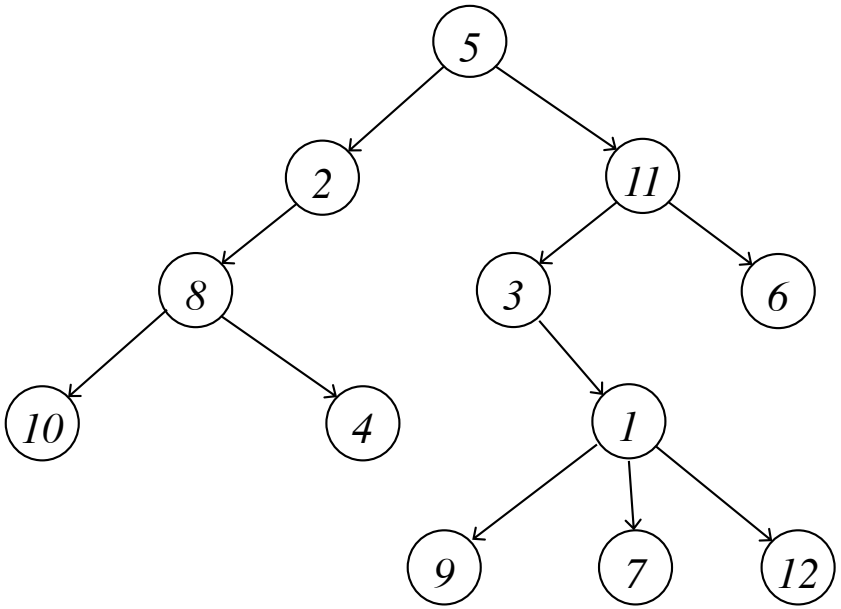
Rooted Trees


5
2
11
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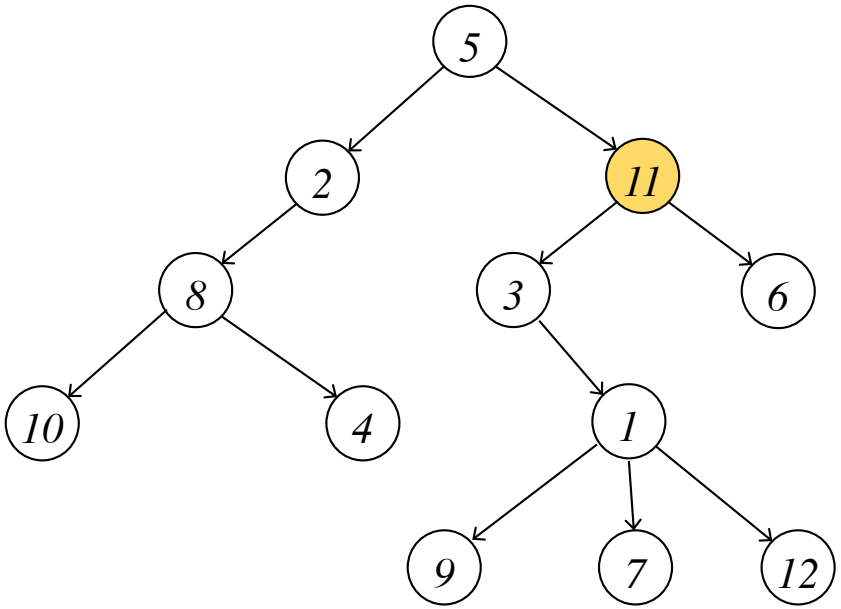
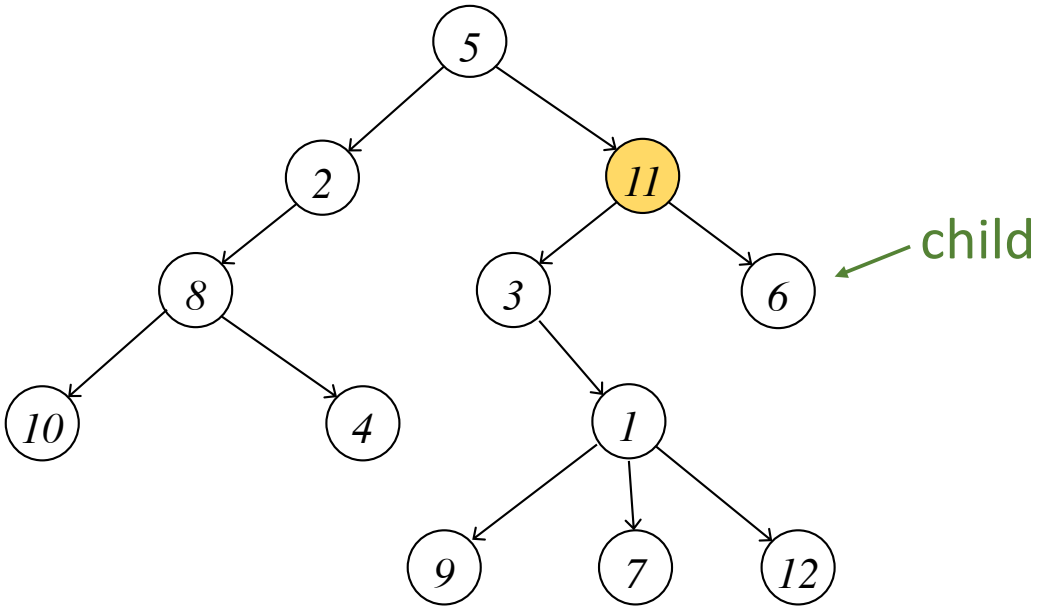
3
6
10
4
1
9
7
12

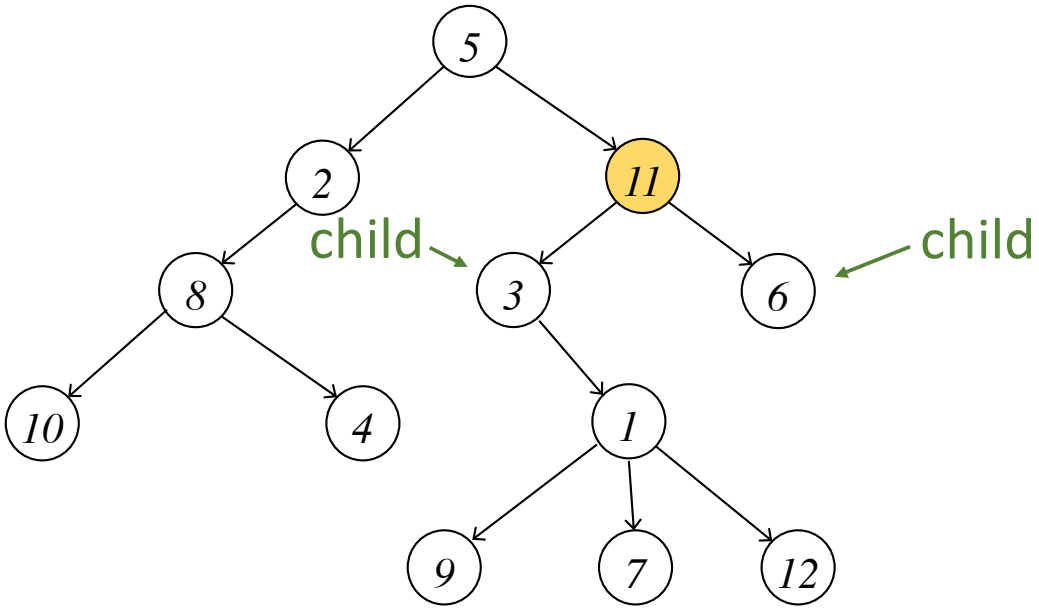
parent-child Relationship



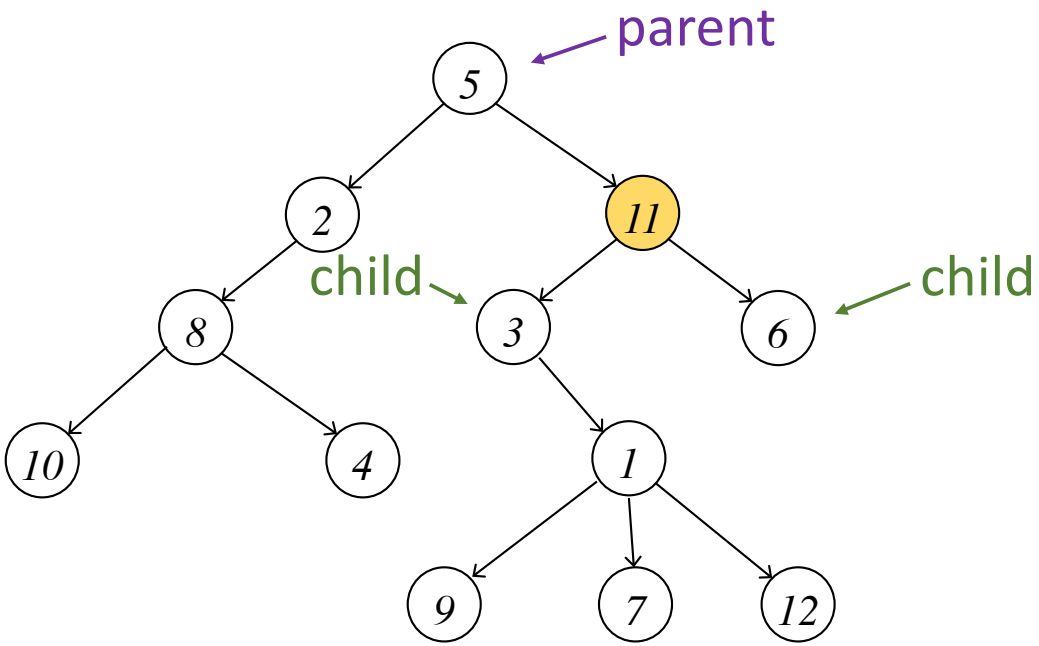
parent-child Relationship



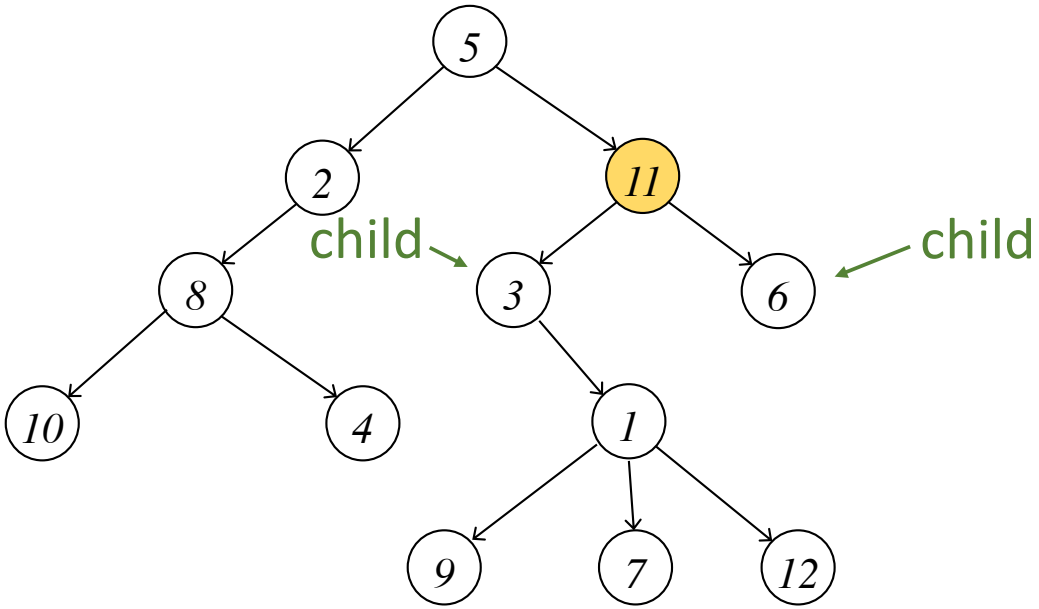
parent-child Relationship



parent-child Relationship



sibling Relationship



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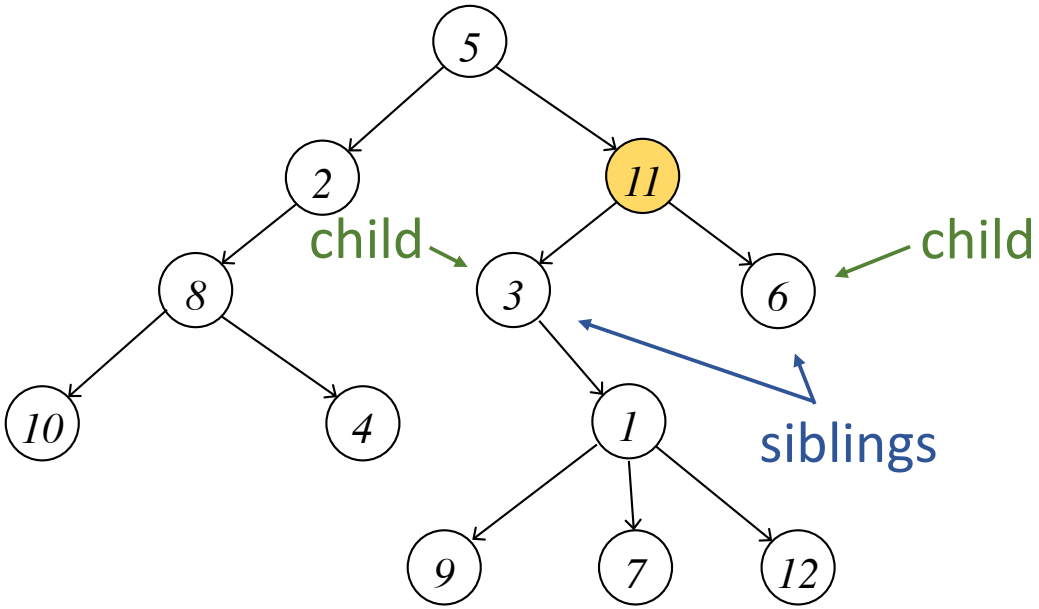

