

CO450 Computer Architectures Week 11 Exercise Handout

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Recap on Truth Tables

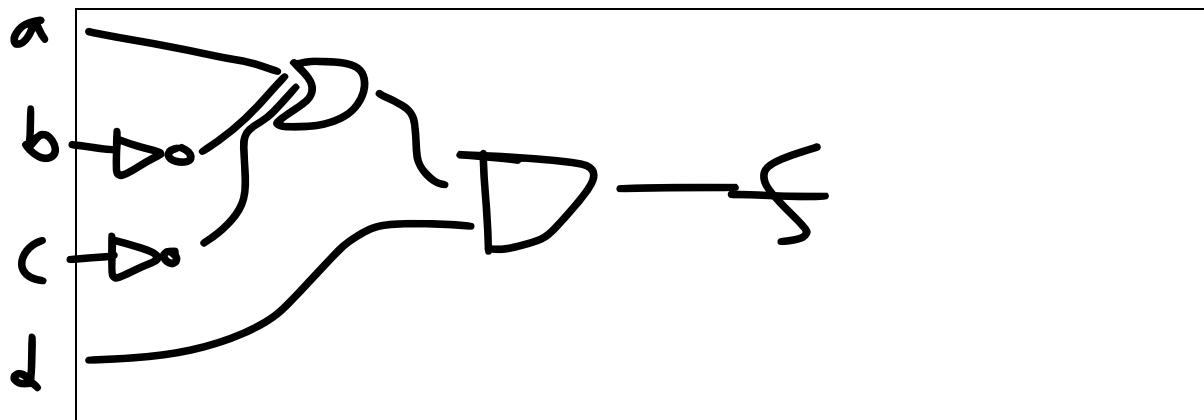
1. Create a Truth Table for the following logic expression:

$$f = \bar{b} \cdot (a \cdot b) + (\bar{c} \cdot d)$$

Recap on Building Logic Circuits from Expressions

1. Create a Logic Circuit Design for the following logic expression:

$$f = (a + \bar{b} + \bar{c}) \cdot d$$



Recap on SoP and PoS Expressions and Circuit Design from a Specification

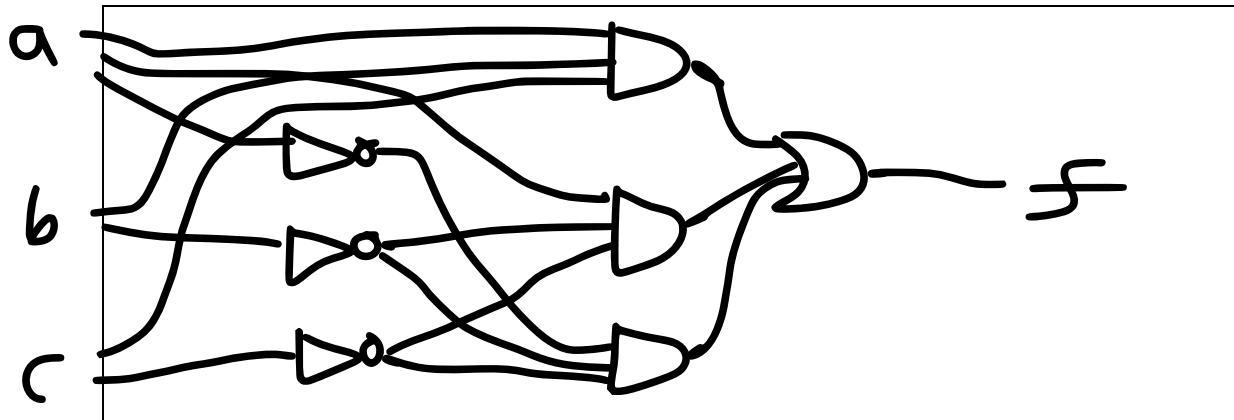
1. What is the fully defined SoP expression for the following specification truth table?

a	b	c	f	Minterms
0	0	0	1	$\bar{a} \cdot \bar{b} \cdot \bar{c}$
1	0	0	1	$a \cdot \bar{b} \cdot \bar{c}$
0	1	0	0	
1	1	0	0	
0	0	1	0	
1	0	1	0	
0	1	1	0	
1	1	1	1	$a \cdot b \cdot c$

