FE670 Algorithmic Trading Strategies

Lecture 8. Transaction Costs and Trade Execution

Sheung Yin Kevin Mo

Stevens Institute of Technology

October 18, 2018

Outline

1. Transaction Costs
2. Liquidity and Transaction Costs
3. Market Impact Measurements and Empirical Findings
4. Forecasting and Modeling Market Impact
5. Incorporating Transaction Costs in Asset-Allocation Models

Transaction Costs

- Financial markets are not frictionless and transactions have a cost associated to them. Costs are incurred when buying or selling securities in the form of, for example, brokerage commissions, bid-ask spreads, taxes, and market impact costs.
- In recent years, portfolio managers have started to more carefully consider transaction costs. Partly, this is due to the flat performance of equities, often just in the single digit, after the period in the 1990s where the stock market returned about 20% per year.
- One way of describing transaction costs is to categorize them in terms of *explicit costs* such as brokerage and taxes, and *implicit costs*, which inlude market impact costs, price movement risk, and opportunity cost.

Taxonomy of Transaction Costs

Portfolio managers and traders need to be able to effectively model and minimize the impact of trading costs on their portfolios and trades.

- *Market impact costs* is, broadly speaking, the price an investor has to pay for obtaining liquidity in the market.
- *Price movement risk* is the risk that the price of an asset changes from the time the investor decides to transact in the asset until the transaction actually takes place.
- *Opportunity cost* is the cost suffered when a trade is not executed.
- *Explicit costs* are those costs that are observable and known up front such as commissions, fees, and taxes.
- *Implicit costs* on the other hand, are non-observable and not known in advance such as market impact and opportunity cost.

Taxonomy of Transaction Costs

Probably the easiest way to describe transaction costs is to categorize them in terms of fixed versus variable costs, and explicit versus implicit transaction costs.

* Fixed transaction costs are independent of factors such as trade size and market conditions. In contrast, variable transaction costs depend on some or all of these factors.

	Fixed	Variable
Explicit	Commissions	Bid-Ask Spreads
	Fees	Taxes
Implicit		Delay Cost
		Price Movement Risk
		Market Impact Costs
		Timing Risk
		Opportunity Cost

Trading commissions and fees, taxes, and bid-ask spreads are explicit transaction costs, which are also referred to as observable transaction costs.

- **Commissions and Fees** Commissions are paid to brokers to execute trades. Normally, commissions on securities and trades are negotiable. When the ownership over an asset is transferred, the investor is charged a *transfer fee*.
- **Taxes** The most common taxes are *capital gain tax* and *tax on dividends*. The tax law distinguishes between two types of capital gain taxes: *short-term* and *long-term*. The former is according to the investor's tax bracket, whereas the latter currently stands at 15%.
- **Bid-Ask Spreads** The bid-ask spread is the immediate transaction cost that the market charges anyone for the privilege of trading. High immediate liquidity is synonymous with small spreads.

Investment delay, market impact cost, price movement risk, market timing, and opportunity cost are implicit transaction costs, which are also referred to as non-observable transaction costs.

- **Investment Delay** Normally, there is a delay between the time when the portfolio manager makes a buy/sell decision of a security and when the actual trade is brought to the market by a trader. If the price of the security changes during this time, the price change (possibly adjusted for general market moves) represents the *investment delay cost*, or the cost of not being able to execute immediately.

  We note that this cost depends on the investment strategy. For example, modern quantitative trading systems that automatically submit an electronic order after generating a trading decision are exposed to smaller delay costs. Some practitioners view the investment delay cost as part of the opportunity cost discussed later in this chapter.

- **Market Impact Cost** The *market impact cost* of a transaction is the deviation of the transaction price from the market (mid) price that would have prevailed had the trade not occurred. We note that the market impact of a trade can be negative if, for example, a trader buys at a price below the no-trade price (i.e., the price that would have prevailed had the trade not taken place). In general, liquidity providers experience negative costs while liquidity demanders will face positive costs.

  We distinguish between two different kinds of market impact costs, temporary and permanent. Total market impact cost is computed as the sum of the two. The temporary market impact cost is of transitory nature and can be seen as the additional liquidity concession necessary for the liquidity provider to take the order.