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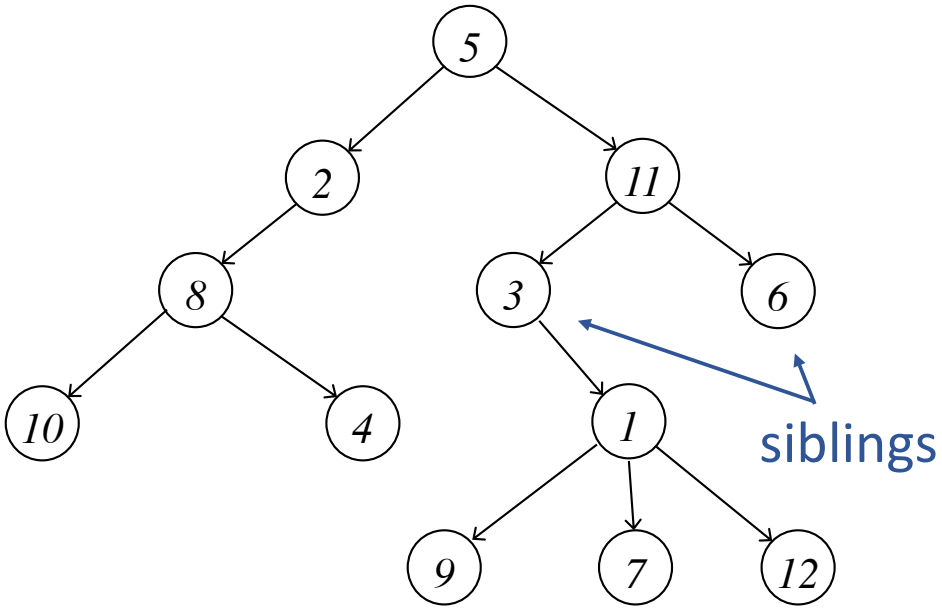
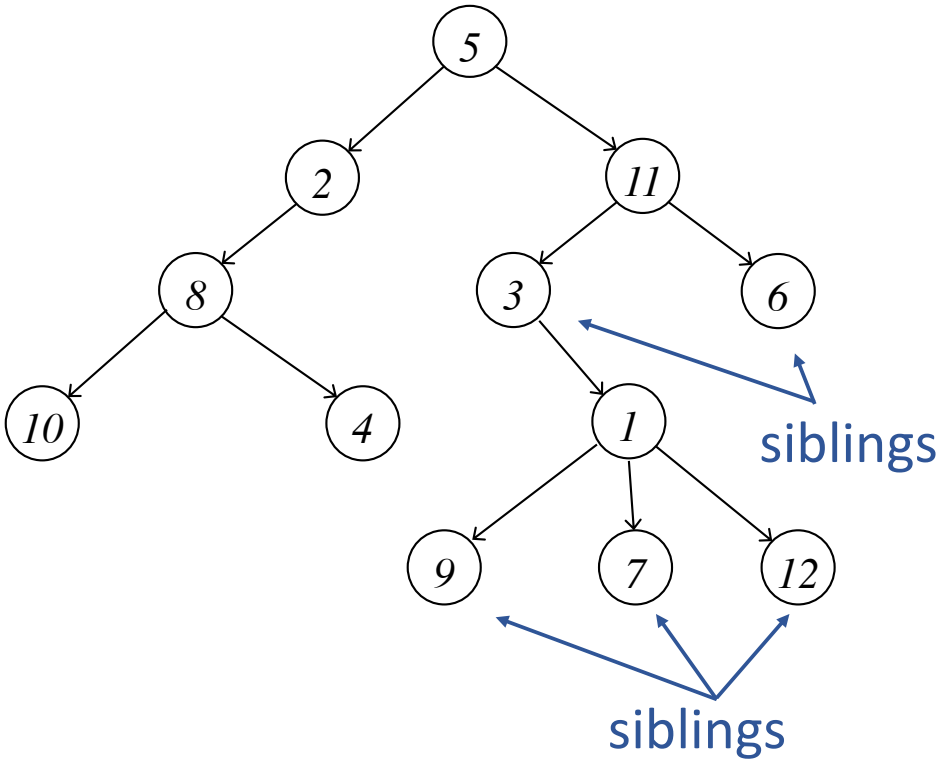
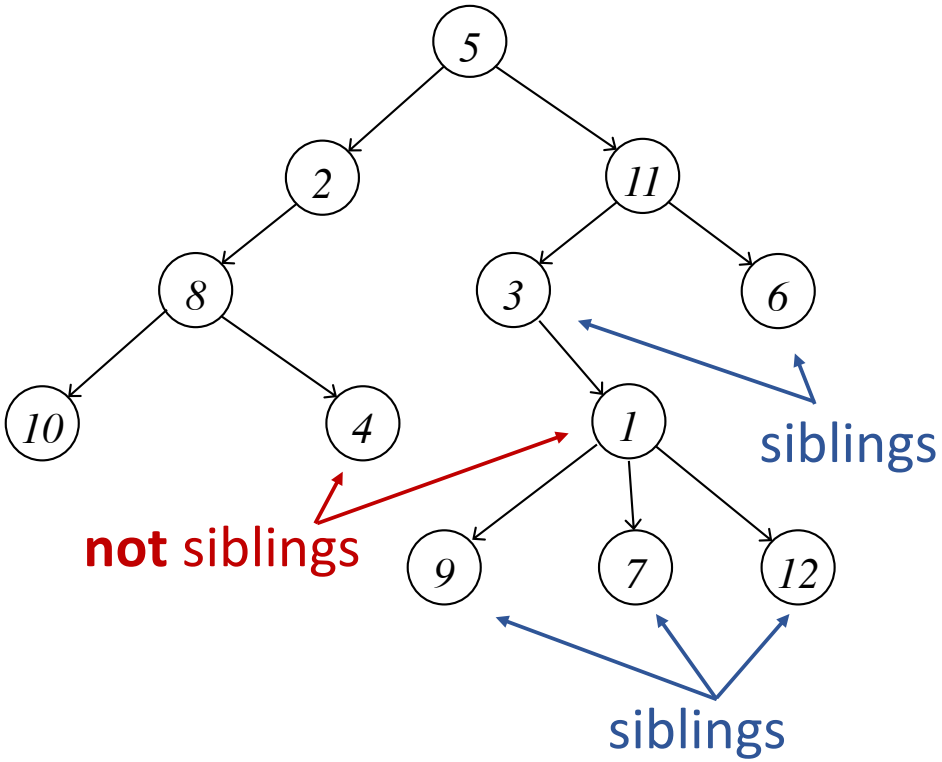


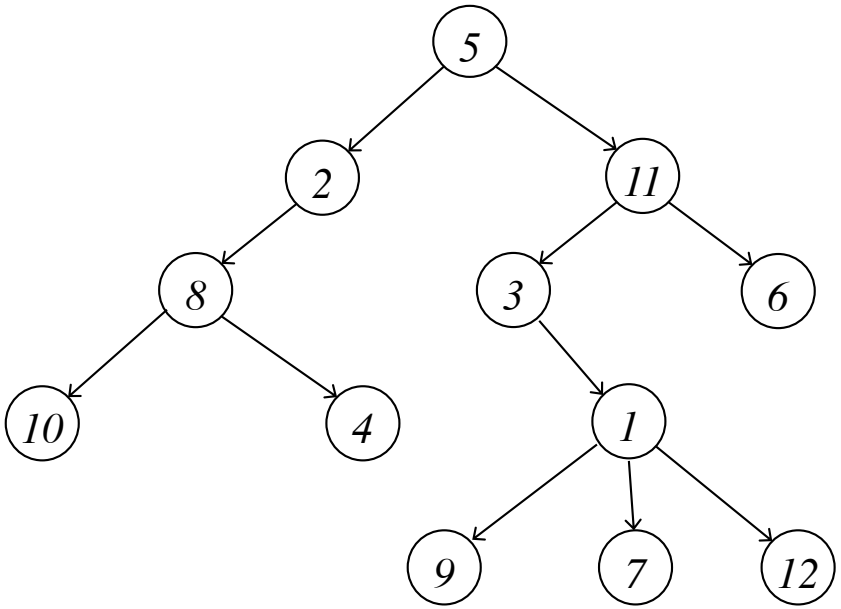
sibling Relationship



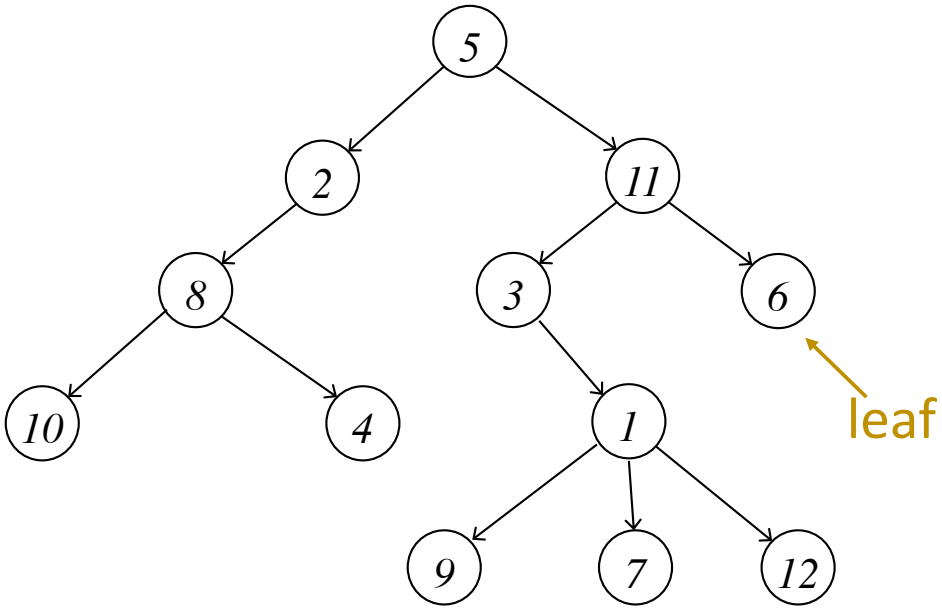
sibling Relationship



Kind of Nodes



Kind of Nodes



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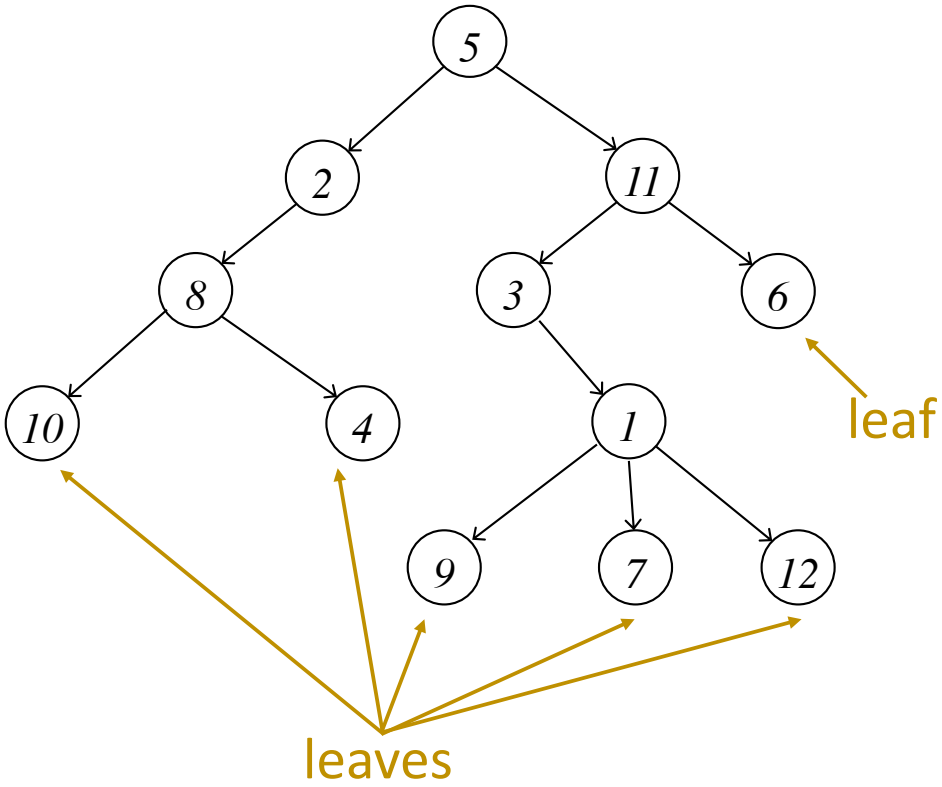

