In [15]:
```julia
using JSON
using DataFrames
using StatsPlots
using Turing
using LinearAlgebra
using Random
```

Multi-level model using football match simulation as an example

In [16]:
```julia
## First, import the data and do some data wrangling

england_league = JSON.parsefile("../data/matches_England.json")

matches_df = DataFrame(home = [], away = [], score_home = [], score_aw
```

0×4 DataFrame

Row	home	away	score_home	score_away
	Any	Any	Any	Any

In [17]:
```julia
# example entry for each game in england_league:  "label" => "Burnley
matches = []
for match in england_league
    push!(matches, split(match["label"], ",")) # "Burnley - AFC Bourne
end

for match in matches
    home, away = split(match[1], " - ")  # "Burnley" # "AFC Bournemout
    score_home, score_away = split(match[2], " - ") # "1" # "2"
    push!(matches_df,[home, away, parse(Int,score_home), parse(Int,sco
end

matches_df

teams = unique(collect(matches_df[:,1]))
```

```
20-element Vector{Any}:
 "Burnley"
 "Crystal Palace"
 "Huddersfield Town"
 "Liverpool"
 "Manchester United"
 "Newcastle United"
 "Southampton"
 "Swansea City"
 "Tottenham Hotspur"
 "West Ham United"
 "Manchester City"
 "Leicester City"
 "Chelsea"
 "Arsenal"
 "Everton"
 "AFC Bournemouth"
 "Watford"
 "West Bromwich Albion"
 "Stoke City"
 "Brighton & Hove Albion"
```

In [ ]:

```
## Now, our model

@model function football_matches(home_teams, away_teams, score_home, s

    # Hyper priors
    μatt ~ Normal(0, 0.1)
    μdef ~ Normal(0, 0.1)
    σatt ~ Exponential(1)
    σdef ~ Exponential(1)
    home ~ Normal(0, 1)

    # Team-specific effects

    att = zeros(length(teams))
    def = zeros(length(teams))

    for i in 1:length(teams)
        att[i] ~ Normal(μatt, σatt)
        def[i] ~ Normal(μdef, σdef)
    end

    #att ~ filldist(Normal(μatt, σatt), length(teams))  # more compact
    #def ~ filldist(Normal(μdef, σdef), length(teams))

    offset = mean(att) + mean(def)

    # the number of matches
    n_matches = length(home_teams)

    # scoring rates θ
    θ_home = Vector{Real}(undef, n_matches)     # or just θ_home = zer
```

```julia
    θ_away = Vector{Real}(undef, n_matches)     # or just θ_away = zer

    # Modeling score-rate and scores for each match
    for i in 1:n_matches
        # scoring rate
        home_team_idx = findfirst(isequal(home_teams[i]), teams)
        away_team_idx = findfirst(isequal(away_teams[i]), teams)

        θ_home[i] = home + att[home_team_idx] + def[away_team_idx] - o
        θ_away[i] = att[away_team_idx] + def[home_team_idx] - offset

        # scores
        score_home[i] ~ Poisson(exp(θ_home[i]))  # To ensure positive
        score_away[i] ~ Poisson(exp(θ_away[i]))
    end
end
```

football_matches (generic function with 2 methods)

In [ ]:

```julia
model = football_matches(matches_df[:,1], matches_df[:,2], matches_df[

posterior = sample(model, NUTS(), 3000)
```

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Sampling   0%|                                         |  ETA: N/A
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│   ϵ = 0.00625
└ @ Turing.Inference /Users/lime/.julia/packages/Turing/vX5F9/src/mcmc/
hmc.jl:213
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Chains MCMC chain (3000×57×1 Array{Float64, 3}):

Iterations        = 1001:1:4000
Number of chains  = 1
Samples per chain = 3000
Wall duration     = 136.29 seconds
Compute duration  = 136.29 seconds
parameters        = μatt, μdef, σatt, σdef, home, att[1], def[1], att[
2], def[2], att[3], def[3], att[4], def[4], att[5], def[5], att[6], de
f[6], att[7], def[7], att[8], def[8], att[9], def[9], att[10], def[10],
att[11], def[11], att[12], def[12], att[13], def[13], att[14], def[14],
att[15], def[15], att[16], def[16], att[17], def[17], att[18], def[18],
att[19], def[19], att[20], def[20]
internals         = lp, n_steps, is_accept, acceptance_rate, log_densit
y, hamiltonian_energy, hamiltonian_energy_error, max_hamiltonian_energy
_error, tree_depth, numerical_error, step_size, nom_step_size

