COMP9417: Homework Set #1

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Question 1

a)

We know from the normal equations for a the gradient of a minimised linear regression is:

$$\hat{\beta}_1 = \frac{\bar{XY} - \bar{X}\bar{Y}}{(\bar{X^2}) - (\bar{X})^2}$$

Which can expand into:

$$\hat{\beta}_1 = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

So now substituting in the transformation, we get:

$$\hat{\beta}_1 = \frac{\frac{1}{n}\sum_{i=1}^{n}(\tilde{X}_i - \bar{X})(Y_i - \bar{Y})}{\frac{1}{n}\sum_{i=1}^{n}(\tilde{X}_i - \bar{X})^2} \tag{1}$$

$$\hat{\beta}_1 = \frac{\frac{1}{n}\sum_{i=1}^{n}(c(X_i + d) - \bar{X})(Y_i - \bar{Y})}{\frac{1}{n}\sum_{i=1}^{n}(c(X_i + d) - \bar{X})^2} \tag{2}$$

$$\ldots \tag{3}$$

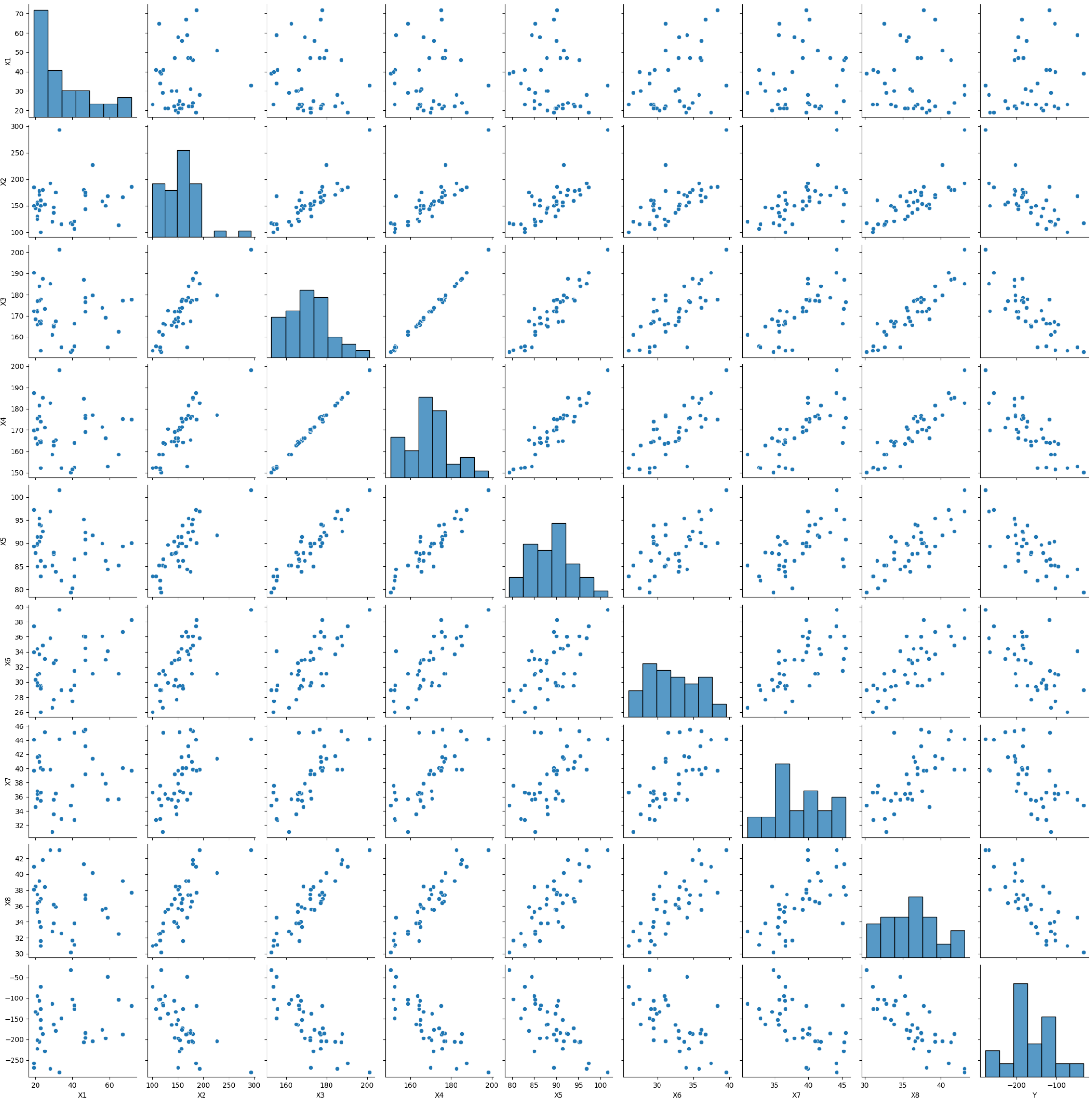
b)

