07 Trajectories

Polynomial Trajectories

$$x(t) = a_0 + \sum_{i=1}^{n} a_i t^i$$

$n=2$ QUADRATIC
$n=3$ CUBIC
$n=4$ QUARTIC
$n=5$ QUINTIC

MORE THAN THIS, IT GETS WEIRD ← "MINIMUM JERK"

IF YOU NEED IT TO BE HIGHER, SPLICE
$n=3$

CUBIC POLYNOMIAL ← NEEDS 4 CONSTRAINTS

$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \qquad 0 \le t \le t_1$

$\dot{x}(t) = a_1 + 2a_2 t + 3a_3 t^2$

4 COEFFICIENTS => 4 CONSTRAINTS

$\ddot{x}(t) = 2a_2 + 6a_3 t$

$x(0) = 0 = a_0 \qquad x(t_1) = x_1 = a_2 t_1^2 + a_3 t_1^3$

$\dot{x}(0) = a_1 = 0 \qquad \dot{x}(t_1) = 0$

More Complicated Trajectories

- Start by identifying constraints

EQNS $\begin{cases} x_1(t) & 0 \le t \le t_1 \\ x_2(t) & t_1 \le t \le t_2 \end{cases}$

CONSTRAINTS

$t=0$	$t=t_1$	$t=t_2$
$x_1(0)=0$	$x_1(t_1) = x_1$	$x_2(t_2) = x_2$
$\dot{x}_1(0)=0$	$x_2(t_1) = x_1$	$\dot{x}_2(t_2) = 0$
	$\dot{x}_1(t_1) = \dot{x}_2(t_1)$	

NO JUMPS IN POSITION

NO JUMP IN VELOCITY

← ASSUMES POLYNOMIAL COMES TO A REST

DEGREES

7 COEFFICIENTS → CUBIC + QUADRATIC
OR
QUARTIC + LINEAR ← THIS WILL NOT WORK (CONSTRAINTS ARE MIXED)
OR
QUINTIC + ZERO ← BORING ($n=0$ IS CONSTANT)

$\therefore n=3$ & $n=2$ (DOESN'T MATTER IF $x_1(t)$ OR $x_2(t)$ IS $n=2$, BECAUSE THERE AREN'T ANY CONSTRAINTS ON ACCELERATIONS)

WE'LL START W/ $x_1(t)$ BEING CUBIC

$x_1(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ $\quad 0 \le t < t_1$

$x_2(t) = b_0 + b_1 t + b_2 t^2$ $\quad t_1 < t < t_2$

THESE COEFFICIENTS WILL CHANGE IF YOU CHANGE t_1

BAD!

SO WRITE THIS IN TIME-SHIFT FORM

CORRECT :)

$x_1(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ $\quad 0 \le t \le t_1$

$x_2(t) = b_0 + b_1(t - t_1) + b_2(t - t_1)^2$ $\quad t_1 < t < t_2$

$\dot{x}_1(t) = a_1 t + 2a_2 t + 3a_3 t^2$

$\dot{x}_2(t) = b_1 + 2b_2(t - t_1)$

UNARY CONSTRAINT = INVOLVES ONLY 1 POLYNOMIAL

BINARY CONSTRAINT = EQUALITY BETWEEN POLYNOMIALS

$x_1(t_1) = x_2(t_1) = x_1$

THIS IS TWO CONSTRAINTS

$x_1(t_1) = x_1$

$x_2(t_1) = x_1$

ONE CONSTRAINT

$x_1(t_1) = x_2(t_1)$

- DIFFERENT, BECAUSE THIS MEANS YOU DON'T CARE WHAT THE FINAL VALUE IS

UNARY CONSTRAINTS

$x_1(0) = 0 = a_0$ $\quad x_1(t_1) = x1 = a_2 t_1^2 + a_3 t_1^3$ $\quad x_2(t_2) = x2 = x1 + b_1(t_2 - t_1) + b_2(t_2 - t_1) =$

$\dot{x}_1(0) = 0 = a_1$ $\quad x_2(t_1) = x1 = b_0$ $\quad \dot{x}_2(t_2) = 0 = b_1 + 2b_2(t_2 - t_1)$

BINARY CONSTRAINT

$2a_2 t_1 + 3a_3 t_1^2 = b_1$

$t_1 - t_1 = 0 \quad \therefore \quad b_2(t_1 - t_1) = 0$

USING THIS NIGHTMARE MESS OF EQNS, YOU CAN DETERMINE ALL COEFFICIENTS