Summary Statistics
  parameters      mean       std      mcse    ess_bulk    ess_tail
rhat   ⋯
      Symbol   Float64   Float64   Float64     Float64     Float64   Fl
oat64  ⋯

        μatt   -0.0106    0.1006    0.0059    286.0900    721.8933
1.0155  ⋯
        μdef   -0.0008    0.0994    0.0059    288.4936    464.7421
1.0031  ⋯
        σatt    0.3849    0.0779    0.0035    621.2962    339.6839
1.0023  ⋯
        σdef    0.2154    0.0612    0.0034    327.5896    117.0460
1.0028  ⋯
        home    0.3380    0.0425    0.0009   2208.4531   2263.3756
```

```
1.0004  ⋯
      att[1]   -0.2672    0.2014    0.0100    409.4098    955.1314
1.0069  ⋯
      def[1]   -0.1680    0.1656    0.0079    421.9315    441.1022
1.0004  ⋯
      att[2]   -0.0720    0.1883    0.0100    355.6188    657.8336
1.0078  ⋯
      def[2]    0.0594    0.1556    0.0078    400.4448   1113.5101
1.0011  ⋯
      att[3]   -0.4594    0.2072    0.0103    402.8109   1040.9419
1.0095  ⋯
      def[3]    0.0862    0.1563    0.0078    406.0630    531.2944
1.0042  ⋯
      att[4]    0.5008    0.1743    0.0101    295.1097    771.1988
1.0159  ⋯
      def[4]   -0.1551    0.1678    0.0081    425.6426    571.6784
0.9998  ⋯
      att[5]    0.2910    0.1777    0.0102    306.1388    709.1792
1.0109  ⋯
      def[5]   -0.3300    0.1843    0.0084    445.9616    167.5677
0.9999  ⋯
      att[6]   -0.1918    0.1969    0.0104    360.3907    940.1000
1.0115  ⋯
      def[6]   -0.0497    0.1541    0.0069    500.0782    991.6641
1.0016  ⋯
        ⋮          ⋮         ⋮         ⋮          ⋮           ⋮          ⋮
⋱
                                                           1 column and 28 row
s omitted

Quantiles
  parameters      2.5%     25.0%     50.0%     75.0%     97.5%
      Symbol   Float64   Float64   Float64   Float64   Float64