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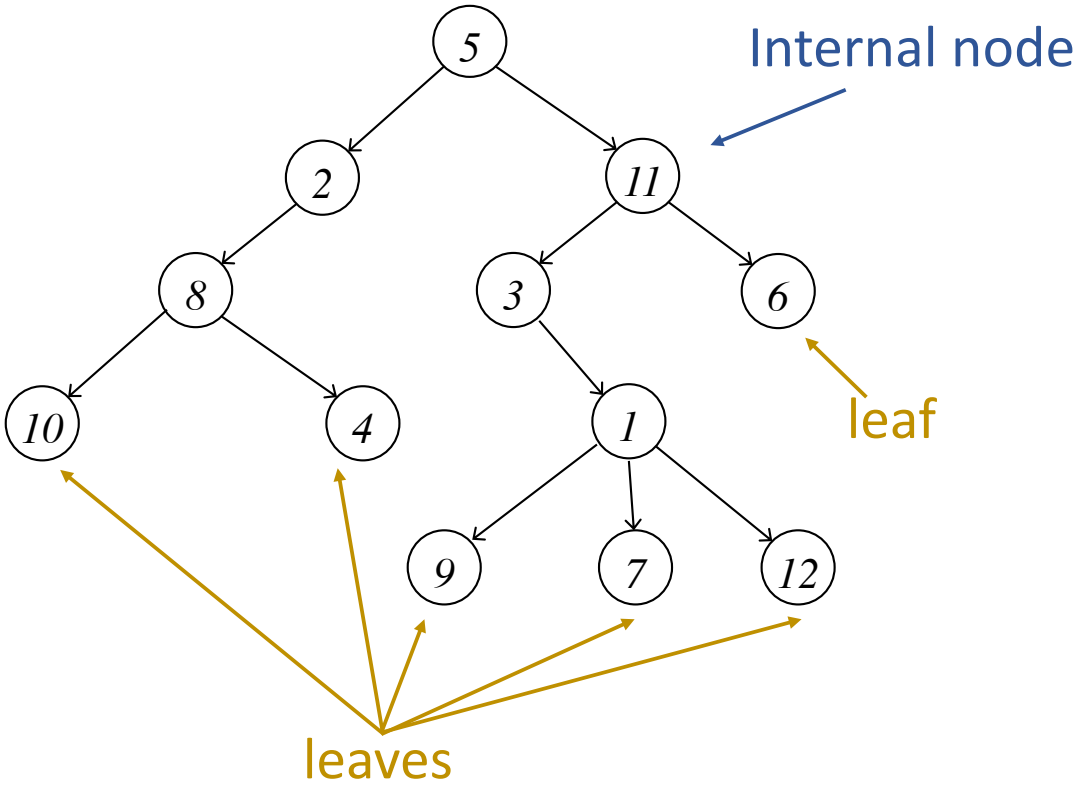
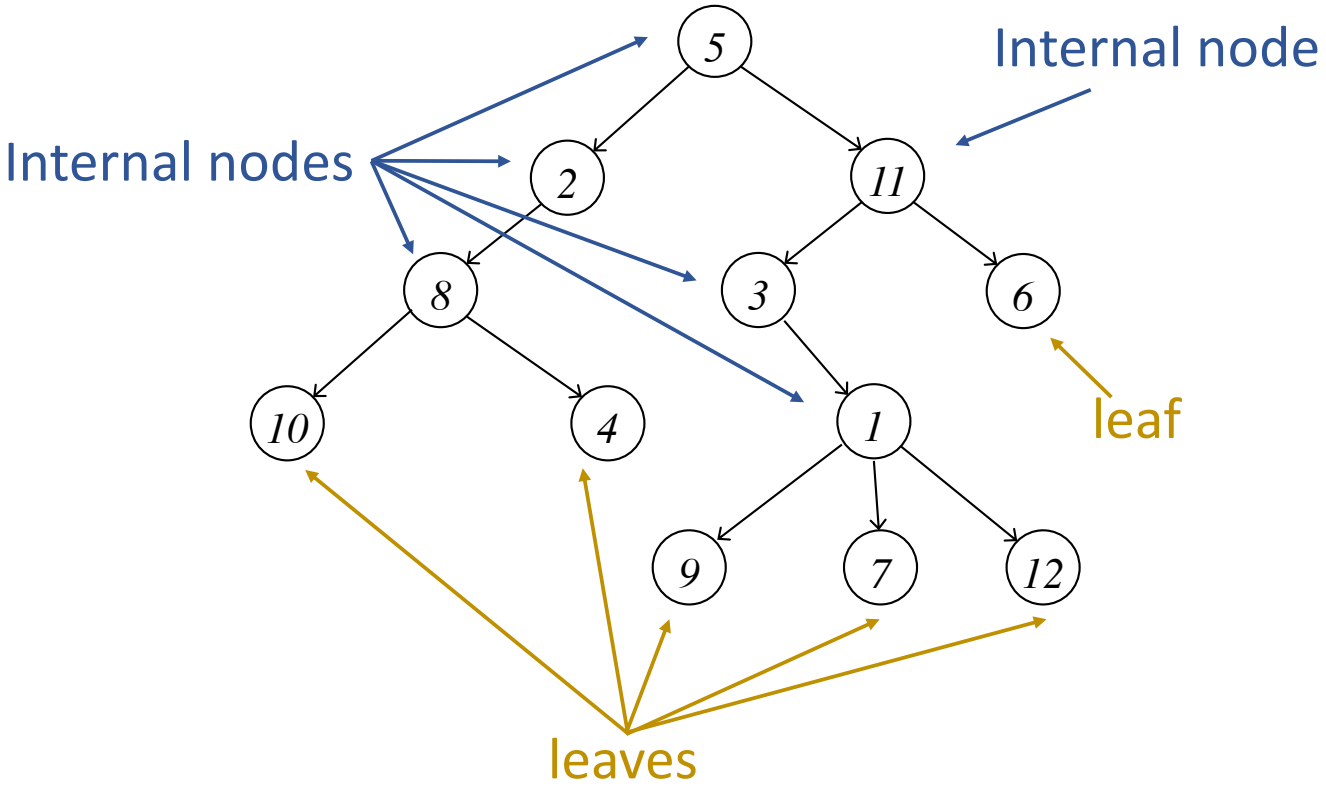
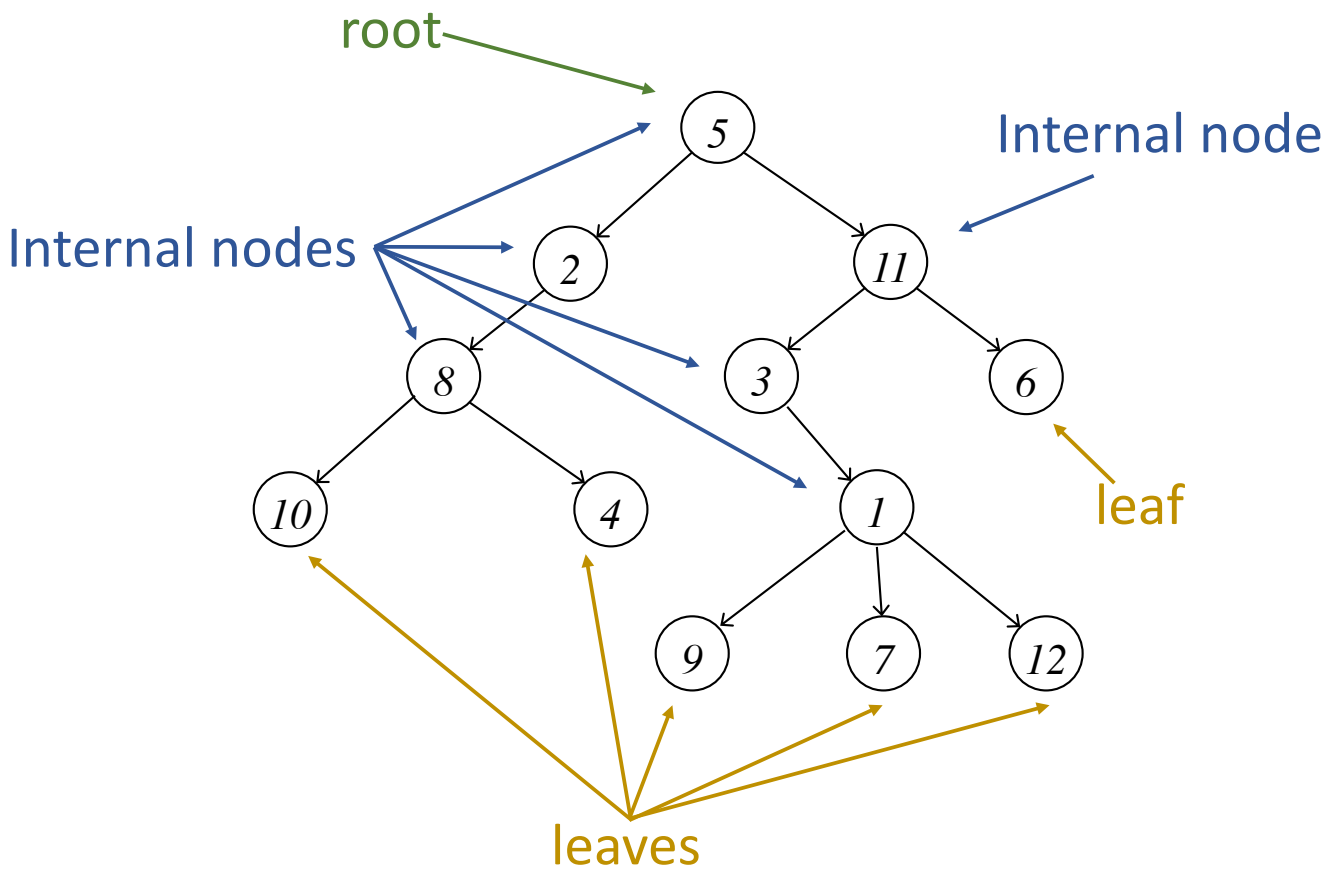


