Problem Chosen	2021	Team Control Number
A	MCM/ICM Summary Sheet	**2119503**

Fungicraft

Summary

The decomposition of plant materials and wood fibers by fungi has a huge impact on the natural environment. In this essay, we study the effects of environmental factors on the decomposition of fungi, studies the competitive behavior patterns among fungi, and finds out the best biological combinations in various environments.

In Model A, we make some preparations for the next step. We collected and tabulated the data we needed for our research on climate patterns. At the same time, in order to facilitate data processing, we have done normalization processing for all the data.

In Model B, we simplified the model considering the decomposition efficiency of fungi. We analyzed the binary relationship between the decomposition efficiency, fungus growth rate and fungus humidity tolerance, and finally obtained the basic model of the decomposition efficiency of fungus by the least square method and transforming the data into a linear relationship.

In Model C, we also take into account the effects of the environment on fungi. The main influence of the environment is divided into four aspects: temperature, humidity, temperature difference and humidity difference. The effect is mainly divided into direct effect, that is, direct effect on the unit decomposition efficiency of fungi, and indirect effect, that is, influence on the growth rate of fungi. We're using a skewed distribution to process the data.

In Model D, we consider the interaction of multiple fungi in coexistence. We use the competitiveness model and the aggression and self-preservation model respectively, and use the continuous Markov chain to model the time series, and get a set of differential equations of the fungal colony area in time, and finally get the distribution trend and decomposition efficiency of different colony combinations in time.Finally, we do the sensitivity analyses to examine our model and find our model is stable and accords with the reality.

Keywords:

Contents

1 Introduction

1.1 Background

The carbon cycle is a critical component for all life on the Earth, which can describe the process of the flow and exchange of carbon throughtout the geochemical cycle of the planet. There's no doubt that the decomposition of carbon must be an important part in the carbon cycle, which allows carbon to be renewed and reused in other forms. The decomposition of planet material and woody fibers undoubtedly is a vital component of is.

1.2 Literature Review

A recent research on wood decomposition influenced by fungtal activities has identified several fungi traits that could contribute to the decomposition rates and also researched connections between several certain traits. They have found that strains of fungi with slow gorwth rate are more liely to survive and be adapted to the changes of the environment.

When exploring the relationship of two traits of interest, the growth rate and the moisture tolerance, with the rate of decomposition, the pattern how these funji interact and how their interactions influence the decomposition rate are still to be solved. It still remains unsloved that how other external factors affect the decomposition rate.

1.3 Restatement of the Problem

We are required to establish mathmatical models to describes the decomposition of ground litter and woody fibers through fungal activities and considering following factors:

- **Multiple Species**The model should incorporate the interactions between different species of fungi in the presence of multiple species of fungi.
- **Dynamics of Interactions**The dynamics of the interactions should be characterized and described including both short- and long-term trends.
- **The Change of Environment**Besides, the model should include predictions about the growth and states of fungi in different environments such as arboreal, temperature and etc.
- **Biodiversity**Finally, the model should describe how the diversity of fungal communities could impact the overall efficiency of the descompsition. The importance of the biodiversuty should be suggested in the model.

2 Assumptions and Justification

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is properly justified.

- **A microbial community is treated as a single homogeneous group**. The fungi community consists of microbial communities of various species of fungi. To research the activities of various species of fungi, we define a microbial community as a group which consisted of a certain kind of fungi or several functionally disctinct pools.

- **We use a month as minimum unit of time when researching the change of temperature** Climate conditions in a certain region tend not to change much during the course of a month. That is to say that the probability distribution of temperature usually keep nearly the same in a month. Therefore, we use a month as minimun unit of time when researching the change of temperature.

3 Notations

Here we give the notations table to list the variation of parameters we use in this essay:

Table 1: Variation of some parameters

Symbols	Description
DR	The wood decomposition rate by fungi activities
M	Moisture tolerance of Fungi
E	The growth rate of fungi
T	Average temperature of environment
Ch	Average temperature difference of environment
S	Average moisture level of environment
Sh	DAverage moisture difference of environment
t	Optimal temperature for fungi growth
s	Optimal moisture level for fungi growth
ch	Funji's tolerance to temperature
sh	Funji's tolerance to moisture
D_s	The degree that the temperature fits to fujin's optimal temperature
W_s	The degree that the moisture fits to fujin's optimal moisture
D_c	The degree that the temperature difference fits to fujin's optimal one
W_c	The degree that the moisture difference fits to fujin's optimal one
m	Density of hyphal

4 Model A: Preparation Works

4.1 Researach on various Climate Type

To research the climate model, we will focus on five climate type: arid,semi-arid, temperate, arboreal, and tropical rain forests. For each type of climate type, we focus on its temperature, moisture, temperature difference and moisture difference and process these data before we research how they influence the fungi's activities. We can get the data of these five types of climate:

facts / months	J	F	M	A	M	J	J	A	S	O	N	D
Average Temperature	0.24074074	0.31481481	0.48148148	0.74074074	0.85185185	0.96296296	0.96296296	0.96296296	0.92592593	0.66666667	0.48148148	0.27777778
Average Humidity	0.41176471	0.26470588	0.11764706	0.74074074	0	0.01470588	0.07352941	0.05882353	0.07352941	0.17647059	0.39705882	0.32352941
Average Temperature Diiference	0.25	0.25	0.28571429	0.32142857	0.28571429	0.28571429	0.28571429	0.28571429	0.28571429	0.14285714	0.28571429	0.25

Figure 1: The relationship between the moisture tolerance of various fungi and the resulting wood decomposition rate.

