Heaps

The *Priority Queue ADT*

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Data Model:

Operations:

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Data Model: A collection of priority-value pairs, that come out in an increasing priorities order

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- ***p.delete_min()***

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- ***p.delete_min()*** removes and returns the pair ***(pri, val)*** with the lowest priority in ***p***, or raises an ***Exception***, if ***p*** is empty.

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- ***p.delete_min()*** removes and returns the pair ***(pri, val)*** with the lowest priority in ***p***, or raises an ***Exception***, if ***p*** is empty.
- ***len(p)*** returns the number of items in ***p***
- ***p.is_empty()*** returns ***True*** if ***p*** is empty, or ***False*** otherwise

The *Priority Queue ADT*

<u>Minimum Priority Queue</u>:

<u>Maximum Priority Queue</u>:

The *Priority Queue ADT*

<u>Minimum Priority Queue</u>:

p = MinPriorityQueue()
len(p)
p.is_empty()
p.insert(pri, val)
p.min()
p.delete_min()

<u>Maximum Priority Queue</u>:

The *Priority Queue ADT*

Minimum Priority Queue:

p = MinPriorityQueue()
len(p)
p.is_empty()
p.insert(pri, val)
p.min()
p.delete_min()

Maximum Priority Queue:

p = MaxPriorityQueue()
len(p)
p.is_empty()
p.insert(pri, val)
p.max()
p.delete_max()

Data Structures for *Priority Queue ADT*

Data Structures for *Priority Queue ADT*

	Min	Insert	Delete Min

Data Structures for *Priority Queue ADT*

	Min	Insert	Delete Min
Unsorted Linked List			
Sorted Linked List			

Data Structures for *Priority Queue ADT*

	Min	Insert	Delete Min
Unsorted Linked List			
Sorted Linked List			
Balanced Search Tree			

Data Structures for *Priority Queue ADT*

	Min	Insert	Delete Min
Unsorted Linked List	$\theta(1)$		
Sorted Linked List			
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	Min	Insert	Delete Min
Unsorted Linked List	$\theta(1)$	$\theta(1)$	
Sorted Linked List			
Balanced Search Tree			

Data Structures for *Priority Queue ADT*

	Min	Insert	Delete Min
Unsorted Linked List	$\theta(1)$	$\theta(1)$	$\theta(n)$
Sorted Linked List			
Balanced Search Tree			

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	Min	Insert	Delete Min
Unsorted Linked List	$\theta(1)$	$\theta(1)$	$\theta(n)$
Sorted Linked List	$\theta(1)$		
Balanced Search Tree			

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	Min	Insert	Delete Min
Unsorted Linked List	$\theta(1)$	$\theta(1)$	$\theta(n)$
Sorted Linked List	$\theta(1)$	$\theta(n)$	
Balanced Search Tree			

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Balanced Search Tree			

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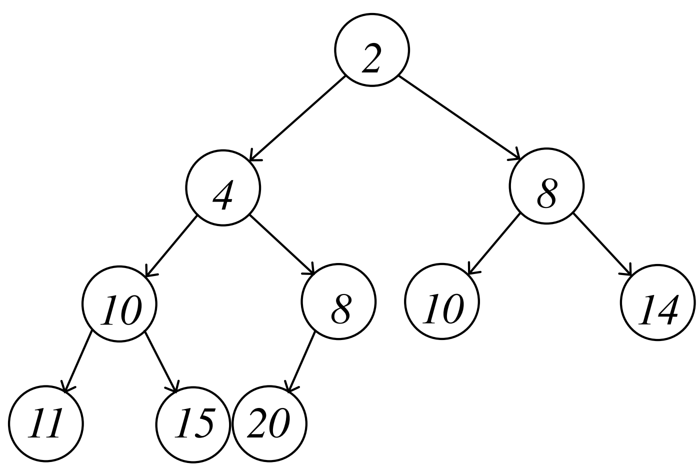
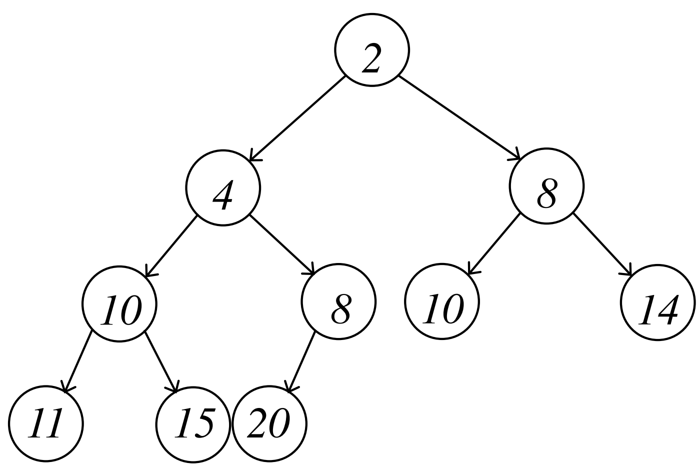
Heap			

Heap



Heap

Let T be a binary tree.

