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## 1 Basic

### 1.1 vimrc

```
"This file should be placed at ~/.vimrc"
se nu ai hls et ru ic is sc cul
se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a
syntax on
hi cursorline cterm=none ctermbg=89
set bg=dark
inoremap {<CR> {<CR><Esc>ko<tab>
"Select
region and then type :Hash to hash your selection."
```

```
"Useful for verifying that there aren't mistypes."
ca Hash w !cpp -dD -P -fpreprocessed
\| tr -d '[:space:]' \| md5sum \| cut -c-6
ca Hash w !cpp -dD -P \| sed "/^#.*"
$/d" \| tr -d '[:space:]' \| md5sum \| cut -c-6
ca Hash w !g++ -E - \| sed "/^#.*"
$/d" \| tr -d '[:space:]' \| md5sum \| cut -c-6
```

### 1.2 readchar [a419b9]

```
inline char readchar() {
    static const size_t bufsize = 65536;
    static char buf[bufsize];
    static char *p = buf, *end = buf;
    if (p == end) end = buf +
        fread_unlocked(buf, 1, bufsize, stdin), p = buf;
    return *p++;
}
```

### 1.3 BigIntIO [d9afcb]

```
__int128 read() {
    __int128 x = 0, f = 1;
    char ch = getchar();
    while (ch < '0' || ch > '9') {
        if (ch == '-') f = -1;
        ch = getchar();
    }
    while (ch >= '0' && ch <= '9') {
        x = x * 10 + ch - '0';
        ch = getchar();
    }
    return x * f;
}
void print(__int128 x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) print(x / 10);
    putchar(x % 10 + '0');
}
bool cmp(__int128 x, __int128 y) { return x > y; }
```

### 1.4 Black Magic [0d8b5f]

```
#include <bits/stdc++.h>
#include <bits/extc++.h>
#include <ext/rope>
using namespace __gnu_pbds;
using namespace __gnu_cxx;
#include <ext/pb_ds/assoc_container.hpp>
typedef
    tree<int, null_type, std::less<int>, rb_tree_tag,
        tree_order_statistics_node_update> tree_set;
typedef cc_hash_table<int, int> umap;
typedef priority_queue<int> heap;

int main() {
    // random
    mt19937 rng(chrono::
        steady_clock::now().time_since_epoch().count());
    int get_rand(int l, int r) { return
        uniform_int_distribution<int>(l, r)(rng); }
    shuffle(v.begin(), v.end(), rng);
    // rb tree
    tree_set s;
    s.insert(71); s.insert(22);
    assert(*s.find_by_order
        (0) == 22); assert(*s.find_by_order(1) == 71);
    assert(s.order_of_key
        (22) == 0); assert(s.order_of_key(71) == 1);
    s.erase(22);
    assert(*s.find_by_order
        (0) == 71); assert(s.order_of_key(71) == 0);
    // mergable heap
    heap a, b; a.join(b);
    // persistant
    rope<char> r[2];
    r[1] = r[0];
    std::string st = "abc";
    r[1].insert(0, st.c_str());
    r[1].erase(1, 1);
    std::cout << r[1].substr(0, 2) << std::endl;
    return 0;
}
```

## 1.5 Pragma Optimization [eac636]

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8040)
```

## 1.6 Bitset [cb5d05]

```
#include<bits/stdc++.h>
using namespace std;

int main () {
    bitset<4> bit;
    bit.all(); // all bit is true, ret tru;
    bit.any(); // any bit is true, ret true
    bit.none(); // all bit is false, ret true
    bit.count();
    bit.to_string('0','1');//with parameter
    bit.reset(); // set all to true
    bit.set(); // set all to false
    std::bitset<8> b3{0}, b4{42};
    std::hash<std::bitset<8>> hash_fn8;
    hash_fn8(b3); hash_fn8(b4);
}
```

# 2 Graph

## 2.1 BCC Vertex\* [740acb]

```
struct BCC { // 0-base
    int n, dft, nbcc;
    vector<int> low, dfn, bln, stk, is_ap, cir;
    vector<vector<int>> G, bcc, nG;
    void make_bcc(int u) {
        bcc.emplace_back(1, u);
        for (; stk.back() != u; stk.pop_back())
            bln[stk.back()] = nbcc, bcc[nbcc].pb(stk.back());
        stk.pop_back(), bln[u] = nbcc++;
    }
    void dfs(int u, int f) {
        int child = 0;
        low[u] = dfn[u] = ++dft, stk.pb(u);
        for (int v : G[u])
            if (!dfn[v]) {
                dfs(v, u), ++child;
                low[u] = min(low[u], low[v]);
                if (dfn[u] <= low[v]) {
                    is_ap[u] = 1, bln[u] = nbcc;
                    make_bcc(v), bcc.back().pb(u);
                }
            } else if (dfn[v] < dfn[u] && v != f)
                low[u] = min(low[u], dfn[v]);
        if (f == -1 && child < 2) is_ap[u] = 0;
        if (f == -1 && child == 0) make_bcc(u);
    }
    BCC(int _n): n(_n), dft(),
        nbcc(), low(n), dfn(n), bln(n), is_ap(n), G(n) {}
    void add_edge(int u, int v) {
        G[u].pb(v), G[v].pb(u);
    }
    void solve() {
        for (int i = 0; i < n; ++i)
            if (!dfn[i]) dfs(i, -1);
    }
    void block_cut_tree() {
        cir.resize(nbcc);
        for (int i = 0; i < n; ++i)
            if (is_ap[i])
                bln[i] = nbcc++;
        cir.resize(nbcc, 1), nG.resize(nbcc);
        for (int i = 0; i < nbcc && !cir[i]; ++i)
            for (int j : bcc[i])
                if (is_ap[j])
                    nG[i].pb(bln[j]), nG[bln[j]].pb(i);
    } // up to 2 * n - 2 nodes!! bln[i] for id
};
```

## 2.2 Bridge\* [4da29a]

```
struct ECC { // 0-base
    int n, dft, ecnt, necc;
    vector<int> low, dfn, bln, is_bridge, stk;
    vector<vector<pii>> G;
    void dfs(int u, int f) {
        dfn[u] = low[u] = ++dft, stk.pb(u);
        for (auto [v, e] : G[u])
```

```
            if (!dfn[v])
                dfs(v, e), low[u] = min(low[u], low[v]);
            else if (e != f)
                low[u] = min(low[u], dfn[v]);
        if (low[u] == dfn[u]) {
            if (f != -1) is_bridge[f] = 1;
            for (; stk.back() != u; stk.pop_back())
                bln[stk.back()] = necc;
            bln[u] = necc++, stk.pop_back();
        }
    }
    ECC(int _n): n(_n), dft(),
        ecnt(), necc(), low(n), dfn(n), bln(n), G(n) {}
    void add_edge(int u, int v) {
        G[u].pb(pii(v, ecnt)), G[v].pb(pii(u, ecnt++));
    }
    void solve() {
        is_bridge.resize(ecnt);
        for (int i = 0; i < n; ++i)
            if (!dfn[i]) dfs(i, -1);
    }
}; // ecc_id(i): bln[i]
```

## 2.3 SCC\* [4057dc]

```
struct SCC { // 0-base
    int n, dft, nscc;
    vector<int> low, dfn, bln, instack, stk;
    vector<vector<int>> G;
    void dfs(int u) {
        low[u] = dfn[u] = ++dft;
        instack[u] = 1, stk.pb(u);
        for (int v : G[u])
            if (!dfn[v])
                dfs(v), low[u] = min(low[u], low[v]);
            else if (instack[v] && dfn[v] < dfn[u])
                low[u] = min(low[u], dfn[v]);
        if (low[u] == dfn[u]) {
            for (; stk.back() != u; stk.pop_back())
                bln[stk.back()] = nscc, instack[stk.back()] = 0;
            instack[u] = 0, bln[u] = nscc++, stk.pop_back();
        }
    }
    SCC(int _n): n(_n), dft(), nscc(),
        low(n), dfn(n), bln(n), instack(n), G(n) {}
    void add_edge(int u, int v) {
        G[u].pb(v);
    }
    void solve() {
        for (int i = 0; i < n; ++i)
            if (!dfn[i]) dfs(i);
    }
}; // scc_id(i): bln[i]
```

## 2.4 2SAT\* [f5630a]

```
struct SAT { // 0-base
    int n;
    vector<bool> istrue;
    SCC scc;
    SAT(int _n): n(_n), istrue(n + n), scc(n + n) {}
    int rv(int a) {
        return a >= n ? a - n : a + n;
    }
    void add_clause(int a, int b) {
        scc.add_edge(rv(a), b), scc.add_edge(rv(b), a);
    }
    bool solve() {
        scc.solve();
        for (int i = 0; i < n; ++i) {
            if (scc.bln[i] == scc.bln[i + n]) return false;
            istrue[i] = scc.bln[i] < scc.bln[i + n];
            istrue[i + n] = !istrue[i];
        }
        return true;
    }
};
```

## 2.5 MinimumMeanCycle\* [3e5d2b]

```
ll read[N][N]; // input here
struct MinimumMeanCycle {
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for (int i = 2; i <= L; ++i)
```

```

    for (int k = 0; k < n; ++k)
        for (int j = 0; j < n; ++j)
            dp[i][j] =
                min(dp[i - 1][k] + road[k][j], dp[i][j]);
    for (int i = 0; i < n; ++i) {
        if (dp[L][i] >= INF) continue;
        ll ta = 0, tb = 1;
        for (int j = 1; j < n; ++j)
            if (dp[j][i] < INF &&
                ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                ta = dp[L][i] - dp[j][i], tb = L - j;
        if (ta == 0) continue;
        if (a == -1 || a * tb > ta * b) a = ta, b = tb;
    }
    if (a != -1) {
        ll g = __gcd(a, b);
        return pll(a / g, b / g);
    }
    return pll(-1LL, -1LL);
}
void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
}
};

```

## 2.6 Virtual Tree\* [1b641b]

```

vector<int> vG[N];
int top, st[N];

void insert(int u) {
    if (top == -1) return st[++top] = u, void();
    int p = LCA(st[top], u);
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if (st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
    // do something
    reset(v[0]);
}

```

## 2.7 Maximum Clique Dyn\* [d50aa9]

```

struct MaxClique { // fast when N <= 100
    bitset<N> G[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void add_edge(int u, int v) {
        G[u][v] = G[v][u] = 1;
    }
    void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
        if (l < 4) {
            for (int i : r) d[i] = (G[i] & mask).count();
            sort(ALL(r),
                [&](int x, int y) { return d[x] > d[y]; });
        }
        vector<int> c(SZ(r));
        int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
        cs[1].reset(), cs[2].reset();
        for (int p : r) {
            int k = 1;
            while ((cs[k] & G[p]).any()) ++k;
            if (k > rgt) cs[++rgt + 1].reset();
            cs[k][p] = 1;
            if (k < lft) r[tp++] = p;
        }
        for (int k = lft; k <= rgt; ++k)

```

```

        for (int p = cs[k]._Find_first
            (); p < N; p = cs[k]._Find_next(p))
            r[tp] = p, c[tp] = k, ++tp;
        dfs(r, c, l + 1, mask);
    }
    void dfs(vector<
        int> &r, vector<int> &c, int l, bitset<N> mask) {
        while (!r.empty()) {
            int p = r.back();
            r.pop_back(), mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr;
            for (int i : r) if (G[p][i]) nr.pb(i);
            if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
            else if (q > ans) ans = q, copy_n(cur, q, sol);
            c.pop_back(), --q;
        }
    }
    int solve() {
        vector<int> r(n);
        ans = q = 0, iota(ALL(r), 0);
        pre_dfs(r, 0, bitset<N>(string(n, '1')));
        return ans;
    }
};

```

## 2.8 Minimum Steiner Tree\* [62d6fb]

```

struct SteinerTree { // 0-base
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcst[N]; // the cost of vertexs
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) {
            fill_n(dst[i], n, INF);
            dst[i][i] = vcst[i] = 0;
        }
    }
    void chmin(int &x, int val) {
        x = min(x, val);
    }
    void add_edge(int ui, int vi, int wi) {
        chmin(dst[ui][vi], wi);
    }
    void shortest_path() {
        for (int k = 0; k < n; ++k)
            for (int i = 0; i < n; ++i)
                for (int j = 0; j < n; ++j)
                    chmin(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int> &ter) {
        shortest_path();
        int t = SZ(ter), full = (1 << t) - 1;
        for (int i = 0; i <= full; ++i)
            fill_n(dp[i], n, INF);
        copy_n(vkst, n, dp[0]);
        for (int msk = 1; msk <= full; ++msk) {
            if (!(msk & (msk - 1))) {
                int who = __lg(msk);
                for (int i = 0; i < n; ++i)
                    dp[msk][i] = vcst[ter[who]] + dst[ter[who]][i];
            }
            for (int i = 0; i < n; ++i)
                for (int sub = (msk - 1) & msk; sub; sub = (sub - 1) & msk)
                    chmin(dp[msk][i],
                        dp[sub][i] + dp[msk ^ sub][i] - vcst[i]);
            for (int i = 0; i < n; ++i) {
                tdst[i] = INF;
                for (int j = 0; j < n; ++j)
                    chmin(tdst[i], dp[msk][j] + dst[j][i]);
            }
            copy_n(tdst, n, dp[msk]);
        }
        return *min_element(dp[full], dp[full] + n);
    }
}; // O(V 3^T + V^2 2^T)

```

## 2.9 Dominator Tree\* [2b8b32]

```

struct dominator_tree { // 1-base
    vector<int> G[N], rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N]; // dominator_tree

```

```

void init(int _n) {
    n = _n;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), rG[i].clear();
}
void add_edge(int u, int v) {
    G[u].pb(v), rG[v].pb(u);
}
void dfs(int u) {
    id[dfn[u] = ++Time] = u;
    for (auto v : G[u])
        if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
}
int find(int y, int x) {
    if (y <= x) return y;
    int tmp = find(pa[y], x);
    if (semi[best[y]] > semi[best[pa[y]]])
        best[y] = best[pa[y]];
    return pa[y] = tmp;
}
void tarjan(int root) {
    Time = 0;
    for (int i = 1; i <= n; ++i) {
        dfn[i] = idom[i] = 0;
        tree[i].clear();
        best[i] = semi[i] = i;
    }
    dfs(root);
    for (int i = Time; i > 1; --i) {
        int u = id[i];
        for (auto v : rG[u])
            if (v == dfn[v]) {
                find(v, i);
                semi[i] = min(semi[i], semi[best[v]]);
            }
        tree[semi[i]].pb(i);
        for (auto v : tree[pa[i]]) {
            find(v, pa[i]);
            idom[v] =
                semi[best[v]] == pa[i] ? pa[i] : best[v];
        }
        tree[pa[i]].clear();
    }
    for (int i = 2; i <= Time; ++i) {
        if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
        tree[id[idom[i]]].pb(id[i]);
    }
}
};

```

## 2.10 Four Cycle [584a52]

```

int main() {
    cin.tie(nullptr) -> sync_with_stdio(false);
    cin >> n >> m;
    for (int i = 1; i <= m; ++i) {
        int u, v;
        cin >> u >> v;
        E[u].push_back(v);
        E[v].push_back(u);
        deg[u]++, deg[v]++;
    }
    for (int u = 1; u <= n; u++)
        for (int v : E[u])
            if (deg[u] > deg[v] || (deg[u] == deg[v] && u > v)) E1[u].push_back(v);
    for (int a = 1; a <= n; a++) {
        for (int b : E1[a])
            for (int c : E[b]) {
                if (deg[a] < deg[c]
                    || (deg[a] == deg[c] && a <= c)) continue;
                total += cnt[c]++;
            }
        for (int b : E1[a])
            for (int c : E[b]) cnt[c] = 0;
    }
    cout << total << '\n';
    return 0;
}

```

## 2.11 Minimum Clique Cover\* [879472]

```

struct Clique_Cover { // 0-base, O(n2^n)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
    }
};

```