facts / months	J	F	M	A	M	J	J	A	S	O	N	D
Average Temperature	9	11	14	17	25	27	28	31	26	21	18	10
Average Humidity	71	68	66	57	66	77	77	71	65	61	63	60
Average Temperature Diiference	5	6	6	7	7	6	5	3	5	6	4	4

Figure 2: The relationship between the moisture tolerance of various fungi and the resulting wood decomposition rate.

facts / months	J	F	M	A	M	J	J	A	S	O	N	D
Average Temperature	0.11111111	0.18518519	0.2962963	0.40740741	0.7037037	0.77777778	0.81481481	0.92592593	0.74074074	0.55555556	0.44444444	0.14814815
Average Humidity	0.79411765	0.75	0.72058824	0.58823529	0.72058824	0.88235294	0.88235294	0.79411765	0.70588235	0.64705882	0.67647059	0.63235294
Average Temperature Diiference	0.14285714	0.21428571	0.21428571	0.28571429	0.28571429	0.21428571	0.88235294	0	0.14285714	0.21428571	0.07142857	0.07142857

Figure 3: The relationship between the moisture tolerance of various fungi and the resulting wood decomposition rate.

facts / months	J	F	M	A	M	J	J	A	S	O	N	D
Average Temperature	0.92592593	0.88888889	0.81481481	0.81481481	0.81481481	0.77777778	0.81481481	0.85185185	0.88888889	0.96296296	0.92592593	0.92592593
Average Humidity	0.55882353	0.67647059	0.82352941	0.83823529	0.83823529	0.73529412	0.61764706	0.55882353	0.54411765	0.52941176	0.48529412	0.48529412
Average Temperature Diiference	0.57142857	0.5	0.42857143	0.5	0.5	0.57142857	0.71428571	0.85714286	1	1	0.64285714	0.71428571

Figure 4: The relationship between the moisture tolerance of various fungi and the resulting wood decomposition rate.

facts / months	J	F	M	A	M	J	J	A	S	O	N	D
Average Temperature	0.03703704	0.03703704	0.25925926	0.2962963	0.44444444	0.66666667	0.77777778	0.7037037	0.55555556	0.40740741	0.22222222	0
Average Humidity	0.92647059	0.91176471	0.95588235	0.89705882	0.97058824	0.92647059	0.92647059	1	0.95588235	0.92647059	0.83823529	0.91176471
Average Temperature Diiference	0.35714286	0.35714286	0.5	0.5	0.5	0.57142857	0.57142857	0.5	0.5	0.57142857	0.42857143	0.28571429

Figure 5: The relationship between the moisture tolerance of various fungi and the resulting wood decomposition rate.

4.2 Data processing

For the data we can find ususally have different dimensions and are usually on different order of magnitude, we normalize all the data to make the necessary data on the same order, which makes them more reasonable and avoids the situations where there are sensitive data.

$$x' = \frac{x-a}{b-a}$$

In the normalization equation, b and a are the maximum and minimum of a group of data.

5 Model B: Wood Descomposition by Fungi

5.1 Model Overview

In this section, we establish a model to suggest how a certain specie of fungi compose wood. We focus on two traits of a fungus: the growth rate of the fungus and the fungus' tolerance to moisture. As has been identified in the research, these two traits determine the rate of wood composition by the fungus. Based on the experiment data of the research done and considering generalized binary relation, we could aprroximate the relationships between these two traits and the decomposition rate with several equations by fitting these data through linear regression.

5.2 The Moisture Tolerance of the Fungi

The links between the decomposition rate and the moisture tolerance of a certain specie of fungi has been researched in the research and can be protrayed with a figure:

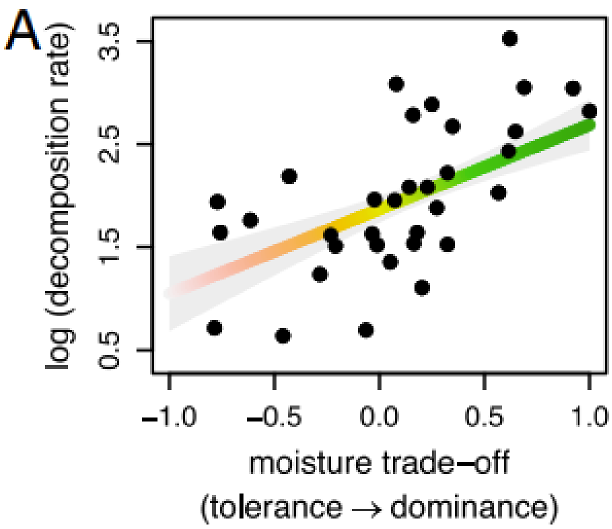

Figure 6: The relationship between the moisture tolerance of various fungi and the resulting wood decomposition rate.

Then we can draw the conclusion that there is a linear relationship bwtween the moisture tolerance and the decomposition and describe this relationship with several equations at least squares principle.

First, we can get the linear relationship for these two factor form based on the figure:

$$\log DR = C_1 M + C_2$$

where C_1 and C_2 are regression coefficient and:

$$e^b = 10$$

Therefore, we can get that $b \approx 2.3025$

And we can convert this expression and get:

$$DR = e^{b(C_1 M + C_2)}$$

$$DR = e^{bC_1 M + C_3}$$

where:

$$bC_2 = C_3$$

Finally, we can get the regression coefficents when the residual is minimal. The coefficient $C_1 \approx 1.312$.