The permanent market impact cost, however, reflects the persistent price change that results as the market adjusts to the information content of the trade. Intuitively, a sell transaction reveals to the market that the security may be overvalued, whereas a buy transaction signals that the security may be undervalued.

Traders can decrease the temporary market impact by extending the trading horizon of an order. For example, a trader executing a less urgent order can buy/sell his position in smaller portions over a period and make sure that each portion only constitutes a small percentage of the average volume. However, this comes at the price of increased opportunity costs, delay costs, and price movement risk.

Price Movement Risk In general, the stock market exhibits a positive drift that gives rise to price movement risk. A trade that goes in the same direction as the general market or an individual security is exposed to price risk. In practice, it can be difficult to separate price movement risk from the market impact cost.

Market Timing Costs The market timing costs are due to the movement in the price of a security at the time of the transaction that can be attributed to other market participants or general market volatility.

Opportunity Costs The cost of not transacting represents an opportunity cost. For example, when a certain trade fails to execute, the portfolio manager misses an opportunity. Commonly, this cost is defined as the difference in performance between a portfolio manager's desired investment and his actual investment after transaction costs.

- Following the completion of an institutional trade, suppose that the ticker tape for XYZ stock reveals that 6,000 shares of XYZ stock were purchased at $82.00.

  XYZ Trade Decomposition: Equity manager wants to buy 10,000 shares of XYZ at current price of $80. Trade desk releases 8,000 shares to broker when price is $81. Broker purchases 6,000 shares of XYZ stock at $82 plus $0.045 (per share) commission. XYZ stock jumps to $85, and remainder of order is canceled. 15 days later the price of XYZ stock is $88. [Frank J. Fabozzi, et al. *Equity Portfolio Management*, 1999, p. 309]
- The commission charge is the easiest to identify - namely, $0.045 per share, or $270 on the purchase of 6,000 shares.
- Since the trade desk did not release the order to buy XYZ stock until it was selling for $81, the assessed trader timing cost is $1 per share. Also the market impact cost is $1 per XYZ share traded, as the stock was selling for $81 when the order was received by the broker - just prior to execution of the 6,000 shares at $82.
- The opportunity cost - resulting from unexecuted shares - of the equity trade is more difficult to estimate. Assuming that the movement of XYZ stock price from $80 to $88 can be largely attributed to information used by the equity manager, it appears that the value the investment idea was 10%($88/$80 - 1) over a 15-day trading interval. The opportunity cost of not purchasing 4,000 is 4%(10%×40%)

Liquidity and Transaction Costs

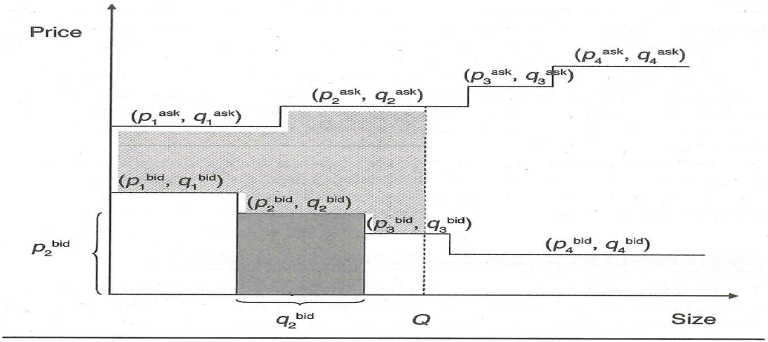
- Liquidity is created by agents transacting in the financial markets when they buy and sell securities. Market makers and brokers/dealers do not create liquidity; they are intermediaries who facilitate trade execution and maintain an orderly market.

- Liquidity and transaction costs are interrelated. A highly liquid market is one where large transactions can be immediately executed without incurring high transaction costs.

- The market impact cost varies with transaction size: the larger the trade size the larger the impact cost. Impact costs are not constant in time, but vary throughout the day as traders change the limit orders that they have in the limit order book. There are many different limit order types such as pegging orders, discretionary limit orders, IOC orders, and fleeting orders.

- Exhibit 11.3 The Supply and Demand Schedule of a Security
  At any given instant, the list of orders sitting in the limit order book embodies the liquidity that exists at a particular point in time.
  The limit order book reveals the prevailing supply and demand in the market.