```

    fill_n(E, n, 0), fill_n(co, 1 << n, 0);
}
void add_edge(int u, int v) {
    E[u] |= 1 << v, E[v] |= 1 << u;
}
int solve() {
    for (int i = 0; i < n; ++i)
        co[1 << i] = E[i] | (1 << i);
    co[0] = (1 << n) - 1;
    dp[0] = (n & 1) * 2 - 1;
    for (int i = 1; i < (1 << n); ++i) {
        int t = i & -i;
        dp[i] = -dp[i ^ t];
        co[i] = co[i ^ t] & co[t];
    }
    for (int i = 0; i < (1 << n); ++i)
        co[i] = (co[i] & i) == i;
    fwt(co, 1 << n, 1);
    for (int ans = 1; ans < n; ++ans) {
        int sum = 0; // probabilistic
        for (int i = 0; i < (1 << n); ++i)
            sum += (dp[i] * co[i]);
        if (sum) return ans;
    }
    return n;
}
};

```

## 2.12 NumberofMaximalClique\* [11fa26]

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsu = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsu++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsu, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
};

```

## 3 Data Structure

### 3.1 Discrete Trick

```

vector<int> val;
// build
sort(ALL
    (val)), val.resize(unique(ALL(val)) - val.begin());
// index of x
upper_bound(ALL(val), x) - val.begin();
// max idx <= x
upper_bound(ALL(val), x) - val.begin();
// max idx < x
lower_bound(ALL(val), x) - val.begin();

```

### 3.2 BIT kth\* [e39485]

```

int bit[N + 1]; // N = 2 ^ k
int query_kth(int k) {

```

```

int res = 0;
for (int i = N >> 1; i >= 1; i >>= 1)
    if (bit[res + i] < k)
        k -= bit[res += i];
return res + 1;
}

```

### 3.3 Interval Container\* [c54d29]

```

/* Add and
   remove intervals from a set of disjoint intervals.
   * Will merge the added interval with
     any overlapping intervals in the set when adding.
   * Intervals are [inclusive, exclusive). */
set<pii>::
    iterator addInterval(set<pii>& is, int L, int R) {
        if (L == R) return is.end();
        auto it = is.lower_bound({L, R}), before = it;
        while (it != is.end() && it->X <= R) {
            R = max(R, it->Y);
            before = it = is.erase(it);
        }
        if (it != is.begin() && (--it)->Y >= L) {
            L = min(L, it->X);
            R = max(R, it->Y);
            is.erase(it);
        }
        return is.insert(before, pii(L, R));
    }
void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->Y;
    if (it->X == L) is.erase(it);
    else (int&)it->Y = L;
    if (R != r2) is.emplace(R, r2);
}

```

### 3.4 Leftist Tree [e91538]

```

struct node {
    ll v, data, sz, sum;
    node *l, *r;
    node(ll k)
        : v(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll V(node *p) { return p ? p->v : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (V(a->r) > V(a->l)) swap(a->r, a->l);
    a->v = V(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}
void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}

```

### 3.5 Heavy light Decomposition\* [b004ae]

```

struct Heavy_light_Decomposition { // 1-base
    int n, ulink[N], deep[N], mxson[N], w[N], pa[N];
    int t, pl[N], data[N], val[N]; // val: vertex data
    vector<int> G[N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), mxson[i] = 0;
    }
    void add_edge(int a, int b) {
        G[a].pb(b), G[b].pb(a);
    }
    void dfs(int u, int f, int d) {
        w[u] = 1, pa[u] = f, deep[u] = d++;
        for (int &i : G[u])
            if (i != f) {
                dfs(i, u, d), w[u] += w[i];
                if (w[mxson[u]] < w[i]) mxson[u] = i;
            }
    }
    void cut(int u, int link) {

```

```

        data[pl[u] = ++t] = val[u], ulink[u] = link;
        if (!mxson[u]) return;
        cut(mxson[u], link);
        for (int i : G[u])
            if (i != pa[u] && i != mxson[u])
                cut(i, i);
    }
    void build() { dfs(1, 1, 1), cut(1, 1), /*build*/; }
    int query(int a, int b) {
        int ta = ulink[a], tb = ulink[b], res = 0;
        while (ta != tb) {
            if (deep[ta] > deep[tb]) swap(ta, tb), swap(a, b);
            // query(pl[tb], pl[b])
            tb = ulink[b = pa[tb]];
        }
        if (pl[a] > pl[b]) swap(a, b);
        // query(pl[a], pl[b])
    }
};

```

### 3.6 Centroid Decomposition\* [5a24da]

```

struct Cent_Dec { // 1-base
    vector<pll> G[N];
    pll info[N]; // store info. of itself
    pll upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    ll dis[lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
        for (int i = 1; i <= n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pll(b, w)), G[b].pb(pll(a, w));
    }
    void get_cent(
        int u, int f, int &mx, int &c, int num) {
        int mxsz = 0;
        sz[u] = 1;
        for (pll e : G[u])
            if (!done[e.X] && e.X != f) {
                get_cent(e.X, u, mx, c, num);
                sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
            }
        if (mx > max(mxsz, num - sz[u]))
            mx = max(mxsz, num - sz[u]), c = u;
    }
    void dfs(int u, int f, ll d, int org) {
        // if required, add self info or climbing info
        dis[layer[org]][u] = d;
        for (pll e : G[u])
            if (!done[e.X] && e.X != f)
                dfs(e.X, u, d + e.Y, org);
    }
    int cut(int u, int f, int num) {
        int mx = 1e9, c = 0, lc;
        get_cent(u, f, mx, c, num);
        done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
        for (pll e : G[c])
            if (!done[e.X]) {
                if (sz[e.X] > sz[c])
                    lc = cut(e.X, c, num - sz[c]);
                else lc = cut(e.X, c, sz[e.X]);
                upinfo[lc] = pll(), dfs(e.X, c, e.Y, c);
            }
        return done[c] = 0, c;
    }
    void build() { cut(1, 0, n); }
    void modify(int u) {
        for (int a = u, ly = layer[a]; a;
            a = pa[a], --ly) {
            info[a].X += dis[ly][u], ++info[a].Y;
            if (pa[a])
                upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
        }
    }
    ll query(int u) {
        ll rt = 0;
        for (int a = u, ly = layer[a]; a;
            a = pa[a], --ly) {
            rt += info[a].X + info[a].Y * dis[ly][u];
            if (pa[a])
                rt -=
                    upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
        }
    }
};

```



```

    return rt;
}
};

```

### 3.7 LiChaoST\* [4a4bee]

```

struct L {
    ll m, k, id;
    L() : id(-1) {}
    L(ll a, ll b, ll c) : m(a), k(b), id(c) {}
    ll at(ll x) { return m * x + k; }
};
class LiChao { // maintain max
private:
    int n; vector<L> nodes;
    void insert(int l, int r, int rt, L ln) {
        int m = (l + r) >> 1;
        if (nodes[rt].id == -1)
            return nodes[rt] = ln, void();
        bool atLeft = nodes[rt].at(l) < ln.at(l);
        if (nodes[rt].at(m) < ln.at(m))
            atLeft ^= 1, swap(nodes[rt], ln);
        if (r - l == 1) return;
        if (atLeft) insert(l, m, rt << 1, ln);
        else insert(m, r, rt << 1 | 1, ln);
    }
    ll query(int l, int r, int rt, ll x) {
        int m = (l + r) >> 1; ll ret = -INF;
        if (nodes[rt].id != -1) ret = nodes[rt].at(x);
        if (r - l == 1) return ret;
        if (x < m) return max(ret, query(l, m, rt << 1, x));
        return max(ret, query(m, r, rt << 1 | 1, x));
    }
public:
    LiChao(int n_) : n(n_), nodes(n * 4) {}
    void insert(L ln) { insert(0, n, 1, ln); }
    ll query(ll x) { return query(0, n, 1, x); }
};

```

### 3.8 Link cut tree\* [a35b5d]

```

struct Splay { // xor-sum
    static Splay nil;
    Splay *ch[2], *f;
    int val, sum, rev, size;
    Splay(int _val = 0) : val(_val), sum(_val), rev(0), size(1) {
        f = ch[0] = ch[1] = &nil; }
    bool isr() { return f->ch[0] != this && f->ch[1] != this; }
    int dir() { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void give_tag(int r) {
        if (r) swap(ch[0], ch[1]), rev ^= 1;
    }
    void push() {
        if (ch[0] != &nil) ch[0]->give_tag(rev);
        if (ch[1] != &nil) ch[1]->give_tag(rev);
        rev = 0;
    }
    void pull() {
        // take care of the nil!
        size = ch[0]->size + ch[1]->size + 1;
        sum = ch[0]->sum ^ ch[1]->sum ^ val;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(), x->pull();
}
void splay(Splay *x) {
    vector<Splay*> splayVec;
    for (Splay *q = x;; q = q->f) {

```

```

        splayVec.pb(q);
        if (q->isr()) break;
    }
    reverse(ALL(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir() == x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
Splay* access(Splay *x) {
    Splay *q = nil;
    for (; x != nil; x = x->f)
        splay(x), x->setCh(q, 1), q = x;
    return q;
}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x) {
    root_path(x), x->give_tag(1);
    x->push(), x->pull();
}
void split(Splay *x, Splay *y) {
    chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
    root_path(x), chroot(y);
    x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
    split(x, y);
    if (y->size != 5) return;
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
Splay* get_root(Splay *x) {
    for (root_path(x); x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    return get_root(x) == get_root(y);
}
Splay* lca(Splay *x, Splay *y) {
    access(x), root_path(y);
    if (y->f == nil) return y;
    return y->f;
}
void change(Splay *x, int val) {
    splay(x), x->val = val, x->pull();
}
int query(Splay *x, Splay *y) {
    split(x, y);
    return y->sum;
}
}

```

### 3.9 KDTree [375ca2]

```

namespace kdt {
    int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn],
        yl[maxn], yr[maxn];
    point p[maxn];
    int build(int l, int r, int dep = 0) {
        if (l == r) return -1;
        function<bool(const point &, const point &)> f =
            [dep](const point &a, const point &b) {
                if (dep & 1) return a.x < b.x;
                else return a.y < b.y;
            };
        int m = (l + r) >> 1;
        nth_element(p + l, p + m, p + r, f);
        xl[m] = xr[m] = p[m].x;
        yl[m] = yr[m] = p[m].y;
        lc[m] = build(l, m, dep + 1);
        if (~lc[m]) {
            xl[m] = min(xl[m], xl[lc[m]]);
            xr[m] = max(xr[m], xr[lc[m]]);
            yl[m] = min(yl[m], yl[lc[m]]);
            yr[m] = max(yr[m], yr[lc[m]]);
        }
        rc[m] = build(m + 1, r, dep + 1);
        if (~rc[m]) {
            xl[m] = min(xl[m], xl[rc[m]]);
            xr[m] = max(xr[m], xr[rc[m]]);
            yl[m] = min(yl[m], yl[rc[m]]);
            yr[m] = max(yr[m], yr[rc[m]]);
        }
    }
}

```

```

    yr[m] = max(yr[m], yr[rc[m]]);
}
return m;
}
bool bound(const point &q, int o, long long d) {
    double ds = sqrt(d + 1.0);
    if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
        q.y < yl[o] - ds || q.y > yr[o] + ds)
        return false;
    return true;
}
long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 1ll * (a.x - b.x) +
        (a.y - b.y) * 1ll * (a.y - b.y);
}
void dfs(
    const point &q, long long &d, int o, int dep = 0) {
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1) && q.x < p[o].x ||
        !(dep & 1) && q.y < p[o].y) {
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
}
void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
}
long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
}
// namespace kdt

```

### 3.10 Treap [5ab1a1]

```

struct node {
    int data, sz;
    node *l, *r;
    node(int k) : data(k), sz(1), l(0), r(0) {}
    void up() {
        sz = 1;
        if (l) sz += l->sz;
        if (r) sz += r->sz;
    }
    void down() {}
};
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(),
            a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}

```

```

bool erase(node *&o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
    node *a, *b;
    split(o, a, b, k);
    o = merge(a, merge(new node(k), b));
}
void interval(node *&o, int l, int r) {
    node *a, *b, *c;
    split2(o, a, b, l - 1), split2(b, b, c, r);
    // operate
    o = merge(a, merge(b, c));
}

```

## 4 Flow/Matching

### 4.1 Dinic [98fb3a]

```

struct MaxFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[MAXN];
    int s, t, dis[MAXN], cur[MAXN], n;
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < (int)G[u].size(); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df;
                    G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int tmp = q.front();
            q.pop();
            for (auto &u : G[tmp])
                if (!dis[u.to] && u.flow != u.cap) {
                    q.push(u.to);
                    dis[u.to] = dis[tmp] + 1;
                }
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(cur, n, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void reset() {
        for (int i = 0; i < n; ++i)
            for (auto &j : G[i]) j.flow = 0;
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, (int)G[v].size()});
        G[v].pb(edge{u, 0, 0, (int)G[u].size() - 1});
    }
};

```

## 4.2 Bipartite Matching\* [784535]

```

struct Bipartite_Matching { // 0-base
    int mp[N], mq[N], dis[N + 1], cur[N], l, r;
    vector<int> G[N + 1];
    bool dfs(int u) {
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            int e = G[u][i];
            if (mq[e] == 1
                || (dis[mq[e]] == dis[u] + 1 && dfs(mq[e])))
                return mp[mq[e] = u] = e, 1;
        }
        return dis[u] = -1, 0;
    }
    bool bfs() {
        queue<int> q;
        fill_n(dis, l + 1, -1);
        for (int i = 0; i < l; ++i)
            if (!mp[i])
                q.push(i), dis[i] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int e : G[u])
                if (!dis[mq[e]])
                    q.push(mq[e]), dis[mq[e]] = dis[u] + 1;
        }
        return dis[l] != -1;
    }
    int matching() {
        int res = 0;
        fill_n(mp, l, -1), fill_n(mq, r, 1);
        while (bfs()) {
            fill_n(cur, l, 0);
            for (int i = 0; i < l; ++i)
                res += (!mp[i] && dfs(i));
        }
        return res; // (i, mp[i] != -1)
    }
    void add_edge(int s, int t) { G[s].pb(t); }
    void init(int _l, int _r) {
        l = _l, r = _r;
        for (int i = 0; i <= l; ++i)
            G[i].clear();
    }
};

```

## 4.3 Kuhn Munkres\* [4b3863]

```

struct KM { // 0-base, maximum matching
    ll w[N][N], hl[N], hr[N], slk[N];
    int fl[N], fr[N], pre[N], qu[N], ql, qr, n;
    bool vl[N], vr[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            fill_n(w[i], n, -INF);
    }
    void add_edge(int a, int b, ll wei) {
        w[a][b] = wei;
    }
    bool Check(int x) {
        if (vl[x] == 1, ~fl[x])
            return vr[qu[qr++] = fl[x]] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {
        fill_n(slk, n, INF), fill_n(vl, n, 0), fill_n(vr, n, 0);
        ql = qr = 0, qu[qr++] = s, vr[s] = 1;
        for (ll d;;) {
            while (ql < qr)
                for (int x = 0, y = qu[ql++]; x < n; ++x)
                    if (!vl[x] && slk
                        [x] >= (d = hl[x] + hr[y] - w[x][y])) {
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!Check(x)) return;
                    }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) hl[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
        }
    }
};

```

```

        for (int x = 0; x < n; ++x)
            if (!vl[x] && !slk[x] && !Check(x)) return;
    }
    ll solve() {
        fill_n(fl, n, -1), fill_n(fr, n, -1), fill_n(hr, n, 0);
        for (int i = 0; i < n; ++i)
            hl[i] = *max_element(w[i], w[i] + n);
        for (int i = 0; i < n; ++i) bfs(i);
        ll res = 0;
        for (int i = 0; i < n; ++i) res += w[i][fl[i]];
        return res;
    }
};

```

## 4.4 MincostMaxflow\* [1c78db]

```

struct MinCostMaxFlow { // 0-base
    struct Edge {
        ll from, to, cap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    int inq[N], n, s, t;
    ll dis[N], up[N], pot[N];
    bool BellmanFord() {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, ll d, ll cap, Edge *e) {
            if (cap > 0 && dis[u] > d) {
                dis[u] = d, up[u] = cap, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, INF, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u]) {
                ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
                relax
                    (e.to, d2, min(up[u], e.cap - e.flow), &e);
            }
        }
        return dis[t] != INF;
    }
    void solve(int _s
        , int _t, ll &flow, ll &cost, bool neg = true) {
        s = _s, t = _t, flow = 0, cost = 0;
        if (neg) BellmanFord(), copy_n(dis, n, pot);
        for (; BellmanFord(); copy_n(dis, n, pot)) {
            for (int
                i = 0; i < n; ++i) dis[i] += pot[i] - pot[s];
            flow += up[t], cost += up[t] * dis[t];
            for (int i = t; past[i]; i = past[i]->from) {
                auto &e = *past[i];
                e.flow += up[t], G[e.to][e.rev].flow -= up[t];
            }
        }
    }
    void init(int _n) {
        n = _n, fill_n(pot, n, 0);
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(ll a, ll b, ll cap, ll cost) {
        G[a].pb(Edge{a, b, cap, 0, cost, SZ(G[b])});
        G[b].pb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
    }
};