Question 2

See Github repository here for all of the python code used in this question.

a)

```
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt

df = pd.read_csv("./data/data.csv")
sns.pairplot(df)
plt.savefig("./outputs/q2a_pairplots.png")
```



The pairs plots shows a scatter plots between variables in the dataset, while the histogram on the diagonal shows the distribution of each variable. For example, the plot on the fourth row in the second column is the scatter plot of X3 on the y-axis and X2 on the x-axis. Such a matrix is useful for seeing correlations between variables. This is important in linear regression to prevent multicollinearity between the variables, which can skew the coefficients of the regression model and lead to unreliable statistical inferences.

b)

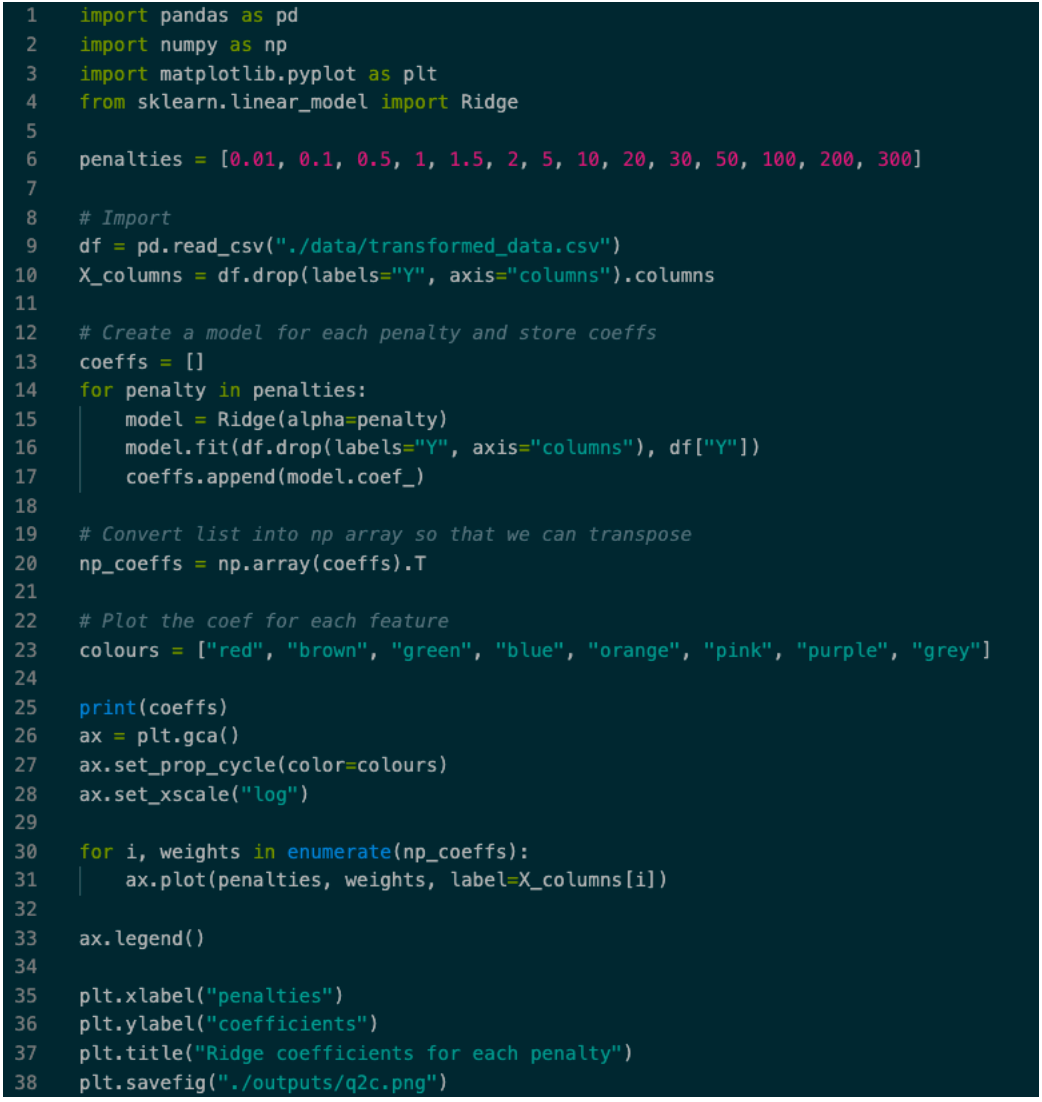
The code in **q2b.py** will produce:

```
Sum of squares for each transformed feature:
X1: 38.000000000000014
X2: 38.0
X3: 38.000000000000014
X4: 37.999999999999986
X5: 37.99999999999998
X6: 38.0
X7: 38.00000000000001
X8: 37.999999999999986
```

These should all be 38 exactly however they are not due to floating point errors.