EXHIBIT 11.3 The Supply and Demand Schedule of a Security



- We sort the bid and ask prices, $p_1^{bid}, \ldots, p_k^{bid}$ and $p_1^{ask}, \ldots, p_l^{ask}$, (from the most to the least competitive) and the corresponding order quantities $q_1^{bid}, \ldots, q_k^{bid}$ and $p_1^{ask}, \ldots, p_l^{ask}$.

  We then combine the sorted bid and ask prices into a supply and demand schedule. For example, the block p_2^{bid}, q_2^{ask} represents the second best sell limit order with price p_2^{bid} and quantity q_2^{ask}.

- We note that unless there is a gap between the bid (demand) and the ask (supply) sides, there will be a match between a seller and buyer, and a trade would occur. The larger the gap, the lower the liquidity and the market participants' desire to trade.

  For a trade of size Q, we can define its liquidity as the reciprocal of the area between the supply and demand curves up to Q.

Market Impact Measurements and Empirical Findings

- The problem with measuring implicit transaction costs is that the true measure, which is the difference between the price of the stock in the absence of a money manager's trade and the execution price, is not observable.

- Furthermore, the execution price is dependent on supply and demand conditions at the margin. Thus, the execution price may be influenced by competitive traders who demand immediate execution or by other investors with similar motives for trading.

- In general, this cost is the difference between the execution price and some appropriate benchmark, a so-called *fair market benchmark*, which is the price that would have prevailed had the trade not taken place.

- Practitioners have identified three different basic approaches to measure the market impact:
  1. *Pretrade measures* use prices occuring before or at the decision to trade as the benchmark, such as the opening price on the same-day or the closing price on the previous day.
  2. Posttrade measures use prices occuring after the decision to trade as the benchmark, such as the closing price of the trading day or the opening price on the next day.
  3. *Same-day* or *average measures* use average prices of a large number of trades during the day of the decision to trade, such as the *volume-weighted average price* (VWAP) calculated over all transactions in the security on the trade day.

- We denote by χ the indicator function that takes on the value 1 or -1 if an order is a buy or sell order, respectively. Formally, we now express the three types of measures of *market impact* (MI) as follows:

$$\mathbf{MI}_{pre} = \left(\frac{p^{ex}}{p^{pre}} - 1\right)\chi$$

$$\mathbf{MI}_{post} = \left(\frac{p^{ex}}{p^{post}} - 1\right)\chi$$

$$\mathbf{MI}_{VWAP} = \left(\frac{\sum_{i=1}^{k} V_i \cdot_i^{ex}}{\sum_{i=1}^{k} V_i} / p^{pre} - 1\right)\chi$$

where p^{ex}, p^{pre}, and p^{post} denote the execution price, pretrade price, and post trade price of the stock, and k denotes the number of transactions in a particular security on the trade date.

- Using this definition, for a stock with market impact MI the resulting *market impact cost* for a trade of size V, MIC, is given by

$$\mathbf{MIC} = \mathbf{MI} \cdot \mathbf{V}$$

- It is also common to adjust market impact for general market movements. For example, the pretrade market impact with market adjustment would take the form

$$\mathbf{MI}_{pre} = \left(\frac{p^{ex}}{p^{pre}} - \frac{p_M^{ex}}{p_M^{pre}} \right)$$

where p_M^{ex} represent the value of the index at the time of the execution, and p_M^{pre} the price of the index at the time before the trade.

- Market adjusted market impact for the posttrade and same-day trade benchmarks are calculated in an analogous fashion.

- As we analyze a portfolio's return over time an important question to ask is whether we can attribute good/bad performance to investment profits/losses or to trading profits/losses.

- In order to better understand a portfolio's performance it can be useful to decompose investment decisions from order execution. This the basic idea behind the *implementation shortfall approach*.

- In the implementation shortfall approach we assume that there is a separation between investment and trading decisions. The portfolio manager makes decisions with respect to the investment strategy (i.e. what should be bought, sold, and held). Subsequently, these decisions are implemented by the traders.

- By comparing the actual portfolio profit/loss (P/L) with the performance of a hypothetical paper portfolio in which all trades are made at hypothetical market prices, we can get an estimate of the implementation shortfall.