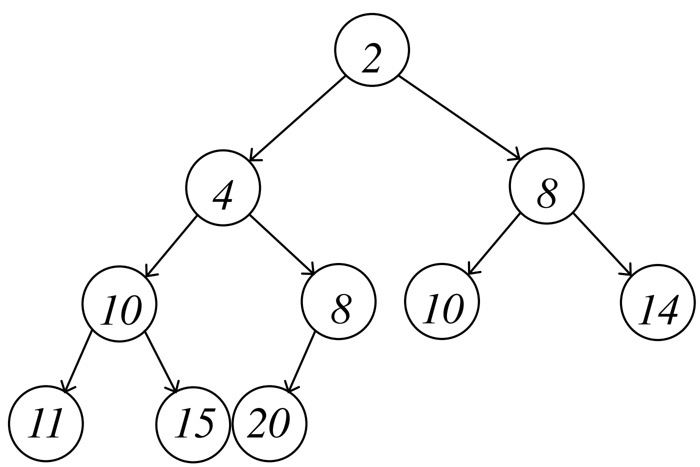


Heap

Let T be a binary tree. We say that T is a *Heap* if it satisfies:

1.

2.



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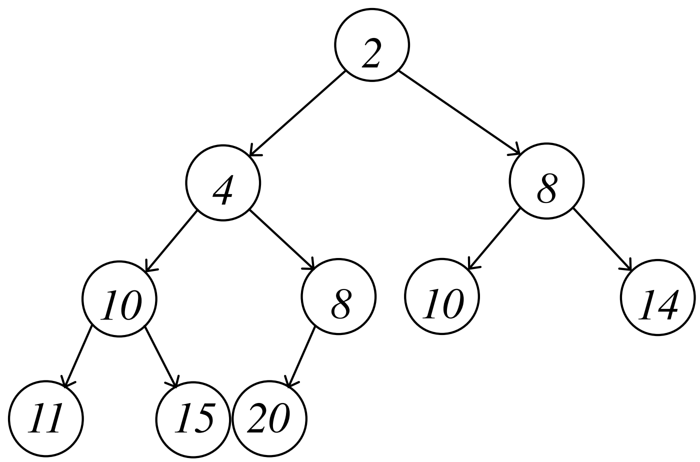

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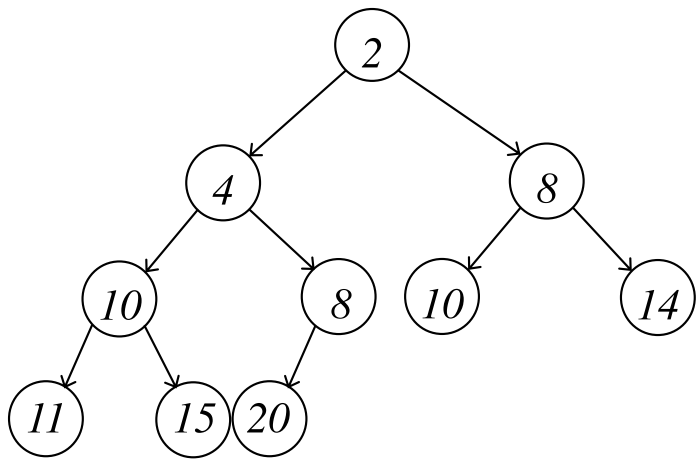
2. Nearly-complete property