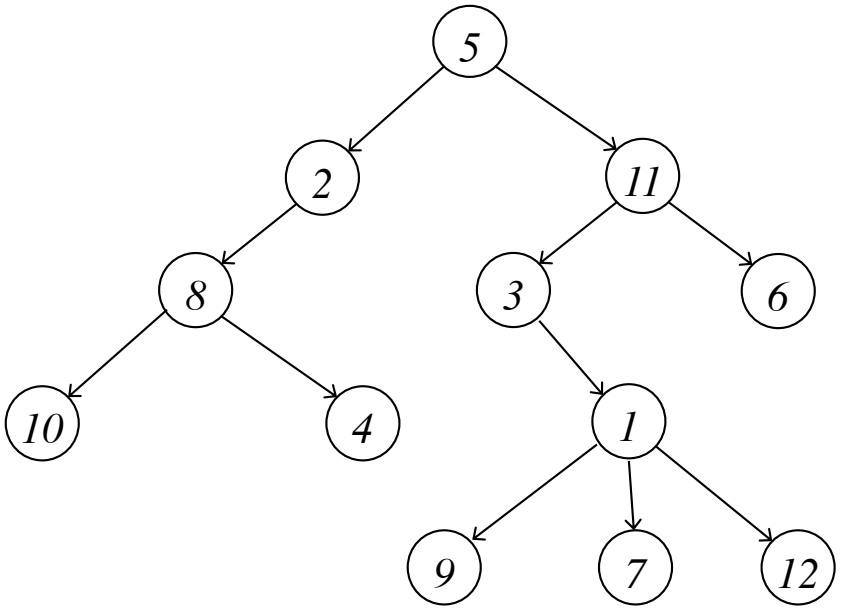
Kind of Nodes



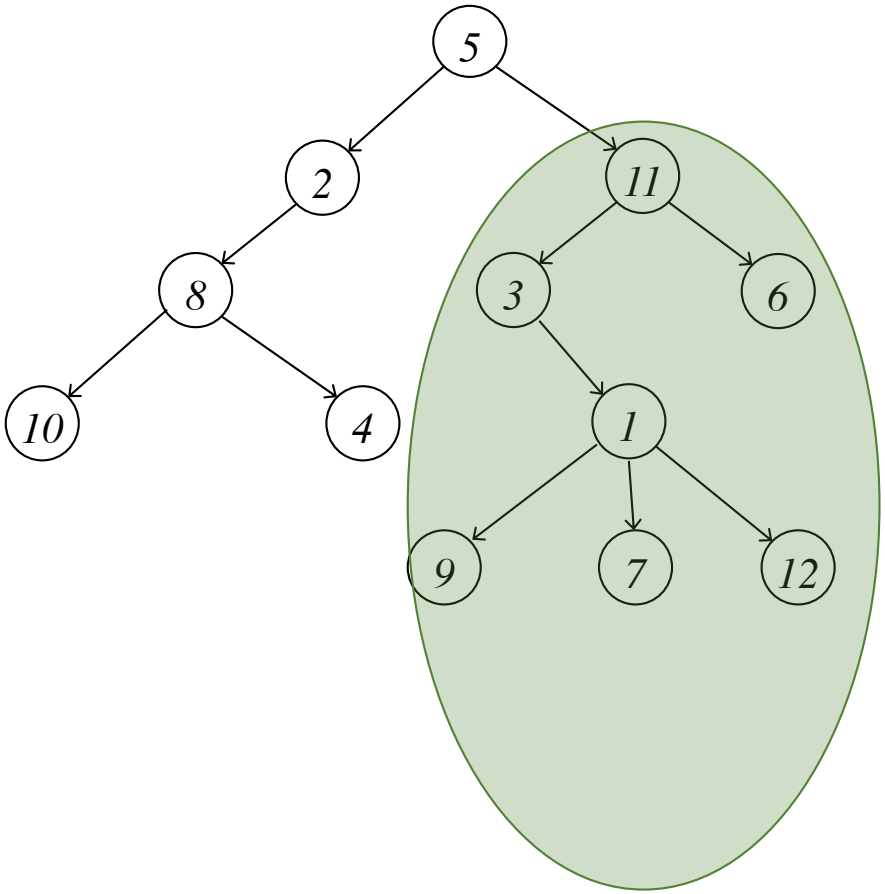
Kind of Nodes



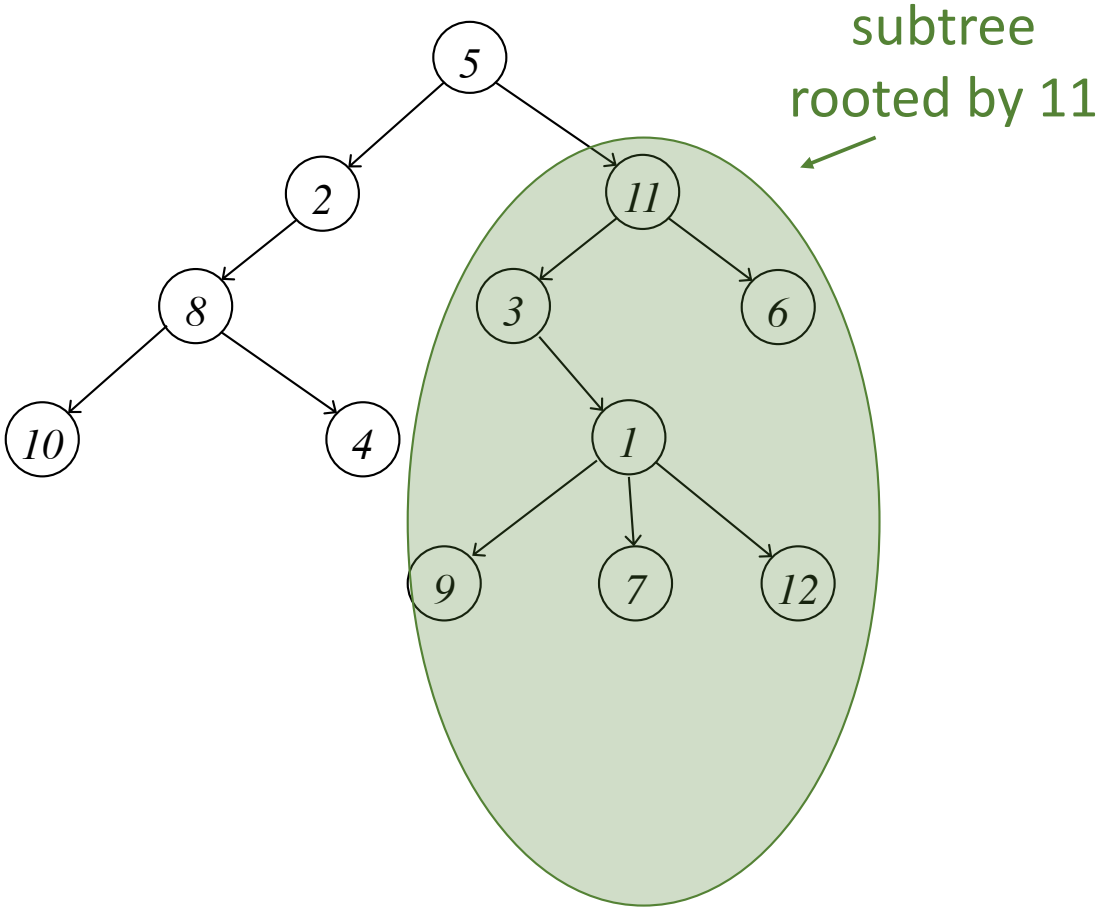
Subtrees



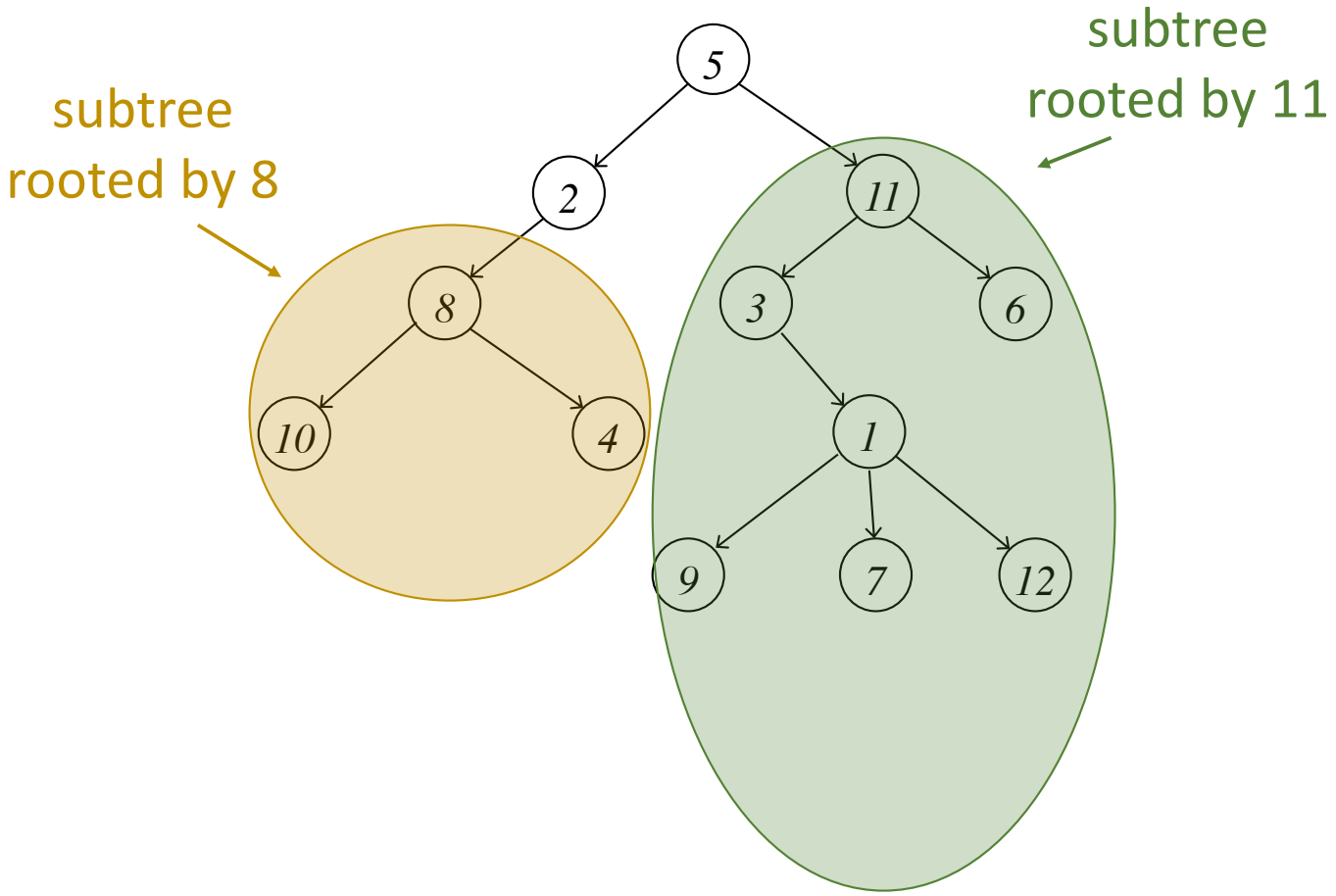
Subtrees



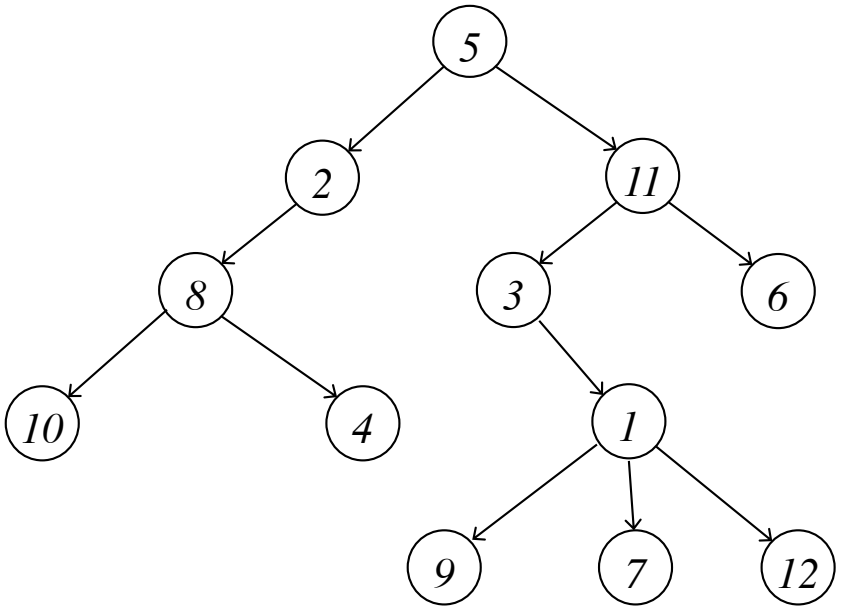
Subtrees



Subtrees

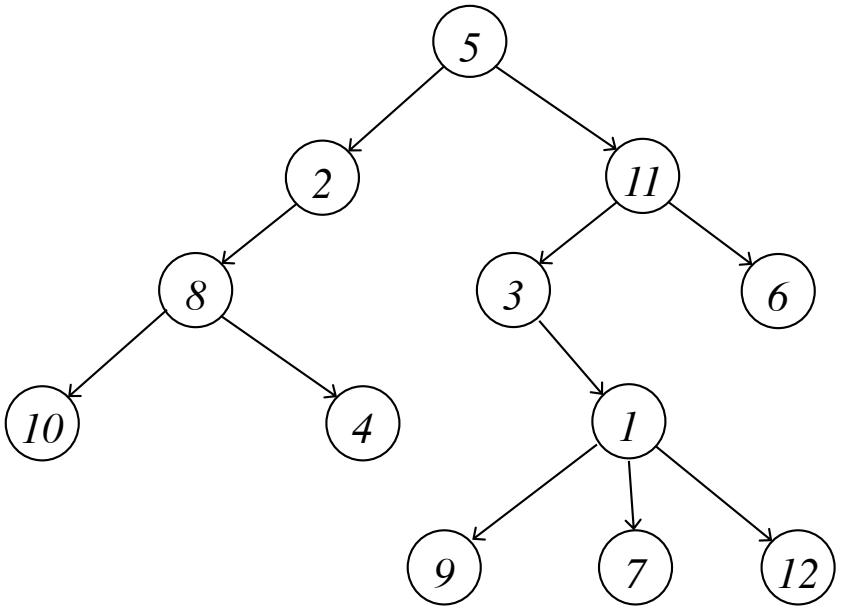


Definitions



Definitions

Edge



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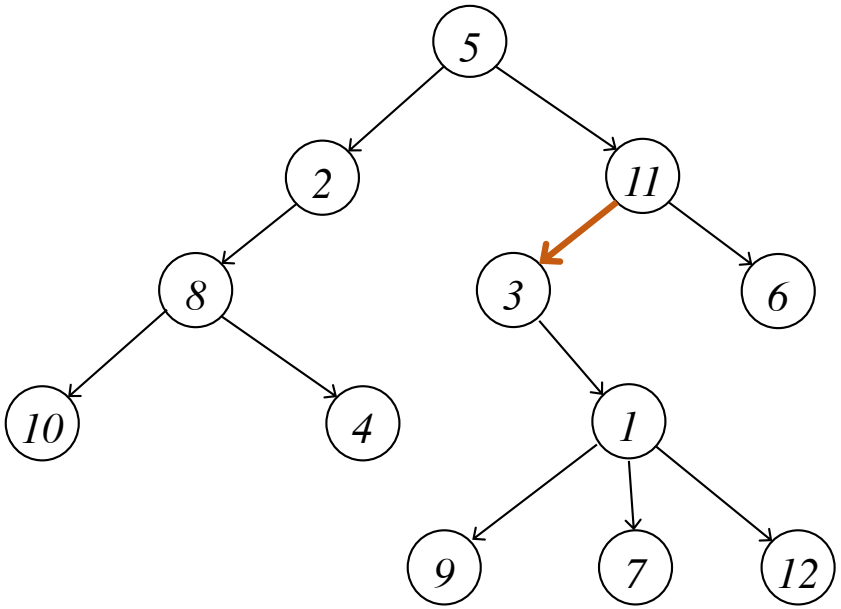
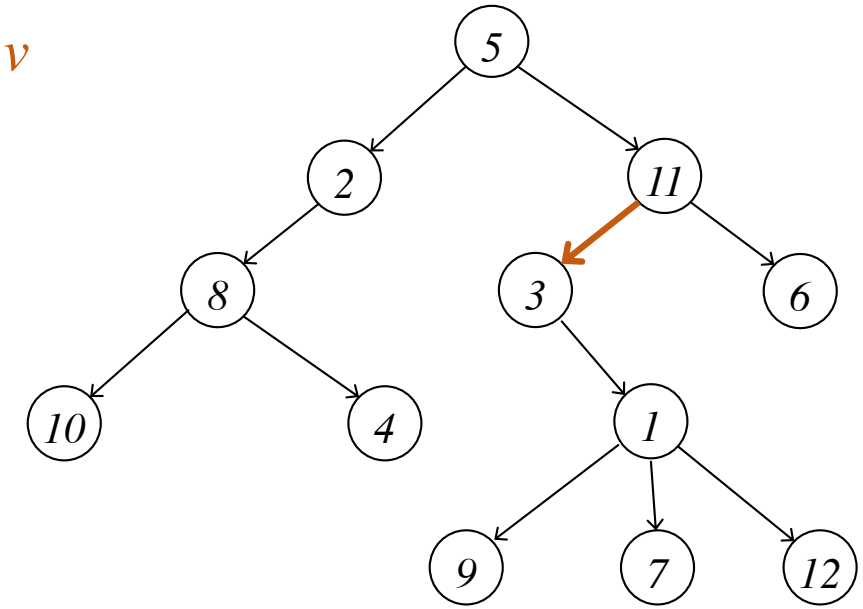


Definitions

Edge

(u, v) is an edge, if u is the parent of v



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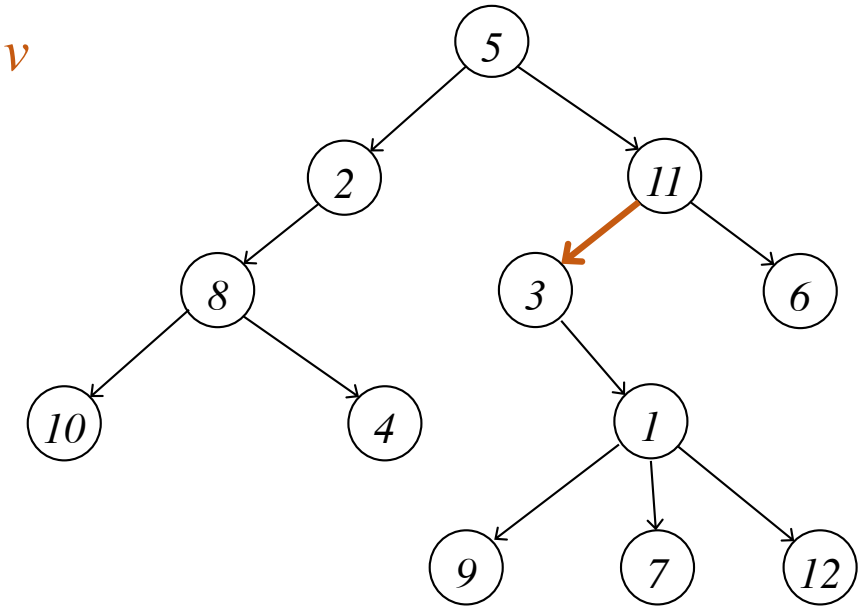
Example: $(11, 3)$ is an edge



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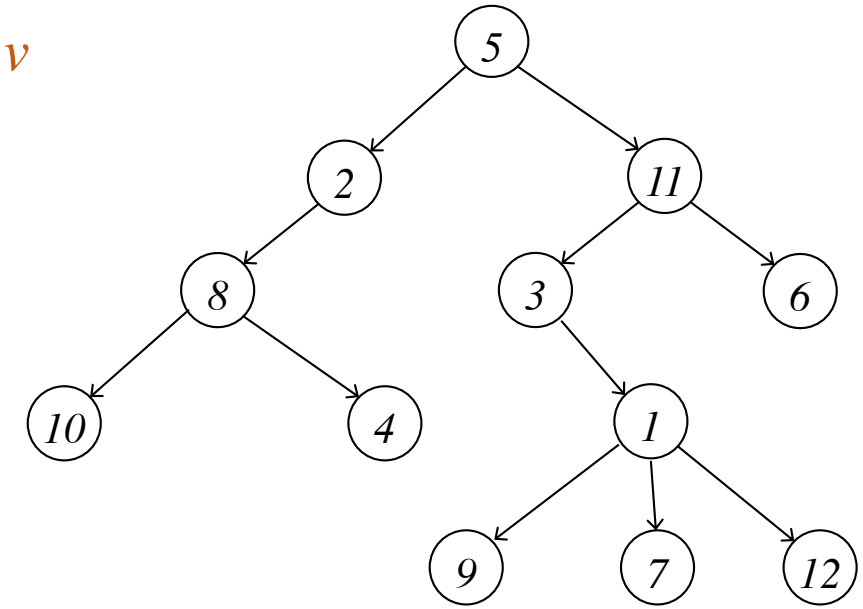