Forecasting and Modeling Market Impact

- We now describe a general methodology for constructing forecasting models for market impact. These types of models are very useful in predicting the resulting trading costs of specific trading strategies and in devising optimal trading approaches.

- The methodology is a linear factor based approach where market impact is the dependent variable. We distinguish between *trade-based* and *asset-based* independent variables or forecasting factors.

Trade-Based Factors

Some examples of trade-based factors include:

- Trade size
- Relative trade size
- Price of market liquidity
- Type of trade (information or noninformation trade)
- Efficiency and trading style of the investor
- Specific characteristics of the market or the exchange
- Time of trade submission and trade timing
- Order type

* Probably the most important market impact forecasting variables are based on absolute or relative trade size.

** Markets and exchanges in each country are different, and so are the resulting market microstructure. Forecasting variables can be used to capture specific market characteristics such as liquidity, efficiency, and institutional features.

*** Buy and sell orders have significantly different market impact costs. Separate models for buy and sell orders are needed.

Asset-Based Factors

- Some examples of asset-based factors are:
  - Price momentum
  - Price volatility
  - Market capitalization
  - Growth versus value
  - Specific industry or sector characteristics

* For a stock that is exhibiting positive price momentum, a buy order is liquidity demanding and it is, therefore, likely that it will have higher market impact cost than a sell order.

** Trades in high volatility stocks result in higher permanent price effects. Trades have a tendency to contain more information when volatility is high.

*** Large-cap stocks are more actively traded and therefore more liquid in comparison to small-cap stocks.

**** Different market sectors show different trading behaviors (e.g. the energy sector vs. non-energy sectors).

A Factor-Based Market Impact Model

- One of the most common approaches in practice and in the literature in modeling market impact is through a linear factor model of the form:

$$\mathbf{MI}_t = \alpha + \sum_{i=1}^{I} \beta_i x_i + \epsilon_i$$

where $\alpha, and \beta_i$ are the factor loadings and x_i are the factors. Frequently, the error term ϵ_i is assumed to be independently and identically distributed.

- Then the resulting market impact cost of a trade of (dollar) size V is the give by

$$\mathbf{MIC}_t = \mathbf{MI}_t \cdot \mathbf{V}$$

However, extension of this model including conditional volatility specifications are also possible.

Forecasting and Modeling Market Impact

- By analyzing both the mean and the volatility of the market impact, we can better understand and manage the trade-off between the two. For example, Bikker and Spierdijk use a specification where the error terms are jointly and serially uncorrelated with mean zero, satisfying

$$Var(\epsilon_t) = exp\left(\gamma + \sum_{j=1}^{J} \delta_j z_j\right)$$

  where γ, δ_j, and z_j are the volatility, factor loadings, and factors, respectively.

  Although the market impact function is linear, this of course does not mean that the dependent variables have to be linear. One of the earliest studies performed by Loeb showed that the market impact is proportional to the square root of the trade size ($V^{3/2}$).

A Factor-Based Market Impact Model

- Chen, Stanzl, and Watanabe suggest to model the nonlinear effects of trade size (dollar trade size V) in a market impact model by using the Box-Cox transformation, that is

$$MI(V_t) = \alpha_b + \beta_b \frac{V_t^{\lambda_b} - 1}{\lambda_b} + \epsilon_t \text{ and } MI(V_\tau) = \alpha_s + \beta_s \frac{V_\tau^{\lambda_s} - 1}{\lambda_s} + \epsilon_\tau$$

  where t, and τ represent the time of transaction for the buys and the sells respectively.

  In this specification, they assumed that ϵ_t and ϵ_τ are independent and identically distributed with mean zero and variance σ^2. The parameter $\alpha_b, \beta_b, \lambda_b, \alpha_s, \beta_s$ and λ_s were then estimated from market data by nonlinear least squares for each individual stock. We assume $\lambda_b, \lambda_s \in [0, 1]$ in order for the market impact for buys to be concave and sells to be convex.