```

## 4.5 Maximum Simple Graph Matching\* [0fe1c3]

```

struct Matching { // 0-base
    queue<int> q; int n;
    vector<int> fa, s, vis, pre, match;
    vector<vector<int>>> G;
    int Find(int u)
    { return u == fa[u] ? u : fa[u] = Find(fa[u]); }
    int LCA(int x, int y) {
        static int tk = 0; tk++; x = Find(x); y = Find(y);
        for (; swap(x, y) if (x != n) {
            if (vis[x] == tk) return x;
            vis[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
    void Blossom(int x, int y, int l) {

```



```

    for (; Find(x) != 1; x = pre[y]) {
        pre[x] = y, y = match[x];
        if (s[y] == 1) q.push(y), s[y] = 0;
        for (int z: {x, y}) if (fa[z] == z) fa[z] = 1;
    }
}
bool Bfs(int r) {
    iota(ALL(fa), 0); fill(ALL(s), -1);
    q = queue<int>(); q.push(r); s[r] = 0;
    for (; !q.empty(); q.pop()) {
        for (int x = q.front(); int u : G[x])
            if (s[u] == -1) {
                if (pre[u] = x, s[u] = 1, match[u] == n) {
                    for (int a = u, b = x, last;
                        b != n; a = last, b = pre[a])
                        last = match[b], match[b] = a, match[a] = b;
                    return true;
                }
                q.push(match[u]); s[match[u]] = 0;
            }
        else if (!s[u] && Find(u) != Find(x)) {
            int l = LCA(u, x);
            Blossom(x, u, l); Blossom(u, x, l);
        }
    }
    return false;
}
Matching(int _n) : n(_n), fa(n + 1), s(n + 1), vis
    (n + 1), pre(n + 1, n), match(n + 1, n), G(n) {}
void add_edge(int u, int v)
{ G[u].pb(v), G[v].pb(u); }
int solve() {
    int ans = 0;
    for (int x = 0; x < n; ++x)
        if (match[x] == n) ans += Bfs(x);
    return ans;
} // match[x] == n means not matched
};

```

#### 4.6 Maximum Weight Matching\* [lec446]

```

#define REP(i, l, r) for (int i=(l); i<=(r); ++i)
struct WeightGraph { // 1-based
    struct edge { int u, v, w; }; int n, nx;
    vector<int> lab; vector<vector<edge>> g;
    vector<int> slk, match, st, pa, S, vis;
    vector<vector<int>> flo, flo_from; queue<int> q;
    WeightGraph(int n_) : n(n_), nx(n * 2), lab(nx + 1),
        g(nx + 1, vector<edge>(nx + 1)), slk(nx + 1),
        flo(nx + 1, flo_from(nx + 1, vector(n + 1, 0))) {
        match = st = pa = S = vis = slk;
        REP(u, 1, n) REP(v, 1, n) g[u][v] = {u, v, 0};
    }
    int E(edge e)
    { return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2; }
    void update_slk(int u, int x, int &s)
    { if (!s || E(g[u][x]) < E(g[s][x])) s = u; }
    void set_slk(int x) {
        slk[x] = 0;
        REP(u, 1, n)
            if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
                update_slk(u, x, slk[x]);
    }
    void q_push(int x) {
        if (x <= n) q.push(x);
        else for (int y : flo[x]) q.push(y);
    }
    void set_st(int x, int b) {
        st[x] = b;
        if (x > n) for (int y : flo[x]) set_st(y, b);
    }
    vector<int> split_flo(auto &f, int xr) {
        auto it = find(ALL(f), xr);
        if (auto pr = it - f.begin(); pr % 2 == 1)
            reverse(1 + ALL(f), it = f.end() - pr);
        auto res = vector(f.begin(), it);
        return f.erase(f.begin(), it), res;
    }
    void set_match(int u, int v) {
        match[u] = g[u][v].v;
        if (u <= n) return;
        int xr = flo_from[u][g[u][v].u];
        auto &f = flo[u], z = split_flo(f, xr);
        REP(i, 0, SZ(z) - 1) set_match(z[i], z[i ^ 1]);
        set_match(xr, v); f.insert(f.end(), ALL(z));
    }
    void augment(int u, int v) {

```

```

        for (;;) {
            int xnv = st[match[u]]; set_match(u, v);
            if (!xnv) return;
            set_match(v = xnv, u = st[pa[xnv]]);
        }
    }
    int lca(int u, int v) {
        static int t = 0; ++t;
        for (++t; u || v; swap(u, v)) if (u) {
            if (vis[u] == t) return u;
            vis[u] = t, u = st[match[u]];
            if (u) u = st[pa[u]];
        }
        return 0;
    }
    void add_blossom(int u, int o, int v) {
        int b = find(n + 1 + ALL(st), 0) - begin(st);
        lab[b] = 0, S[b] = 0, match[b] = match[o];
        vector<int> f = {o};
        for (int t : {u, v}) {
            reverse(1 + ALL(f));
            for (int x = t, y; x != o; x = st[pa[y]])
                f.pb(x), f.pb(y = st[match[x]]), q_push(y);
        }
        flo[b] = f; set_st(b, b);
        REP(x, 1, nx) g[b][x].w = g[x][b].w = 0;
        fill(ALL(flo_from[b]), 0);
        for (int xs : flo[b]) {
            REP(x, 1, nx)
                if (g[b][x].w == 0 || E(g[xs][x]) < E(g[b][x]))
                    g[b][x] = g[xs][x], g[x][b] = g[x][xs];
            REP(x, 1, n)
                if (flo_from[xs][x]) flo_from[b][x] = xs;
        }
        set_slk(b);
    }
    void expand_blossom(int b) {
        for (int x : flo[b]) set_st(x, x);
        int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
        for (int x : split_flo(flo[b], xr)) {
            if (xs == -1) { xs = x; continue; }
            pa[xs] = g[x][xs].u, S[xs] = 1, S[x] = 0;
            slk[xs] = 0, set_slk(x), q_push(x), xs = -1;
        }
        for (int x : flo[b])
            if (x == xr) S[x] = 1, pa[x] = pa[b];
            else S[x] = -1, set_slk(x);
        st[b] = 0;
    }
    bool on_found_edge(const edge &e) {
        if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
            int nu = st[match[v]]; pa[v] = e.u; S[v] = 1;
            slk[v] = slk[nu] = S[nu] = 0; q_push(nu);
        }
        else if (S[v] == 0) {
            if (int o = lca(u, v)) add_blossom(u, o, v);
            else return augment(u, v), augment(v, u), true;
        }
        return false;
    }
    bool matching() {
        fill(ALL(S), -1), fill(ALL(slk), 0);
        q = queue<int>();
        REP(x, 1, nx) if (st[x] == x && !match[x])
            pa[x] = S[x] = 0, q_push(x);
        if (q.empty()) return false;
        for (;;) {
            while (SZ(q)) {
                int u = q.front(); q.pop();
                if (S[st[u]] == 1) continue;
                REP(v, 1, n)
                    if (g[u][v].w > 0 && st[u] != st[v]) {
                        if (E(g[u][v]) != 0)
                            update_slk(u, st[v], slk[st[v]]);
                        else if
                            (on_found_edge(g[u][v])) return true;
                    }
            }
            int d = INF;
            REP(b, n + 1, nx) if (st[b] == b && S[b] == 1)
                d = min(d, lab[b] / 2);
            REP(x, 1, nx)
                if (int
                    s = slk[x]; st[x] == x && s && S[x] <= 0)
                    d = min(d, E(g[s][x]) / (S[x] + 2));
            REP(u, 1, n)
                if (S[st[u]] == 1) lab[u] += d;

```

```

    else if (S[st[u]] == 0) {
        if (lab[u] <= d) return false;
        lab[u] -= d;
    }
    REP(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
        lab[b] += d * (2 - 4 * S[b]);
    REP(x, 1, nx)
        if (int s = slk[x]; st[x] == x &&
            s && st[s] != x && E(g[s][x]) == 0)
            if (on_found_edge(g[s][x])) return true;
    REP(b, n + 1, nx)
        if (st[b] == b && S[b] == 1 && lab[b] == 0)
            expand_blossom(b);
}
return false;
}
pair<ll, int> solve() {
    fill(ALL(match), 0);
    REP(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    REP(u, 1, n) REP(v, 1, n) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
    }
    fill(ALL(lab), w_max);
    int n_matches = 0; ll tot_weight = 0;
    while (matching()) ++n_matches;
    REP(u, 1, n) if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void add_edge(int u, int v, int w)
{ g[u][v].w = g[v][u].w = w; }
};

```

#### 4.7 SW-mincut [6621f5]

```

struct SW{ // global min cut, O(V^3)
#define REP for (int i = 0; i < n; ++i)
static const int MXN = 514, INF = 2147483647;
int vst[MXN], edge[MXN][MXN], wei[MXN];
void init(int n) {
    REP fill_n(edge[i], n, 0);
}
void addEdge(int u, int v, int w){
    edge[u][v] += w; edge[v][u] += w;
}
int search(int &s, int &t, int n){
    fill_n(vst, n, 0), fill_n(wei, n, 0);
    s = t = -1;
    int mx, cur;
    for (int j = 0; j < n; ++j) {
        mx = -1, cur = 0;
        REP if (wei[i] > mx) cur = i, mx = wei[i];
        vst[cur] = 1, wei[cur] = -1;
        s = t; t = cur;
        REP if (!vst[i]) wei[i] += edge[cur][i];
    }
    return mx;
}
int solve(int n) {
    int res = INF;
    for (int x, y; n > 1; n--){
        res = min(res, search(x, y, n));
        REP edge[i][x] = (edge[x][i] += edge[y][i]);
        REP {
            edge[y][i] = edge[n - 1][i];
            edge[i][y] = edge[i][n - 1];
        } // edge[y][y] = 0;
    }
    return res;
}
} sw;

```

#### 4.8 BoundedFlow\*(Dinic\*) [4a793f]

```

struct BoundedFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n + 2; ++i)
            G[i].clear(), cnt[i] = 0;
    }
};

```

```

void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
}
void add_edge(int u, int v, int cap) {
    G[u].pb(edge{v, cap, 0, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
}
int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
        edge &e = G[u][i];
        if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
            int df = dfs(e.to, min(e.cap - e.flow, cap));
            if (df) {
                e.flow += df, G[e.to][e.rev].flow -= df;
                return df;
            }
        }
    }
    dis[u] = -1;
    return 0;
}
bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (edge &e : G[u])
            if (!dis[e.to] && e.flow != e.cap)
                q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
}
int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
        fill_n(cur, n + 3, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}
bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            add_edge(n + 1, i, cnt[i]), sum += cnt[i];
        else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            G[n + 1].pop_back(), G[i].pop_back();
        else if (cnt[i] < 0)
            G[i].pop_back(), G[n + 2].pop_back();
    return sum != -1;
}
int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
}
};

```

#### 4.9 Gomory Hu tree\* [11be99]

```

MaxFlow Dinic;
int g[MXN];
void GomoryHu(int n) { // 0-base
    fill_n(g, n, 0);
    for (int i = 1; i < n; ++i) {
        Dinic.reset();
        add_edge(i, g[i], Dinic.maxflow(i, g[i]));
        for (int j = i + 1; j <= n; ++j)
            if (g[j] == g[i] && ~Dinic.dis[j])
                g[j] = i;
    }
}

```

#### 4.10 Minimum Cost Circulation\* [ba97cf]

```

struct MinCostCirculation { // 0-base
    struct Edge {

```

```

11 from, to, cap, fcap, flow, cost, rev;
} *past[N];
vector<Edge> G[N];
11 dis[N], inq[N], n;
void BellmanFord(int s) {
    fill_n(dis, n, INF), fill_n(inq, n, 0);
    queue<int> q;
    auto relax = [&](int u, 11 d, Edge *e) {
        if (dis[u] > d) {
            dis[u] = d, past[u] = e;
            if (!inq[u]) inq[u] = 1, q.push(u);
        }
    };
    relax(s, 0, 0);
    while (!q.empty()) {
        int u = q.front();
        q.pop(), inq[u] = 0;
        for (auto &e : G[u])
            if (e.cap > e.flow)
                relax(e.to, dis[u] + e.cost, &e);
    }
}
void try_edge(Edge &cur) {
    if (cur.cap > cur.flow) return ++cur.cap, void();
    BellmanFord(cur.to);
    if (dis[cur.from] + cur.cost < 0) {
        ++cur.flow, --G[cur.to][cur.rev].flow;
        for (int i = cur.from; past[i]; i = past[i]->from) {
            auto &e = *past[i];
            ++e.flow, --G[e.to][e.rev].flow;
        }
        ++cur.cap;
    }
}
void solve(int mxlg) {
    for (int b = mxlg; b >= 0; --b) {
        for (int i = 0; i < n; ++i)
            for (auto &e : G[i])
                e.cap *= 2, e.flow *= 2;
        for (int i = 0; i < n; ++i)
            for (auto &e : G[i])
                if (e.fcap >> b & 1)
                    try_edge(e);
    }
}
void init(int _n) { n = _n;
    for (int i = 0; i < n; ++i) G[i].clear();
}
void add_edge(11 a, 11 b, 11 cap, 11 cost) {
    G[a].pb(Edge{a, b, 0, cap, 0, cost, SZ(G[b]) + (a == b)});
    G[b].pb(Edge{b, a, 0, 0, 0, -cost, SZ(G[a]) - 1});
}
} mcmf; // O(VE * ElogC)

```

## 4.11 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  - Constructs supersource  $S$  and sink  $T$ .
  - For each edge  $(x, y, l, u)$ , connect  $x \rightarrow y$  with capacity  $u - l$ .
  - For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  - If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
    - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
  - The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.
- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X, Y)$ 
  - Redirect every edge:  $y \rightarrow x$  if  $(x, y) \in M, x \rightarrow y$  otherwise.
  - DFS from unmatched vertices in  $X$ .
  - $x \in X$  is chosen iff  $x$  is unvisited.
  - $y \in Y$  is chosen iff  $y$  is visited.
- Minimum cost cyclic flow
  - Construct supersource  $S$  and sink  $T$
  - For each edge  $(x, y, c)$ , connect  $x \rightarrow y$  with  $(cost, cap) = (c, 1)$  if  $c > 0$ , otherwise connect  $y \rightarrow x$  with  $(cost, cap) = (-c, 1)$
  - For each edge with  $c < 0$ , sum these cost as  $K$ , then increase  $d(y)$  by 1, decreased  $d(x)$  by 1

- For each vertex  $v$  with  $d(v) > 0$ , connect  $S \rightarrow v$  with  $(cost, cap) = (0, d(v))$
  - For each vertex  $v$  with  $d(v) < 0$ , connect  $v \rightarrow T$  with  $(cost, cap) = (0, -d(v))$
  - Flow from  $S$  to  $T$ , the answer is the cost of the flow  $C + K$
- Maximum density induced subgraph
    - Binary search on answer, suppose we're checking answer  $T$
    - Construct a max flow model, let  $K$  be the sum of all weights
    - Connect sources  $s \rightarrow v, v \in G$  with capacity  $K$
    - For each edge  $(u, v, w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$
    - For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
    - $T$  is a valid answer if the maximum flow  $f < K|V|$
  - Minimum weight edge cover
    - For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u, v)$ .
    - Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
    - Find the minimum weight perfect matching on  $G'$ .
  - Project selection problem
    - If  $p_v > 0$ , create edge  $(s, v)$  with capacity  $p_v$ ; otherwise, create edge  $(v, t)$  with capacity  $-p_v$ .
    - Create edge  $(u, v)$  with capacity  $w$  with  $w$  being the cost of choosing  $u$  without choosing  $v$ .
    - The min cut is equivalent to the maximum profit of a subset of projects.
  - Dual of minimum cost maximum flow
    - Capacity  $c_{uv}$ , Flow  $f_{uv}$ , Cost  $w_{uv}$ , Required Flow difference for vertex  $b_u$ .
    - If all  $w_{uv}$  are integers, then optimal solution can happen when all  $p_u$  are integers.