The correct fully defined SoP expression is:

$$(\bar{a} \bar{b} \bar{c}) + (a \bar{b} \bar{c}) + (a b c)$$

The Circuit Design is:



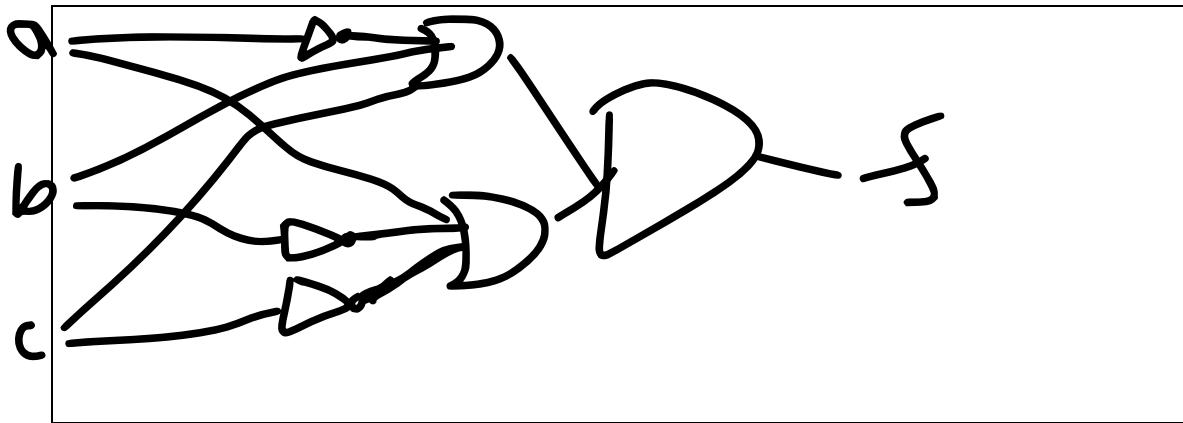
2. What is the fully defined PoS expression and Circuit Design for the following specification truth table?

a	b	c	f	Maxterms
0	0	0	1	
1	0	0	0	$\bar{a} + b + c$
0	1	0	1	
1	1	0	1	
0	0	1	1	
1	0	1	1	
0	1	1	0	$a + \bar{b} + \bar{c}$
1	1	1	1	

The correct fully defined PoS expression is:

$$(\bar{a} + b + c) \cdot (a + \bar{b} + \bar{c})$$

The Circuit Design is:



Recap on Specified Truth Tables from SoP and PoS Expressions

1. What was the specified truth table (f) that was used to develop the following SoP expression?

$$f = (a \cdot b \cdot c) + (\bar{a} \cdot b \cdot \bar{c}) + (\bar{a} \cdot \bar{b} \cdot c)$$

a	b	c	f
0	0	0	0
1	0	0	0
0	1	0	1
1	1	0	0
0	0	1	1
1	0	1	0
0	1	1	0
1	1	1	1

2. What was the specified truth table (f) that was used to develop the following PoS expression?

$$f = (a + \bar{b} + c) \cdot (a + b + \bar{c}) \cdot (\bar{a} + \bar{b} + \bar{c})$$

a	b	c	f
0	0	0	1
1	0	0	1
0	1	0	0
1	1	0	1
0	0	1	0
1	0	1	1
0	1	1	1
1	1	1	0

Drawing out a SoP or PoS Karnaugh Map (K-map)

1. Draw a two variable K-map in the SoP form with inputs labelled as: $a \ b$

Then enter the correct binary/hex number, created by the inputs, for each minterm.

We have done this first one for you (red numbers are binary and blue are hex for each minterm).

SoP K- map	\bar{b}	b
\bar{a}	00 0	01 1
a	10 2	11 3

2. Draw a three variable K-map in the SoP form with inputs labelled as: $a \ b \ c$

Then enter the correct binary/hex number, created by the inputs, for each minterm.

SoP K- map	$\bar{b} \cdot \bar{c}$	$\bar{b} \cdot c$	$b \cdot \bar{c}$	$b \cdot c$
\bar{a}	000 001 0 1	011 3	010 2	
a	100 101 4 5	111 7	110 6	

3. Draw a four variable K-map in the SoP form with inputs labelled as: $a \ b \ c \ d$

Then enter the correct binary/hex number, created by the inputs, for each minterm.