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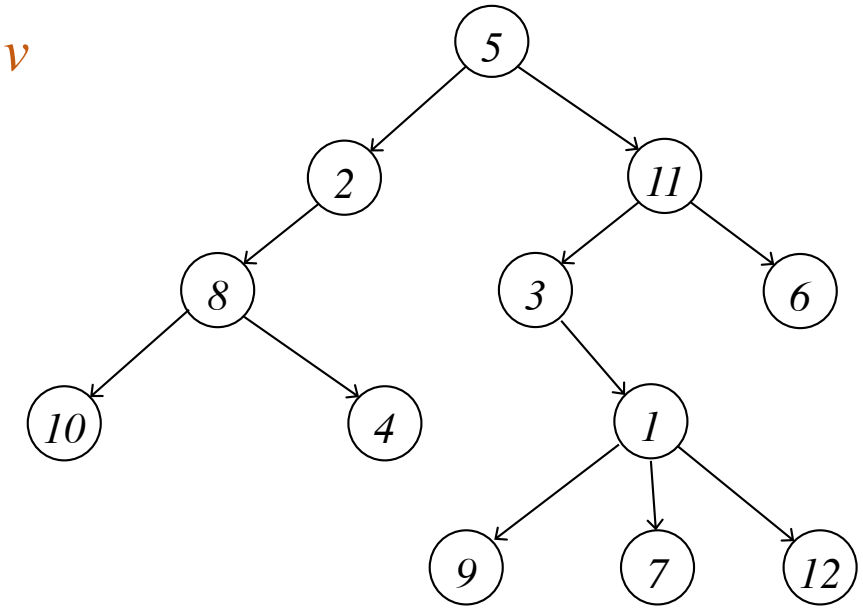

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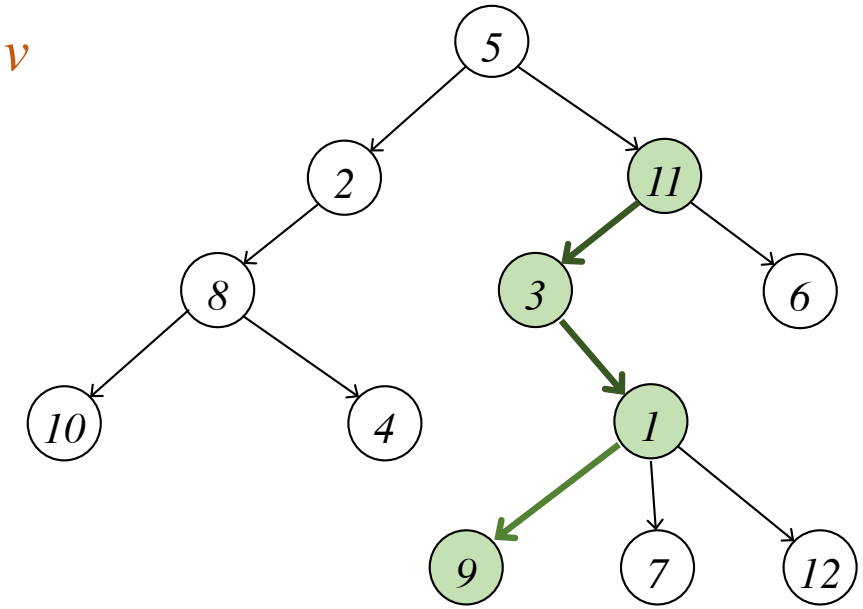

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Example: $(11, 3)$ is an edge

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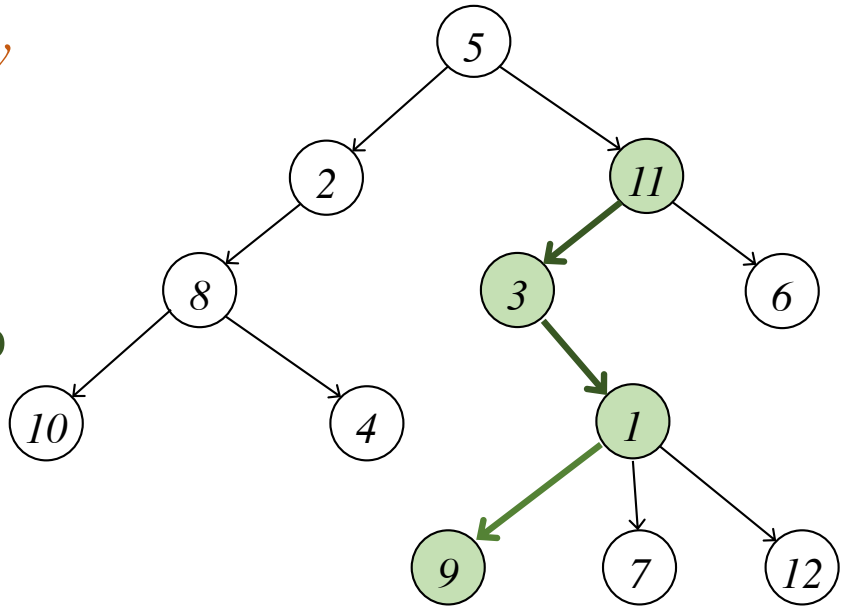
$p=(v_1, v_2, \ldots, v_k)$ is a path, if each two consecutive nodes, forms an edge



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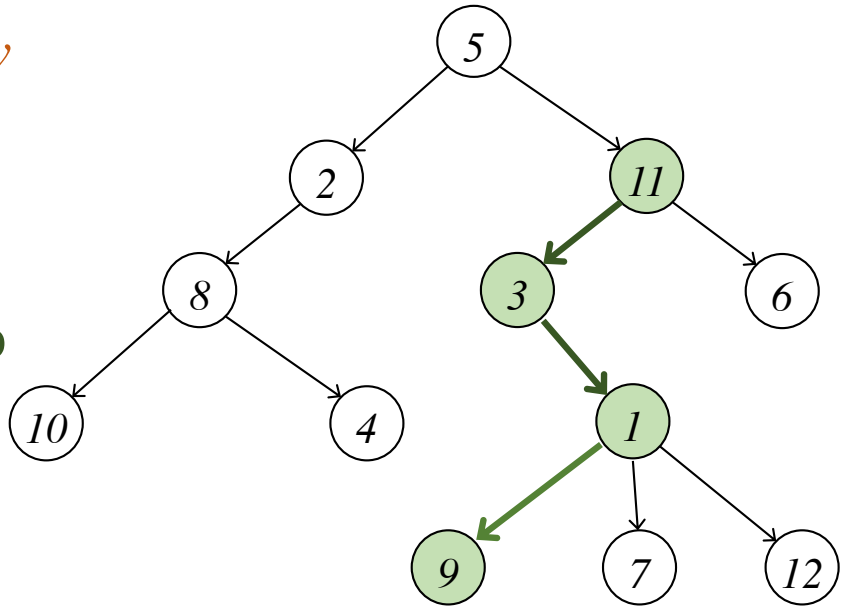
$p=(v_1, v_2, \ldots, v_k)$ is a path, if each two consecutive nodes, forms an edge

Example: $p_1=(11, 3, 1, 9)$ is a path



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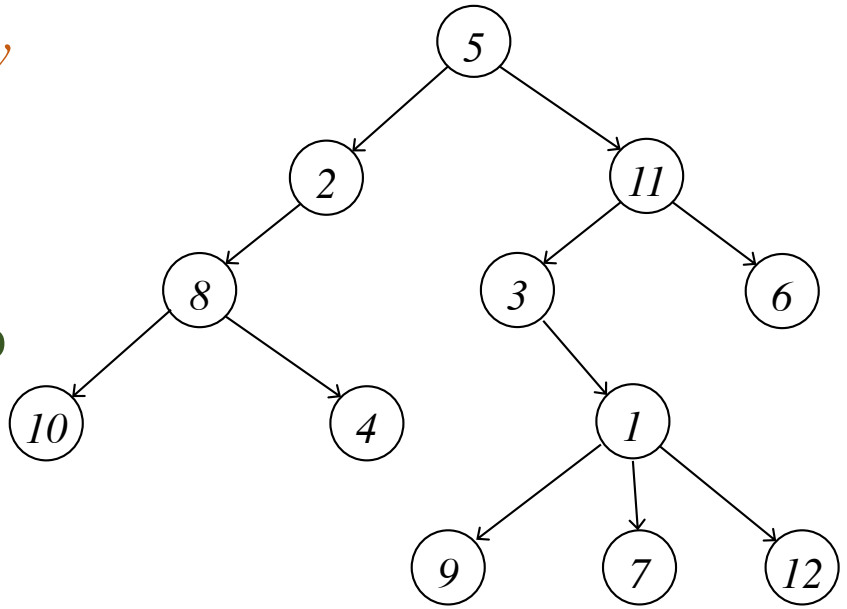

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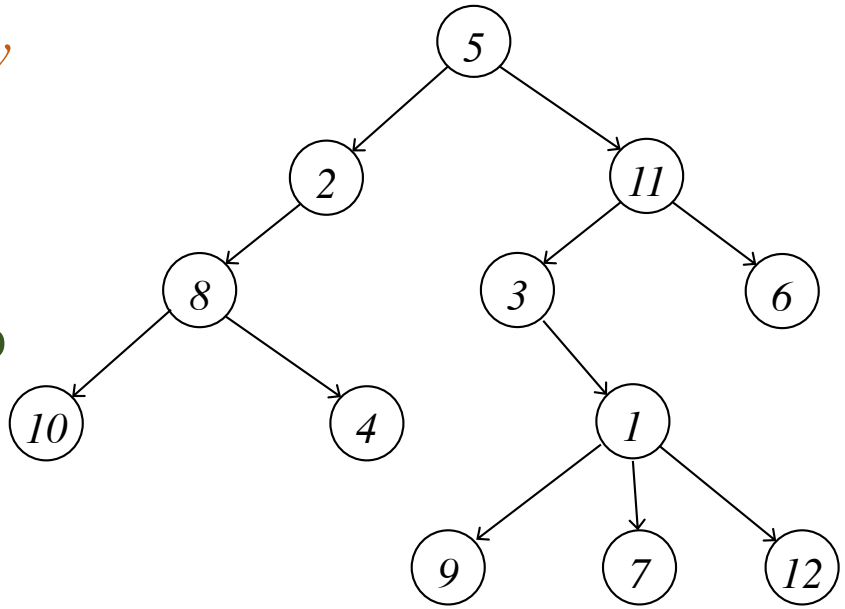
Length of a path



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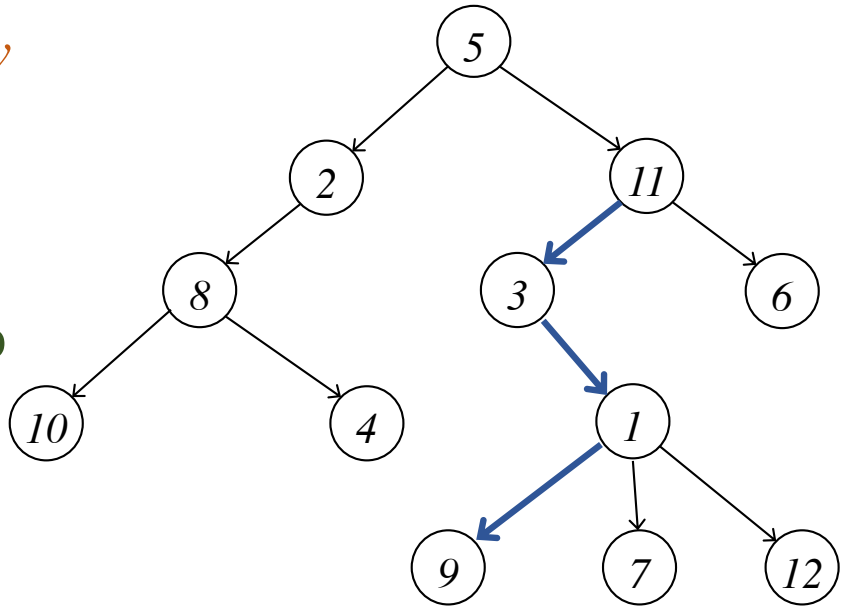

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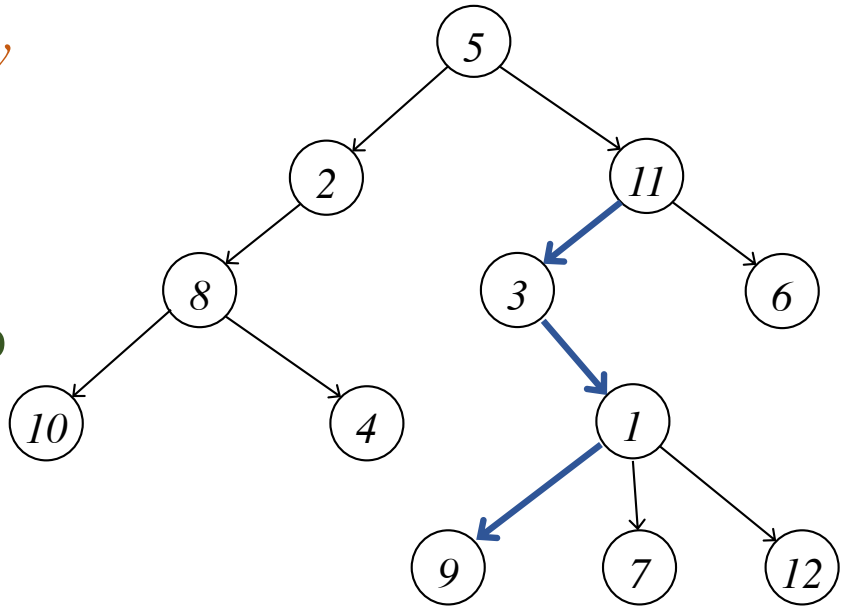
Length of a path

$|p|$= # of edges in p



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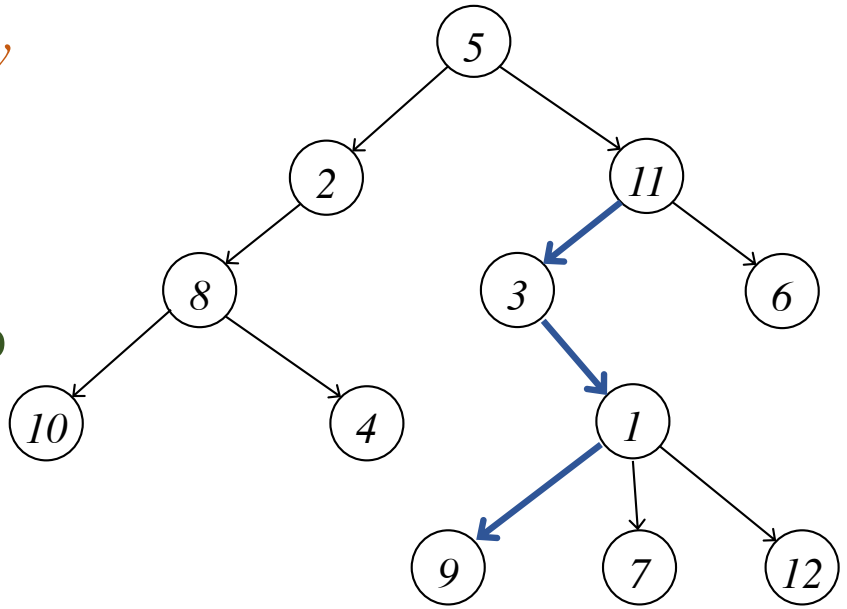
Length of a path