5.3 Linear Regression

To describe the relationships between the moisture tolerance and the the decomposition rate with the experiments data in the terms of equations, regression analysis is perferomed on the data. Through the figure of the fitting data, it can be concluded that the decomposition is linearly dependent on this fungi trait. Therefore we fit the data at the least squares principle, which is a frequently-used principle of lineaar regression.

The method of least squares is a standard approach in regression analysis to approximate the solution of overdetermined systems by minimizing the sum of the squares of the residuals made in the results of every single equation. Given the overdetermined equation:

$$\sum_{j=1}^{n} X_{ij}\beta_j = y_i, i = 1, 2, 3 \ldots, m$$

where *m* is the nunmer of the samples and *n* is the dimension of the vectors, and we can convert this to:

$$X\beta = y$$

$$X = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & \cdots & X_{2n} \\ \vdots & \vdots & & \vdots \\ X_{m1} & X_{m2} & \cdots & X_{mn} \end{bmatrix}, \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}, y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

where *X* is the matrix of data sample, *y* is a soultion for this regression and β is the regression coefficient. When the sum of the second moment norms of the residuals is minimal, we can get the optimal estimate of :

By differentiating to find the minimal value, we can get:

$$X^T X\beta = X^T y$$

When the is a non-singular matrix, we can get the unique solution for:

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T y$$

5.4 The Growth Rate of the Fungi

With the data of 34 species' descomposition rate and growth rate at different temperatures, we can also protray their relationship whith a figure:

From the figure, we can get that the decomposition rate and the growth rate are not linearly related. However, we find that if we process these data with some special method, we can find that there is still a linear relationship between the prossesd data of decomposition rate and the growth rate of the fungi:

$$\ln DR = b_1 E^{b_2} + C_6$$

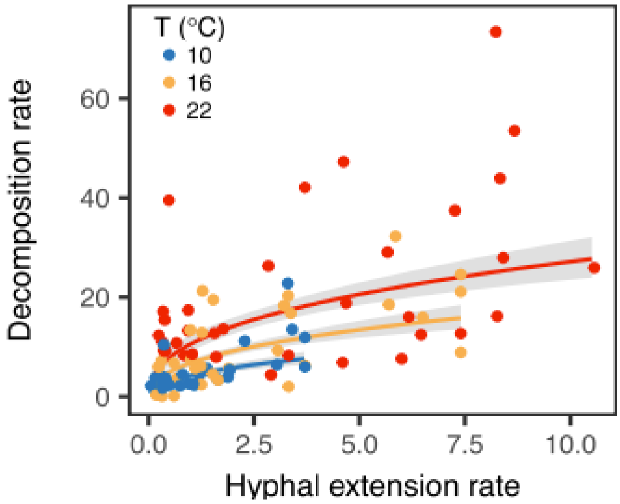

Figure 7: The relationship between the growth rate at differen temperatures and the resulting wood decomposition rate.

$$DR = e^{b_1 E^{b_2} + C_6}$$

For we use nolinear regression expressions, we should process the data before we use the least square method:

$$ddr = \ln(\ln DR)$$

$$B_1 = \ln b_1$$

$$f = \ln E$$

$$ddr = B_1 + b_2 f + \delta$$

Then we can get the regression coefficients b_1 and b_2:

$$b_1 = e^{\Sigma ddr} - b_2 \sum f$$

$$b_2 = \frac{n \sum f \cdot ddr - \sum f \cdot \sum ddr}{n \sum f^2 - (\sum f)^2}$$

Finally, we can get the regression coefficient:

Table 2: Result of Regression Coefficient

Temperature	b_1	b_2
10°C	0.314	1.226
16°C	-5.609	-0.1261
22°C	0.019	1.784

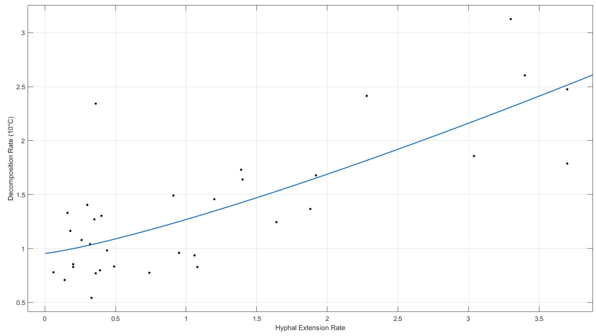

Figure 8: The relationship between the growth rate and the resulting wood decomposition rate at $10°$C.

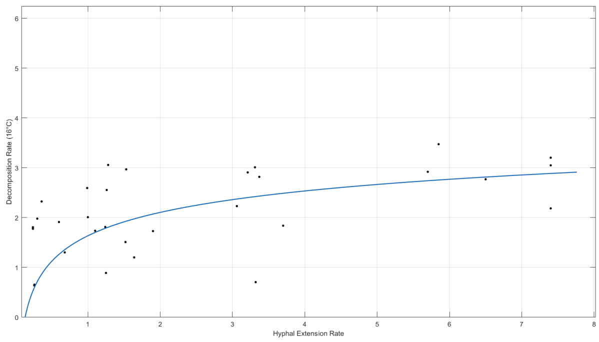

Figure 9: The relationship between the growth rate and the resulting wood decomposition rate at $16°$C.

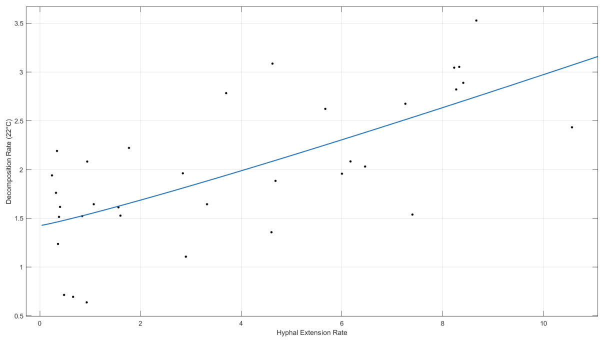

Figure 10: The relationship between the growth rate and the resulting wood decomposition rate at $22°$C.