SoP K- map	$\bar{c} \cdot \bar{d}$	$\bar{c} \cdot d$	$c \cdot \bar{d}$	$c \cdot d$
$\bar{a} \cdot \bar{b}$	0000 0001 0 1	0011 0010 3 2		
$\bar{a} \cdot b$	0100 0101 4 5	0111 0110 7 6		
$a \cdot \bar{b}$	1100 1101 C D	1111 1110 F E		
$a \cdot b$	1000 1001 8 9	1011 1010 B A		

A	10
B	11
C	12
D	13
E	14
F	15

4. Draw a two variable K-map in the PoS form with inputs labelled as: $a \ b$

Then enter the correct binary/hex number, created by the inputs, for each maxterm.

PoS K- map	b	\bar{b}
a	.. 0	01 1
\bar{a}	10 2	11 3

5. Draw a three variable K-map in the PoS form with inputs labelled as: $a \ b \ c$

Then enter the correct binary/hex number, created by the inputs, for each maxterm.

PoS K- map	$b+c$	$b+\bar{c}$	$\bar{b}+c$	$\bar{b}+\bar{c}$
a	000 0	001 1	011 3	010 2
\bar{a}	100 4	101 5	111 7	110 6

6. Draw a four variable K-map in the PoS form with inputs labelled as: $a \ b \ c \ d$

Then enter the correct binary/hex number, created by the inputs, for each maxterm.

PoS K- map	$c+d$	$c+\bar{d}$	$\bar{c}+d$	$\bar{c}+\bar{d}$
$a+b$	0000 0	0001 1	0011 3	0010 2
$\bar{a}+b$	0100 4	0101 5	0111 7	0110 6
$\bar{a}+\bar{b}$	1100 C	1101 D	1111 F	1110 E
$\bar{a}+b$	1000 8	1001 9	1011 B	1010 A

Placing Minterms (SoP) or Maxterms (PoS) on a Karnaugh Map (K-map)

1. Look at the following SoP expression and then map its minterms (1's) to an appropriate K-map:

$$f = (\bar{a} \cdot b \cdot c \cdot d) + (\bar{a} \cdot b \cdot c \cdot \bar{d}) + (a \cdot b \cdot c \cdot d) + (a \cdot b \cdot c \cdot \bar{d})$$

SoP K-map	$\bar{c}\bar{d}$	$\bar{c}d$	$c\bar{d}$	cd
$\bar{a}\bar{b}$	1	0	0	1
$\bar{a}b$	0	0	1	1
$a\bar{b}$	0	1	1	1
ab	1	0	0	1

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a terrible
text editor

2. Look at the following PoS expression and then map its maxterms (0's) to an appropriate K-map:

$$f = (\bar{a} + \cancel{b} + \bar{c} + d) \cdot (\bar{a} + b + \bar{c} + \bar{d}) \cdot (a + b + \bar{c} + d) \cdot (a + \bar{b} + c + \bar{d})$$

PoS K-map	cd	$c\bar{d}$	$\bar{c}\bar{d}$	$\bar{c}d$
$a\bar{b}$	1	1	1	0
$a\bar{b}$	1	0	1	1
$\bar{a}\bar{b}$	1	1	0	1
$\bar{a}b$	1	0	0	0

Karnaugh Map Coordinates to Determine SoP and PoS Expressions

1. Look at the following coordinates and assuming four input variables ($a b c d$) use them to determine the SoP and PoS expressions they represent

$$f = \Sigma m(9BDF)$$

PoS K-map				
		\textcircled{d}	\textcircled{d}	
		\textcircled{c}	\textcircled{b}	

1

SoP K-map		-	--	
- -				
- +		1	1	

P.S

The ~~expression~~ expression from the coordinates provided is:

$$(\bar{a} + \bar{b} + c + \bar{d})(\bar{a} + \bar{b} + \bar{a} + \bar{b})(\bar{a} + b + \bar{c} + \bar{d})(\bar{a} + b + \bar{c} + d)$$

S.P

The ~~expression~~ expression from the coordinates provided is:

