Chapter 6 Trajectory Planning

READ 6.1 - 6.3

Pass 1:

- Trajectory = Path followed, with the time required
  - Can be planned in joint space, or in Cartesian
    - Joint = angles
    - Cartesian = position + orientation
      - more directly allows geometric constraints of external world, BUT involves inverse kinematics :(
- Issues
  - Specific target from initial starting point
  - Obstacle avoidance
  - Staying within manipulator capabilities

- Polynomial trajectories
  - Not higher than n = 5 (too many wiggles)
  - Linear, quadratic (acceleration can be constant), cubic (jerk is constant)
  - Multi-segment
    - Splicing
    - Parabolic blend
      - Ramping up and ramping down is I think what this is talking about
  - Cartesian trajectory planning
    - Piecewise eqns
- Interpolating 3D rotations
  - It looks like you do half rotation on each side
  - Taylor trajectory for orientation

Polynomial Trajectories

STATIONARY

$x(t) = a_0$

$$x(t) = a_0 + \sum_{i=1}^{n} a_i t^n \qquad t_0 < t < t_1$$

LINEAR

SPECIFY:

- BEGINNING POSITION $(\underline{x}_0)$
- VELOCITY $(\underline{x}')$

$a_0 = \underline{x}_0$

$a_1 = \underline{x}'$

VELOCITY CONSTANT

QUADRATIC ← CONSTANT ACCELERATION $2a_2$

SPECIFY:

- ENDPOINT POSITIONS $(\underline{x}_0, \underline{x}_1)$
- INITIAL VELOCITY $(\underline{x}_0')$

$\underline{x}_1 = \underline{x}_0 + \underline{x}_0' t_1 + a_2 t_1^2$

$a_0 = \underline{x}_0$

$a_1 = \underline{x}_0'$

$$a_2 = \frac{\underline{x}_1 - \underline{x}_0 - \underline{x}_0' t_1}{t_1^2}$$

CUBIC ← CONSTANT JERK $(6a_3)$

SPECIFY

- ENDPOINT POSITIONS $(\underline{x}_0 \quad \underline{x}_1)$
- END POINT VELOCITIES $(\underline{x}_0', \underline{x}_1')$

$a_0 = x_0$

$a_1 = x_0'$

$$a_2 = \frac{3(\underline{x}_1 - \underline{x}_0) - 2(\underline{x}_0' + \underline{x}_1')t_1}{t_1^2}$$

$$a_3 = \frac{2(\underline{x}_0 - \underline{x}_1) + (\underline{x}_0' + \underline{x}_1')t_1}{t_1^3}$$

MULTI-SEGMENT TRAJECTORIES

$$\underline{x}_1(t) = a_0 + \sum_{i=1}^{n} a_i t^i \qquad 0 \le t \le t_1$$

$$\underline{x}_2 = b_0 + \sum_{i=1}^{n} b_i t^i \qquad t_1 \le t \le t_2$$

UNARY VS BINARY CONSTRAINTS

POSITION + VELOCITY CONTINUITY

$x_1(t_1) = x_2(t_1)$

$\dot{x}_1(t_1) = \dot{x}_2(t_1)$

APPLICATION LOOKS LIKE...

$$\left.\begin{aligned} x_1(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\ x_2(t) &= b_0 + b_1(t - t_1) + b_2(t - t_1)^2 \end{aligned}\right\}$$

GIVEN 7 CONSTRAINTS $\left(\begin{aligned} & x_1(t_1) = x_2(t_1) = x_1 \\ & \dot{x}_1(t_1) = \dot{x}_2(t_1) \\ & x_1(0) = \dot{x}_1(0) = 0 \\ & x_2(t_2) = x_2 \end{aligned} \right)$

$\Rightarrow$ ONE POLYNOMIAL IS DEGREE 3, THE OTHER IS $n=2$

LINEAR SEGMENT w/ PARABOLIC BLEND