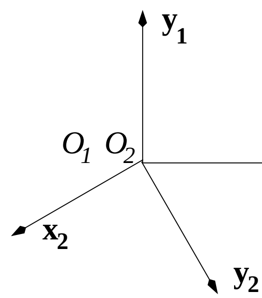
CS 5310/6310 & ME 5220/6220 **Homework 1 Solution** **September 2, 2022**

1. (30 pts) Suppose ${}^1\mathbf{x}_2 = \frac{1}{2}\begin{bmatrix} -\sqrt{3} \\ -1 \end{bmatrix}$.

   (a) (10pts) What is ${}^1\mathbf{y}_2$? Draw a diagram showing the orientation of axes 2 relative to axes 1.



   From the right-hand rule,

$$ {}^1\mathbf{y}_2 = \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} $$

   The dot product shows that these $\mathbf{x}$ and $\mathbf{y}$ axes are perpendicular.

   (b) (10pts) What is the matrix ${}^1\mathbf{R}_2$?

$$ {}^1\mathbf{R}_2 = \begin{bmatrix} {}^1\mathbf{x}_2 \ {}^1\mathbf{y}_2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -\sqrt{3} & 1 \\ -1 & -\sqrt{3} \end{bmatrix} $$

   (c) (10pts) What is the rotation angle θ such that ${}^1\mathbf{R}_2 = \mathbf{R}(\theta)$?

$$ \theta = \text{atan2}(-1, -\sqrt{3}) = 210^o $$

2. (30pts) Suppose ${}^1\mathbf{x}_2 = \frac{1}{5}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$.

   (a) (6pts) What is ${}^1\mathbf{y}_2$?

   We need to find ${}^1\mathbf{y}_2$, by observing that ${}^1\mathbf{x}_2 \cdot {}^1\mathbf{y}_2 = 0$. By inspection, ${}^1\mathbf{y}_2 = \frac{1}{5}\begin{bmatrix} -3 \\ 4 \end{bmatrix}$. The other choice ${}^1\mathbf{y}_2 = \frac{1}{5}\begin{bmatrix} 3 \\ -4 \end{bmatrix}$ does not obey the right-hand rule.

   (b) (6pts) What is ${}^1\mathbf{R}_2$?

$$
{}^1\mathbf{R}_2 = \begin{bmatrix} {}^1\mathbf{x}_2 & {}^1\mathbf{y}_2 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}
$$

   (c) (6pts) What is ${}^2\mathbf{R}_1$?

$$
{}^2\mathbf{R}_1 = {}^1\mathbf{R}_2^T = \frac{1}{5}\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}
$$

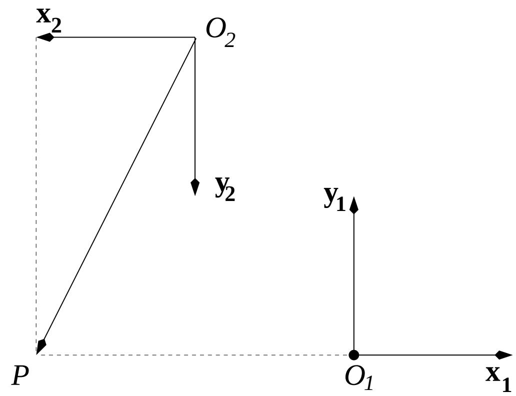
   (d) (6pts) Given ${}^2\mathbf{p} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$, what is the numerical value of ${}^1\mathbf{p}$?

$$
{}^1\mathbf{p} = {}^1\mathbf{R}_2\, {}^2\mathbf{p} = \frac{1}{5}\begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1 \\ -7 \end{bmatrix}
$$

   (e) (6pts) Given ${}^1\mathbf{q} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, what is the numerical value of ${}^2\mathbf{q}$?

$$
{}^2\mathbf{q} = {}^2\mathbf{R}_1\, {}^1\mathbf{q} = \frac{1}{5}\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}
$$

3. (30 pts) Consider the coordinate system 2 in relation to coordinate system 1 below. Suppose O_2 is located at (-1,2) relative to coordinate system 1, and point P is located at (1,2) relative to coordinate system 2. Let $\mathbf{p}_i = P - O_i$. What are the following vectors (i.e., their x and y components)?



(a) (6 pts) ${}^1\mathbf{x}_2$ and ${}^1\mathbf{y}_2$.

Answer: $${}^1\mathbf{x}_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad {}^1\mathbf{y}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

(b) (8 pts) ${}^2\mathbf{p}_2$ and ${}^1\mathbf{p}_2$.

Answer: $${}^2\mathbf{p}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad {}^1\mathbf{p}_2 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

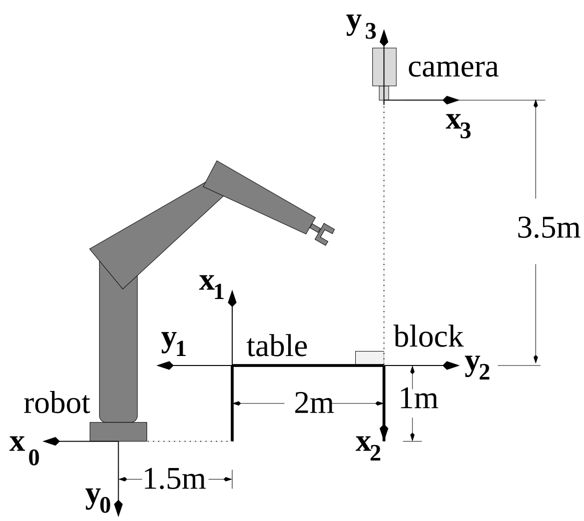
(c) (8 pts) ${}^1\mathbf{p}_1$ and ${}^2\mathbf{p}_1$.

Answer: $${}^1\mathbf{p}_1 = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \quad {}^2\mathbf{p}_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

(d) (8 pts) ${}^1\mathbf{d}_{12}$ and ${}^2\mathbf{d}_{12}$.

Answer: $${}^1\mathbf{d}_{12} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad {}^2\mathbf{d}_{12} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

4. (40 pts) Consider the combination of robot, table, block, and camera in the figure below, with associated coordinate systems as shown.



(a) (28pts) Find ${}^0\mathbf{R}_1$, ${}^1\mathbf{R}_2$, ${}^2\mathbf{R}_3$ and ${}^0\mathbf{R}_3$ by inspection.

$${}^0\mathbf{R}_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad {}^1\mathbf{R}_2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad {}^2\mathbf{R}_3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \qquad {}^0\mathbf{R}_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

(b) (12pts) Suppose ${}^0\mathbf{p} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. By inspection, find ${}^1\mathbf{p}$, ${}^2\mathbf{p}$, and ${}^3\mathbf{p}$.

$${}^1\mathbf{p} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \qquad {}^2\mathbf{p} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \qquad {}^3\mathbf{p} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$