$|p|$= # of edges in p

Example: $|p_1|=3$



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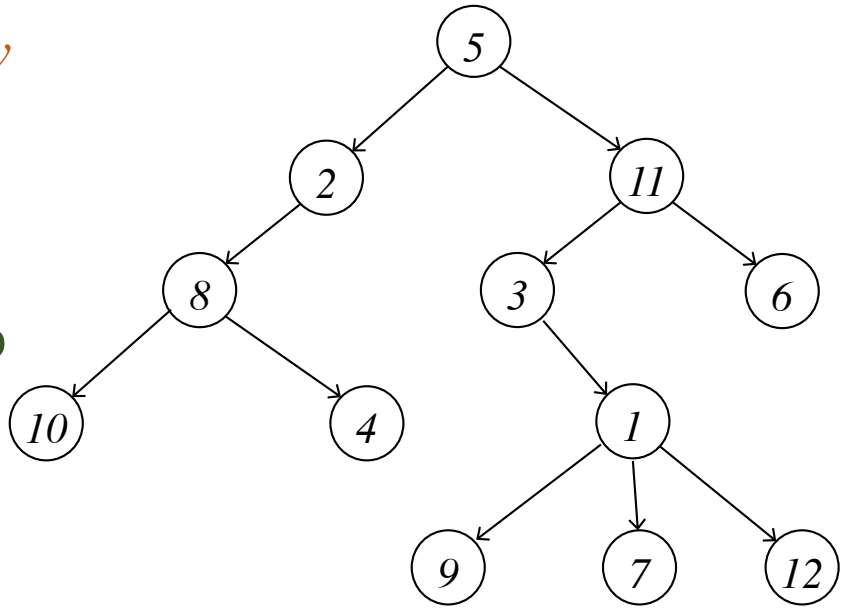
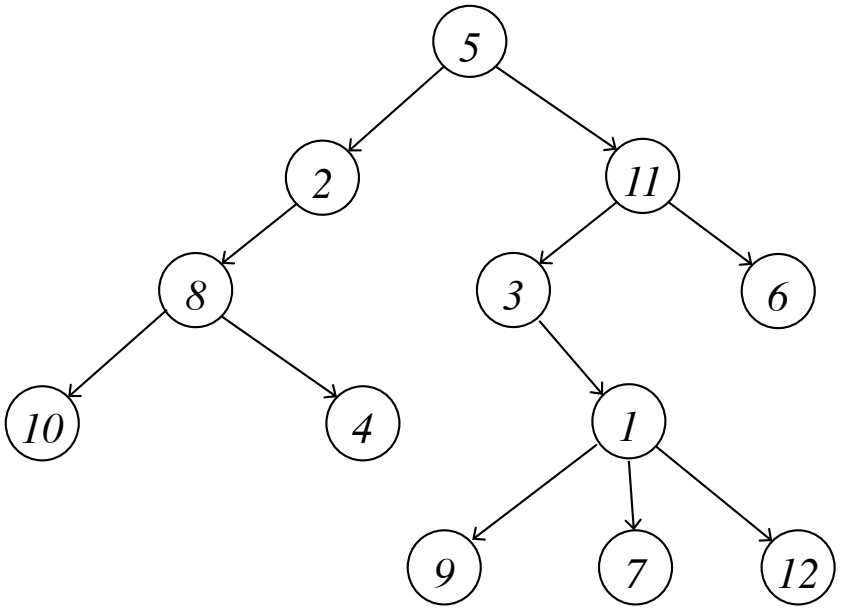
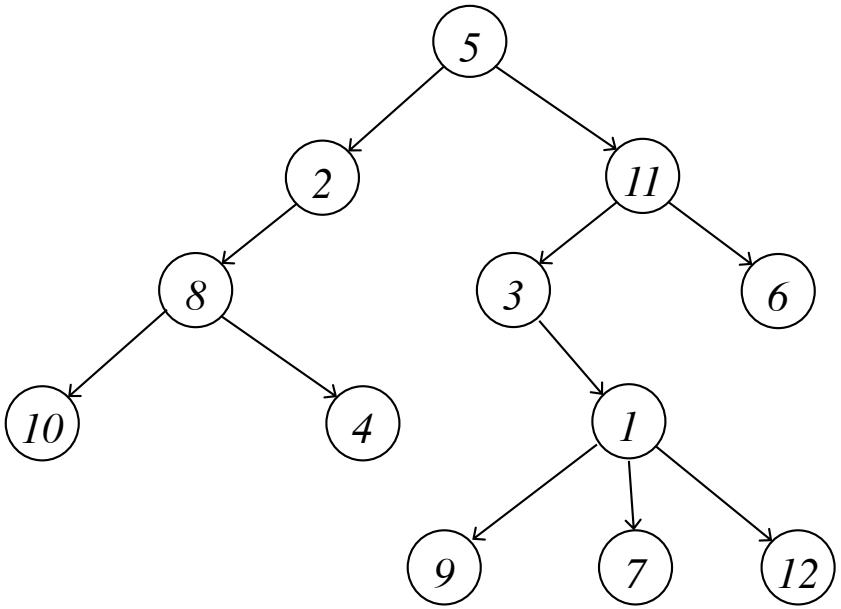


Definitions



Definitions

Ancestor



Definitions

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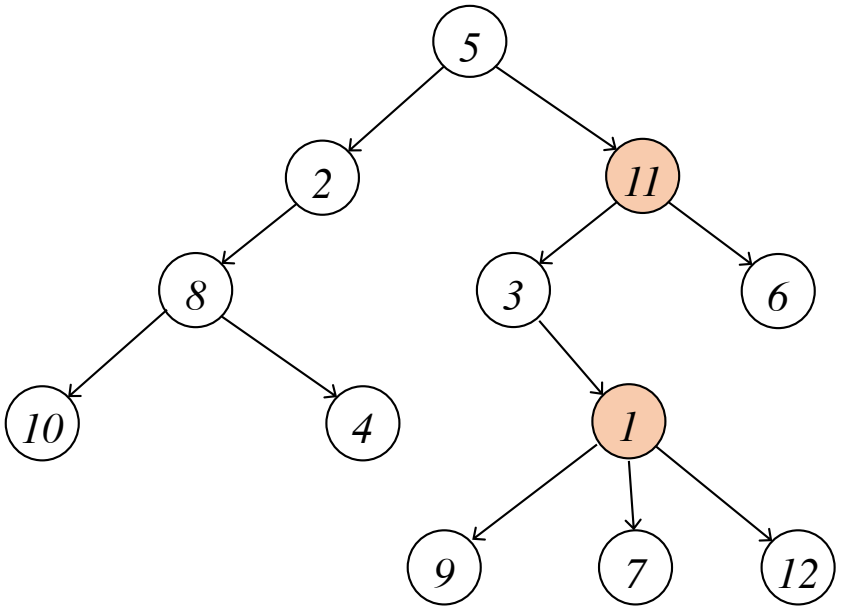

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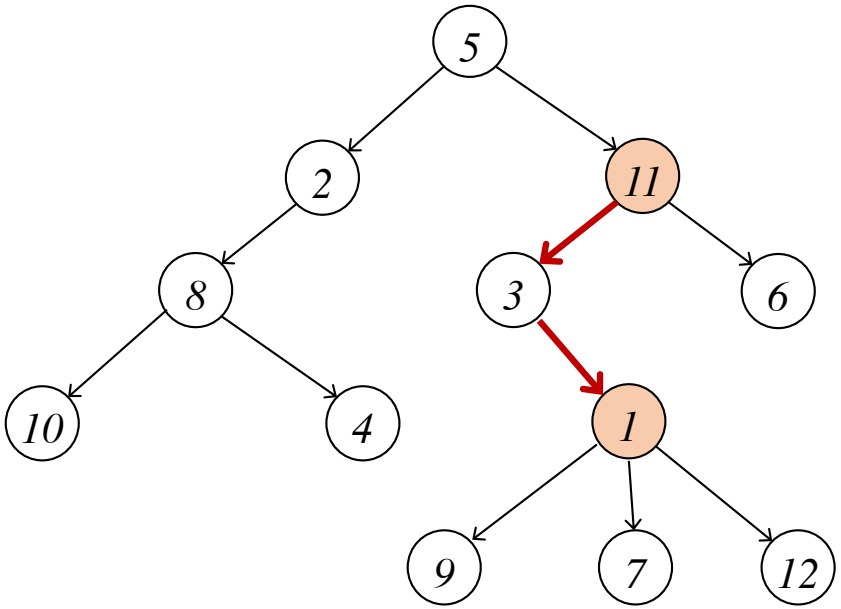
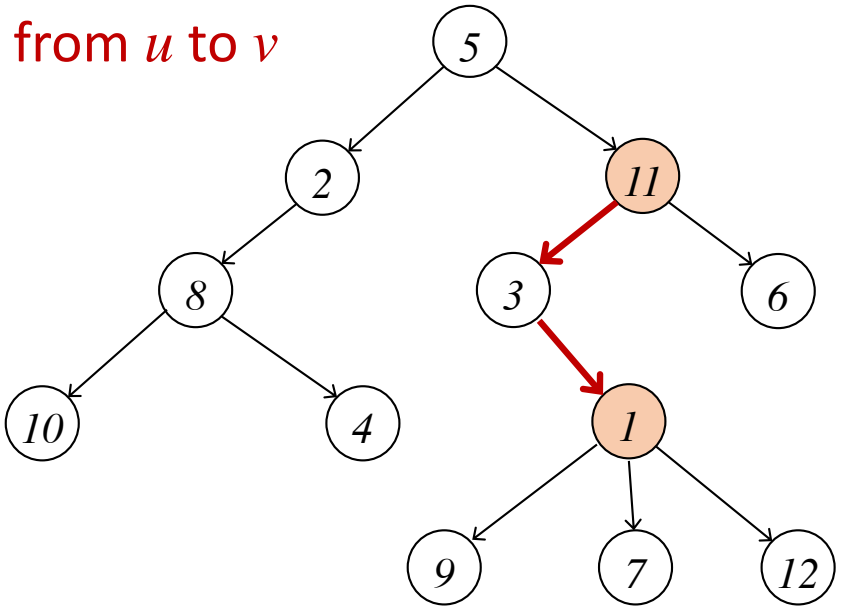
Ancestor



Definitions

Ancestor

u is an ancestor of *v*, if there is a path from *u* to *v*



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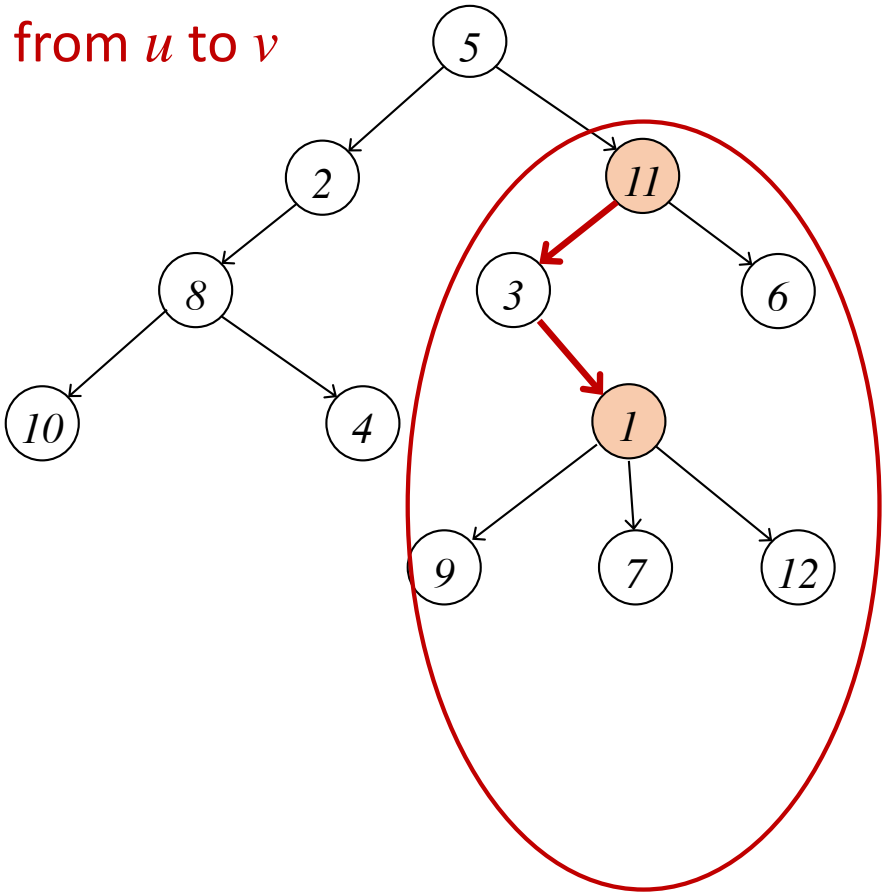
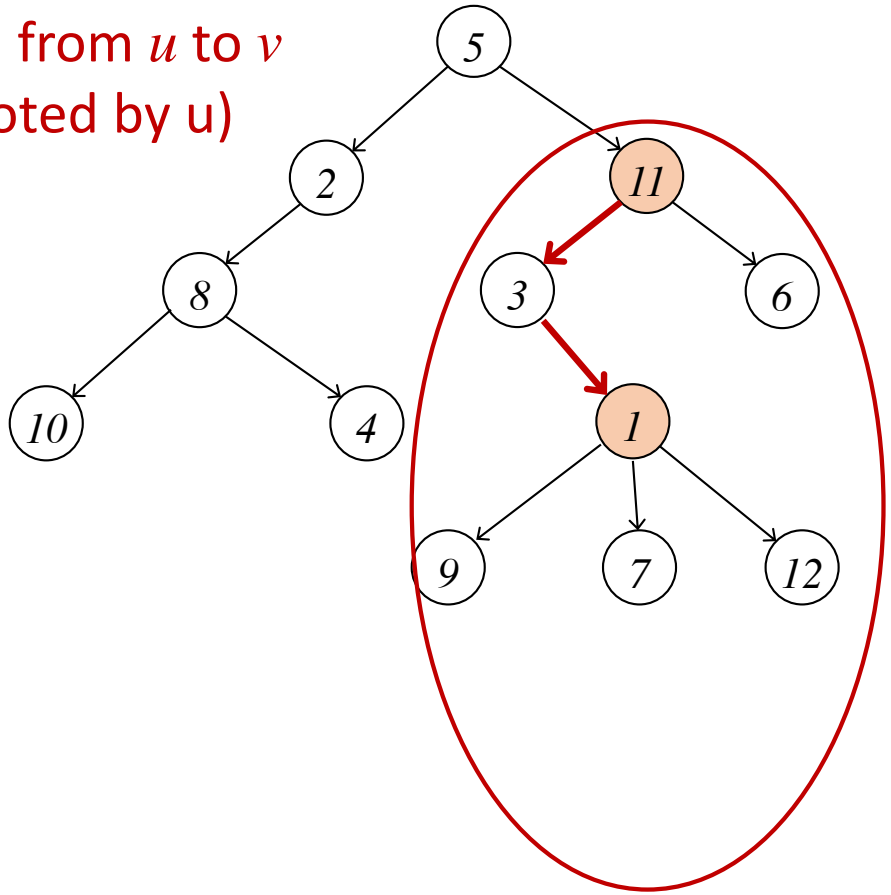