5.5 Evaluation for Regression

After we get the solutions for the regressions, we evaluate the accuracy of our regression models. First, we calculate the sum of errors' squares:

$$SSE = \sum (dr - DR)^2$$

where dr is the value in our regression and DR is orginal data.This index can be used as a proxy of the regression soulution's error.

Then we calculate the sum of the modification values' square:

$$SST = \sum (dr - \overline{dr})^2$$

Then we define the determination coefficient R^2:

$$R^2 = 1 - \frac{SSE}{SST}$$

As R^2 approaching to 1, our model fits the data better.

Then we calculate the evulation coefficient R^2 for moisture tolerance and growth rate: $R \approx 0.864$

Thus we can draw the conclusion that our regression fits the data well and our model is accurate and reliable.

5.6 Model of Wood Descomposition by Fungi

Finally, we combine the model above and give equations to describe how these two traits of fungi determine the descomposition rate at deiierent temperature:

$$\begin{cases} DR = e^{1.312M+0.314E^{1.226}} \\ DR = e^{1.312M-5.609E^{-0.1261}} \\ DR = e^{1.312M+0.019E^{1.784}} \end{cases}$$

6 Model C: Effect of the Environments

6.1 Model Overview

According to the data we get from experiments, we can find that there's still something unreasonable. By observing the figure of the decomposition rate and growth rate of fungi, we can find that temperature also have an effect on the fungi's growth rate which is a trait that determines the decomposition rate. Therefore, we guess that the moisture has the same effect. Considering these factors, we include the environment, which consists of two aspects of temperature and moistre, in our model and it can affect the fungi's ability to decomposing woody fiber directly and indirectly.

6.2 Indirect Effect of Environment

In our model B, we have found that the decomposition rate is highly linearly dependent on the griwth rate of the fungi. Therefor, we can conclude that the environment can affect the decomposition rate by influencing the growth rate of the fungi. To research how environment determine the growth rate, we abstract the environment into a class that contains four attributes: temperature, temperature difference, moisture and moisture difference.

6.2.1 Temperature

By observing experiments data of different species of fungi, we can find a specie of fungi can keep growing at itt maximum growth rate in a certain temperature range and its growth rate will decline if the temperature goes up or down from this range, which fits the skewed distribution or normal distribution. Therefore, we fit the data in the form of beta distribution and gamma distribution, which are two types of skewed distribution.

The model of beta distribution is:

$$E(t) = \frac{t^{\alpha-1}(1-t)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}du} C_1$$

and the model of gamma distribution is:

$$E(t) = \tfrac{\beta^\alpha}{\gamma(\alpha)} t^{\alpha-1} e^{-\beta t} C_2$$

$$\Gamma(\alpha) = \textstyle\int_0^{+\infty} u^{\alpha-1} e^{-u} du$$

We use maximum likelihood method to fitting the function and introduce the log-lokelihood function:

$$\log L(\alpha, \lambda) = (\alpha - 1) \sum_{i=1}^{n} \log x_i - \frac{1}{x} \sum_{i=1}^{n} x_i - n \log \beta(\alpha) - n\alpha \log \lambda$$

We take the first partial derivatives of these expressions with respect to a and $lambda$ respectively and make the derivatibes equal to 0 so that the maximum likelihood function minimum. We can get the differential equations:

$$\frac{\partial \log L(\alpha, \beta)}{\partial \alpha} = \sum_{i=1}^{n} \ln x_i - \frac{\beta'(\alpha)}{\beta(x)} - n \ln \beta = 0$$

Then we covert it to:

$$\frac{\partial \log L(\alpha, \beta)}{\partial \beta} = \frac{1}{b^2} \sum_{i=1}^{n} x_i - n \frac{\alpha}{\beta} = 0$$

There is no analytical solution to the equations, but we can still get the maximum likelihood estimate of $alpha$ and $beta$.

Finally, we can get two models of temperature-affected growth rate.

By comparing the residuals of these two models, we choose distribution as our final model.

Then, we assess the changing of the weather and environment to determine the impact of the temperature of environment. We choose day as the minimum unit of the temeperature distribution model over time and research the probability distribution of temperature in a day. Traditional meteorology research has proved that the temperature distribution during a certain range of time usually fits the normal distribution. Therefore we choose normal distribution to approximate the distribution of temperature and introduce the index of average temperature in a day and average temperature difference in a day.

$$f(x) = \frac{2}{\sqrt{2\pi} \cdot Ch} e^{-\frac{2(x-T)^2}{ch^2}}$$

And we can get that the growth rate of fungi with its growth rate at a certain temperature and the probability of the certain temperature.

$$E = \int E(x) f(x) dx$$

6.2.2 Moisture

For the growth rate in different moisture level, we can get that when the moisture degree is lower that about 90 percentages, the growth rate has an upward trend with the increase of moisture and a sharply downward trend with the decrease of moisture, which fits the skewed-normal distribution. Therefore, we still fit the data to beta distribution and gamma distribution.

By calculating the residuals, we can get the optimal regression model.