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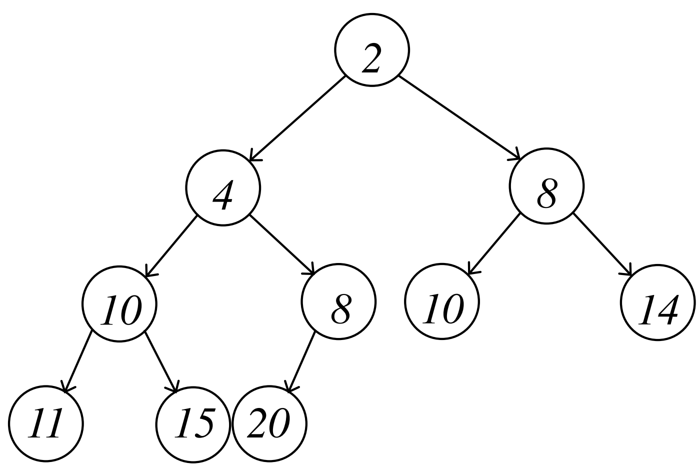

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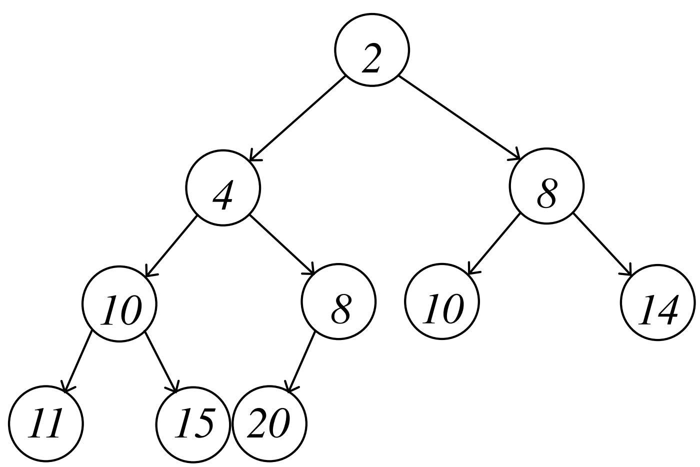

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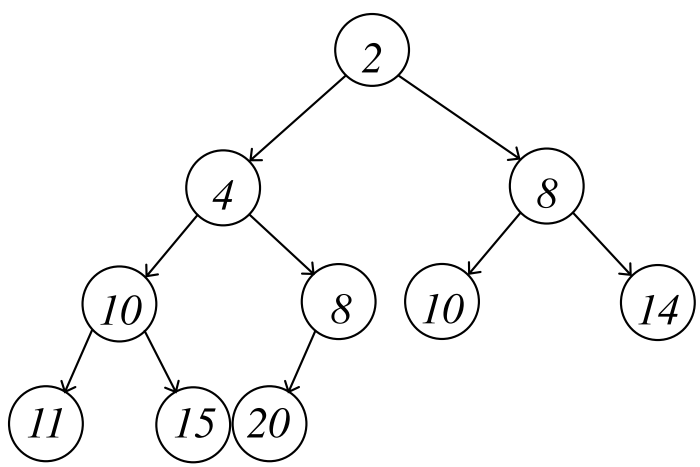

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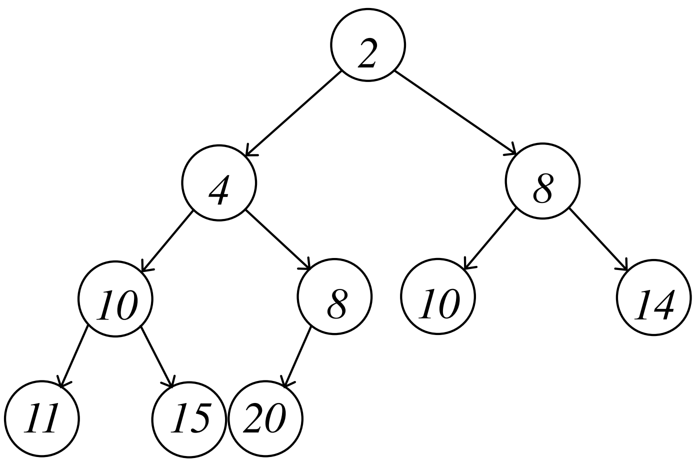

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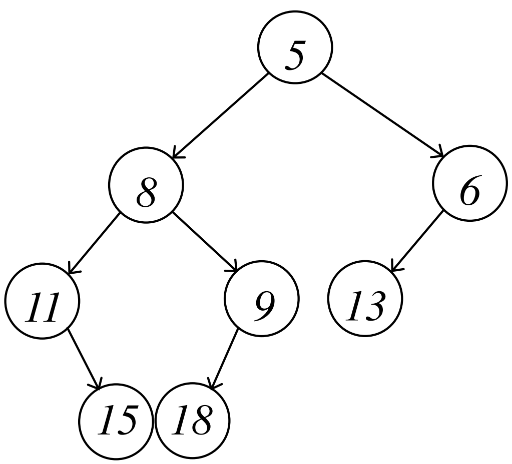