Definitions

Ancestor

u is an ancestor of *v*, if there is a path from *u* to *v*
(alternatively, if v is in the subtree rooted by u)



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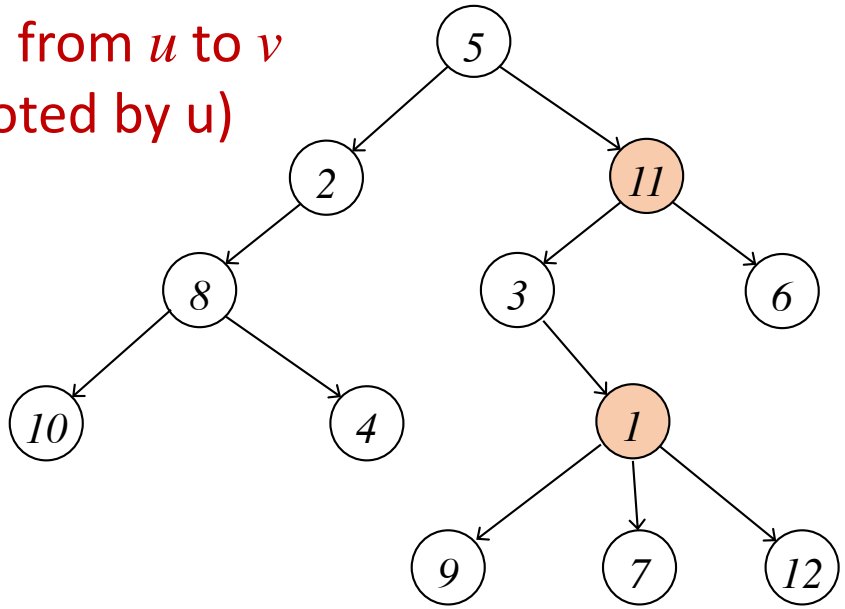
Examples:

•



Definitions

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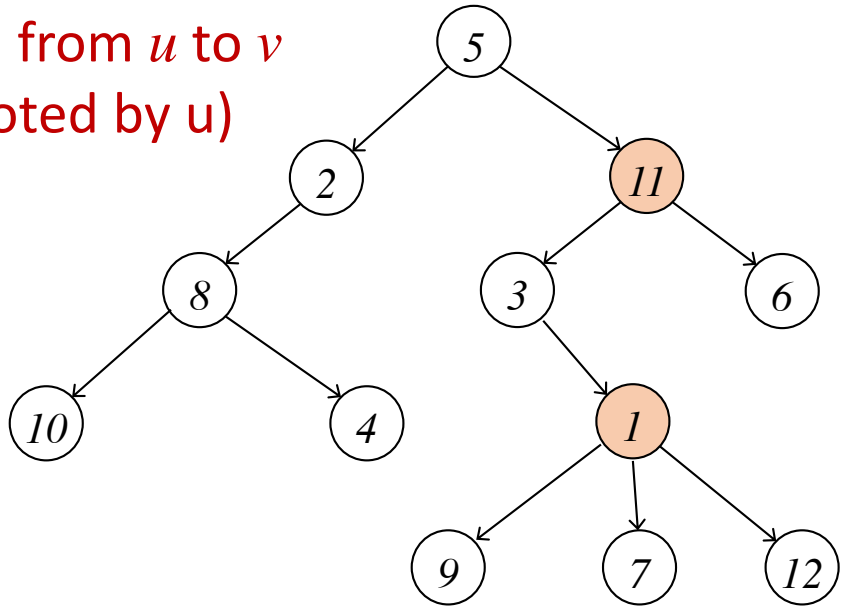
u is an ancestor of *v*, if there is a path from *u* to *v* (alternatively, if v is in the subtree rooted by u)

Examples:

- *11* is an ancestor of *1*



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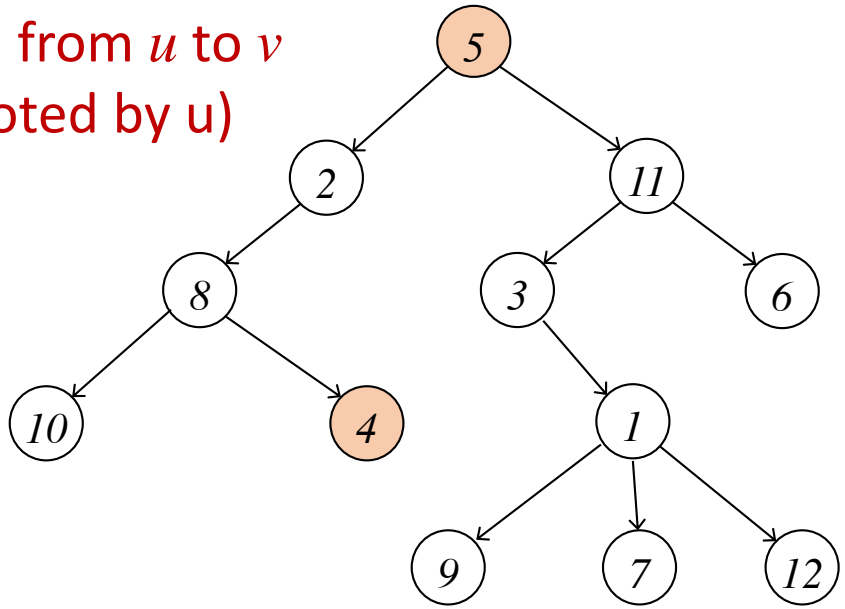
(alternatively, if v is in the subtree rooted by u)
Examples:

- *11* is an ancestor of *1*
-



Definitions

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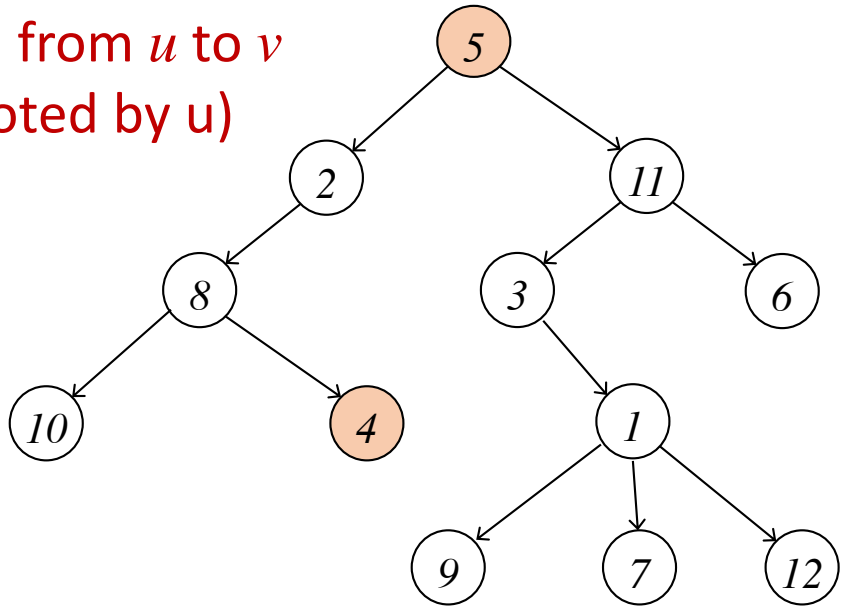
u is an ancestor of *v*, if there is a path from *u* to *v* (alternatively, if v is in the subtree rooted by u)

Examples:

- *11* is an ancestor of *1*
- *5* is an ancestor of *4*



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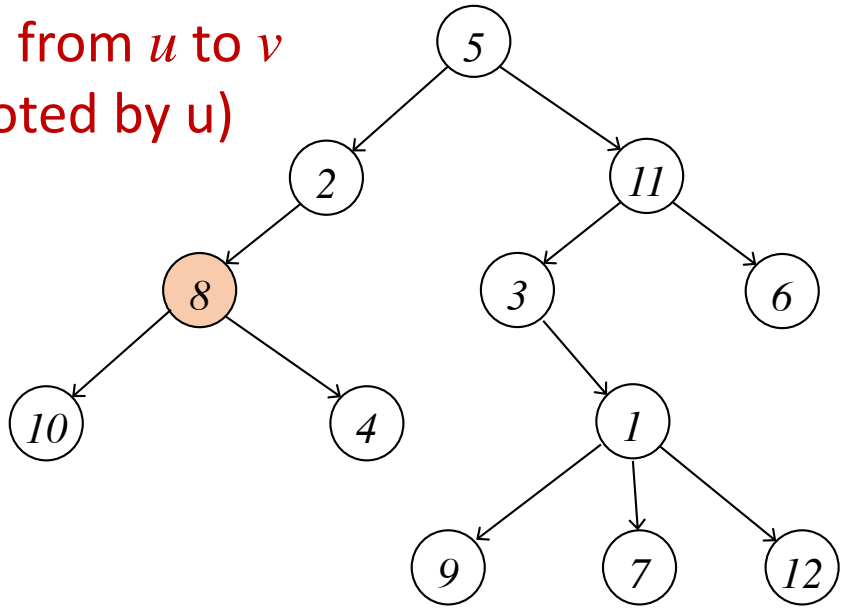

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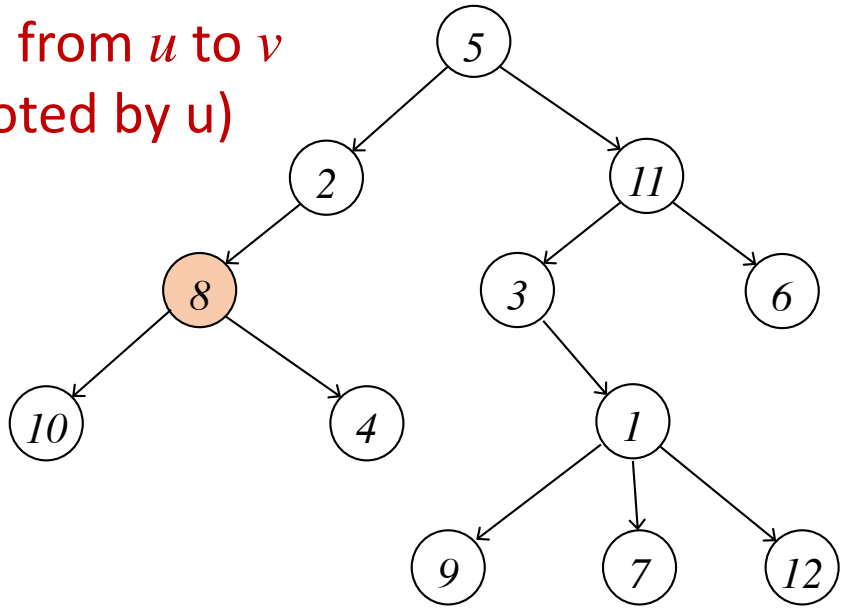

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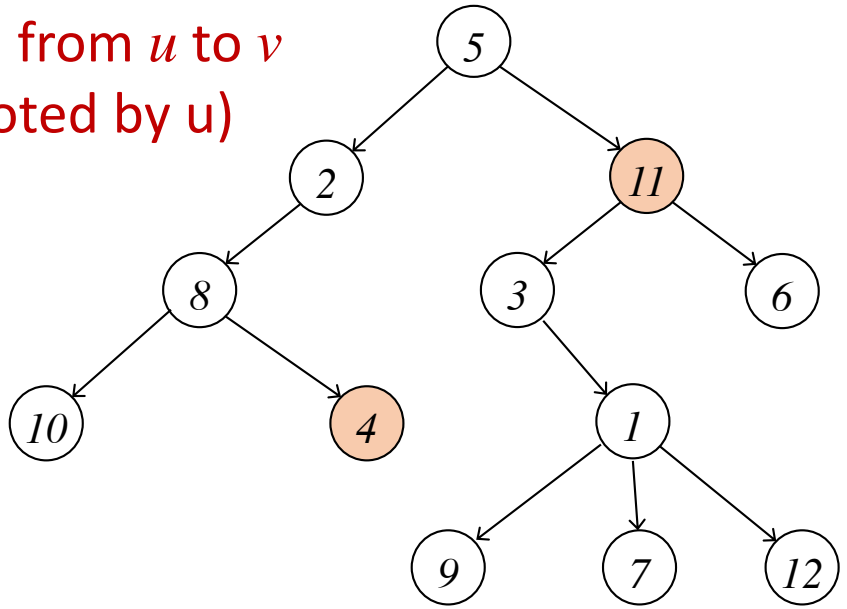