For the difference of temperature and moisture in the environment, for the temperature and moistre in a certain place is random and fit normal distribution, we introduce the index of average temperature difference and average moisture difference.

$$g(x) = \frac{2}{\sqrt{2\pi} \cdot Sh} e^{-\frac{2(x-s)^2}{Sh^2}}$$

Then we can get the expression to describe the relationship between the external moisture level and the growth rate of fungi:

$$E = \int F(x)g(x)dx$$

Combining the effect of temperature and moisture, we can get:

$$E = \int_{-\infty}^{+\infty} [E(x)f(x) + F(x)g(x)]dx$$

which describes the relationship between the external environment and the internal growth rate of the fungi.

6.3 Direct Effect of Environment

Analyzing the data of growth rate and decomposition rate in diferent temperature, we can find that the decomposition rate still changes with the temperature as the growth rate of hyphal growth rate. It tells that the temperature not only indirectly determines the decomposition rate by influencing the hyphal growth rate but also directly influences the decomposition rate.

For the difference between the environment that fungi can be adapted to and the partical environment, we introduce the concept of he degree of fungi's adaptation to the environment.

The adaptation index of moisture and temperature are defined as:

$$D_S = -|S - s|$$

$$W_S = -|T - t|$$

and the adaptation index of moisture difference and temperature difference are defined as:

$$D_c = \frac{S\left(D_s + \frac{Ch-ch}{2}\right) + S\left(\frac{Ch-ch}{2} - D_s\right)}{2}$$

$$W_c = \frac{S\left(W_s + \frac{Sh-sh}{2}\right) + S\left(\frac{Sh-sh}{2} - W_s\right)}{2}$$

where: Commonly, a fungus has different traits to adapt itself to environment in different aspects and these traits usually function respectively, so we can conclude that these four adaptation indexes and establish linear regression model:

$$P = a\left(D_s + 1\right) + b\left(W_s + 1\right) + cD_c + dW_c + \varepsilon$$

In the research done before, we can get the spearman correlation coefficient between some traits of fungi and the decomposition rate and we can get:

$$P = 0.18D_s + 0.31W_s + 0.14D_c - 0.46W_c + 0.49$$

And we can get the decomposition rate model includeing the climate effection after we introduce the index of hyphal density:

$$DR = mEp \int dt$$

7 Model D: Interactions between Fungi

7.1 Model Overview

In the model of fungi's competition, we consider two pattern of models: a comprehensive assessment of fungal competitiveness and assessment considering the offensive ability and defensive ability of fungi respectively. For that every microbial community has an interaction effect on each other and the state evolves continuously from the previous state over time. The sate of a moment only depends on the previous moment. Therefore, we establish the competition model between microbial communuties with differential equations set by using Markov chain over continuous time.

7.2 Model of Overall Competitive Ranking

Competitive ranking represents fungi's ability to obtain resources and survive. We think that the competitive ranking of a specie of microbial community depends on its hyphal density, growth rate, temperature tolerance and moisture tolerance. So we can get the linear regression model of competition model:

$$G = \alpha_1 M + \alpha_2 E + \alpha_3 S + \alpha_4 D$$

First, we determine each factor's relative weight on the carrying capacity according to the consequence and get a weight matrix A:

$$A = [a_1, a_2, a_3, a_4]$$

However, this weight matrix is completely based on our subjective extrapolation, which may have deviation with the reality. Therefore, we construct another relatively objective weight matrix to correct this one.

Based on model A, we have got a set of objective weight of these factors. We calculate a specific factor's weight based on its frequency of occurrence in passages on carrying capacity on an authoritative natural science website, for a factor having more effect on the carrying capacity must be mentioned repeatedly and emphasized in articles. In this way, we calculate the weight of the factor i in this way:

$$n_i = \frac{k_i}{\sum_{j=1}^{4} k_j}$$

Then, we construct the factors set and the evaluation level set:

$$U = [u_1, u_2, u_3, u_4]$$

where U_1 represents the hyphal density, U_2 represents the growth rate, U_3 represents the moisture tolerance and U_4 represents the temperature tolerance.

In this way, we construct the fuzzy evaluation matrix:

$$V = [v_1, v_2, v_3, v_4]$$

$$\mathrm{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix}$$

$$\sum_{j=1}^{4} r_{ji} = 1$$
$$j \in [1, 4]$$

With the data got by the crawler, we can calculate ther, which is the factors' fuzzy affiliation to the weight value from 0.1 to 0.9.

For a set of distributed data, we can only get that it fits a normal distribution without its mean value and variance or a binomial distribution without its mean value. Therefore, we use maximum likelihood estimation to get a set of data to complete the model to make its result like the data we get through the crawler.

$$argmaxp(x;)$$

In the equation, $p(x;\mu)$ is the optimal likelihood function which calculates the model's possibility to give crawler's data with the parameter μ. We assume that every parameter is standalone and get:

$$P\left(\mathrm{x}_1, \ldots, \mathrm{x}_n ; \mu\right)=\prod_{i=1}^n p\left(\mathrm{x}_i ; \mu\right)$$

We put the result of data to the model and get the value of r. Finally, we combine the equations and get the fuzzy comprehensive evaluation vector. According to the evaluation theory of the fuzzy set theory, we can express the fuzzy comprehensive evaluation vector in the form of fuzzy matrix:

$$S=W \cdot A=\left[\begin{array}{llll} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{array}\right]\left[\begin{array}{llll} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{array}\right]=\left[S_1, S_2, S_3, S_4\right]$$

The S are the final corrected weight. And, finally we get the weights:

Therefore, we obtained the final evaluation model of fungal competitive ranking:

$$G=0.29 M+0.57 E+0.11 S+0.05 D$$

We then suggest that the mutual predation between the two fungi depends on the state and competitiveness of the two species. We take our competitiveness assessment model into our time-continuous Markov chain to obtain the final differential equations.Taking all above into consideration, we can get that we assume a situation where n species of various fungi grow together.