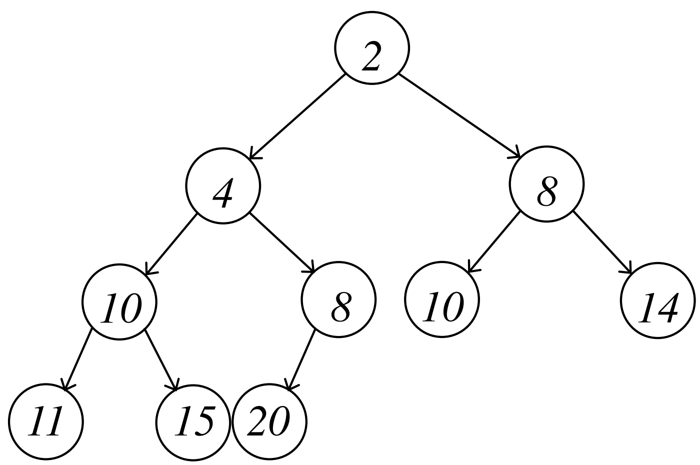

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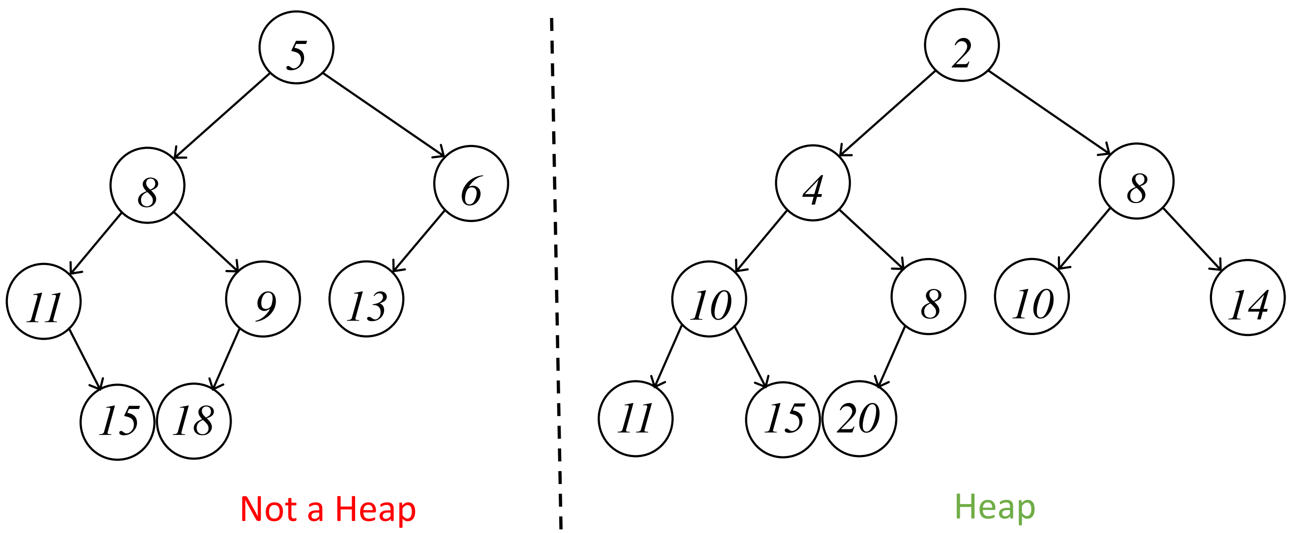

