HW 5

Counting:

1. 1m, 2e, 1t, 1i, 1n, 1g $\binom{6}{5}+\binom{5}{3}=6+\frac{5\cdot4}{2}=\boxed{16}$, $6\cdot5!+\frac{5\cdot4}{2}\cdot\frac{5!}{2!}=11\cdot5!=\boxed{1320}$
2. 2 pairs: $\binom{13}{2}\cdot\binom{4}{2}\cdot\binom{4}{2}\cdot44=\frac{13\cdot12}{2}\cdot6\cdot6\cdot44=\boxed{123552}$, same color: $\frac{(13\cdot2)\cdot(12\cdot2)}{2}\cdot44=\binom{13}{2}\cdot4\cdot44=\boxed{13728}$
3. $2\cdot\binom{11}{6}=924$
4. (Assuming songs are differentiable) $6^{16}+(16\cdot6^{15})$; if songs are indifferentiable, $\binom{\overset{16+6-1}{21}}{5}+\binom{20}{5}$
5. Ways to form BST of 2: 2
   Ways to form BST of 3: $3\cdot(2)$
   Ways to form BST of 4: $4\cdot(3(2))$, etc, so given BST of 2 (3&9), ways to make 12 is $\boxed{12!/2}$, assuming node values don't have to be ints.
   If node values have to be ints, $2\cdot5!\cdot3!=\boxed{1440}$
6. $10-3=7$, $\binom{7+4-1}{4-1}=\binom{10}{3}=\boxed{120}$

Probability:

1. $\binom{21}{13}/(21^{13}/13!)=13!\cdot\binom{21}{13}/21^{13}$
2. 100000 possible #'s, Assuming no leading zeroes, $4\cdot4\cdot5+4\cdot4\cdot5\cdot7+4\cdot4\cdot5\cdot7\cdot6=16\cdot5\cdot(1+7+42)=80\cdot50=$ 4000 that meet criteria; if leading zeroes, 5000, probability $=\frac{1}{25}$, or $\frac{1}{20}$. 7 #'s that meet: $\binom{10}{7}\cdot\frac{24^3}{25^{10}}=\binom{10}{7}\left(\frac{24}{25}\right)^3\left(\frac{1}{25}\right)^7$, or $\binom{10}{7}\cdot\frac{19^3}{20^{10}}=\binom{10}{7}\left(\frac{19}{20}\right)^3\left(\frac{1}{20}\right)^7$
3. $P(A\&B)=3\cdot\frac{1}{6^3}$, $P(B)=\frac{1}{36}$, $P(A)=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}+3\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot(1-\frac{1}{2})=\frac{1}{2}$, $P(A\&B)=P(A)P(B)$, so independent
4. Straights: A-5 to 10-A, 10 possibilities (excluding suits), so $10\cdot4^5$ including suits, probability is $\frac{4^5\cdot10}{\binom{52}{5}}$
5. $P(\text{super plays \& win } 3/5)=\frac{65}{100}\cdot\binom{5}{3}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2$, $P(\text{super no play \& win } 3/5)=\frac{35}{100}\cdot\binom{5}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^2$, sum is $10\cdot(65\cdot75^3\cdot25^2+35\cdot40^3\cdot60^2)/100^6$; conditional probability: $\frac{65\cdot75^3\cdot25^2}{65\cdot75^3\cdot25^2+35\cdot40^3\cdot60^2}=\frac{13\cdot15^3\cdot5^2}{13\cdot15^3\cdot5^2+7\cdot8^3\cdot12^2}$
6. $\frac{37}{40}$ (same as probability that any randomly picked ball was cardinal)