The area of each specie's microbial community are represented as X_1, X_2...and X_n. The growth rate of microbial community X_i is represented as D_i, the moisture tolerance is represented as S_i and the hyphal density is represented as M_i.

$$
\begin{aligned}
\frac{\partial x_1}{\partial t} =& D_1 + k \sum_{i=1}^{n} (G_i - G_1)\, x_1 x_i \\
& \cdots\cdots \\
\frac{\partial x_p}{\partial t} =& D_p + k \sum_{i=1}^{n} (G_i - G_p)\, x_p x_i \\
& \cdots\cdots \\
\frac{\partial x_n}{\partial t} =& D_n + k \sum_{i=1}^{n} (G_i - G_n)\, x_n x_i
\end{aligned}
$$

Therefore, we can also get the final decomposition efficiency model

$$Q_i = M_i x_i p$$

where:

$$P = 0.18D_s + 0.31W_s + 0.14D_c - 0.46W_c + 0.49$$

$$Q = \sum_{i=1}^{n} Q_i = \sum_{i=1}^{n} M_i x_i p$$

In this Model, we also consider the influence of external environment, that is, using the relationship between growth rate and external environment in Model C:

$$D_i = \int_{-\infty}^{+\infty} [E(x)f(x) + F(x)g(x)]dx$$

7.3 Model of Offend and Defend

In this model, we divide the competitive ranking into two sections: the offensive ability and defensive ability of fungi respectively instead of researching the competitive ranking of funji totally. We think that the offensive ability enable a specie of funji to gain more resources at presence of other sepcies when the resouces are limited and keep them grow rapidly, which is positive dependent on the growth rate of a specie of funji. The defensive ability help themselves survive hard conditions, which is positive dependent on the moisture tolerance of the fungi. Taking all above into consideration, we can get that we assume a situation where n species of various fungi grow together. The area of each specie's microbial community are represented as X_1, X_2...and X_n. The growth rate of microbial community X_i is represented as D_i, the moisture tolerance is represented as S_i and the hyphal density is represented as M_i. We use

Markov chains in continuous time for establishing model and get the following differential equations:

$$\frac{\partial X_1}{\partial t} = D_1 + k_1 \sum_{i=1}^{n} \left(D_1 - S_i\right) x_1 x_i - k_2 \sum_{i=1}^{n} \left(D_i - S_1\right) x_i x_1 + \left(k_2 - k_1\right) \left(D_1 - S_1\right) X_1^2$$

$$\ldots\ldots$$

$$\frac{\partial X_1}{\partial t} = D_P + k_1 \sum_{i=1}^{n} \left(D_1 - S_i\right) x_p x_i - k_2 \sum_{i=1}^{n} \left(D_i - S_p\right) x_i x_1 + \left(k_2 - k_1\right) \left(D_p - S_p\right) X_p^2$$

$$\ldots\ldots$$

$$\frac{\partial X_n}{\partial t} = D_n + k_1 \sum_{i=1}^{n} \left(D_n - S_i\right) x_n x_i - k_2 \sum_{i=1}^{n} \left(D_i - S_n\right) x_i x_n + \left(k_2 - k_1\right) \left(D_n - S_n\right) X_n^2$$

This differential equations don't have analytical solution, but we can get the trend of colony size's changing with time. At the same time, we can get the decomposition ability of a microbial community. For microbial community i, we can get its ability of decomposing:

$$Q_i = M_i x_i p$$

where:

$$P = 0.18D_s + 0.31W_s + 0.14D_c - 0.46W_c + 0.49$$

8 Model Analysis and Sensitivity Analysis

8.1 The Influence of Temperature

For the first sensitivity analysis, we considered changing the values of the mean temperature and temperature difference to check whether our probability distribution model of temperature would remain stable in the case of sudden temperature changes.According to the results of the images, our fitting of the temperature probability model is stable and reasonable.

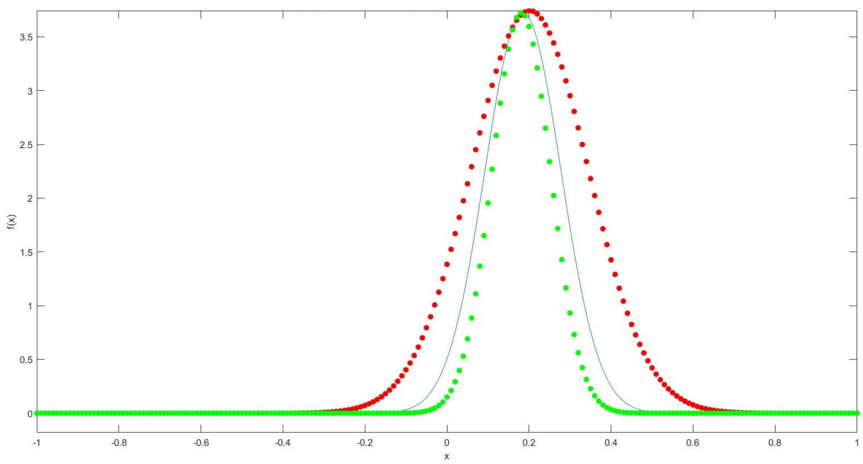

Figure 11: Sevsitivity Analysis on Temperature

8.2 The Influence of Moisture

For the second sensitivity analysis, we considered our humidity model.In the face of the sudden increase of average humidity and humidity difference caused by sudden rainfall, the image results show that our humidity model still has a high fitting interpretation rate, which is stable and feasible.