Not

2. Look at the following coordinates and assuming four input variables ($a b c d$) use them to determine the SoP and PoS expressions they represent

$$f = \Sigma m(8E5A)$$

PoS K-map				
		O		
				O
	O			O

SoP K-map				
				I
				I
		I		I

The SoP expression from the coordinates provided is:

$$(\bar{a}b\bar{c}d) + (\bar{a}\bar{b}\bar{c}\bar{d}) + (ab\bar{c}\bar{d}) + (\bar{a}\bar{b}\bar{c}\bar{d})$$

The PoS expression from the coordinates provided is:

in or

Identifying Prime Implicants in Karnaugh Maps

1. Create K-maps for the following SoP and PoS expressions and then circle the prime implicants in each

$$f = (\bar{a} \cdot b \cdot \bar{c} \cdot \bar{d}) + (a \cdot b \cdot \bar{c} \cdot \bar{d}) + (a \cdot \bar{b} \cdot \bar{c} \cdot \bar{d}) + (a \cdot \bar{b} \cdot \bar{c} \cdot d) + (\bar{a} \cdot b \cdot c \cdot \bar{d})$$

$$f = (a + b + c + d) \cdot (a + b + c + \bar{d}) \cdot (a + \bar{b} + c + \bar{d}) \cdot (a + \bar{b} + c + d) \cdot (a + b + \bar{c} + d)$$

PoS K-map	-	-	-	-	-
	0	0		0	
	0	0			
-					
-					

SoP K-map	-	-	-	-	-
-	-				
-		1			1
-		1			
-	1	1	1		

Simplifying SoP and PoS Expressions using Karnaugh Maps

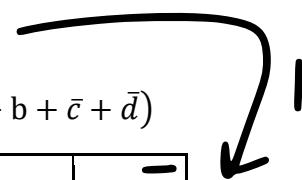
1. Simplify the following SoP and PoS expressions using K-maps

$$f = (a \cdot b \cdot c \cdot d) + (a \cdot b \cdot \bar{c} \cdot d) + (a \cdot \bar{b} \cdot \bar{c} \cdot d) + (a \cdot \bar{b} \cdot c \cdot d) + (\bar{a} \cdot \bar{b} \cdot c \cdot d)$$

$$f = (a + \bar{b} + c + d) \cdot (a + \bar{b} + c + \bar{d}) \cdot (a + b + c + \bar{d}) \cdot (a + b + \bar{c} + \bar{d}) \cdot (\bar{a} + b + \bar{c} + \bar{d})$$

o ↤

PoS K-map	$c+d$	$c+d$	$c+d$	-	-
$c+b$		$\textcircled{0}$	$\textcircled{0}$		
$c+\bar{b}$	$\textcircled{0}$	$\textcircled{0}$			
$\bar{a}+b$			$\textcircled{0}$		



SoP K-map	-	-	$c+d$	$c+d$	-
$a+b$				$ $	$ $
$\bar{a}+b$			$ $	$ $	
$\bar{a}+\bar{b}$			$ $	$ $	

The simplified SoP expression is:

$$(a \cdot d) + (\bar{b} \cdot c \cdot d)$$

The simplified PoS expression is:

$$(a + \bar{b} + c) \cdot (a + b + \bar{d}) \cdot (b + \bar{c} + \bar{d})$$

The Answers

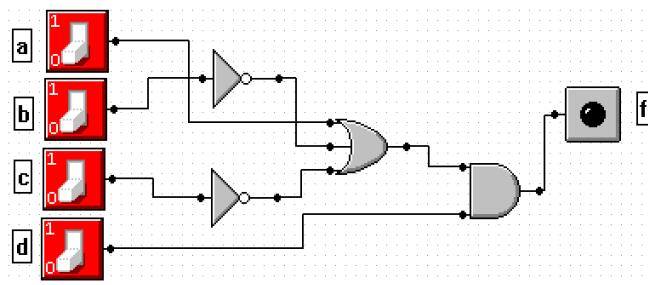
Recap on Truth Tables

1.