$$\min \sum_{uv} w_{uv} f_{uv} - f_{uv} \geq -c_{uv} \Leftrightarrow \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv})$$

$$\sum_v f_{vu} - \sum_v f_{uv} = -b_u \quad p_u \geq 0$$

## 4.12 matching

- 最大匹配 + 最小邊覆蓋 =  $V$
- 最大獨立集 + 最小點覆蓋 =  $V$
- 最大匹配 = 最小點覆蓋
- 最小路徑覆蓋數 =  $V$  - 最大匹配數

## 5 String

### 5.1 KMP [5a0728]

```

int F[MXN];
vector<int> match(string A, string B) {
    vector<int> ans;
    F[0] = -1, F[1] = 0;
    for (int i = 1, j = 0; i < SZ(B); F[++i] = ++j) {
        if (B[i] == B[j]) F[i] = F[j]; // optimize
        while (j != -1 && B[i] != B[j]) j = F[j];
    }
    for (int i = 0, j = 0; i < SZ(A); ++i) {
        while (j != -1 && A[i] != B[j]) j = F[j];
        if (++j == SZ(B)) ans.pb(i + 1 - j), j = F[j];
    }
    return ans;
}

```

### 5.2 Z-value\* [b47c17]

```

int z[MXN];
void make_z(const string &s) {
    int l = 0, r = 0;
    for (int i = 1; i < SZ(s); ++i) {
        for (z[i] = max(0, min(r - i + 1, z[i - l]));
            i + z[i] < SZ(s) && s[i + z[i]] == s[z[i]];
            ++z[i]);
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}

```

### 5.3 Manacher\* [1ad8ef]

```

int z[MXN]; // 0-base
/* center i: radius z[i * 2 + 1] / 2
   center i, i + 1: radius z[i * 2 + 2] / 2
   both aba, abba have radius 2 */
void Manacher(string tmp) {
    string s = "%";
    int l = 0, r = 0;
    for (char c : tmp) s.pb(c), s.pb('%');
    for (int i = 0; i < SZ(s); ++i) {
        z[i] = r > i ? min(z[i * 2 - i], r - i) : 1;
        while (i - z[i] >= 0 && i + z[i] < SZ(s)
            && s[i + z[i]] == s[i - z[i]]) ++z[i];
    }
}

```

```

    if (z[i] + i > r) r = z[i] + i, l = i;
}
}

```

## 5.4 SAIS\* [6f26bc]

```

auto sais(const auto &s) {
    const int n = SZ(s), z = ranges::max(s) + 1;
    if (n == 1) return vector{0};
    vector<int> c(z); for (int x : s) ++c[x];
    partial_sum(ALL(c), begin(c));
    vector<int> sa(n); auto I = views::iota(0, n);
    vector<bool> t(n, true);
    for (int i = n - 2; i >= 0; --i)
        t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    auto is_lms = views::filter([&t](int x) {
        return x && t[x] && !t[x - 1];
    });
    auto induce = [&] {
        for (auto x = c; int y : sa)
            if (y-- && (!t[y] || sa[x[s[y] - 1]++] = y);
        for (auto x = c; int y : sa | views::reverse)
            if (y-- && t[y] || sa[-x[s[y]]] = y;
    };
    vector<int> lms, q(n); lms.reserve(n);
    for (auto x = c; int i : I | is_lms)
        q[i] = SZ(lms), lms.pb(sa[-x[s[i]]] = i);
    induce(); vector<int> ns(SZ(lms));
    for (int j = -1, nz = 0; int i : sa | is_lms) {
        if (j >= 0) {
            int len = min({n - i, n - j, lms[q[i] + 1] - i});
            ns[q[i]] = nz += lexicographical_compare(
                begin(s) + j, begin(s) + j + len,
                begin(s) + i, begin(s) + i + len);
        }
        j = i;
    }
    fill(ALL(sa), 0); auto nsa = sais(ns);
    for (auto x = c; int y : nsa | views::reverse)
        y = lms[y], sa[-x[s[y]]] = y;
    return induce(), sa;
}
// sa[i]: sa[i]-th suffix
// is the i-th lexicographically smallest suffix.
// hi[i]: LCP of suffix sa[i] and suffix sa[i - 1].
struct Suffix {
    int n; vector<int> sa, hi, ra;
    Suffix
        (const auto &s, int _n) : n(_n), hi(n), ra(n) {
        vector<int> s(n + 1); // s[n] = 0;
        copy_n(s, n, begin(s)); // s shouldn't contain 0
        sa = sais(s); sa.erase(sa.begin());
        for (int i = 0; i < n; ++i) ra[sa[i]] = i;
        for (int i = 0, h = 0; i < n; ++i) {
            if (!ra[i]) { h = 0; continue; }
            for (int j = sa[ra[i] - 1]; max
                (i, j) + h < n && s[i + h] == s[j + h];) ++h;
            hi[ra[i]] = h ? h - 1 : 0;
        }
    }
};

```

## 5.5 Aho-Corasick Automatan\* [794a77]

```

struct AC_Automatan {
    int nx[len][sigma], fl[len], cnt[len], ord[len], top;
    int rxn[len][sigma]; // node actually be reached
    int newnode() {
        fill_n(nx[top], sigma, -1);
        return top++;
    }
    void init() { top = 1, newnode(); }
    int input(string &s) {
        int X = 1;
        for (char c : s) {
            if (!nx[X][c - 'A']) nx[X][c - 'A'] = newnode();
            X = nx[X][c - 'A'];
        }
        return X; // return the end node of string
    }
    void make_fl() {
        queue<int> q;
        q.push(1), fl[1] = 0;
        for (int t = 0; !q.empty(); ) {
            int R = q.front();
            q.pop(), ord[t++] = R;

```

```

        for (int i = 0; i < sigma; ++i)
            if (~nx[R][i]) {
                int X = rxn[R][i] = nx[R][i], Z = fl[R];
                for (; Z && !nx[Z][i]; ) Z = fl[Z];
                fl[X] = Z ? nx[Z][i] : 1, q.push(X);
            }
        else rxn[R][i] = R > 1 ? rxn[fl[R]][i] : 1;
    }
}
void solve() {
    for (int i = top - 2; i > 0; --i)
        cnt[fl[ord[i]]] += cnt[ord[i]];
}
} ac;

```

## 5.6 Smallest Rotation [4f469f]

```

string mcp(string s) {
    int n = SZ(s), i = 0, j = 1;
    s += s;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && s[i + k] == s[j + k]) ++k;
        if (s[i + k] <= s[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

## 5.7 De Bruijn sequence\* [a09470]

```

constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
    int C, N, K, L, buf[MAXC * MAXN]; // K <= C^N
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr >= L) return;
        if (t > N) {
            if (N % p) return;
            for (int i = 1; i <= p && ptr < L; ++i)
                out[ptr++] = buf[i];
        } else {
            buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
            for (int j = buf[t - p] + 1; j < C; ++j)
                buf[t] = j, dfs(out, t + 1, t, ptr);
        }
    }
    void solve(int _c, int _n, int _k, int *out) {
        int p = 0;
        C = _c, N = _n, K = _k, L = N + K - 1;
        dfs(out, 1, 1, p);
        if (p < L) fill(out + p, out + L, 0);
    }
} dbs;

```

## 5.8 Extended SAM\* [64c3b7]

```

struct exSAM {
    int len[N * 2], link[N * 2]; // maxlength, suflink
    int next[N * 2][CNUM], tot; // [0, tot), root = 0
    int lenSorted[N * 2]; // topo. order
    int cnt[N * 2]; // occurrence
    int newnode() {
        fill_n(next[tot], CNUM, 0);
        len[tot] = cnt[tot] = link[tot] = 0;
        return tot++;
    }
    void init() { tot = 0, newnode(), link[0] = -1; }
    int insertSAM(int last, int c) {
        int cur = next[last][c];
        len[cur] = len[last] + 1;
        int p = link[last];
        while (p != -1 && !next[p][c])
            next[p][c] = cur, p = link[p];
        if (p == -1) return link[cur] = 0, cur;
        int q = next[p][c];
        if (len
            [p] + 1 == len[q]) return link[cur] = q, cur;
        int clone = newnode();
        for (int i = 0; i < CNUM; ++i)
            next[
                clone][i] = len[next[q][i]] ? next[q][i] : 0;
        len[clone] = len[p] + 1;
        while (p != -1 && next[p][c] == q)
            next[p][c] = clone, p = link[p];
        link[link[cur] = clone] = link[q];
    }

```

```

    link[q] = clone;
    return cur;
}
void insert(const string &s) {
    int cur = 0;
    for (auto ch : s) {
        int &nxt = next[cur][int(ch - 'a')];
        if (!nxt) nxt = newnode();
        cnt[cur = nxt] += 1;
    }
}
void build() {
    queue<int> q;
    q.push(0);
    while (!q.empty()) {
        int cur = q.front();
        q.pop();
        for (int i = 0; i < CNUM; ++i)
            if (next[cur][i])
                q.push(insertSAM(cur, i));
    }
    vector<int> lc(tot);
    for (int i = 1; i < tot; ++i) ++lc[len[i]];
    partial_sum(ALL(lc), lc.begin());
    for (int i = 1; i < tot; ++i) lenSorted[--lc[len[i]]] = i;
}
void solve() {
    for (int i = tot - 2; i >= 0; --i)
        cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
}
};

```

## 5.9 PalTree\* [d7d2cf]

```

struct palindromic_tree {
    struct node {
        int next[26], fail, len;
        int cnt, num; // cnt: appear times, num: number of
                        // pal. suf.
        node(int l = 0) : fail(0), len(l), cnt(0), num(0) {
            for (int i = 0; i < 26; ++i) next[i] = 0;
        }
    };
    vector<node> St;
    vector<char> s;
    int last, n;
    palindromic_tree() : St(2), last(1), n(0) {
        St[0].fail = 1, St[1].len = -1, s.pb(-1);
    }
    inline void clear() {
        St.clear(), s.clear(), last = 1, n = 0;
        St.pb(0), St.pb(-1);
        St[0].fail = 1, s.pb(-1);
    }
    inline int get_fail(int x) {
        while (s[n - St[x].len - 1] != s[n])
            x = St[x].fail;
        return x;
    }
    inline void add(int c) {
        s.push_back(c - 'a'), ++n;
        int cur = get_fail(last);
        if (!St[cur].next[c]) {
            int now = SZ(St);
            St.pb(St[cur].len + 2);
            St[now].fail =
                St[get_fail(St[cur].fail)].next[c];
            St[cur].next[c] = now;
            St[now].num = St[St[now].fail].num + 1;
        }
        last = St[cur].next[c], ++St[last].cnt;
    }
    inline void count() { // counting cnt
        auto i = St.rbegin();
        for (; i != St.rend(); ++i) {
            St[i->fail].cnt += i->cnt;
        }
    }
    inline int size() { // The number of diff. pal.
        return SZ(St) - 2;
    }
};

```

## 5.10 Main Lorentz [615b8f]

```
vector<pair<int, int>> rep[kN]; // 0-base [1, r]
```

```

void main_lorentz(const string &s, int sft = 0) {
    const int n = s.size();
    if (n == 1) return;
    const int nu = n / 2, nv = n - nu;
    const string u = s.substr(0, nu), v = s.substr(nu,
        ru(u.rbegin(), u.rend()), rv(v.rbegin(), v.rend()));
    main_lorentz(u, sft), main_lorentz(v, sft + nu);
    const auto z1 = Zalgo(ru), z2 = Zalgo(v + '#' + u),
        z3 = Zalgo(ru + '#' + rv), z4 = Zalgo(v);
    auto get_z = [](const vector<int> &z, int i) {
        return
            (0 <= i and i < (int)z.size()) ? z[i] : 0;
    };
    auto add_rep
        = [&](bool left, int c, int l, int k1, int k2) {
            const
                int L = max(1, l - k2), R = min(l - left, k1);
            if (L > R) return;
            if (left)
                rep[l].emplace_back(sft + c - R, sft + c - L);
            else
                rep[l].emplace_back
                    (sft + c - R - l + 1, sft + c - L - l + 1);
        };
    for (int cntr = 0; cntr < n; cntr++) {
        int l, k1, k2;
        if (cntr < nu) {
            l = nu - cntr;
            k1 = get_z(z1, nu - cntr);
            k2 = get_z(z2, nv + 1 + cntr);
        } else {
            l = cntr - nu + 1;
            k1 = get_z(z3, nu + 1 + nv - 1 - (cntr - nu));
            k2 = get_z(z4, (cntr - nu) + 1);
        }
        if (k1 + k2 >= l)
            add_rep(cntr < nu, cntr, l, k1, k2);
    }
} // p \in [1, r] => s[p, p + i) = s[p + i, p + 2i)

```

## 6 Math

### 6.1 ax+by=gcd(only exgcd \*) [7b833d]

```

pll exgcd(ll a, ll b) {
    if (b == 0) return pll(1, 0);
    ll p = a / b;
    pll q = exgcd(b, a % b);
    return pll(q.Y, q.X - q.Y * p);
}
/* ax+by=res, let x be minimum non-negative
g, p = gcd(a, b), exgcd(a, b) * res / g
if p.X < 0: t = (abs(p.X) + b / g - 1) / (b / g)
else: t = -(p.X / (b / g))
p += (b / g, -a / g) * t */

```

### 6.2 Floor and Ceil [692c04]

```

int floor(int a, int b)
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
int ceil(int a, int b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }

```

### 6.3 Floor Enumeration [7cbcdf]

```

// enumerating x = floor(n / i), [1, r]
for (int l = 1, r; l <= n; l = r + 1) {
    int x = n / l;
    r = n / x;
}

```

### 6.4 Mod Min [9118e1]

```

// min{k | 1 <= ((ak) mod m) <= r}, no solution -> -1
ll mod_min(ll a, ll m, ll l, ll r) {
    if (a == 0) return l ? -1 : 0;
    if (ll k = (l + a - 1) / a; k * a <= r)
        return k;
    ll b = m / a, c = m % a;
    if (ll y = mod_min(c, a, a - r % a, a - l % a))
        return (l + y * c + a - 1) / a + y * b;
    return -1;
}

```

### 6.5 Linear Mod Inverse [5a4cbf]

```

inv[1] = 1;
for (int i = 2; i
    <= N; ++i) inv[i] = ((mod - mod / i) * inv[mod % i]) % mod;

```



## 6.6 Linear Filter Mu [ac2ac3]

```
void getMu() {
    mu[1] = 1;
    for (int i = 2; i <= n; ++i) {
        if (!flg[i]) p[++tot] = i, mu[i] = -1;
        for (int j = 1; j <= tot && i * p[j] <= n; ++j) {
            flg[i * p[j]] = 1;
            if (i % p[j] == 0) {
                mu[i * p[j]] = 0;
                break;
            }
            mu[i * p[j]] = -mu[i];
        }
    }
}
```

## 6.7 Gaussian integer gcd [763e59]

```
cpx gaussian_gcd(cpx a, cpx b) {
#define rnd
    (a, b) ((a >= 0 ? a * 2 + b : a * 2 - b) / (b * 2))
    ll c = a.real() * b.real() + a.imag() * b.imag();
    ll d = a.imag() * b.real() - a.real() * b.imag();
    ll r = b.real() * b.real() + b.imag() * b.imag();
    if (c % r == 0 && d % r == 0) return b;
    return gaussian_gcd
        (b, a - cpx(rnd(c, r), rnd(d, r)) * b);
}
```

## 6.8 GaussElimination [6308be]

```
void GAS(V<V<double>>&vc) {
    int len = vc.size();
    for (int i = 0; i < len; ++i) {
        int idx = find_if(vc.begin() + i, vc.end(), [&](
            auto&v) { return v[i] != 0; }) - vc.begin();
        if (idx == len) continue;
        if (i != idx) swap(vc[idx], vc[i]);
        double pivot = vc[i][i];
        for_each(vc[i].begin(), vc[i].end(), [&](auto &a) { a /= pivot; });
        for (int j = 0; j < len; ++j) {
            if (i == j) continue;
            if (vc[j][i] != 0) {
                double mul = vc[j][i] / vc[i][i];
                transform(vc[j].begin(), vc[j].end(),
                    vc[i].begin(), vc[j].begin(),
                    [&](auto &a, auto &b) {
                        return a - b * mul;
                    });
            }
        }
    }
};
```

## 6.9 floor sum\* [49de67]