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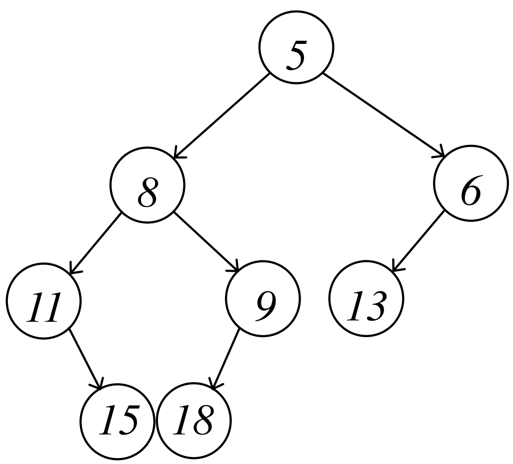
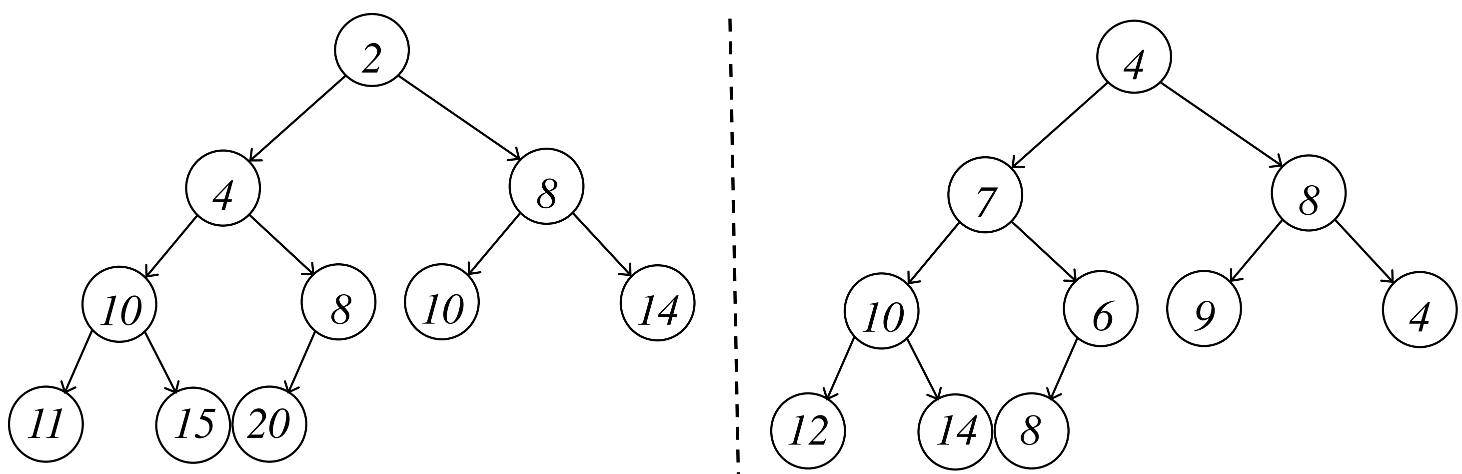


Not a Heap



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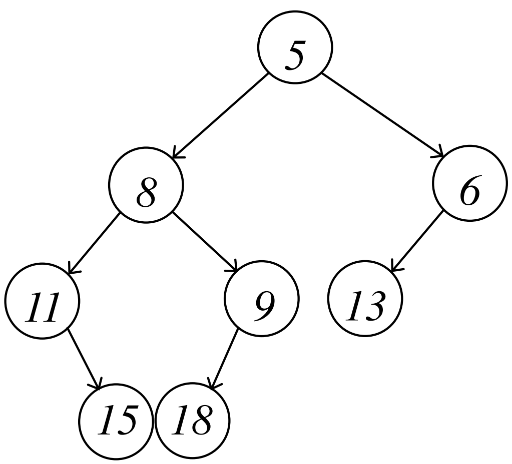

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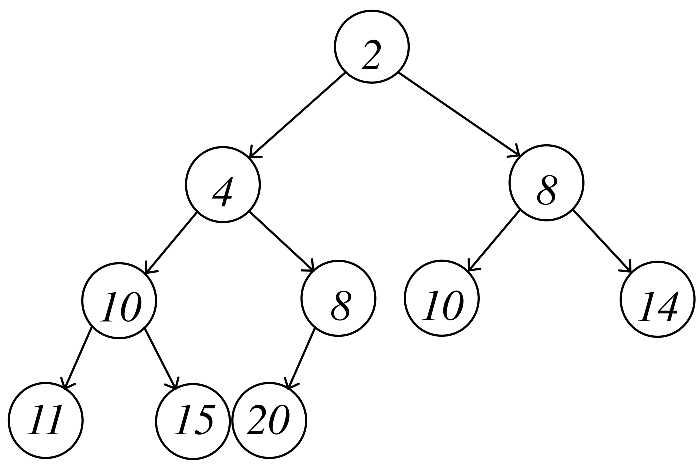