a	b	c	d	\bar{b}	$a \cdot b$	$\bar{b} \cdot (a \cdot b)$	$c \cdot d$	$\bar{c} \cdot \bar{d}$	f
0	0	0	0	1	0	0	0	1	1
1	0	0	0	1	0	0	0	1	1
0	1	0	0	0	0	0	0	1	1
1	1	0	0	0	1	0	0	1	1
0	0	1	0	1	0	0	0	1	1
1	0	1	0	1	0	0	0	1	1
0	1	1	0	0	0	0	0	1	1
1	1	1	0	0	1	0	0	1	1
0	0	0	1	1	0	0	0	1	1
1	0	0	1	1	0	0	0	1	1
0	1	0	1	0	0	0	0	1	1
1	1	0	1	0	1	0	0	1	1
0	0	1	1	1	0	0	1	0	0
1	0	1	1	1	0	0	1	0	0
0	1	1	1	0	0	0	1	0	0
1	1	1	1	0	1	0	1	0	0

Recap on Building Logic Circuits from Expressions

1. $f = (a + \bar{b} + \bar{c}) \cdot d$

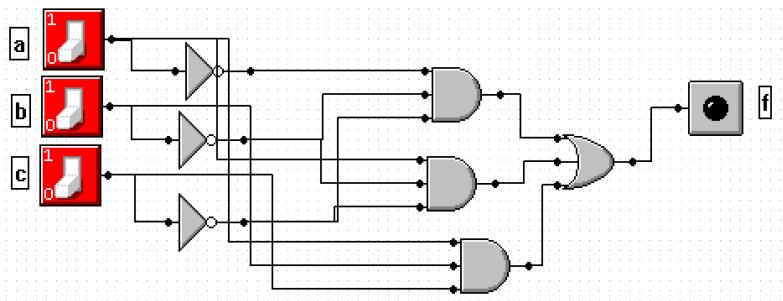


Recap on SoP and Pos Expressions and Circuit Design from a Specification

1.

a	b	c	f	Minterms
0	0	0	1	$(\bar{a} \cdot \bar{b} \cdot \bar{c})$
1	0	0	1	$(a \cdot \bar{b} \cdot \bar{c})$
0	1	0	0	
1	1	0	0	
0	0	1	0	
1	0	1	0	
0	1	1	0	
1	1	1	1	$(a \cdot b \cdot c)$

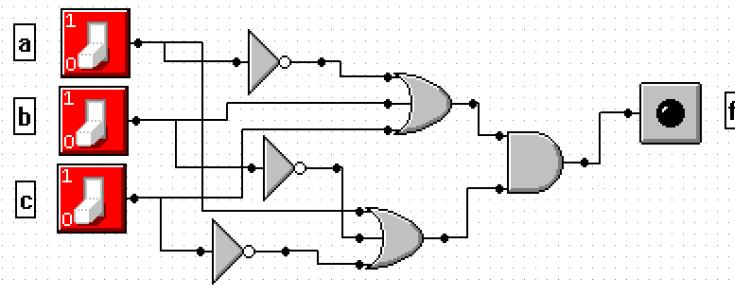
$$f = (\bar{a} \cdot \bar{b} \cdot \bar{c}) + (a \cdot \bar{b} \cdot \bar{c}) + (a \cdot b \cdot c)$$



2.

a	b	c	f	Maxterms
0	0	0	1	
1	0	0	0	$(\bar{a} + b + c)$
0	1	0	1	
1	1	0	1	
0	0	1	1	
1	0	1	1	
0	1	1	0	$(a + \bar{b} + \bar{c})$
1	1	1	1	

$$f = (\bar{a} + b + c) \cdot (a + \bar{b} + \bar{c})$$



Recap on Specified Truth Tables from SoP and PoS Expressions

$$1. f = (a \cdot b \cdot c) + (\bar{a} \cdot b \cdot \bar{c}) + (\bar{a} \cdot \bar{b} \cdot c)$$

a	b	c	f
0	0	0	0
1	0	0	0
0	1	0	1
1	1	0	0
0	0	1	1
1	0	1	0
0	1	1	0
1	1	1	1

$$2. f = (a + \bar{b} + c) \cdot (a + b + \bar{c}) \cdot (\bar{a} + \bar{b} + \bar{c})$$

a	b	c	f
0	0	0	1
1	0	0	1
0	1	0	0
1	1	0	1
0	0	1	0
1	0	1	1
0	1	1	1
1	1	1	0

Drawing out a SoP or PoS Karnaugh Map (K-map)

1.