```
ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m)
        ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m)
        ans += n * (b / m), b %= m;
    ll y_max
        = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
} // sum^{n-1}_0 floor((a * i + b) / m) in log(n + m + a + b)
```

## 6.10 Miller Rabin\* [06308c]

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : primes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a, ll n) {
    if ((a = a % n) == 0) return 1;
    if (n % 2 == 0) return n == 2;
    ll tmp = (n - 1) / ((n - 1) & (1 - n));
    ll t = __lg(((n - 1) & (1 - n))), x = 1;
    for (; tmp; tmp >>= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
```

```
while (--t)
    if ((x = mul(x, x, n)) == n - 1) return 1;
return 0;
}
```

## 6.11 Fraction [4ab37a]

```
struct fraction {
    ll n, d;
    fraction
        (const ll &n=0, const ll &d=1): n(_n), d(_d) {
        ll t = gcd(n, d);
        n /= t, d /= t;
        if (d < 0) n = -n, d = -d;
    }
    fraction operator-() const
    { return fraction(-n, d); }
    fraction operator+(const fraction &b) const
    { return fraction(n * b.d + b.n * d, d * b.d); }
    fraction operator-(const fraction &b) const
    { return fraction(n * b.d - b.n * d, d * b.d); }
    fraction operator*(const fraction &b) const
    { return fraction(n * b.n, d * b.d); }
    fraction operator/(const fraction &b) const
    { return fraction(n * b.d, d * b.n); }
    void print() {
        cout << n;
        if (d != 1) cout << "/" << d;
    }
};
```

## 6.12 Simultaneous Equations [a231be]

```
struct matrix { //m variables, n equations
    int n, m;
    fraction M[MAXN][MAXN + 1], sol[MAXN];
    int solve() { // -1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m) continue;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                fraction tmp = -M[j][piv] / M[i][piv];
                for (int k = 0; k <=
                    m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
            }
        }
        int rank = 0;
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m && M[i][m].n) return -1;
            else if (piv
                < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
        }
        return rank;
    }
};
```

## 6.13 Pollard Rho\* [fdef9b]

```
map<ll, int> cnt;
void PollardRho(ll n) {
    if (n == 1) return;
    if (prime(n)) return ++cnt[n], void();
    if (n % 2
        == 0) return PollardRho(n / 2), ++cnt[2], void();
    ll x = 2, y = 2, d = 1, p = 1;
#define f(x, n, p) ((mul(x, x, n) + p) % n)
    while (true) {
        if (d != n && d != 1) {
            PollardRho(n / d);
            PollardRho(d);
            return;
        }
        if (d == n) ++p;
        x = f(x, n, p), y = f(f(y, n, p), n, p);
        d = gcd(abs(x - y), n);
    }
}
```

## 6.14 Simplex Algorithm [6b4566]

```
const int MAXN = 11000, MAXM = 405;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXN];
double d[MAXN][MAXM], x[MAXN];
```

```

int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(int n, int m){
    +hm;
    fill_n(d[n], m + 1, 0);
    fill_n(d[n + 1], m + 1, 0);
    iota(ix, ix + n + m, 0);
    int r = n, s = m - 1;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    copy_n(c, m - 1, d[n]);
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
            swap(ix[s], ix[r + m]);
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
                if (d[n + 1][j] > eps ||
                    (d[n + 1][j] > -eps && d[n][j] > eps))
                    s = j;
            }
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
            if (r < 0 ||
                (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps ||
                (dd < eps && ix[r + m] > ix[i + m]))
                r = i;
        }
        if (r < 0) return -1; // not bounded
    }
    if (d[n + 1][m] < -eps) return -1; // not executable
    double ans = 0;
    fill_n(x, m, 0);
    for (int i = m; i < n + m; ++i) { // the missing enumerated x[i] = 0
        if (ix[i] < m - 1) {
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i - m][m];
        }
    }
    return ans;
}

```

### 6.14.1 Construction

| Primal                                      | Dual   |
|---|--|
| Maximize $c^T x$ s.t. $Ax \leq b, x \geq 0$ | Minimize $b^T y$ s.t. $A^T y \geq c, y \geq 0$ |
| Maximize $c^T x$ s.t. $Ax \leq b$           | Minimize $b^T y$ s.t. $A^T y = c, y \geq 0$    |
| Maximize $c^T x$ s.t. $Ax = b, x \geq 0$    | Minimize $b^T y$ s.t. $A^T y \geq c$           |

$\bar{x}$  and  $\bar{y}$  are optimal if and only if for all  $i \in [1, n]$ , either  $\bar{x}_i = 0$  or  $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$

holds and for all  $i \in [1, m]$  either  $\bar{y}_i = 0$  or  $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$  holds.

1. In case of minimization, let  $c'_i = -c_i$
2.  $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3.  $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$ 
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If  $x_i$  has no lower bound, replace  $x_i$  with  $x_i - x'_i$

### 6.15 chineseRemainder [a53b6d]

```

ll solve(ll x1, ll m1, ll x2, ll m2) {
    ll g = gcd(m1, m2);
    if ((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pll p = exgcd(m1, m2);
    ll lcm = m1 * m2 * g;
    ll res = p.first * (x2 - x1) * m1 + x1;
    // be careful with overflow
}

```

```

return (res % lcm + lcm) % lcm;
}

```

### 6.16 Factorial without prime factor\* [c324f3]

```

// O(p^k + log^2 n), pk = p^k
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
    prod[0] = 1;
    for (int i = 1; i <= pk; ++i)
        if (i % p) prod[i] = prod[i - 1] * i % pk;
        else prod[i] = prod[i - 1];
    ll rt = 1;
    for (; n; n /= p) {
        rt = rt * mpow(prod[pk], n / pk, pk) % pk;
        rt = rt * prod[n % pk] % pk;
    }
    return rt;
} // (n! without factor p) % p^k

```

### 6.17 PiCount\* [cad6d4]

```

ll PrimeCount(ll n) { // n ~ 10^13 => < 2s
    if (n <= 1) return 0;
    int v = sqrt(n), s = (v + 1) / 2, pc = 0;
    vector<int> smalls(v + 1), skip(v + 1), roughs(s);
    vector<ll> larges(s);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    for (int i = 0; i < s; ++i) {
        roughs[i] = 2 * i + 1;
        larges[i] = (n / (2 * i + 1) + 1) / 2;
    }
    for (int p = 3; p <= v; ++p) {
        if (smalls[p] > smalls[p - 1]) {
            int q = p * p;
            ++pc;
            if (1LL * q * q > n) break;
            skip[p] = 1;
            for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
            int ns = 0;
            for (int k = 0; k < s; ++k) {
                int i = roughs[k];
                if (skip[i]) continue;
                ll d = 1LL * i * p;
                larges[ns] = larges[k] - (d <= v ? larges[smalls[d] - pc] : smalls[n / d]) + pc;
                roughs[ns++] = i;
            }
            s = ns;
            for (int j = v / p; j >= p; --j) {
                int c =
                    smalls[j] - pc, e = min(j * p + p, v + 1);
                for (int i = j * p; i < e; ++i) smalls[i] -= c;
            }
        }
        for (int k = 1; k < s; ++k) {
            const ll m = n / roughs[k];
            ll t = larges[k] - (pc + k - 1);
            for (int l = 1; l < k; ++l) {
                int p = roughs[l];
                if (1LL * p * p > m) break;
                t -= smalls[m / p] - (pc + l - 1);
            }
            larges[0] -= t;
        }
        return larges[0];
    }
}

```

### 6.18 Discrete Log\* [da27bf]

```

int DiscreteLog(int s, int x, int y, int m) {
    constexpr int kStep = 32000;
    unordered_map<int, int> p;
    int b = 1;
    for (int i = 0; i < kStep; ++i) {
        p[y] = i;
        y = 1LL * y * x % m;
        b = 1LL * b * x % m;
    }
    for (int i = 0; i < m + 10; i += kStep) {
        s = 1LL * s * b % m;
        if (p.find(s) != p.end()) return i + kStep - p[s];
    }
    return -1;
}
int DiscreteLog(int x, int y, int m) {
}

```

```

if (m == 1) return 0;
int s = 1;
for (int i = 0; i < 100; ++i) {
    if (s == y) return i;
    s = 1LL * s * x % m;
}
if (s == y) return 100;
int p = 100 + DiscreteLog(s, x, y, m);
if (fpow(x, p, m) != y) return -1;
return p;
}

```

## 6.19 Berlekamp Massey [3eb6fa]

```

template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
    vector<T> d(SZ(output) + 1), me, he;
    for (int f = 0, i = 1; i <= SZ(output); ++i) {
        for (int j = 0; j < SZ(me); ++j)
            d[i] += output[i - j - 2] * me[j];
        if ((d[i] - output[i - 1]) == 0) continue;
        if (me.empty()) {
            me.resize(f + i);
            continue;
        }
        vector<T> o(i - f - 1);
        T k = -d[i] / d[f]; o.pb(-k);
        for (T x : he) o.pb(x * k);
        o.resize(max(SZ(o), SZ(me)));
        for (int j = 0; j < SZ(me); ++j) o[j] += me[j];
        if (i - f + SZ(he) >= SZ(me)) he = me, f = i;
        me = o;
    }
    return me;
}

```

## 6.20 Primes

```

/* 12721 13331 14341 75577 123457 222557
   556679 999983 109774749 1076767633 100102021
   999997771 1001010013 1000512343 987654361 999991231
   999888733 98789101 987777733 999991921 100101333
   1010102101 1000000000039 100000000000037
   2305843009213693951 4611686018427387847
   9223372036854775783 18446744073709551557 */

```

## 6.21 Theorem

Lucas's Theorem

For non-negative integer  $n, m$  and prime  $P$ ,  
 $C(m, n) \bmod P = C(m/M, n/M) * C(m \% M, n \% M) \bmod P$   
 $= \text{mult}_i(C(m\_i, n\_i))$   
 where  $m\_i$  is the  $i$ -th digit of  $m$  in base  $P$ .

Kirchhoff's theorem

$A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$   
 Deleting any one row, one column, and call the  $\det(A)$

Nth Catalan recursive function:

$C_0 = 1, C_{n+1} = C_n * 2(2n+1)/(n+2)$

Mobius Formula

$u(n) = 1$ , if  $n = 1$   
 $(-1)^m$ , 若  $n$  無平方數因數, 且  $n = p_1 * p_2 * p_3 * \dots * p_k$   
 $0$ , 若  $n$  有大於 1 的平方數因數

-Property

1. (積性函數)  $u(a)u(b) = u(ab)$
2.  $\sum_{d|n} u(d) = [n == 1]$

Mobius Inversion Formula

if  $f(n) = \sum_{d|n} g(d)$   
 then  $g(n) = \sum_{d|n} u(d) f(n/d)$   
 $= \sum_{d|n} u(d) f(n/d)$

-Application

the number/power of  $\gcd(i, j) = k$

-Trick

分塊,  $O(\sqrt{n})$

Chinese Remainder Theorem ( $m_i$  兩兩互質)

```

x = a_1 (mod m_1)
x = a_2 (mod m_2)
....
x = a_i (mod m_i)
construct a solution:
Let M = m_1 * m_2 * m_3 * ... * m_n
Let M_i = M / m_i

```

```

t_i = 1 / M_i
t_i * M_i = 1 (mod m_i)

```

solution  $x = a_1 * t_1 * M_1 + a_2 * t_2 * M_2 + \dots + a_n * t_n * M_n + k * M$   
 $= k * M + a_i * t_i * M_i$ ,  $k$  is positive integer.  
 under mod  $M$ , there is one solution  $x = a_i * t_i * M_i$

Burnside's lemma

$|G| * |X/G| = \sum (|X^g|)$  where  $g \in G$

總方法數: 每一種旋轉下不動點的個數總和除以旋轉的方法數

## 6.22 Estimation

|                 |     |     |     |      |      |       |        |       |        |        |     |     |         |     |       |
|-----------------|-----|-----|-----|------|------|-------|--------|-------|--------|--------|-----|-----|---------|-----|-------|
| $n$             | 2   | 3   | 4   | 5    | 6    | 7     | 8      | 9     | 20     | 30     | 40  | 50  | 100     |     |       |
| $p(n)$          | 2   | 3   | 5   | 7    | 11   | 15    | 22     | 30    | 62     | 75     | 60  | 42  | 2e5 2e8 |     |       |
| $n$             | 100 | 1e3 | 1e6 | 1e9  | 1e12 | 1e15  | 1e18   |       |        |        |     |     |         |     |       |
| $d(i)$          | 12  | 32  | 240 | 1344 | 6720 | 26880 | 103680 |       |        |        |     |     |         |     |       |
| $n$             | 1   | 2   | 3   | 4    | 5    | 6     | 7      | 8     | 9      | 10     | 11  | 12  | 13      | 14  | 15    |
| $\binom{2n}{n}$ | 2   | 6   | 20  | 70   | 252  | 924   | 3432   | 12870 | 48620  | 184756 | 7e5 | 2e6 | 1e7     | 4e7 | 1.5e8 |
| $n$             | 2   | 3   | 4   | 5    | 6    | 7     | 8      | 9     | 10     | 11     | 12  | 13  |         |     |       |
| $B_n$           | 2   | 5   | 15  | 52   | 203  | 877   | 4140   | 21147 | 115975 | 7e5    | 4e6 | 3e7 |         |     |       |

## 6.23 General Purpose Numbers

- Bernoulli numbers

$$B_0 = 1, B_1^{\pm} = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + k S(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left( x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x [C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k, j$ : s.s.t.  $\pi(j) > \pi(j+1), k+1, j$ : s.s.t.  $\pi(j) \geq j, k, j$ : s.s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

## 6.24 Tips for Generating Functions

- Ordinary Generating Function  $A(x) = \sum_{i \geq 0} a_i x^i$

- $A(rx) \Rightarrow r^n a_n$
- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$
- $x A(x)' \Rightarrow n a_n$
- $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^n a_i$

- Exponential Generating Function  $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x^i$

- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A^{(k)}(x) \Rightarrow a_{n+k}$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n \binom{n}{i} a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$
- $x A(x) \Rightarrow n a_n$

- Special Generating Function

- $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
- $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{i}{n-1} x^i$

## 7 Polynomial

### 7.1 Fast Fourier Transform [5e2ea2]

```

const int maxn = 131072;
using cplx = complex<double>;
const cplx I = cplx(0, 1);
const double pi = acos(-1);
cplx omega[maxn + 1];

```

```

void prefft() {
    for (int i = 0; i <= maxn; ++i) omega[i] = exp(i * 2 * pi / maxn * I);
}

```

```

void bin(vector<cplx> &a, int n) {
    int lg;
    for (lg = 0; (1 << lg) < n; ++lg); --lg;
    vector<cplx> tmp(n);
    for (int i = 0; i < n; ++i) {
        int to = 0;
        for (int j = 0; (1 << j) <
              n; ++j) to |= (((i >> j) & 1) << (lg - j));
        tmp[to] = a[i];
    }
    for (int i = 0; i < n; ++i) a[i] = tmp[i];
}

void fft(vector<cplx> &a, int n) {
    bin(a, n);
    for (int step = 2; step <= n; step <= 1) {
        int to = step >> 1;
        for (int i = 0; i < n; i += step) {
            for (int k = 0; k < to; ++k) {
                cplx x = a[i
                    + to + k] * omega[maxn / step * k];
                a[i + to + k] = a[i + k] - x;
                a[i + k] += x;
            }
        }
    }
}

void ifft(vector<cplx> &a, int n) {
    fft(a, n);
    reverse(a.begin() + 1, a.end());
    for (int i = 0; i < n; ++i) a[i] /= n;
}

vector<int> multiply(const vector<
    int> &a, const vector<int> &b, bool trim = false) {
    int d = 1;
    while
        (d < max(a.size(), b.size())) d <= 1; d <= 1;
    vector<cplx> pa(d), pb(d);
    for (int i
        = 0; i < a.size(); ++i) pa[i] = cplx(a[i], 0);
    for (int i
        = 0; i < b.size(); ++i) pb[i] = cplx(b[i], 0);
    fft(pa, d); fft(pb, d);
    for (int i = 0; i < d; ++i) pa[i] *= pb[i];
    ifft(pa, d);
    vector<int> r(d);
    for (int
        i = 0; i < d; ++i) r[i] = round(pa[i].real());
    if (trim)
        while (r.size() && r.back() == 0) r.pop_back();
    return r;
}