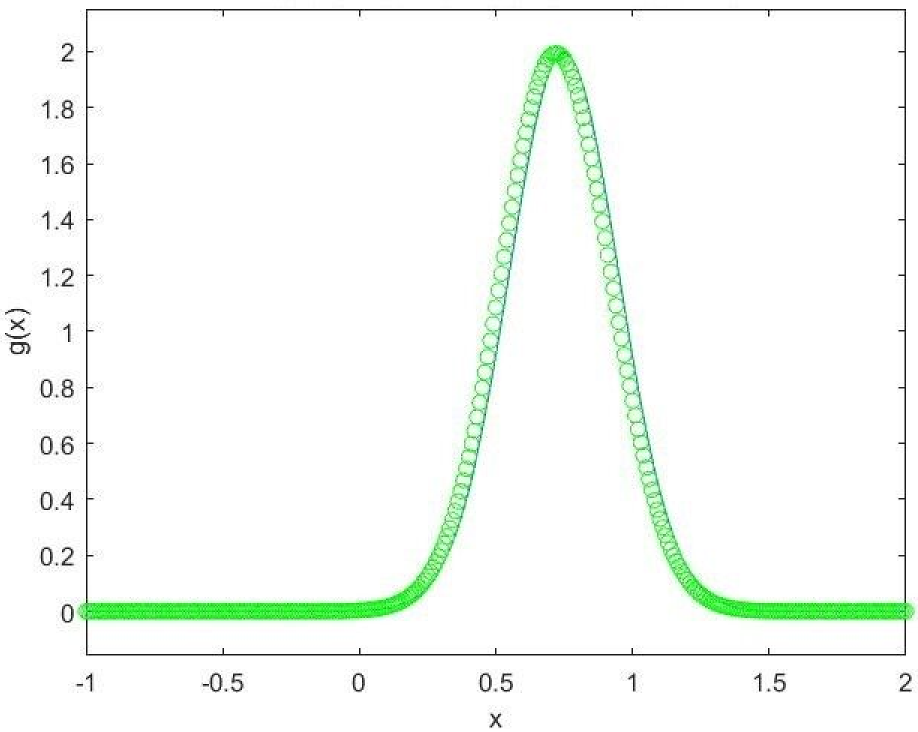

Figure 12: Sevsitivity Analysis on Moisture

9 Strength and Weakness

9.1 Strength

- **Our model works steadily.** The temperature statistical model based on big data is scientific and reasonable, and can pass various statistical tests, including SA validation. The statistical model obtained have a reliable statistical description.

- **Our model has a wide range of application.** Our model did not focus on a datum of a certain area, whereas can be applied to varieties of modes.

9.2 Weakness

- **Our model relies on large amounts of data.** Our model did not focus on a datum of a certain area, whereas can be applied to varieties of modes.

10 Atricle

Fungi's activities play an important role in the decomposition of the woody fiber and ground litter, which is a vital component in the carbon cycle. Here we will talk about some traits of the fungi that determine its wood decomposition rate and what we can do to help accelerate the decomposition of ground litter and wood with the theory we develop based on the previous conclusion. According to the recent research, we can get that some traits of fungi do affect their ability to decompose woody fiber. In this essay, we will focus on the growth rate of fungi and the moisture tolerance of fungi and research how they determine the decomposition of wood. First, we can get an approximate relationship between these factors by researching the experiment data: It' obvious that there is a

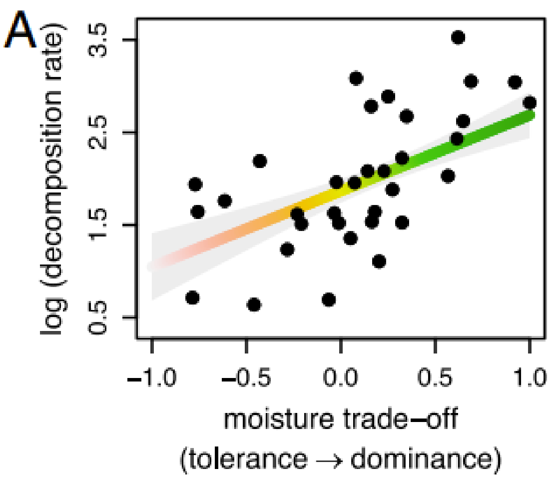

Figure 13: The relationship between the moisture tolerance of various fungi and the resulting wood decomposition rate.

linear relationship between the decomposition rate and the moisture tolerance of fungi. Besides, by observing the data we can find that the data of the fungi's growth rate can also be linear with the decomposition rate after being processed. Therefore, the basic relationship between these traits and the fungi's ability of composition can be described with the expression:

$$\begin{cases} DR = e^{1.312M+0.314E^{1.226}} \\ DR = e^{1.312M-5.609E^{-0.1261}} \\ DR = e^{1.312M+0.019E^{1.784}} \end{cases}$$