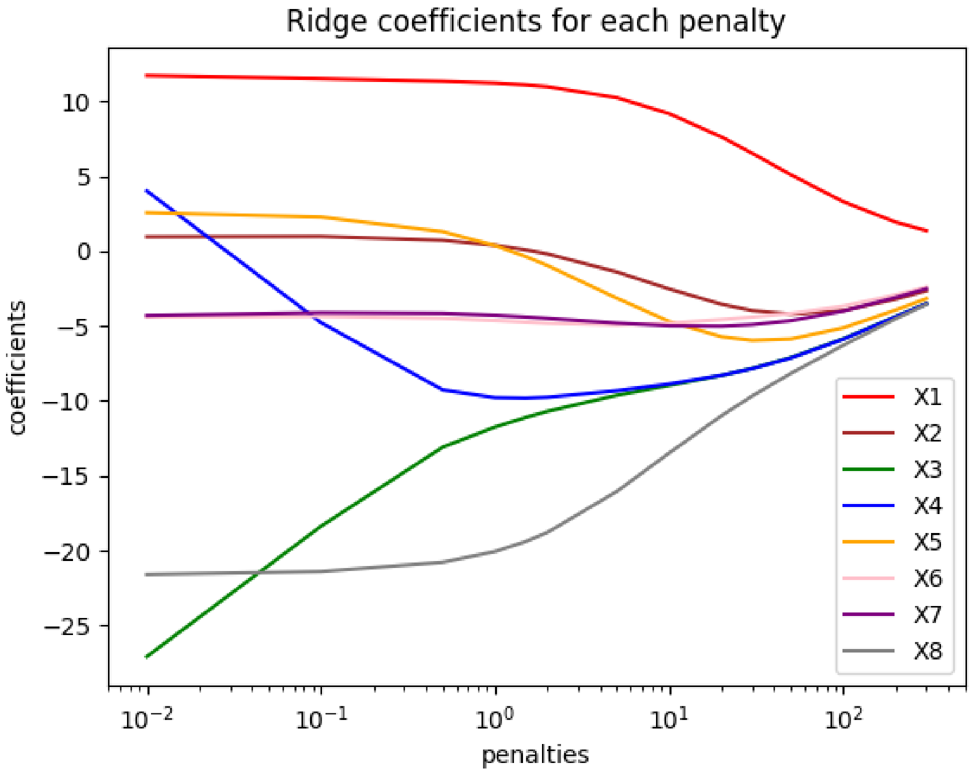
c)

Code used (also see in repository here):



```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import Ridge

penalties = [0.01, 0.1, 0.5, 1, 1.5, 2, 5, 10, 20, 30, 50, 100, 200, 300]

# Import
df = pd.read_csv("./data/transformed_data.csv")
X_columns = df.drop(labels="Y", axis="columns").columns

# Create a model for each penalty and store coeffs
coeffs = []
for penalty in penalties:
    model = Ridge(alpha=penalty)
    model.fit(df.drop(labels="Y", axis="columns"), df["Y"])
    coeffs.append(model.coef_)

# Convert list into np array so that we can transpose
np_coeffs = np.array(coeffs).T

# Plot the coef for each feature
colours = ["red", "brown", "green", "blue", "orange", "pink", "purple", "grey"]

print(coeffs)
ax = plt.gca()
ax.set_prop_cycle(color=colours)
ax.set_xscale("log")

for i, weights in enumerate(np_coeffs):
    ax.plot(penalties, weights, label=X_columns[i])

ax.legend()

plt.xlabel("penalties")
plt.ylabel("coefficients")
plt.title("Ridge coefficients for each penalty")
plt.savefig("./outputs/q2c.png")
```

The following graph is yielded:



As a general trend, the variance in the coefficients decreases as the penalty for the Ridge regression is increased. Also, by penalty=300, the coefficients are clustered around 0. In particular for X3, X4 and X5, these were positively correlated variables and their magnitudes in coefficients were quick to reduce as the penalty increased.

d)

e)