```

| Prime     | Root | Prime      | Root |
|-----------|------|------------|------|
| 7681      | 17   | 167772161  | 3    |
| 12289     | 11   | 104857601  | 3    |
| 40961     | 3    | 985661441  | 3    |
| 65537     | 3    | 998244353  | 3    |
| 786433    | 10   | 1107296257 | 10   |
| 5767169   | 3    | 2013265921 | 31   |
| 7340033   | 3    | 2810183681 | 11   |
| 23068673  | 3    | 2885681153 | 3    |
| 469762049 | 3    | 605028353  | 3    |

## 7.2 Number Theory Transform\* [7d51db]

```

vector<int> omega;
void Init() {
    omega.resize(kN + 1);
    long long x = fpow(kRoot, (Mod - 1) / kN);
    omega[0] = 1;
    for (int i = 1; i <= kN; ++i) {
        omega[i] = 1LL * omega[i - 1] * x % kMod;
    }
}

void Transform(vector<int> &v, int n) {
    BitReverse(v, n);
    for (int s = 2; s <= n; s <= 1) {
        int z = s >> 1;
        for (int i = 0; i < n; i += s) {
            for (int k = 0; k < z; ++k) {
                int x = 1LL
                    * v[i + k + z] * omega[kN / s * k] % kMod;
                v[i + k + z] = (v[i + k] + kMod - x) % kMod;
                (v[i + k] += x) %= kMod;
            }
        }
    }
}

```

```

}
}
void InverseTransform(vector<int> &v, int n) {
    Transform(v, n);
    for (int i = 1; i < n / 2; ++i) swap(v[i], v[n - i]);
    const int kInv = fpow(n, kMod - 2);
    for (int i
        = 0; i < n; ++i) v[i] = 1LL * v[i] * inv % kMod;
}

```

## 7.3 Fast Walsh Transform\* [c9cdb6]

```

/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
    for (int L = 2; L <= n; L <= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}

const int N = 21;
int f[
    N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N];
void
    subset_convolution(int *a, int *b, int *c, int L) {
    // c_k = \sum_{i | j = k, i & j = 0} a_i * b_j
    int n = 1 << L;
    for (int i = 1; i < n; ++i)
        ct[i] = ct[i & (i - 1)] + 1;
    for (int i = 0; i < n; ++i)
        f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
    for (int i = 0; i <= L; ++i)
        fwt(f[i], n, 1), fwt(g[i], n, 1);
    for (int i = 0; i <= L; ++i)
        for (int j = 0; j <= i; ++j)
            for (int x = 0; x < n; ++x)
                h[i][x] += f[j][x] * g[i - j][x];
    for (int i = 0; i <= L; ++i)
        fwt(h[i], n, -1);
    for (int i = 0; i < n; ++i)
        c[i] = h[ct[i]][i];
}

```

## 7.4 Polynomial Operation [869cb1]

```

#define
    fi(s, n) for (int i = (int)(s); i < (int)(n); ++i)
template<int MAXN, ll P, ll RT> // MAXN = 2^k
struct Poly : vector<ll> { // coefficients in [0, P)
    using vector<ll>::vector;
    static NTR<MAXN, P, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int m) : vector<ll>(m) {
        copy_n(p.data(), min(p.n(), m), data());
    }
    Poly& irev()
        { return reverse(data(), data() + n()), *this; }
    Poly& isz(int m) { return resize(m), *this; }
    Poly& iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if
            (((*this)[i] += rhs[i]) >= P) (*this)[i] -= P;
        return *this;
    }
    Poly& imul(ll k) {
        fi(0, n()) (*this)[i] = (*this)[i] * k % P;
        return *this;
    }
    Poly Mul(const Poly &rhs) const {
        int m = 1;
        while (m < n() + rhs.n() - 1) m <= 1;
        Poly X(*this, m), Y(rhs, m);
        ntt(X.data(), m), ntt(Y.data(), m);
        fi(0, m) X[i] = X[i] * Y[i] % P;
        ntt(X.data(), m, true);
        return X.isz(n() + rhs.n() - 1);
    }
    Poly Inv() const { // (*this)[0] != 0, 1e5/95ms
        if (n() == 1) return {ntt.minv((*this)[0])};
        int m = 1;
        while (m < n() * 2) m <= 1;
        Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(m);
        Poly Y(*this, m);
        ntt(Xi.data(), m), ntt(Y.data(), m);
        fi(0, m) {

```

```

    Xi[i] *= (2 - Xi[i] * Y[i]) % P;
    if ((Xi[i] % P) < 0) Xi[i] += P;
}
ntt(Xi.data(), m, true);
return Xi.isz(n());
}
Poly Sqrt()
const { // Jacobi((*this)[0], P) = 1, 1e5/235ms
if (n()
== 1) return {QuadraticResidue((*this)[0], P)};
Poly
X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
return
X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 + 1);
}
pair<Poly, Poly> DivMod
(const Poly &rhs) const { // (rhs.)back() != 0
if (n() < rhs.n()) return {{0}, *this};
const int m = n() - rhs.n() + 1;
Poly X(rhs); X.irev().isz(m);
Poly Y(*this); Y.irev().isz(m);
Poly Q = Y.Mul(X.Inv()).isz(m).irev();
X = rhs.Mul(Q), Y = *this;
fi(0, n()) if ((Y[i] - X[i]) < 0) Y[i] += P;
return {Q, Y.isz(max(1, rhs.n() - 1))};
}
Poly Dx() const {
Poly ret(n() - 1);
fi(0,
ret.n()) ret[i] = (i + 1) * (*this)[i + 1] % P;
return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
Poly ret(n() + 1);
fi(0, n())
ret[i + 1] = ntt.minv(i + 1) * (*this)[i] % P;
return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
Poly Y = Mul(rhs).isz(n() + nn - 1);
return Poly(Y.data() + n() - 1, Y.data() + Y.n());
}
vector<ll> _eval(const
vector<ll> &x, const vector<Poly> &up) const {
const int m = (int)x.size();
if (!m) return {};
vector<Poly> down(m * 2);
// down[1] = DivMod(up[1]).second;
// fi(2, m *
2) down[i] = down[i / 2].DivMod(up[i]).second;
down[1] = Poly(up[1])
.irev().isz(n()).Inv().irev()._tmul(m, *this);
fi(2, m * 2) down[i]
= up[i ^ 1]._tmul(up[i].n() - 1, down[i / 2]);
vector<ll> y(m);
fi(0, m) y[i] = down[m + i][0];
return y;
}
static vector<Poly> _tree1(const vector<ll> &x) {
const int m = (int)x.size();
vector<Poly> up(m * 2);
fi(0, m) up[m + i] = {(x[i] ? P - x[i] : 0), 1};
for (int i = m - 1; i
> 0; --i) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
return up;
}
vector
<ll> Eval(const vector<ll> &x) const { // 1e5, 1s
auto up = _tree1(x); return _eval(x, up);
}
static Poly Interpolate(const vector
<ll> &x, const vector<ll> &y) { // 1e5, 1.4s
const int m = (int)x.size();
vector<Poly> up = _tree1(x), down(m * 2);
vector<ll> z = up[1].Dx()._eval(x, up);
fi(0, m) z[i] = y[i] * ntt.minv(z[i]) % P;
fi(0, m) down[m + i] = {z[i]};
for (int i = m -
1; i > 0; --i) down[i] = down[i * 2].Mul(up[i
* 2 + 1]).iadd(down[i * 2 + 1].Mul(up[i * 2]));
return down[1];
}
Poly Ln() const { // (*this)[0] == 1, 1e5/170ms
return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // (*this)[0] == 0, 1e5/360ms

```

```

if (n() == 1) return {1};
Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
Poly Y = X.Ln(); Y[0] = P - 1;
fi(0, n())
if ((Y[i] = (*this)[i] - Y[i]) < 0) Y[i] += P;
return X.Mul(Y).isz(n());
}
// M := P(P - 1). If k >= M, k := k % M + M.
Poly Pow(ll k) const {
int nz = 0;
while (nz < n() && !(*this)[nz]) ++nz;
if (nz * min(k, (ll)n()) >= n()) return Poly(n());
if (!k) return Poly({1}, n());
Poly X(data() + nz, data() + nz + n() - nz * k);
const ll c = ntt.mpow(X[0], k % (P - 1));
return X.Ln().imul
(k % P).Exp().imul(c).irev().isz(n()).irev();
}
static ll
LinearRecursion(const vector<ll> &a, const vector
<ll> &coef, ll n) { // a_n = \sum c_j a_{n-j}
const int k = (int)a.size();
assert((int)coef.size() == k + 1);
Poly C(k + 1), W(Poly({1}, k), M = {0, 1});
fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
C[k] = 1;
while (n) {
if (n % 2) W = W.Mul(M).DivMod(C).second;
n /= 2, M = M.Mul(M).DivMod(C).second;
}
ll ret = 0;
fi(0, k) ret = (ret + W[i] * a[i]) % P;
return ret;
}
};
#undef fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

## 7.5 Value Polynomial [96cde9]

```

struct Poly {
mint base; // f(x) = poly[x - base]
vector<mint> poly;
Poly(mint b = 0, mint x = 0): base(b), poly(1, x) {}
mint get_val(const mint &x) {
if (x >= base && x < base + SZ(poly))
return poly[x - base];
mint rt = 0;
vector<mint> lmul(SZ(poly), 1), rmul(SZ(poly), 1);
for (int i = 1; i < SZ(poly); ++i)
lmul[i] = lmul[i - 1] * (x - (base + i - 1));
for (int i = SZ(poly) - 2; i >= 0; --i)
rmul[i] = rmul[i + 1] * (x - (base + i + 1));
for (int i = 0; i < SZ(poly); ++i)
rt += poly[i] * ifac[i] * inegfac
[SZ(poly) - 1 - i] * lmul[i] * rmul[i];
return rt;
}
void raise() { // g(x) = sigma{base:x} f(x)
if (SZ(poly) == 1 && poly[0] == 0)
return;
mint nw = get_val(base + SZ(poly));
poly.pb(nw);
for (int i = 1; i < SZ(poly); ++i)
poly[i] += poly[i - 1];
}
};

```

## 7.6 Newton's Method

Given  $F(x)$  where

$$F(x) = \sum_{i=0}^{\infty} \alpha_i (x - \beta)^i$$

for  $\beta$  being some constant. Polynomial  $P$  such that  $F(P) = 0$  can be found iteratively. Denote by  $Q_k$  the polynomials such that  $F(Q_k) = 0 \pmod{x^{2^k}}$ , then

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

## 8 Geometry

### 8.1 Basic [e38806]

```

bool same
(double a, double b) { return abs(a - b) < eps; }

```



```

struct P {
    double x, y;
    P() : x(0), y(0) {}
    P(double x, double y) : x(x), y(y) {}
    P operator + (P b) { return P(x + b.x, y + b.y); }
    P operator - (P b) { return P(x - b.x, y - b.y); }
    P operator * (double b) { return P(x * b, y * b); }
    P operator / (double b) { return P(x / b, y / b); }
    double operator * (P b) { return x * b.x + y * b.y; }
    double operator ^ (P b) { return x * b.y - y * b.x; }
    double abs() { return hypot(x, y); }
    P unit() { return *this / abs(); }
    P rot(double o) {
        double c = cos(o), s = sin(o);
        return P(c * x - s * y, s * x + c * y);
    }
    double angle() { return atan2(y, x); }
};

struct L {
    // ax + by + c = 0
    double a, b, c, o;
    P pa, pb;
    L() : a(0), b(0), c(0), o(0), pa(), pb() {}
    L(P pa, P pb) : a(pa.y - pb.y), b(pb.x - pa.x),
        c(pa ^ pb), o(atan2(-a, b)), pa(pa), pb(pb) {}
    P project(P p) { return pa + (pb - pa).unit() * ((pb - pa) * (p - pa) / (pb - pa).abs()); }
    P reflect(P p) { return p + (project(p) - p) * 2; }
    double get_ratio(P p) { return (p - pa) * (pb - pa) / ((pb - pa).abs() * (pb - pa).abs()); }
    bool inside(P p) { return min(pa.x, pb.x) <= p.x && p.x <= max(pa.x, pb.x) && min(pa.y, pb.y) <= p.y && p.y <= max(pa.y, pb.y) && same(a * p.x + b * p.y, -c); }
};

bool SegmentIntersect(P p1, P p2, P p3, P p4) {
    if (max(p1.x, p2.x) < min(p3.x, p4.x) ||
        max(p3.x, p4.x) < min(p1.x, p2.x)) return false;
    if (max(p1.y, p2.y) < min(p3.y, p4.y) ||
        max(p3.y, p4.y) < min(p1.y, p2.y)) return false;
    return sign((p3 - p1) ^ (p4 - p1)) * sign((p3 - p2) ^ (p4 - p2)) <= 0 &&
        sign((p1 - p3) ^ (p2 - p3)) * sign((p1 - p4) ^ (p2 - p4)) <= 0;
}

bool parallel(L x, L y) { return same(x.a * y.b, x.b * y.a); }

P Intersect(L x, L y) { return P(-x.b * y.c + x.c * y.b, x.a * y.c - x.c * y.a) / (-x.a * y.b + x.b * y.a); }

```

## 8.2 KD Tree [375ca2]

```

namespace kdt {
int root, lc[maxn],
    rc[maxn], xl[maxn], xr[maxn], yl[maxn], yr[maxn];
point p[maxn];
int build(int l, int r, int dep = 0) {
    if (l == r) return -1;
    function<bool(const point &, const point &>> f = [dep](const point &a, const point &b) {
        if (dep & 1) return a.x < b.x;
        else return a.y < b.y;
    };
    int m = (l + r) >> 1;
    nth_element(p + l, p + m, p + r, f);
    xl[m] = xr[m] = p[m].x;
    yl[m] = yr[m] = p[m].y;
    lc[m] = build(l, m, dep + 1);
    if (~lc[m]) {
        xl[m] = min(xl[m], xl[lc[m]]);
        xr[m] = max(xr[m], xr[lc[m]]);
        yl[m] = min(yl[m], yl[lc[m]]);
        yr[m] = max(yr[m], yr[lc[m]]);
    }
    rc[m] = build(m + 1, r, dep + 1);
    if (~rc[m]) {
        xl[m] = min(xl[m], xl[rc[m]]);
        xr[m] = max(xr[m], xr[rc[m]]);
        yl[m] = min(yl[m], yl[rc[m]]);
        yr[m] = max(yr[m], yr[rc[m]]);
    }
}

```

```

    return m;
}

bool bound(const point &q, int o, long long d) {
    double ds = sqrt(d + 1.0);
    if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
        q.y < yl[o] - ds || q.y > yr[o] + ds) return false;
    return true;
}

long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 1ll * (a.x - b.x) +
        (a.y - b.y) * 1ll * (a.y - b.y);
}

void dfs(
    const point &q, long long &d, int o, int dep = 0) {
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1)
        && q.x < p[o].x || !(dep & 1) && q.y < p[o].y) {
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
}

void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
}

long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
}
}

```

## 8.3 Sector Area [c41fb7]

```

// calc area of sector which include a, b
double SectorArea(P a, P b, double r) {
    double o = atan2(a.y, a.x) - atan2(b.y, b.x);
    while (o <= 0) o += 2 * pi;
    while (o >= 2 * pi) o -= 2 * pi;
    o = min(o, 2 * pi - o);
    return r * r * o / 2;
}

```

## 8.4 Half Plane Intersection [f7274e]

```

bool jizz(L l1, L l2, L l3) {
    P p = Intersect(l2, l3);
    return ((l1.pb - l1.pa) ^ (p - l1.pa)) < -eps;
}

bool cmp(const L &a, const L &b) {
    return same(
        a.o, b.o) ? (((b.pb - b.pa) ^ (a.pb - b.pa)) > eps) : a.o < b.o;
}