        μatt   -0.2005   -0.0769   -0.0106    0.0569    0.1855
        μdef   -0.2022   -0.0655    0.0009    0.0668    0.1866
        σatt    0.2633    0.3281    0.3729    0.4302    0.5561
        σdef    0.0804    0.1757    0.2123    0.2506    0.3494
        home    0.2516    0.3101    0.3398    0.3673    0.4177
      att[1]   -0.6797   -0.3978   -0.2636   -0.1333    0.1242
      def[1]   -0.4877   -0.2831   -0.1685   -0.0515    0.1470
      att[2]   -0.4397   -0.1989   -0.0713    0.0565    0.2913
      def[2]   -0.2495   -0.0467    0.0640    0.1688    0.3637
      att[3]   -0.8748   -0.5914   -0.4588   -0.3222   -0.0598
      def[3]   -0.2396   -0.0133    0.0914    0.1876    0.3825
      att[4]    0.1529    0.3881    0.5069    0.6142    0.8376
      def[4]   -0.4933   -0.2669   -0.1504   -0.0392    0.1603
      att[5]   -0.0623    0.1790    0.2905    0.4048    0.6297
      def[5]   -0.6938   -0.4537   -0.3274   -0.2027    0.0213
      att[6]   -0.5740   -0.3206   -0.1891   -0.0596    0.1925
      def[6]   -0.3658   -0.1516   -0.0457    0.0545    0.2445
        ⋮         ⋮         ⋮         ⋮         ⋮         ⋮
```

28 rows omitted

In [20]:
```
posterior_df=DataFrame(posterior)
```

3000×59 DataFrame *2975 rows omitted*

Row	iteration	chain	μatt	μdef	σatt	σdef	home
	Int64	Int64	Float64	Float64	Float64	Float64	Float6
1	1001	1	-0.0340967	0.0488872	0.371805	0.341548	0.41
2	1002	1	0.120224	0.0379764	0.442336	0.104175	0.318
3	1003	1	0.120224	0.0379764	0.442336	0.104175	0.318
4	1004	1	0.14776	0.0602034	0.431715	0.100922	0.329
5	1005	1	0.14776	0.0602034	0.431715	0.100922	0.329
6	1006	1	0.14776	0.0602034	0.431715	0.100922	0.329
7	1007	1	0.132189	0.0570596	0.422664	0.106005	0.358
8	1008	1	0.140849	0.0218837	0.340951	0.149594	0.302
9	1009	1	0.000211701	0.00791739	0.55847	0.0972845	0.407
10	1010	1	-0.0436966	-0.0107217	0.500536	0.108407	0.4
11	1011	1	0.198782	0.0145961	0.331272	0.195929	0.260
12	1012	1	-0.00990157	0.133114	0.629202	0.263415	0.343
13	1013	1	0.143073	0.0929881	0.423556	0.258046	0.315
⋮	⋮	⋮	⋮	⋮	⋮	⋮	
2989	3989	1	-0.152038	0.000856577	0.322627	0.227176	0.2
2990	3990	1	0.0130078	0.0471447	0.487366	0.20001	0.379
2991	3991	1	-0.120245	-0.0915178	0.35016	0.21365	0.365
2992	3992	1	-0.0803213	-0.0308569	0.334981	0.158142	0.40
2993	3993	1	-0.0574519	-0.0464465	0.347634	0.225209	0.308
2994	3994	1	0.10894	-0.106624	0.42737	0.23224	0.381
2995	3995	1	0.00348895	-0.0755697	0.39041	0.162871	0.317
2996	3996	1	0.0239133	-0.128041	0.357514	0.194984	0.395
2997	3997	1	0.180213	-0.0651568	0.314539	0.16265	0.37
2998	3998	1	0.0579803	0.0572436	0.451031	0.199167	0.369
2999	3999	1	0.176402	0.0165819	0.444763	0.227071	0.329
3000	4000	1	-0.0406021	-0.0143352	0.464827	0.241888	0.33

In [21]:

```
DataFrames.transform!(posterior_df, AsTable(Between("att[1]","att[20]"
```

```
DataFrames.transform!(posterior_df, AsTable(Between("def[1]","def[20]"
DataFrames.transform!(posterior_df, AsTable([:att_mean,:def_mean]) =>
```

3000×62 DataFrame *2975 rows omitted*

Row	iteration	chain	μatt	μdef	σatt	σdef	home
	Int64	Int64	Float64	Float64	Float64	Float64	Float6
1	1001	1	-0.0340967	0.0488872	0.371805	0.341548	0.41
2	1002	1	0.120224	0.0379764	0.442336	0.104175	0.318
3	1003	1	0.120224	0.0379764	0.442336	0.104175	0.318
4	1004	1	0.14776	0.0602034	0.431715	0.100922	0.329
5	1005	1	0.14776	0.0602034	0.431715	0.100922	0.329
6	1006	1	0.14776	0.0602034	0.431715	0.100922	0.329
7	1007	1	0.132189	0.0570596	0.422664	0.106005	0.358
8	1008	1	0.140849	0.0218837	0.340951	0.149594	0.302
9	1009	1	0.000211701	0.00791739	0.55847	0.0972845	0.407
10	1010	1	-0.0436966	-0.0107217	0.500536	0.108407	0.4
11	1011	1	0.198782	0.0145961	0.331272	0.195929	0.260
12	1012	1	-0.00990157	0.133114	0.629202	0.263415	0.343
13	1013	1	0.143073	0.0929881	0.423556	0.258046	0.315
⋮	⋮	⋮	⋮	⋮	⋮	⋮	
2989	3989	1	-0.152038	0.000856577	0.322627	0.227176	0.2
2990	3990	1	0.0130078	0.0471447	0.487366	0.20001	0.379
2991	3991	1	-0.120245	-0.0915178	0.35016	0.21365	0.365
2992	3992	1	-0.0803213	-0.0308569	0.334981	0.158142	0.40
2993	3993	1	-0.0574519	-0.0464465	0.347634	0.225209	0.308
2994	3994	1	0.10894	-0.106624	0.42737	0.23224	0.381
2995	3995	1	0.00348895	-0.0755697	0.39041	0.162871	0.317
2996	3996	1	0.0239133	-0.128041	0.357514	0.194984	0.395
2997	3997	1	0.180213	-0.0651568	0.314539	0.16265	0.37
2998	3998	1	0.0579803	0.0572436	0.451031	0.199167	0.369
2999	3999	1	0.176402	0.0165819	0.444763	0.227071	0.329
3000	4000	1	-0.0406021	-0.0143352	0.464827	0.241888	0.33

In [22]:
```
# For this example, we are interested in a pair of teams (no need to u
```

