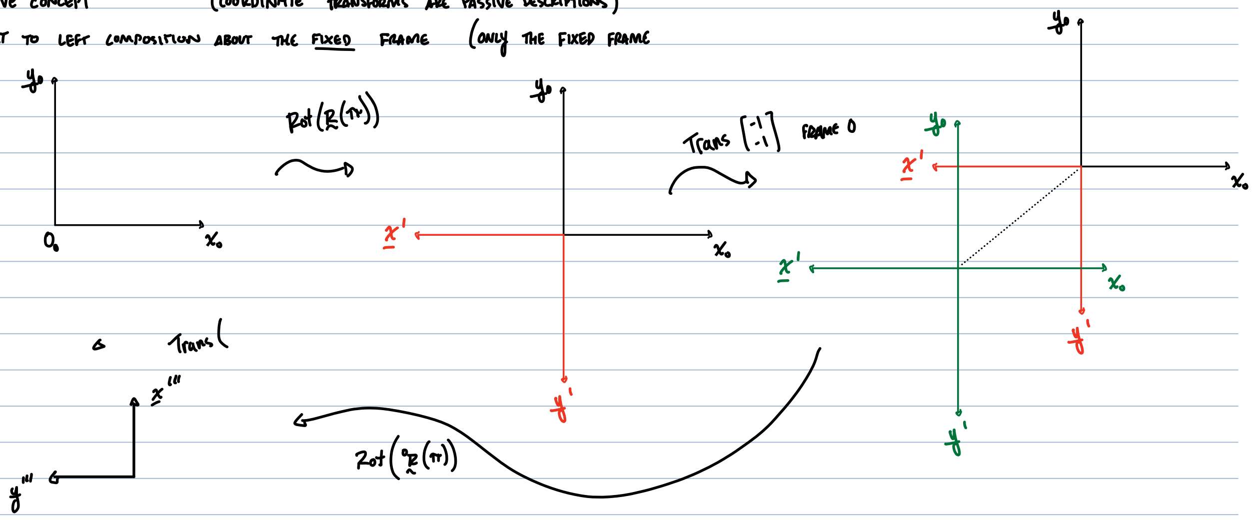
EXPRESS THIS IN TERMS OF Trans( ) & Rot( )

- THIS CAN HELP W/VISUALIZATION

$${}^0\underset{\sim}{T}_2 = \underbrace{\text{Trans}({}^0d_{01})\,\text{Rot}\left(R\left(\tfrac{-\pi}{2}\right)\right)}_{\text{COORDINATE SYS } 1}\underbrace{\text{Trans}({}^1\underset{\sim}{d}_{12})\,\text{Rot}(R(\pi))}_{\text{COORDINATE SYS } 2}$$

Operators :

- Active concept (coordinate transforms are passive descriptions)
- Right to left composition about the fixed frame (only the fixed frame



1. Rot($^0R(\pi)$)
2. Trans($^0\begin{bmatrix}-1\\-1\end{bmatrix}$)
3. Rot($^0R(\frac{-\pi}{2})$)
4. Trans($^0\begin{bmatrix}2\\1\end{bmatrix}$)

} Always in the fixed frame

USUALLY WHERE THE CODE IS :)

SRC = SOURCE CODE (PYTHON FUNCTIONS)

dat = DATA FILE

BIN =