SoP K- map	\bar{b}	b
\bar{a}	00 0	01 1
a	10 2	11 3

2.

SoP K- map	$\bar{b} \cdot \bar{c}$	$\bar{b} \cdot c$	$b \cdot c$	$b \cdot \bar{c}$
\bar{a}	000 0	001 1	011 3	010 2
a	100 4	101 5	111 7	110 6

3.

SoP K-map	$\bar{c} \cdot \bar{d}$	$\bar{c} \cdot d$	$c \cdot d$	$c \cdot \bar{d}$
$\bar{a} \cdot \bar{b}$	0000 0	0001 1	0011 3	0010 2
$\bar{a} \cdot b$	0100 4	0101 5	0111 7	0110 6
$a \cdot b$	1100 C	1101 D	1111 F	1110 E
$a \cdot \bar{b}$	1000 8	1001 9	1011 B	1010 A

4.

PoS K-map	b	\bar{b}
a	00 0	01 1
\bar{a}	10 2	11 3

5.

PoS K-map	$b + c$	$b + \bar{c}$	$\bar{b} + \bar{c}$	$\bar{b} + c$
a	000 0	001 1	011 3	010 2
\bar{a}	100 4	101 5	111 7	110 6

6.

PoS K-map	$c + d$	$c + \bar{d}$	$\bar{c} + \bar{d}$	$\bar{c} + d$
$a + b$	0000 0	001 1	011 3	010 2
$a + \bar{b}$	0100 4	0101 5	0111 7	0110 6
$\bar{a} + \bar{b}$	1100 C	1101 D	1111 F	1110 E
$\bar{a} + b$	1000 8	1001 9	1011 B	1010 A

Placing Minterms (SoP) or Maxterms (PoS) on a Karnaugh Map (K-map)

$$1. f = (\bar{a} \cdot b \cdot c \cdot d) + (\bar{a} \cdot b \cdot c \cdot \bar{d}) + (a \cdot b \cdot c \cdot d) + (a \cdot b \cdot c \cdot \bar{d})$$

SoP K-map	$\bar{c} \cdot \bar{d}$	$\bar{c} \cdot d$	$c \cdot d$	$c \cdot \bar{d}$
$\bar{a} \cdot \bar{b}$	0	0	0	0
$\bar{a} \cdot b$	0	0	1	1
$a \cdot b$	0	0	1	1
$a \cdot \bar{b}$	0	0	0	0

$$2. f = (\bar{a} + b + \bar{c} + d) \cdot (\bar{a} + b + \bar{c} + \bar{d}) \cdot (a + b + \bar{c} + d) \cdot (a + \bar{b} + c + \bar{d})$$

PoS K-map	$c + d$	$c + \bar{d}$	$\bar{c} + \bar{d}$	$\bar{c} + d$
$a + b$	1	1	1	0
$a + \bar{b}$	1	0	1	1
$\bar{a} + \bar{b}$	1	1	1	1
$\bar{a} + b$	1	1	0	0

Karnaugh Map Coordinates to Determine SoP and PoS Expressions

$$1. f = \Sigma m(9BDF)$$

PoS K-map	$c + d$	$c + \bar{d}$	$\bar{c} + \bar{d}$	$\bar{c} + d$
$a + b$	1	1	1	1
$a + \bar{b}$	1	1	1	1
$\bar{a} + \bar{b}$	1	0	0	1
$\bar{a} + b$	1	0	0	1

SoP K-map	$\bar{c} \cdot \bar{d}$	$\bar{c} \cdot d$	$c \cdot d$	$c \cdot \bar{d}$
$\bar{a} \cdot \bar{b}$	0	0	0	0
$\bar{a} \cdot b$	0	0	0	0
$a \cdot b$	0	1	1	0
$a \cdot \bar{b}$	0	1	1	0

$$f = (\bar{a} + \bar{b} + c + \bar{d}) \cdot (\bar{a} + \bar{b} + \bar{c} + \bar{d}) \cdot (\bar{a} + b + c + \bar{d}) \cdot (\bar{a} + b + \bar{c} + \bar{d})$$

$$f = (a \cdot b \cdot \bar{c} \cdot d) + (a \cdot b \cdot c \cdot d) + (a \cdot \bar{b} \cdot \bar{c} \cdot d) + (a \cdot \bar{b} \cdot c \cdot d)$$

$$2. f = \Sigma m(8E5A)$$

PoS K-map	$c + d$	$c + \bar{d}$	$\bar{c} + \bar{d}$	$\bar{c} + d$
$a + b$	1	1	1	1
$a + \bar{b}$	1	0	1	1
$\bar{a} + \bar{b}$	1	1	1	0
$\bar{a} + b$	0	1	1	0