```
teamA = "Manchester City"
teamB = "Manchester United"

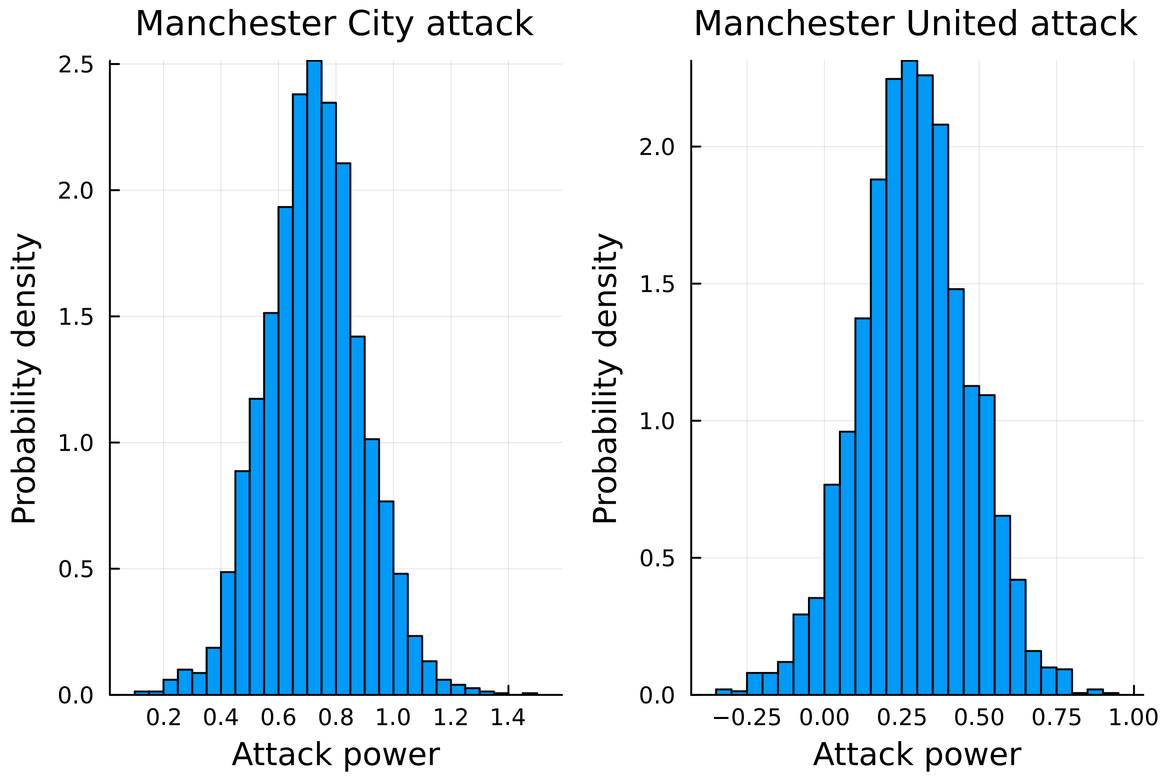
teamA_id = findfirst(isequal(teamA), teams)
teamB_id = findfirst(isequal(teamB), teams)

teamA_att_post = posterior_df[:,"att[$teamA_id]"]
teamA_def_post = posterior_df[:,"def[$teamA_id]"]