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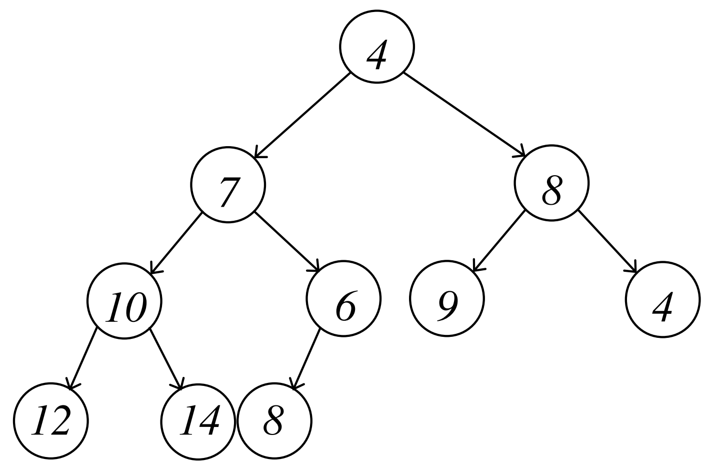

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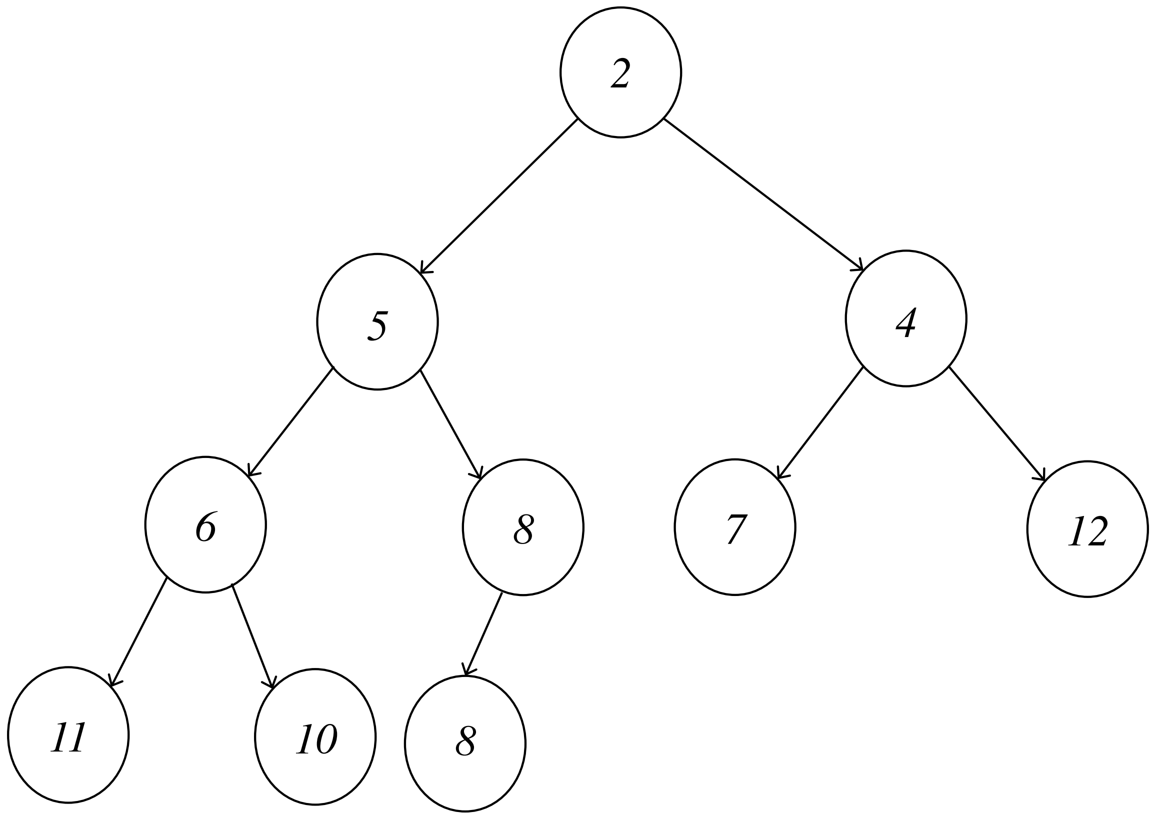
2
5
4
6
8
7
12
11
10
8

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Unsorted Linked List	$\theta(1)$	$\theta(1)$	$\theta(n)$
Sorted Linked List	$\theta(1)$	$\theta(n)$	$\theta(1)$
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Heap			

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Heap	$\theta(1)$	$\theta(\log(n))$	$\theta(\log(n))$