A Factor-Based Market Impact Model

- We now want see how transaction costs models can be incorporated into standard asset-allocation models. For simplicity, we will use the mean-variance model to describe the basic approach. It is straightforward to extend this approach into other frameworks.
  The first description of an extension of the mean-variance framework that include transaction costs is given by

$$\max_{w} \left(w'\boldsymbol{\mu} - \lambda w'\boldsymbol{\Sigma} w - \lambda_{TC} \cdot TC \right)$$

  subject to $I'w = 1, I = [1, 1, .., 1]'$ where TC denotes a transaction cost penalty function λ_{TC} a transaction cost aversion parameter.
- In other words, the objective is to maximize expected return less the cost of risk and transaction costs. The transaction costs term in the utility function introduces resistance or friction in the re-balancing process.

A Factor-Based Market Impact Model

- Transaction costs models can involve complicated nonlinear functions. Portfolio managers frequently employ approximations of the transaction cost penalty function in the mean-variance framework.

One of the most common simplifications to the transaction cost penalty function is to assume that it is a separable function dependent only on the portfolio weights w, or more specifically on the portion to be traded $x = w - w_0$, where w_0 is the original portfolio and w is the new portfolio after rebalancing. Mathematically, we can express this as

$$TC(x) = \sum_{i=1}^{N} TC_i(x_i)$$

where TC_i' is the transaction cost function for security i and x_i is the portion of security i to be traded.

A Factor-Based Market Impact Model

- The transaction cost function TC_i is often parameterized as a quadratic function of the form

$$TC_i(x_i) = \alpha_i \cdot \chi_{\{x_i \neq 0\}} + \beta_i |x_i| + \gamma_i |x_i|^2$$

where the coefficients α_i, β_i and γ_i maybe different for each asset, and $\chi_{\{x \neq 0\}}$ is the indicator function that is equal to one when $x_i \neq 0$ and zero otherwise.

When all $\alpha_i = 0$, the resulting optimization problem is a quadratic optimization problem of the form

$$\max_{w} \left(w' \boldsymbol{\mu} - \lambda w' \boldsymbol{\Sigma} w - \lambda_{TC} (\beta' |\mathbf{x}| + |\mathbf{x}|' \boldsymbol{\Gamma} |\mathbf{x}|) \right)$$

subject to the usual constraints, where $\boldsymbol{\beta}' = (\beta_1, ..., \beta_N)$ and $\boldsymbol{\Gamma}$ is diagonal matrix with diagonal elements $\gamma_1, ..., \gamma_N$.

A Factor-Based Market Impact Model

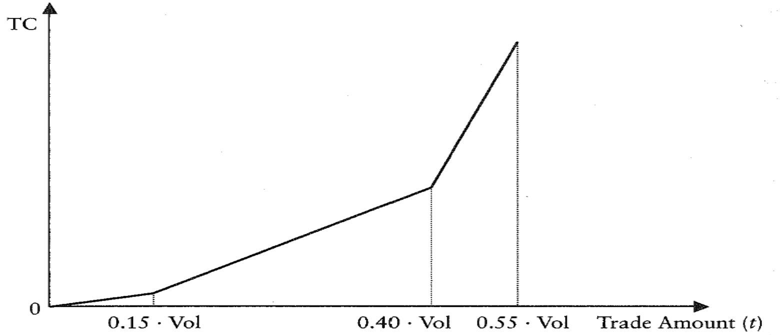
- Alternatively, piecewise-linear approximations to transaction cost function models can be used. An example of a piecewise-linear function of transaction costs for a trade of size t of a particular security is illustrated in Exhibit 11.4. The transaction cost function illustrated in the graph assumes that the rate of increase of transaction costs (reflected in the slop of the function) changes at certain threshold points.

$$TC(x) = \begin{cases} s_1 x, & 0 \le x \le 0.15 \cdot Vol \\ s_1(0.15 \cdot Vol) + s_2(x - 0.15 \cdot Vol), & 0.15 \cdot Vol \le x \le 0.40 \cdot Vol \\ s_1(0.15 \cdot Vol) + s_2(x - 0.25 \cdot Vol) + s_3(x - 0.40 \cdot Vol), & 0.40 \cdot Vol \le x \le 0.55 \cdot Vol \end{cases}$$

  where s_1, s_2, s_3 are the slopes of the three linear segments on the graph.
- Including piecewise-linear functions for transaction costs in the objective function of the mean-variance portfolio optimization problem is straightforward.