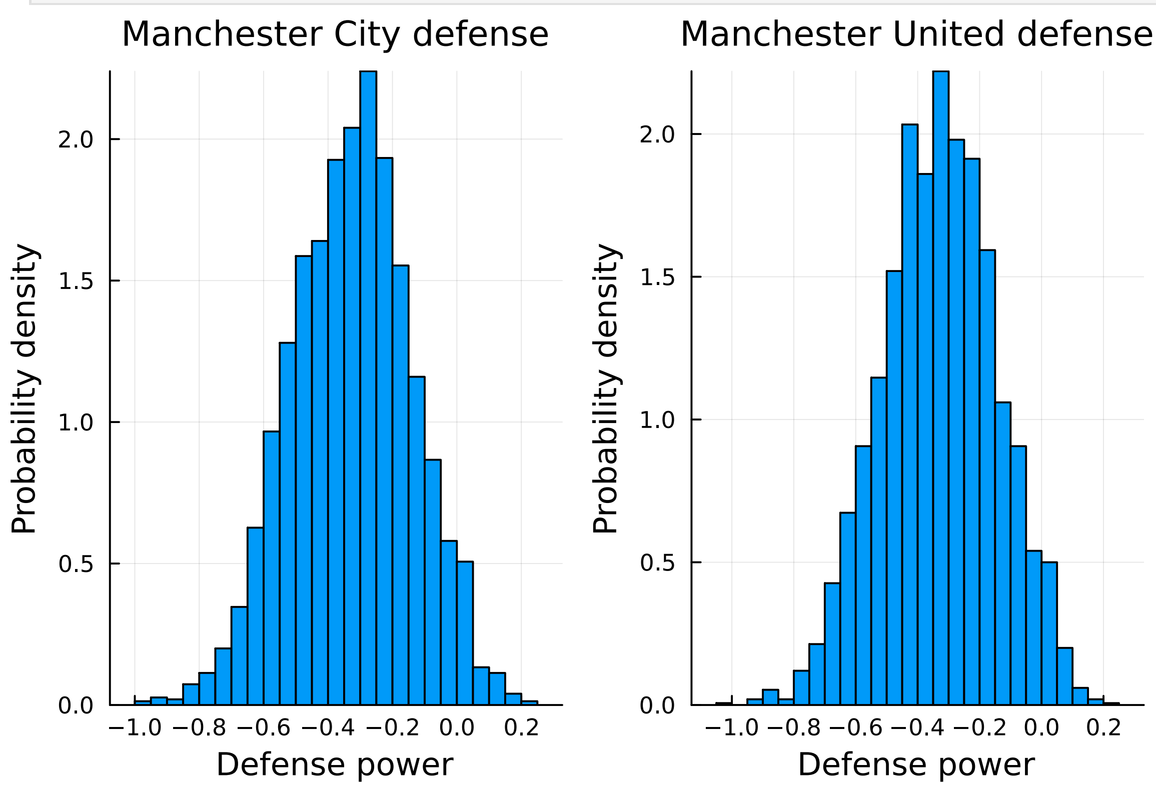
teamB_att_post = posterior_df[:,"att[$teamB_id]"]
teamB_def_post = posterior_df[:,"def[$teamB_id]"]
```

```
3000-element Vector{Float64}:
 -0.31659686077845584
  0.010909344846748792
  0.010909344846748792
 -0.005602923863187977
 -0.005602923863187977
 -0.005602923863187977
 -0.12470192624633054
 -0.23946056821044293
 -0.17673201401029642
 -0.12582298768177858
  ⋮
 -0.28907656726617503
 -0.600319104560645
 -0.25490945685899014
 -0.4968341292721567
 -0.227480226431094
 -0.7068032570364412
 -0.04770233420003743
 -0.03970682618439173
 -0.5052118038597926
```

In [23]:
```
ha1 = histogram(teamA_att_post, title=teamA*" attack", titlefontsize =
ha2 = histogram(teamB_att_post, title=teamB*" attack", titlefontsize =
plot(ha1, ha2, layout=(1,2));
xlabel!("Attack power");
ylabel!("Probability density")
```



In [24]:
```
hd1 = histogram(teamA_def_post, title=teamA*" defense", titlefontsize
hd2 = histogram(teamB_def_post, title=teamB*" defense", titlefontsize
plot(hd1, hd2, layout=(1,2));
xlabel!("Defense power");
ylabel!("Probability density")
```



Mini Project

Consult the lecture notes.

In [ ]:

```
Random.seed!(205579184)
# hint: let's simulate 500 hypothetical finals (then you will have a t

# first leg: teamA is the home team and teamB is the away team

θ_home = posterior_df[:,:home] + posterior_df[:,"att[$teamA_id]"] + po
θ_away = posterior_df[:,"att[$teamB_id]"] + posterior_df[:,"def[$teamA

teamA_score = rand.(Poisson.(exp.(θ_home)),500)
teamB_score = rand.(Poisson.(exp.(θ_away)),500)