// available area for L l is (l.pb - l.pa) ^ (p - l.pa) > 0
vector<P> HPI(vector<L> &ls) {
    sort(ls.begin(), ls.end(), cmp);
    vector<L> pls(1, ls[0]);
    for (int i = 0; i < (int) ls.size(); ++i) if (!
        same(ls[i].o, pls.back().o)) pls.push_back(ls[i]);
    deque<int> dq; dq.push_back(0); dq.push_back(1);
#define meow(a, b, c
    ) while(dq.size() > 1u && jizz(pls[a], pls[b], pls[c]))
    for (int i = 2; i < (int) pls.size(); ++i) {
        meow(i, dq.back(), dq[dq.size() - 2]) dq.pop_back();
        meow(i, dq[0], dq[1]) dq.pop_front();
        dq.push_back(i);
    }
    meow(dq
        .front(), dq.back(), dq[dq.size() - 2]) dq.pop_back();
    meow(dq.back(), dq[0], dq[1]) dq.pop_front();
    if (dq.size() < 3u) return vector<P>(); // no solution or solution is not a convex
    vector<P> rt;
    for (int i = 0; i < (int) dq.size(); ++i) rt.push_back(
        Intersect(pls[dq[i]], pls[dq[(i + 1) % dq.size()]]));
    return rt;
}

```

## 8.5 Rotating Sweep Line [0411f0]

```

void rotatingSweepLine(vector<pair<int,int>> &ps){
    int n=int(ps.size());
    vector<int> id(n),pos(n);
    vector<pair<int,int>> line(n*(n-1)/2);
    int m=-1;
    for(int i=0;i<n;++i)for
        (int j=i+1;j<n;++j)line[++m]=make_pair(i,j); ++m;
    sort(line.begin(),line.end(),[&](const
        pair<int,int> &a,const pair<int,int> &b)->bool{
        if(ps
            [a.first].first==ps[a.second].first)return 0;
        if(ps
            [b.first].first==ps[b.second].first)return 1;
        return (double
            )(ps[a.first].second-ps[a.second].second)/(ps
            [a.first].first-ps[a.second].first) < (double
            )(ps[b.first].second-ps[b.second].second
            )/(ps[b.first].first-ps[b.second].first);
        });
    for(int i=0;i<n;++i)id[i]=i;
    sort(id.begin(),id.end(),[&](const
        int &a,const int &b){ return ps[a]<ps[b]; });
    for(int i=0;i<n;++i)pos[id[i]]=i;

    for(int i=0;i<n;++i){
        auto l=line[i];
        // meow
        tie(pos[l.first],pos[l.second],
            id[pos[l.first]],id[pos[l.second]])=make_tuple
            (pos[l.second],pos[l.first],l.second,l.first);
    }
}

```

## 8.6 Triangle Center [4e8ee9]

```

Point TriangleCircumCenter(Point a, Point b, Point c) {
    Point res;
    double a1 = atan2(b.y - a.y, b.x - a.x) + pi / 2;
    double a2 = atan2(c.y - b.y, c.x - b.x) + pi / 2;
    double ax = (a.x + b.x) / 2;
    double ay = (a.y + b.y) / 2;
    double bx = (c.x + b.x) / 2;
    double by = (c.y + b.y) / 2;
    double r1 = (sin(a2) * (ax - bx) + cos(a2) * (by
        - ay)) / (sin(a1) * cos(a2) - sin(a2) * cos(a1));
    return Point(ax + r1 * cos(a1), ay + r1 * sin(a1));
}

Point TriangleMassCenter(Point a, Point b, Point c) {
    return (a + b + c) / 3.0;
}

Point TriangleOrthoCenter(Point a, Point b, Point c) {
    return TriangleMassCenter(a, b
        , c) * 3.0 - TriangleCircumCenter(a, b, c) * 2.0;
}

Point TriangleInnerCenter(Point a, Point b, Point c) {
    Point res;
    double la = len(b - c);
    double lb = len(a - c);
    double lc = len(a - b);
    res.x = (
        la * a.x + lb * b.x + lc * c.x) / (la + lb + lc);
    res.y = (
        la * a.y + lb * b.y + lc * c.y) / (la + lb + lc);
    return res;
}

```

## 8.7 Polygon Center [ee6ff0]

```

Point BaryCenter(vector<Point> &p, int n) {
    Point res(0, 0);
    double s = 0.0, t;
    for (int i = 1; i < p.size() - 1; i++) {
        t = Cross(p[i] - p[0], p[i + 1] - p[0]) / 2;
        s += t;
        res.x += (p[0].x + p[i].x + p[i + 1].x) * t;
        res.y += (p[0].y + p[i].y + p[i + 1].y) * t;
    }
    res.x /= (3 * s);
    res.y /= (3 * s);
    return res;
}

```

## 8.8 Maximum Triangle [3a6d38]

```

double ConvexHullMaxTriangleArea
    (Point p[], int res[], int chnum) {
    double area = 0, tmp;
    res[chnum] = res[0];
    for (int i = 0, j = 1, k = 2; i < chnum; i++) {
        while (fabs(Cross(p[
            res[j]] - p[res[i]], p[res[(k + 1) % chnum]] -
            p[res[i]])) > fabs(Cross(p[res[j]] - p[res[i]],
            p[res[k]] - p[res[i]]))) k = (k + 1) % chnum;
        tmp = fabs(Cross(
            p[res[j]] - p[res[i]], p[res[k]] - p[res[i]]));
        if (tmp > area) area = tmp;
        while (fabs(Cross(p[
            res[(j + 1) % chnum]] - p[res[i]], p[res[k]] -
            p[res[i]])) > fabs(Cross(p[res[j]] - p[res[i]],
            p[res[k]] - p[res[i]]))) j = (j + 1) % chnum;
        tmp = fabs(Cross(
            p[res[j]] - p[res[i]], p[res[k]] - p[res[i]]));
        if (tmp > area) area = tmp;
    }
    return area / 2;
}

```

## 8.9 Point in Polygon [0a9a66]

```

int pip(vector<P> ps, P p) {
    int c = 0;
    for (int i = 0; i < ps.size(); ++i) {
        int a = i, b = (i + 1) % ps.size();
        L l(ps[a], ps[b]);
        P q = l.project(p);
        if ((p - q).abs() < eps && l.inside(q)) return 1;
        if (same(ps[
            a].y, ps[b].y) && same(ps[a].y, p.y)) continue;
        if (ps[a].y > ps[b].y) swap(a, b);
        if (ps[a].y <= p.y && p.y <
            ps[b].y && p.x <= ps[a].x + (ps[b].x - ps[a].x
            ) / (ps[b].y - ps[a].y) * (p.y - ps[a].y)) ++c;
    }
    return (c & 1) * 2;
}

```

## 8.10 Circle [466c44]

```

struct C {
    P c;
    double r;
    C(P c = P(0, 0), double r = 0) : c(c), r(r) {}
};

vector<P> Intersect(C a, C b) {
    if (a.r > b.r) swap(a, b);
    double d = (a.c - b.c).abs();
    vector<P> p;
    if (same(a.r + b.r,
        d)) p.push_back(a.c + (b.c - a.c).unit() * a.r);
    else if (a.r + b.r > d && d + a.r >= b.r) {
        double o = acos
            ((sq(a.r) + sq(d) - sq(b.r)) / (2 * a.r * d));
        P i = (b.c - a.c).unit();
        p.push_back(a.c + i.rot(o) * a.r);
        p.push_back(a.c + i.rot(-o) * a.r);
    }
    return p;
}

double IntersectArea(C a, C b) {
    if (a.r > b.r) swap(a, b);
    double d = (a.c - b.c).abs();
    if (d >= a.r + b.r - eps) return 0;
    if (d + a.r <= b.r + eps) return sq(a.r) * acos(-1);
    double p = acos
        ((sq(a.r) + sq(d) - sq(b.r)) / (2 * a.r * d));
    double q = acos
        ((sq(b.r) + sq(d) - sq(a.r)) / (2 * b.r * d));
    return p * sq(a.r) + q * sq(b.r) - a.r * d * sin(p);
}

```

```

// remove second
    level if to get points for line (defalut: segment)
vector<P> CircleCrossLine(P a, P b, P o, double r) {
    double x = b.x - a.x, y = b.y - a.y, A = sq(x) + sq(y)
        , B = 2 * x * (a.x - o.x) + 2 * y * (a.y - o.y);
    double C = sq(a.x - o.x
        ) + sq(a.y - o.y) - sq(r), d = B * B - 4 * A * C;
    vector<P> t;
    if (d >= -eps) {
        d = max(0., d);
        double i = (-B - sqrt(d)) / (2 * A);

```

```

    double j = (-B + sqrt(d)) / (2 * A);
    if (i - 1.0 <= eps && i >=
        -eps) t.emplace_back(a.x + i * x, a.y + i * y);
    if (j - 1.0 <= eps && j >=
        -eps) t.emplace_back(a.x + j * x, a.y + j * y);
}
return t;
}
// calc area
// intersect by circle with radius r and triangle OAB
double AreaOfCircleTriangle(P a, P b, double r) {
    bool ina = a.abs() < r, inb = b.abs() < r;
    auto p = CircleCrossLine(a, b, P(0, 0), r);
    if (ina) {
        if (inb) return abs(a ^ b) / 2;
        return SectorArea(b, p[0], r) + abs(a ^ p[0]) / 2;
    }
    if (inb) return
        SectorArea(p[0], a, r) + abs(p[0] ^ b) / 2;
    if (p.size() == 2u) return SectorArea(a, p[0], r)
        + SectorArea(p[1], b, r) + abs(p[0] ^ p[1]) / 2;
    else return SectorArea(a, b, r);
}
// for any triangle
double AreaOfCircleTriangle(vector<P> ps, double r) {
    double ans = 0;
    for (int i = 0; i < 3; ++i) {
        int j = (i + 1) % 3;
        double o = atan2
            (ps[i].y, ps[i].x) - atan2(ps[j].y, ps[j].x);
        if (o >= pi) o = o - 2 * pi;
        if (o <= -pi) o = o + 2 * pi;
        ans += AreaOfCircleTriangle
            (ps[i], ps[j], r) * (o >= 0 ? 1 : -1);
    }
    return abs(ans);
}

```

## 8.11 Tangent of Circles and Points to Circle [19eb58]

```

vector<L> tangent(C a, C b) {
#define Pij \
    P i = (b.c - a.c).unit() * a.r, j = P(i.y, -i.x);\
    z.emplace_back(a.c + i, a.c + i + j);
#define deo(I,J) \
    double d = (a
        .c - b.c).abs(), e = a.r I b.r, o = acos(e / d);\
    P i =
        (b.c - a.c).unit(), j = i.rot(o), k = i.rot(-o);\
    z.emplace_back(a.c + j * a.r, b.c J j * b.r);\
    z.emplace_back(a.c + k * a.r, b.c J k * b.r);
    if (a.r < b.r) swap(a, b);
    vector<L> z;
    if ((a.c - b.c).abs() + b.r < a.r) return z;
    else if (same((a.c - b.c).abs() + b.r, a.r)) { Pij; }
    else {
        deo(+,+);
        if (same(d, a.r + b.r)) { Pij; }
        else if (d > a.r + b.r) { deo(+,-); }
    }
    return z;
}

vector<L> tangent(C c, P p) {
    vector<L> z;
    double d = (p - c.c).abs();
    if (same(d, c.r)) {
        P i = (p - c.c).rot(pi / 2);
        z.emplace_back(p, p + i);
    } else if (d > c.r) {
        double o = acos(c.r / d);
        P i = (p - c.c).unit
            (), j = i.rot(o) * c.r, k = i.rot(-o) * c.r;
        z.emplace_back(c.c + j, p);
        z.emplace_back(c.c + k, p);
    }
    return z;
}

```

## 8.12 Area of Union of Circles [490636]

```

vector<pair<double, double>> CoverSegment(C &a, C &b) {
    double d = (a.c - b.c).abs();
    vector<pair<double, double>> res;
    if (same(a.r + b.r, d)) ;
    else if (d <= abs(a.r - b.r) + eps) {

```

```

        if (a.r < b.r) res.emplace_back(0, 2 * pi);
    } else if (d < abs(a.r + b.r) - eps) {
        double o = acos((sq(a.r) + sq(d) - sq(b
            .r)) / (2 * a.r * d)), z = (b.c - a.c).angle();
        if (z < 0) z += 2 * pi;
        double l = z - o, r = z + o;
        if (l < 0) l += 2 * pi;
        if (r > 2 * pi) r -= 2 * pi;
        if (l > r) res.emplace_back
            (l, 2 * pi), res.emplace_back(0, r);
        else res.emplace_back(l, r);
    }
    return res;
}
double CircleUnionArea
    (vector<C> c) { // circle should be identical
    int n = c.size();
    double a = 0, w;
    for (int i = 0; w = 0, i < n; ++i) {
        vector<pair<double, double>> s = {{2 * pi, 9}}, z;
        for (int j = 0; j < n; ++j) if (i != j) {
            z = CoverSegment(c[i], c[j]);
            for (auto &e : z) s.push_back(e);
        }
        sort(s.begin(), s.end());
        auto F = [&](double t) { return c[i].r * (c[i].r *
            t + c[i].c.x * sin(t) - c[i].c.y * cos(t)); };
        for (auto &e : s) {
            if (e.first > w) a += F(e.first) - F(w);
            w = max(w, e.second);
        }
    }
    return a * 0.5;
}

```

## 8.13 Minimum Distance of 2 Polygons [7eb8bb]

```

// p, q is convex
double TwoConvexHullMinDist
    (Point P[], Point Q[], int n, int m) {
    int YMinP = 0, YMaxQ = 0;
    double tmp, ans = 999999999;
    for (i =
        0; i < n; ++i) if (P[i].y < P[YMinP].y) YMinP = i;
    for (i =
        0; i < m; ++i) if (Q[i].y > Q[YMaxQ].y) YMaxQ = i;
    P[n] = P[0], Q[m] = Q[0];
    for (int i = 0; i < n; ++i) {
        while (tmp = Cross(
            Q[YMaxQ + 1] - P[YMinP + 1], P[YMinP] - P[YMinP
                + 1]) > Cross(Q[YMaxQ] - P[YMinP + 1], P[YMinP
                    ] - P[YMinP + 1])) YMaxQ = (YMaxQ + 1) % m;
        if (tmp < 0) ans = min(ans, PointToSegDist
            (P[YMinP], P[YMinP + 1], Q[YMaxQ]));
        else ans = min(ans, TwoSegMinDist(P[
            YMinP], P[YMinP + 1], Q[YMaxQ], Q[YMaxQ + 1]));
        YMinP = (YMinP + 1) % n;
    }
    return ans;
}

```

## 8.14 2D Convex Hull [65eaab]

```

bool operator<(const P &a, const P &b) {
    return same(a.x, b.x) ? a.y < b.y : a.x < b.x;
}
bool operator>(const P &a, const P &b) {
    return same(a.x, b.x) ? a.y > b.y : a.x > b.x;
}

#define crx(a, b, c) ((b - a) ^ (c - a))

vector<P> convex(vector<P> ps) {
    vector<P> p;
    sort(ps.begin(), ps.end(), [&](P a, P b) { return
        same(a.x, b.x) ? a.y < b.y : a.x < b.x; });
    for (int i = 0; i < ps.size(); ++i) {
        while (p.size() >= 2 && crx(p[p.size() -
            2], ps[i], p[p.size() - 1]) >= 0) p.pop_back();
        p.push_back(ps[i]);
    }
    int t = p.size();
    for (int i = (int)p.size() - 2; i >= 0; --i) {
        while (p.size() > t && crx(p[p.size() -
            2], ps[i], p[p.size() - 1]) >= 0) p.pop_back();
        p.push_back(ps[i]);
    }
}

```

```

    p.pop_back();
    return p;
}

int sgn(double
    x) { return same(x, 0) ? 0 : x > 0 ? 1 : -1; }

P isLL(P p1, P p2, P q1, P q2) {
    double a = crx(q1, q2, p1), b = -crx(q1, q2, p2);
    return (p1 * b + p2 * a) / (a + b);
}

struct CH {
    int n;
    vector<P> p, u, d;
    CH() {}
    CH(vector<P> ps) : p(ps) {
        n = ps.size();
        rotate(p.begin(), min_element(p.begin(), p.end()), p.end());
        auto t = max_element(p.begin(), p.end());
        d = vector<P>(p.begin(), next(t));
        u = vector<P>(t, p.end()); u.push_back(p[0]);
    }
    int find(vector<P> &v, P d) {
        int l = 0, r = v.size();
        while (l + 5 < r) {
            int L = (l * 2 + r) / 3, R = (l + r * 2) / 3;
            if (v[L] * d > v[R] * d) r = R;
            else l = L;
        }
        int x = l;
        for (int i = l + 1; i < r; ++i) if (v[i] * d > v[x] * d) x = i;
        return x;
    }
    int findFarest(P v) {
        if (v.y > 0 || v.y == 0 && v.x > 0) return
            ((int)d.size() - 1 + find(u, v)) % p.size();
        return find(d, v);
    }
}