SoP K-map	$\bar{c} \cdot \bar{d}$	$\bar{c} \cdot d$	$c \cdot d$	$c \cdot \bar{d}$
$\bar{a} \cdot \bar{b}$	0	0	0	0
$\bar{a} \cdot b$	0	1	0	0
$a \cdot b$	0	0	0	1
$a \cdot \bar{b}$	1	0	0	1

$$f = (\bar{a} + b + c + d) \cdot (a + \bar{b} + c + \bar{d}) \cdot (\bar{a} + \bar{b} + \bar{c} + d) \cdot (\bar{a} + b + \bar{c} + d)$$

$$f = (\bar{a} \cdot b \cdot \bar{c} \cdot \bar{d}) + (\bar{a} \cdot b \cdot \bar{c} \cdot d) + (a \cdot b \cdot c \cdot \bar{d}) + (a \cdot \bar{b} \cdot c \cdot \bar{d})$$

Identifying Prime Implicants in Karnaugh Maps

1.

$$f = (\bar{a} \cdot b \cdot \bar{c} \cdot \bar{d}) + (a \cdot b \cdot \bar{c} \cdot \bar{d}) + (a \cdot \bar{b} \cdot \bar{c} \cdot \bar{d}) + (a \cdot \bar{b} \cdot \bar{c} \cdot d) + (\bar{a} \cdot b \cdot c \cdot \bar{d})$$

$$f = (a + b + c + d) \cdot (a + b + c + \bar{d}) \cdot (a + \bar{b} + c + \bar{d}) \cdot (a + \bar{b} + c + d) \cdot (a + b + \bar{c} + d)$$

PoS K-map	$c + d$	$c + \bar{d}$	$\bar{c} + \bar{d}$	$\bar{c} + d$
$a + b$	0	0		0
$a + \bar{b}$	0	0		
$\bar{a} + \bar{b}$				
$\bar{a} + b$				

SoP K-map	$\bar{c} \cdot \bar{d}$	$\bar{c} \cdot d$	$c \cdot d$	$c \cdot \bar{d}$
$\bar{a} \cdot \bar{b}$				
$\bar{a} \cdot b$	1			1
$a \cdot b$	1			
$a \cdot \bar{b}$	1	1		

Simplifying SoP and PoS Expressions using Karnaugh Maps

1.

$$f = (a \cdot b \cdot c \cdot d) + (a \cdot b \cdot \bar{c} \cdot d) + (a \cdot \bar{b} \cdot \bar{c} \cdot d) + (a \cdot \bar{b} \cdot c \cdot d) + (\bar{a} \cdot \bar{b} \cdot c \cdot d)$$

$$f = (a + \bar{b} + c + d) \cdot (a + \bar{b} + c + \bar{d}) \cdot (a + b + c + \bar{d}) \cdot (a + b + \bar{c} + \bar{d}) \cdot (\bar{a} + b + \bar{c} + \bar{d})$$

PoS K-map	$c + d$	$c + \bar{d}$	$\bar{c} + \bar{d}$	$\bar{c} + d$
$a + b$		0	0	
$a + \bar{b}$	0	0		
$\bar{a} + \bar{b}$				
$\bar{a} + b$			0	

SoP K-map	$\bar{c} \cdot \bar{d}$	$\bar{c} \cdot d$	$c \cdot d$	$c \cdot \bar{d}$
$\bar{a} \cdot \bar{b}$			1	
$\bar{a} \cdot b$				
$a \cdot b$		1	1	
$a \cdot \bar{b}$	1		1	

$$f = (a \cdot d) + (\bar{b} \cdot c \cdot d)$$

$$f = (a + \bar{b} + c) \cdot (a + b + \bar{d}) \cdot (b + \bar{c} + \bar{d})$$