# second leg: teamA is the away team and teamB is the home team

θ_home = posterior_df[:,:home] + posterior_df[:,"att[$teamB_id]"] + po
θ_away = posterior_df[:,"att[$teamA_id]"] + posterior_df[:,"def[$teamB

teamA_score += rand.(Poisson.(exp.(θ_away)),500)  # add the first-leg
teamB_score += rand.(Poisson.(exp.(θ_home)),500)

# transform into long column vectors
teamA_score = vcat(teamA_score...)
teamB_score = vcat(teamB_score...)

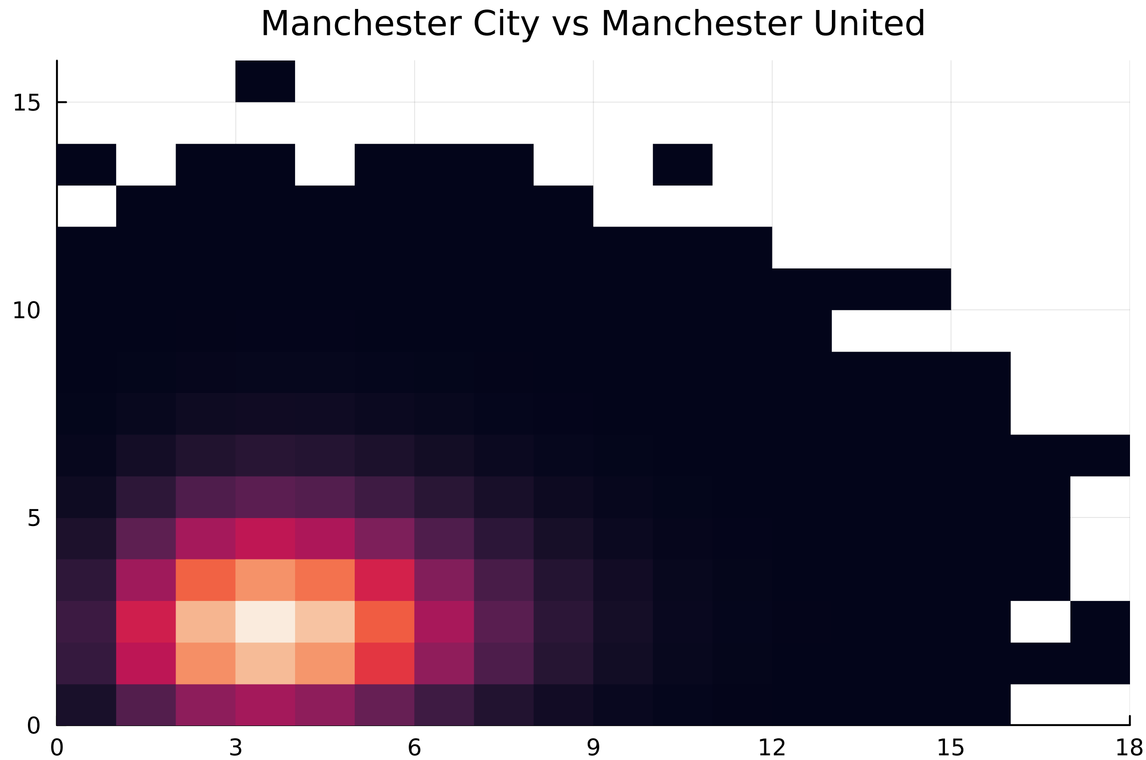
display(histogram2d(teamA_score, teamB_score, title=teamA*" vs "*teamB
# https://docs.juliaplots.org/dev/generated/colorschemes/

# Winning probabilities
winning_prob_A = sum(teamA_score .> teamB_score) / length(teamA_score)
println("Winning probability of "*teamA*" against "*teamB*" is "*strin

winning_prob_B = sum(teamA_score .< teamB_score) / length(teamA_score)
println("Winning probability of "*teamB*" against "*teamA*" is "*strin

draw_prob = sum(teamA_score .== teamB_score) / length(teamA_score)
println("Draw probability between "*teamA*" and "*teamB*" is "*string(

println("Sum of probabilities (sanity check): "*string((winning_prob_A
```




```
Winning probability of Manchester City against Manchester United is 61.
504%
Winning probability of Manchester United against Manchester City is 24.
09%
Draw probability between Manchester City and Manchester United is 14.40
5%
Sum of probabilities (sanity check): 100.0%
```