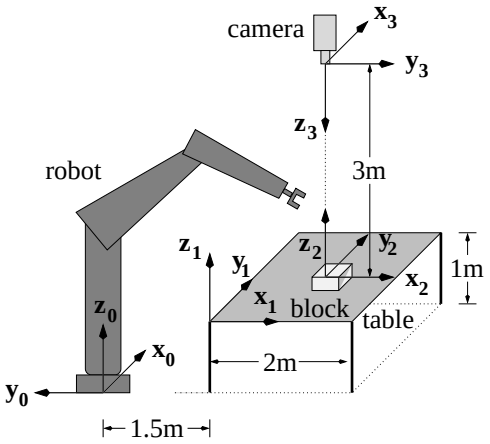
**CS 5310/6310 & ME 5220/6220** **Homework 3** **Due September 21, 2022**

1. (20 pts) This question concerns the difference between an operator and a coordinate transformation viewpoint. Suppose frames 0 and 1 are initially aligned.

   (a) (10 pts) If frame 1 is obtained from frame 0 by a rotation of $\pi/2$ about the $\mathbf{x}_0$-axis, followed by a rotation of $\pi$ about the fixed $\mathbf{y}_0$ axis, find the rotation matrix ${}^0\mathbf{R}_1$ representing the composite transformation. Sketch the initial and final frames.

   (b) (10 pts) If frame 1 is obtained from frame 0 by rotation of $\pi/2$ about the $\mathbf{x}_0$-axis, followed by a rotation of $\pi$ about the current $\mathbf{y}_1$ axis, find the rotation matrix ${}^0\mathbf{R}_1$ representing the composite transformation. Sketch the initial and final frames.

2. (24pts) Consider the combination of robot, table, block, and camera below, with associated coordinate systems as shown. The relative locations of robot, table, block and camera are shown. Find ${}^0\mathbf{T}_1$, ${}^1\mathbf{T}_2$, ${}^2\mathbf{T}_3$, and ${}^0\mathbf{T}_3$ by inspection.




3. (20pts) Let $\mathbf{k} = [2\ -2\ 1]^T/3$, $\theta = \pi/2$. Derive $\mathbf{R}_k(\theta)$.

4. (20pts) Derive the angle $\theta$ and axis $\mathbf{k}$ for

$$\mathbf{R} = \frac{1}{25}\begin{bmatrix} 16 & 12 & 15 \\ 12 & 9 & -20 \\ -15 & 20 & 0 \end{bmatrix}$$

5. (20 pts) This problem is for graduate students. When $\theta = \pi$, the derivation of the axis in the angle-axis formula does not work because of a divide by zero ($\sin\theta$). Develop a new approach to derive the axis for this case. Your answer should have the appropriate number of solutions, and must be robust against special cases (e.g., $k_1 = 0$.).