P get(int l, int r, P a, P b) {
    int s = sgn(crx(a, b, p[l % n]));
    while (l + 1 < r) {
        int m = (l + r) >> 1;
        if (sgn(crx(a, b, p[m % n])) == s) l = m;
        else r = m;
    }
    return isLL(a, b, p[l % n], p[(l + 1) % n]);
}

vector<P> getLineIntersect(P a, P b) {
    int X = findFarest((b - a).rot(pi / 2));
    int Y = findFarest((a - b).rot(pi / 2));
    if (X > Y) swap(X, Y);
    if (sgn(crx(a, b, p[X])) * sgn(crx(a, b, p[Y])) < 0)
        return {get(X, Y, a, b), get(Y, X + n, a, b)};
    return {}; // tangent case falls here
}

void update_tangent(P q, int i, int &a, int &b) {
    if (sgn(crx(q, p[a], p[i])) > 0) a = i;
    if (sgn(crx(q, p[b], p[i])) < 0) b = i;
}

void bs(int l, int r, P q, int &a, int &b) {
    if (l == r) return;
    update_tangent(q, l % n, a, b);
    int s = sgn(crx(q, p[l % n], p[(l + 1) % n]));
    while (l + 1 < r) {
        int m = (l + r) >> 1;
        if (sgn(crx(q, p[m % n], p[(m + 1) % n])) == s) l = m;
        else r = m;
    }
    update_tangent(q, r % n, a, b);
}

int x = l;
for (int i = l + 1; i < r; ++i) if (v[i] * d > v[x] * d) x = i;
return x;
}

int findFarest(P v) {
    if (v.y > 0 || v.y == 0 && v.x > 0) return
        ((int)d.size() - 1 + find(u, v)) % p.size();
    return find(d, v);
}

P get(int l, int r, P a, P b) {

```

```

    int s = sgn(crx(a, b, p[l % n]));
    while (l + 1 < r) {
        int m = (l + r) >> 1;
        if (sgn(crx(a, b, p[m % n])) == s) l = m;
        else r = m;
    }
    return isLL(a, b, p[l % n], p[(l + 1) % n]);
}

vector<P> getIS(P a, P b) {
    int X = findFarest((b - a).spin(pi / 2));
    int Y = findFarest((a - b).spin(pi / 2));
    if (X > Y) swap(X, Y);
    if (sgn(crx(a, b, p[X])) * sgn(crx(a, b, p[Y])) <
        0) return {get(X, Y, a, b), get(Y, X + n, a, b)};
    return {};
}

void update_tangent(P q, int i, int &a, int &b) {
    if (sgn(crx(q, p[a], p[i])) > 0) a = i;
    if (sgn(crx(q, p[b], p[i])) < 0) b = i;
}

void bs(int l, int r, P q, int &a, int &b) {
    if (l == r) return;
    update_tangent(q, l % n, a, b);
    int s = sgn(crx(q, p[l % n], p[(l + 1) % n]));
    while (l + 1 < r) {
        int m = (l + r) >> 1;
        if (sgn(crx(q, p[m % n], p[(m + 1) % n])) == s) l = m;
        else r = m;
    }
    update_tangent(q, r % n, a, b);
}

bool contain(P p) {
    if (p.x < d[0].x || p.x > d.back().x) return 0;
    auto it
        = lower_bound(d.begin(), d.end(), P(p.x, -1e12));
    if (it->x == p.x) {
        if (it->y > p.y) return 0;
    } else if (crx(*prev(it), *it, p) < -eps) return 0;
    it = lower_bound(u.begin(), u.end(), P(p.x, 1e12), greater<P>());
    if (it->x == p.x) {
        if (it->y < p.y) return 0;
    } else if (crx(*prev(it), *it, p) < -eps) return 0;
    return 1;
}

bool get_tangent(P p, int &a, int &b) { // b -> a
    if (contain(p)) return 0;
    a = b = 0;
    int i
        = lower_bound(d.begin(), d.end(), p) - d.begin();
    bs(0, i, p, a, b);
    bs(i, d.size(), p, a, b);
    i = lower_bound(u.begin(), u.end(), p, greater<P>()) - u.begin();
    bs((int)d.size() - 1, (int)d.size() - 1 + i, p, a, b);
    bs((int)d.size() - 1 + i, (int)d.size() - 1 + u.size(), p, a, b);
    return 1;
}
};

```

## 8.15 3D Convex Hull [29e4a9]

```

double
    absvol(const P a, const P b, const P c, const P d) {
    return abs(((b-a)^(c-a))*(d-a))/6;
}

struct convex3D {
    static const int maxn=1010;
    struct T {
        int a,b,c;
        bool res;
        T() {}
        T(int a, int b, int c, bool res=1):a(a),b(b),c(c),res(res) {}
    };
    int n,m;
    P p[maxn];
    T f[maxn*8];
    int id[maxn][maxn];
    bool on(T &t, P &q) {
        return ((p[t.c]-p[t.b])^(p[t.a]-p[t.b]))*(q-p[t.a])>eps;
    }
    void meow(int q, int a, int b) {

```

### 8.16 Minimum Enclosing Circle [fc0e72]

```

circle_min_enclosing(vector<pt> &p) {
    random_shuffle(p.begin(), p.end());
    double r = 0.0;
    pt cent;
    for (int i = 0; i < p.size(); ++i) {
        if (norm2(cent - p[i]) <= r) continue;
        cent = p[i];
        r = 0.0;
        for (int j = 0; j < i; ++j) {
            if (norm2(cent - p[j]) <= r) continue;
            cent = (p[i] + p[j]) / 2;
            r = norm2(p[j] - cent);
            for (int k = 0; k < j; ++k) {
                if (norm2(cent - p[k]) <= r) continue;
            }
        }
    }
}

```

## 8.17 Closest Pair [f6de57]

**9 Else**

```

/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);
    while (r - l > 1) {
        int m = (l + r) / 2;
        if (pred(0, m) ? rv : pred(m, (m + 1) % n)) r = m;
        else l = m;
    }
    return pred(l, r % n) ? l : r % n;
}

```

```

/*
Mo's Algorithm With modification
Block:  $N^{\frac{2}{3}}$ , Complexity:  $N^{\frac{5}{3}}$ 
*/
struct Query {
    int L, R, LBid, RBid, T;
    Query(int l, int r, int t):
        L(l), R(r), LBid(l / blk), RBid(r / blk), T(t) {}
    bool operator<(const Query &q) const {
        if (LBid != q.LBid) return LBid < q.LBid;
        if (RBid != q.RBid) return RBid < q.RBid;
        return T < b.T;
    }
};

void solve(vector<Query> query) {
    sort(ALL(query));
    int L=0, R=0, T=-1;
    for (auto q : query) {
        while (T < q.T) addTime(L, R, ++T); // TODO
        while (T > q.T) subTime(L, R, T--); // TODO
        while (R < q.R) add(arr[++R]); // TODO
        while (L > q.L) add(arr[--L]); // TODO
        while (R > q.R) sub(arr[R--]); // TODO
        while (L < q.L) sub(arr[L++]); // TODO
        // answer query
    }
}

```

```
/*
Mo's Algorithm On Tree
Preprocess:
```



```

1) LCA
2) dfs with in[u] = dft++, out[u] = dft++
3) ord[in[u]] = ord[out[u]] = u
4) bitset<MAXN> inset
*/
struct Query {
    int L, R, LBid, lca;
    Query(int u, int v) {
        int c = LCA(u, v);
        if (c == u || c == v)
            q.lca = -1, q.L = out[c ^ u ^ v], q.R = out[c];
        else if (out[u] < in[v])
            q.lca = c, q.L = out[u], q.R = in[v];
        else
            q.lca = c, q.L = out[v], q.R = in[u];
        q.Lid = q.L / blk;
    }
    bool operator<(const Query &q) const {
        if (LBid != q.LBid) return LBid < q.LBid;
        return R < q.R;
    }
};
void flip(int x) {
    if (inset[x]) sub(arr[x]); // TODO
    else add(arr[x]); // TODO
    inset[x] = ~inset[x];
}
void solve(vector<Query> query) {
    sort(ALL(query));
    int L = 0, R = 0;
    for (auto q : query) {
        while (R < q.R) flip(ord[++R]);
        while (L > q.L) flip(ord[--L]);
        while (R > q.R) flip(ord[R--]);
        while (L < q.L) flip(ord[L++]);
        if (~q.lca) add(arr[q.lca]);
        // answer query
        if (~q.lca) sub(arr[q.lca]);
    }
}

```

## 9.4 Additional Mo's Algorithm Trick

- Mo's Algorithm With Addition Only
  - Sort query same as the normal Mo's algorithm.
  - Foreach query  $[l, r]$ :
  - If  $l/blk = r/blk$ , brute-force.
  - If  $l/blk \neq curL/blk$ , initialize  $curL := (l/blk + 1) \cdot blk$ ,  $curR := curL - 1$
  - If  $r > curR$ , increase  $curR$
  - decrease  $curL$  to fit  $l$ , and then undo after answering
- Mo's Algorithm With Offline Second Time
  - Require: Changing answer  $\equiv$  adding  $f([l, r], r+1)$ .
  - Require:  $f([l, r], r+1) = f([l, r], r+1) - f([l, l], r+1)$ .
  - Part1: Answer all  $f([l, r], r+1)$  first.
  - Part2: Store  $curR \rightarrow R$  for  $curL$  (reduce the space to  $O(N)$ ), and then answer them by the second offline algorithm.
  - Note: You must do the above symmetrically for the left boundaries.

## 9.5 Hilbert Curve [1274a3]

```

ll hilbert(int n, int x, int y) {
    ll res = 0;
    for (int s = n / 2; s; s >>= 1) {
        int rx = (x & s) > 0;
        int ry = (y & s) > 0;
        res += s * 111 * s * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) x = s - 1 - x, y = s - 1 - y;
            swap(x, y);
        }
    }
    return res;
} // n = 2^k

```

## 9.6 Dynamic Convex Trick\* [673ffd]

```

// only works for integer coordinates!! maintain max
struct Line {
    mutable ll a, b, p;
    bool operator
        <(const Line &rhs) const { return a < rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    ll Div(ll a,
        ll b) { return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {

```

```

        if (y == end()) { x->p = kInf; return 0; }
        if (x
            ->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void addline(ll a, ll b) {
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin
            () && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin
            () && (--x)->p >= y->p) isect(x, erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};

```

## 9.7 All LCS\* [78a378]

```

void all_lcs(string s, string t) { // 0-base
    vector<int> h(SZ(t));
    iota(ALL(h), 0);
    for (int a = 0; a < SZ(s); ++a) {
        int v = -1;
        for (int c = 0; c < SZ(t); ++c)
            if (s[a] == t[c] || h[c] < v)
                swap(h[c], v);
        // LCS[s[0, a], t[b, c]] =
        // c - b + 1 - sum([h[i] >= b] | i <= c)
        // h[i] might become -1 !!
    }
}

```

## 9.8 Adaptive Simpson\* [4074b3]

```

template<typename Func, typename d = double>
struct Simpson {
    using pdd = pair<d, d>;
    Func f;
    pdd mix(pdd l, pdd r, optional<d> fm = {}) {
        d h = (r.X - l.X) / 2, v = fm.value_or(f(l.X + h));
        return {v, h / 3 * (l.Y + 4 * v + r.Y)};
    }
    d eval(pdd l, pdd r, d fm, d eps) {
        pdd m((l.X + r.X) / 2, fm);
        d s = mix(l, r, fm).second;
        auto [flm, sl] = mix(l, m);
        auto [fmr, sr] = mix(m, r);
        d delta = sl + sr - s;
        if (abs(delta)
            ) <= 15 * eps) return sl + sr + delta / 15;
        return eval(l, m, flm, eps / 2) +
            eval(m, r, fmr, eps / 2);
    }
    d eval(d l, d r, d eps) {
        return eval
            ({l, f(l)}, {r, f(r)}, f((l + r) / 2), eps);
    }
    d eval2(d l, d r, d eps, int k = 997) {
        d h = (r - l) / k, s = 0;
        for (int i = 0; i < k; ++i, l += h)
            s += eval(l, l + h, eps / k);
        return s;
    }
};
template<typename Func>
Simpson<Func> make_simpson(Func f) { return {f}; }

```

## 9.9 Simulated Annealing [de78c6]

```

double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 100000; ++it) {
    // ans:
    answer, nw: current value, rnd(): mt19937 rnd()
    if (exp(-(nw - ans
        ) / factor) >= (double)(rnd() % base) / base)
        ans = nw;
    factor *= 0.99995;
}

```

## 9.10 Tree Hash\* [34aae5]

```

ull seed;
ull shift(ull x) {

```

```

x ^= x << 13;
x ^= x >> 7;
x ^= x << 17;
return x;
}
ull dfs(int u, int f) {
    ull sum = seed;
    for (int i : G[u])
        if (i != f)
            sum += shift(dfs(i, u));
    return sum;
}

```

## 9.11 Binary Search On Fraction [765c5a]

```

struct Q {
    ll p, q;
    Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p,q <= N
Q frac_bs(ll N) {
    Q lo{0, 1}, hi{1, 0};
    if (pred(lo)) return lo;
    assert(pred(hi));
    bool dir = 1, L = 1, H = 1;
    for (; L || H; dir = !dir) {
        ll len = 0, step = 1;
        for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
            if (Q mid = hi.go(lo, len + step);
                mid.p > N || mid.q > N || dir ^ pred(mid))
                t++;
            else len += step;
        swap(lo, hi = hi.go(lo, len));
        (dir ? L : H) = !!len;
    }
    return dir ? hi : lo;
}

```

## 9.12 Bitset LCS [330ab1]

```

cin >> n >> m;
for (int i = 1, x; i <= n; ++i)
    cin >> x, p[x].set(i);
for (int i = 1, x; i <= m; i++) {
    cin >> x, (g = f) |= p[x];
    f.shiftLeftByOne(), f.set(0);
    ((f = g - f) ^= g) &= g;
}
cout << f.count() << '\n';

```

## 9.13 N Queens Problem [d1fcc]

```

void solve
(vector<int> &ret, int n) { // no sol when n=2,3
    if (n % 6 == 2) {
        for (int i = 2; i <= n; i += 2) ret.pb(i);
        ret.pb(3); ret.pb(1);
        for (int i = 7; i <= n; i += 2) ret.pb(i);
        ret.pb(5);
    } else if (n % 6 == 3) {
        for (int i = 4; i <= n; i += 2) ret.pb(i);
        ret.pb(2);
        for (int i = 5; i <= n; i += 2) ret.pb(i);
        ret.pb(1); ret.pb(3);
    } else {
        for (int i = 2; i <= n; i += 2) ret.pb(i);
        for (int i = 1; i <= n; i += 2) ret.pb(i);
    }
}

```

# 10 Python

## 10.1 Misc

```

# decimal
from decimal import *
setcontext(Context(prec
    =MAX_PREC, Emax=MAX_EMAX, rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction
    ('3.14159').limit_denominator(10).numerator # 22
# N*M
map(int, input().split())
arr2d = [ [ list
    (map(int, input().split())) ] for i in range(N) ]

```

```

#a^b%M
pow(a, b, M)
#random
from random import *
randrange(L, R, step) # [L, R) L+k*step
randint(L, R) # int from [L, R]
choice(list) # pick 1 item from list
choices(list, k) # pick k item
shuffle(list)
Uniform(L, R) # float from [L, R]
#print
print(f"num: {num}, pi: {pi:.2f}, str: {text}")
print(1, 2, 3, 4, sep=' | ', end=' <-- END\n')
# file IO
r = open("filename.in")
a = r.read() # read whole content into one string
w = open("filename.out", "w")
w.write('123\n')
# IO redirection
import sys
sys.stdin = open('filename.in')
sys.stdout = open('filename.out', 'w')
print("123", file=sys.stderr)

```

## 10.2 Stress testing

```

i=0
while true
do
    echo $i
    i=$((i+1))
    python3 gen.py > input.txt
    python3 brute.py < input.txt > brute.txt
    ./sol < input.txt > output.txt
    diff output.txt brute.txt
    if [ $? -ne 0 ]
    then
        break
    fi
done

```