The environment obviously affects fungi to a certain extent. The influence is mainly reflected in the fitness of environment and fungus, and the fitness is related to the characteristics of fungus itself and the temperature and humidity of the environment. For the growth characteristics of fungi themselves, we found that they conform to the skewness distribution, while for the temperature and depth distribution of the environment, we use the normal distribution to build the probability model:

$$E(t) = \frac{t^{\alpha-1}(1-t)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}du}C_1$$

$$E(t) = \tfrac{\beta^{\alpha}}{\gamma(\alpha)}t^{\alpha-1}e^{-\beta t}C_2$$

$$\Gamma(\alpha) = \int_0^{+\infty} u^{\alpha-1}e^{-u}du$$

$$f(x) = \frac{2}{\sqrt{2\pi}\cdot Ch}e^{-\frac{2(x-T)^2}{ch^2}}$$

$$g(x) = \frac{2}{\sqrt{2\pi}\cdot Sh}e^{-\frac{2(x-s)^2}{Sh^2}}$$

However, in the natural world, it's hard to find a microbial community that totally consists of a specific specie of fungi. Different species usually grow together and form a symbiotic or competitive relationship. When several various species grow together, they share the same resources such as water and oxygen, and finally grow towards a state of equilibrium where all species have access to necessary resources. In this symbiotic relationship, because the state of all fungal colonies is continuously changing and constantly affected by other species of fungi, it conforms to the Markov chain in the continuous state and forms a pluralistic dynamic system. This dynamic system will have a large oscillation in the short term, but in a longer time scale, the strains will restrict each other, and finally reach a state of dynamic balance. In this dynamical system, we take into account different weather conditions. In arid and semi-arid regions, we find that a single dominant species can persist. In temperate regions, we find that combinations of a dominant species with high growth rates and environmental adaptability and three or four weaker species tend to produce the most efficient combinations of organisms for decomposition. In tropical rainforest areas, we find that combinations of weaker species are more stable than combinations of stronger species.

References

[1] https://www.worldweatheronline.com/

[2] www.github.com/dsmaynard/fungal_biogeography

[3] https://data.world/data-society/global-climate-change-data/workspace/file?filename=GlobalLandTemperatures%2FGlobalTemperatures.csv

[4] https://www.pnas.org/content/pnas/117/21/11551.full.pdf

[5] https://www.pnas.org/content/pnas/suppl/2020/05/13/1909166117.DCSupplemental/pnas.1909166117.sapp.pdf

[6] https://www.doc88.com/p-9029670237688.html

Paper written and generated via LATEX, free distribution. Graph generated and calculation using MATLAB R2020a and Python.

```
beta regreassion
estimate_betas_roughly <- function(Y, model_matrix, offset_matrix, pseudo_count = 1)
    if(nrow(Y) == 0) return(matrix(numeric(0), nrow = 0, ncol = ncol(model_matrix)))
    qrx <- qr(model_matrix)
    Q <- qr.Q(qrx)
    R <- qr.R(qrx)

    norm_log_count_mat <- t(log((Y / exp(offset_matrix) + pseudo_count)))
    t(solve(R, as.matrix(t(Q) %*% norm_log_count_mat)))
  }

  estimate_betas_fisher_scoring <- function(Y, model_matrix, offset_matrix,
                                            dispersions, beta_mat_init){
    stopifnot(nrow(model_matrix) == ncol(Y))
    stopifnot(nrow(beta_mat_init) == nrow(Y))
    stopifnot(ncol(beta_mat_init) == ncol(model_matrix))
    stopifnot(length(dispersions) == nrow(Y))
    stopifnot(dim(offset_matrix) == dim(Y))


    betaRes <- fitBeta_fisher_scoring(Y, model_matrix, exp(offset_matrix), dispersic
                                  ridge_penalty = 1e-6, tolerance = 1e-8, max_re
1000)

    list(Beta = betaRes$beta_mat, iterations = betaRes$iter, deviances = betaRes$dev
  }

  estimate_betas_roughly_group_wise <- function(Y, offset_matrix, groups){
    norm_Y <- Y / exp(offset_matrix)
    do.call(cbind, lapply(unique(groups), function(gr){
      log(DelayedMatrixStats::rowMeans2(norm_Y, cols = groups == gr))
    }))
  }

  estimate_betas_group_wise <- function(Y, offset_matrix,  dispersions, beta_group_i
    stopifnot(nrow(beta_group_init) == nrow(Y))
    stopifnot(ncol(beta_group_init) == length(unique(groups)))
    stopifnot(length(dispersions) == nrow(Y))
    stopifnot(dim(offset_matrix) == dim(Y))
    stopifnot(is.null(beta_mat_init) != is.null(beta_group_init))
    if(is.null(beta_group_init)){
      first_occurence_in_groups <- match(unique(groups), groups)
      beta_group_init <- beta_mat_init %*% t(model_matrix[first_occurence_in_groups,
    }

    Beta_res_list <- lapply(unique(groups), function(gr){
      betaRes <- fitBeta_one_group(Y[, gr == groups, drop = FALSE],
                                   offset_matrix[, gr == groups, drop = FALSE], thet
                                   beta_start_values = beta_group_init[, gr == uniqu
                                   tolerance = 1e-8, maxIter = 100)
    })
    Beta <- do.call(cbind, lapply(Beta_res_list, function(x) x$beta))
    Iteration_mat <- do.call(cbind, lapply(Beta_res_list, function(x) x$iter))
```

```
  Deviance_mat <- do.call(cbind, lapply(Beta_res_list, function(x) x$deviance))
  Beta <- pmax(Beta, -1e8)
  first_occurence_in_groups <- match(unique(groups), groups)
  if(nrow(Beta) > 0){
    Beta <- t(solve(model_matrix[first_occurence_in_groups, ,drop=FALSE], t(Beta))
  }

  list(Beta = Beta,
       iterations = matrixStats::rowSums2(Iteration_mat),
       deviances = matrixStats::rowSums2(Deviance_mat))
}

estimate_betas_group_wise_optimize_helper <- function(y, offset, theta, lower_bour
  optimize(function(beta){
    sum(dnbinom(y, mu = exp(beta + offset), size = 1/theta, log = TRUE))
  }, lower = lower_bound, upper = upper_bound, maximum = TRUE)$maximum
}
```