- Exhibit 11.4 An Example of Modeling Transaction Costs (TC) as a Piecewise Linear Function of Trade Size t

EXHIBIT 11.4 An Example of Modeling Transaction Costs (TC) as a Piecewise-Linear Function of Trade Size *t*



A Factor-Based Market Impact Model

- We can introduce new decision variables that corresponding to the number of pieces in the piecewise-linear approximation. There are three segments, and we introduce three variables y_1, y_2, y_3 and write the penalty term in the objective function for an individual asset as

$$\lambda_{TC}(s_1 \cdot y_1 + s_2 \cdot y_2 + s_3 \cdot y_3)$$

If there are N assets in the portfolio, the total transaction cost will be the sum of the transaction costs for each individual asset:

$$\lambda_{TC} \sum_{i=1}^{N} (s_{1,i} \cdot y_{1,i} + s_{2,i} \cdot y_{2,i} + s_{3,i} \cdot y_{3,i})$$

In addition, one needs to specify the following constraints on the new variables:

$0 \leq y_{1,i} \leq 0.15 \cdot Vol_i; 0 \leq y_{2,i} \leq 0.25 \cdot Vol_i; 0 \leq y_{3,i} \leq 0.15 \cdot Vol_i$

A Factor-Based Market Impact Model

- One also needs to link the traded amount of asset i to the optimal portfolio allocation. This is done by adding another set of constraints. We introduce variable z_i, one for each asset in the portfolio, that would represent the amount traded (but not the direction of the trade):

$$z_i = y_{1,i} + y_{2,i} + y_{3,i} \text{for each asset} i$$

  and also that z_i equals the change in the portfolio holdings of asset i. The latter condition is imposed by writing the constraint

$$z_i = |w_i - w_{0,i}| \text{for each asset} i$$

  where w_0 and w_i are the initial and the final amount of asset i in the portfolio, respectively.
- Despite the apparent complexity, piecewise-linear approximations for transaction costs are very solver-friendly, and save time in the portfolio optimization.

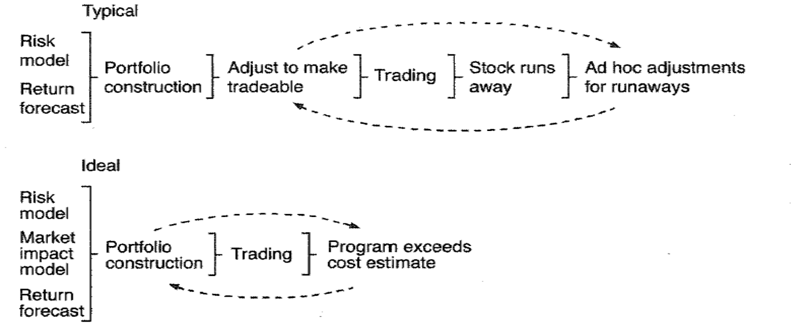
Incorporating Transaction Costs in Asset-Allocation Models

- Equity trading should not be viewed separately from equity portfolio management. One the contrary, the management of equity trading costs is an integral part of any successful investment management strategy. There four key element for a superior investment performance:
  1. Forming realistic return expectations.
  2. Controlling portfolio risk.
  3. Efficient control of trading costs.
  4. Monitoring total investment performance.

Unfortunately, most discussions of equity portfolio management focus solely on the relationship between expected return and portfolio risk - with little if any emphasis on whether the selected securities in the optimal or target portfolio can be acquired in a cost efficient way.

- Exhibit 11.5 Typical versus Ideal Portfolio Management

EXHIBIT 11.5 Typical versus Ideal Portfolio Management



In the typical scenario, portfolio managers engage in fundamental and/or quantitative research to identify investment opportunities - albeit with a measure of investment prudence (risk control) in mind. Upon completion, the portfolio manager reveals the list of securities that form the basis of the target portfolio to the senior trader. At this point, the senior trader informs the portfolio manager of certain nontradable positions-which causes the portfolio manager to adjust the list of securities either by hand or some other ad hoc procedure. This, in turn, causes the investor's portfolio to be suboptimal.

- A better approach to equity portfolio management requires a systematic integration of portfolio management and trading processes. In this context, the returns forecast, risk estimates, and trading cost program are jointly combined in determining the optimal investment portfolio.
- Accordingly, the portfolio manager can incorporate the appropriate trading cost information into the portfolio construction and risk control process.