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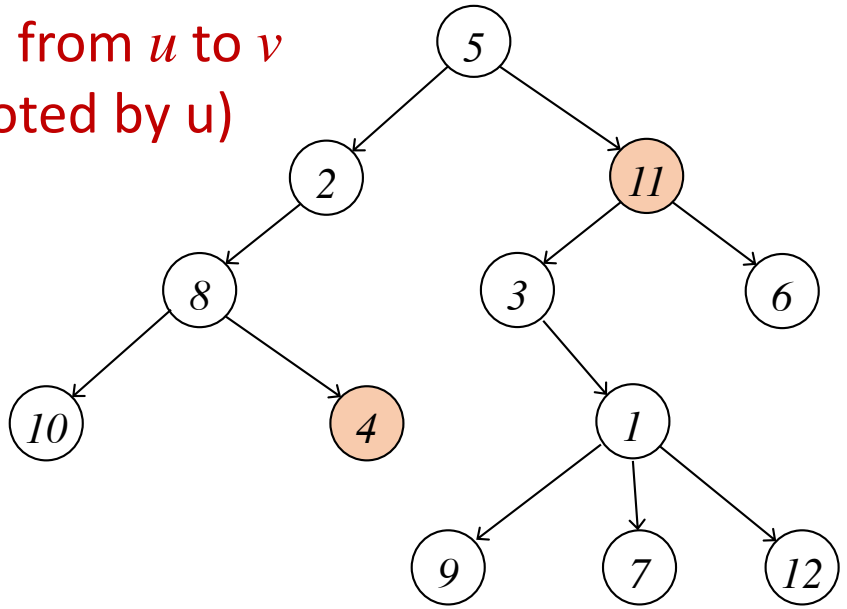

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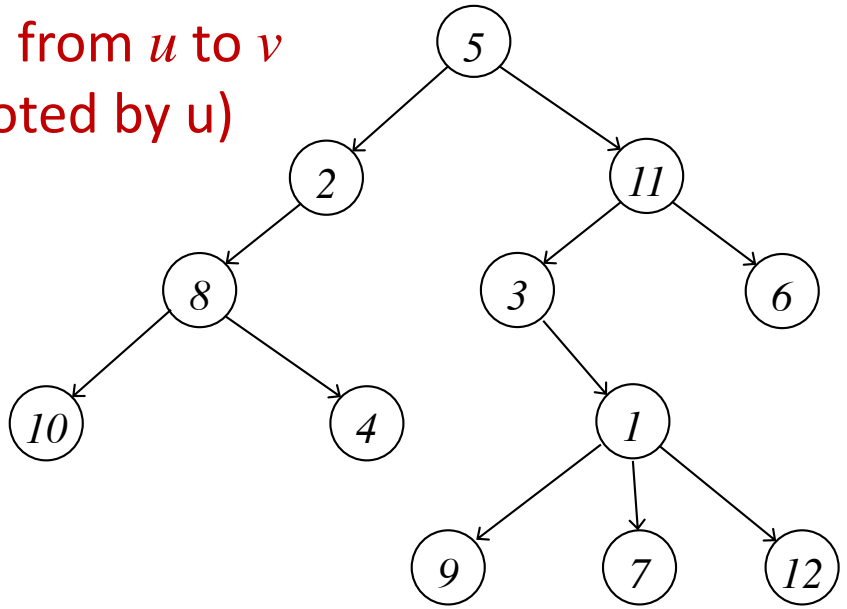

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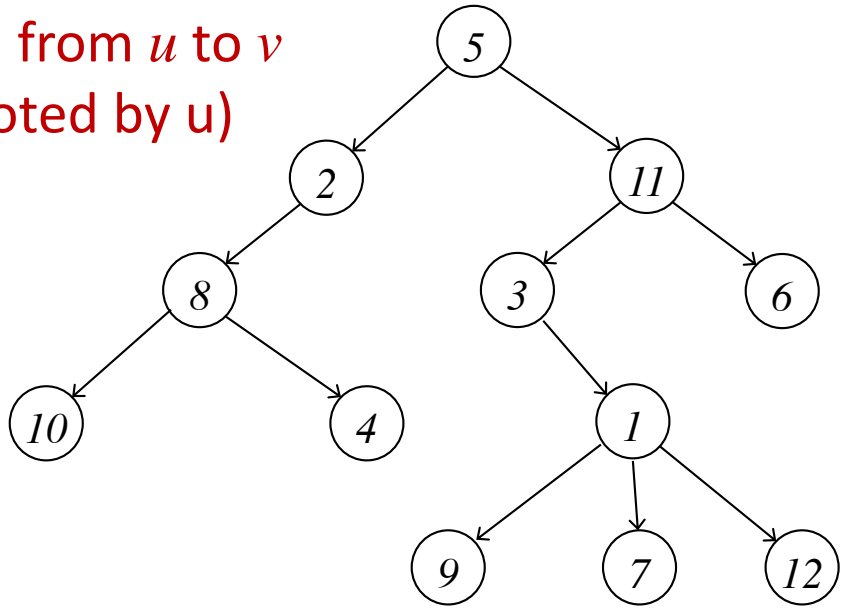

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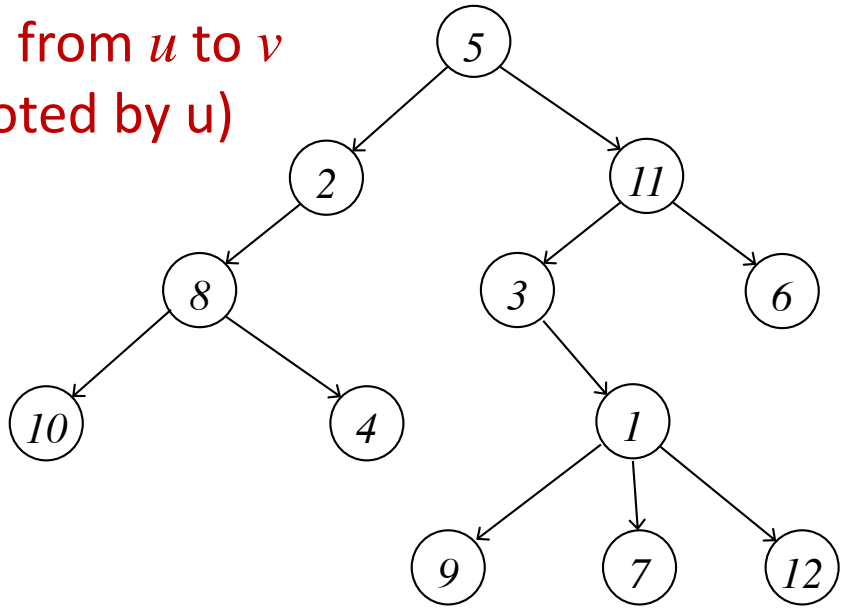

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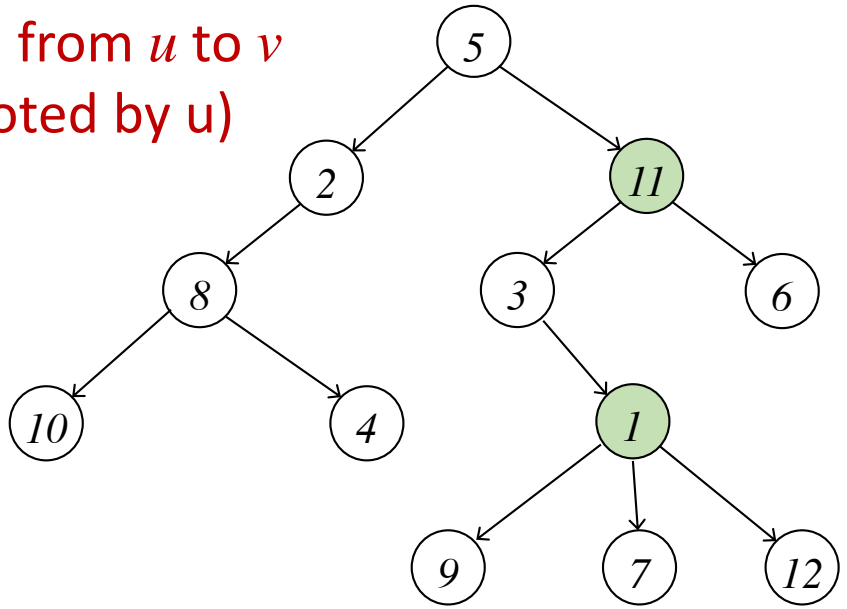

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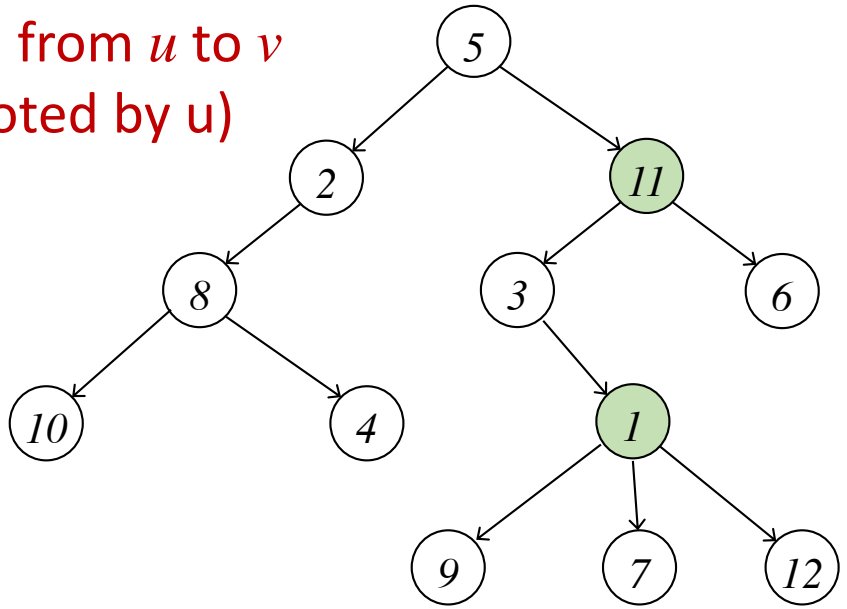

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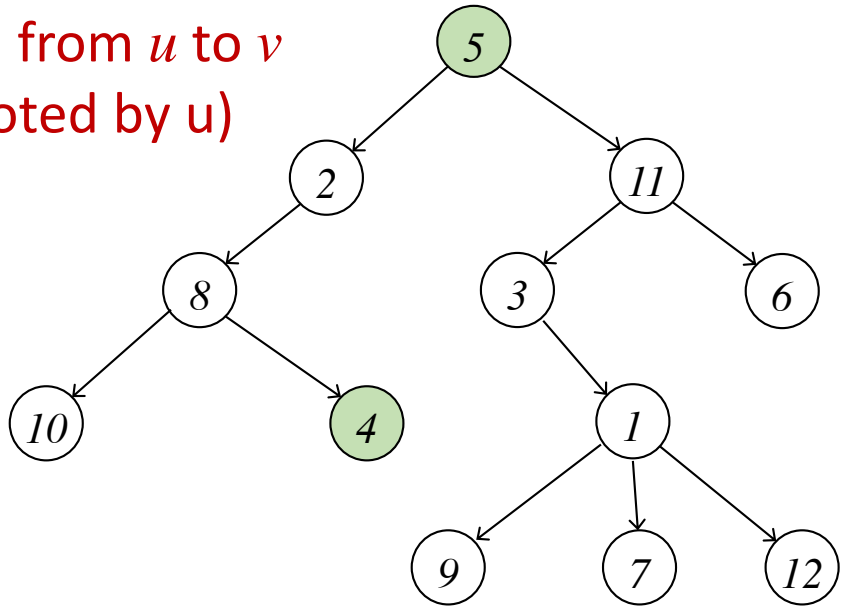

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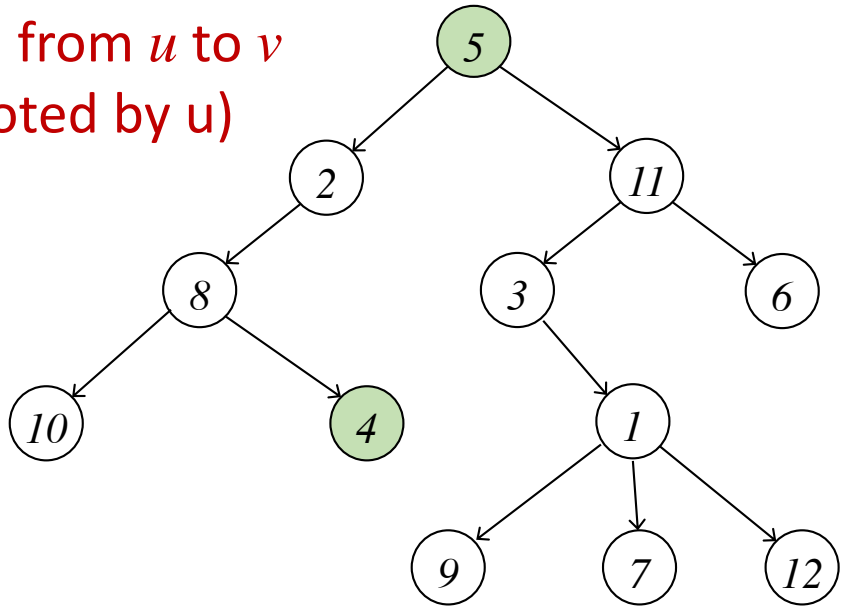

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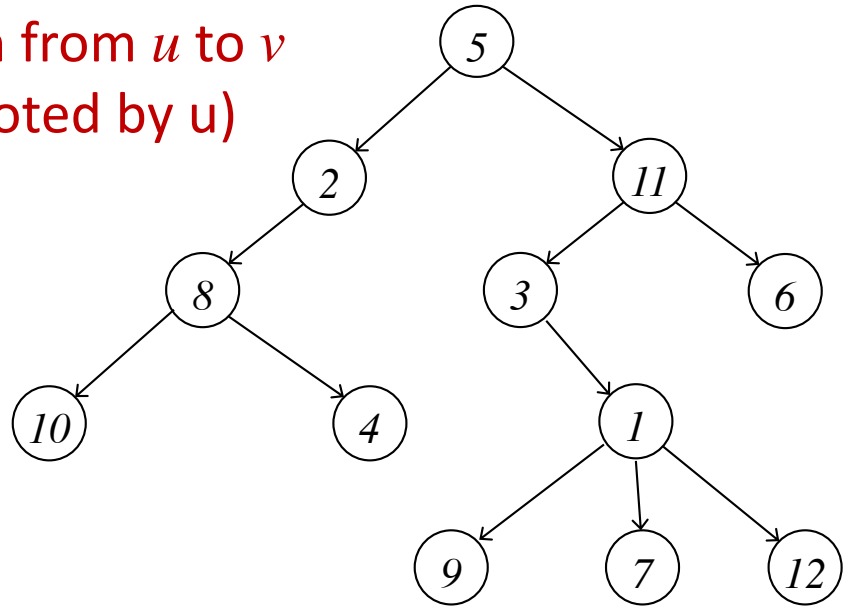

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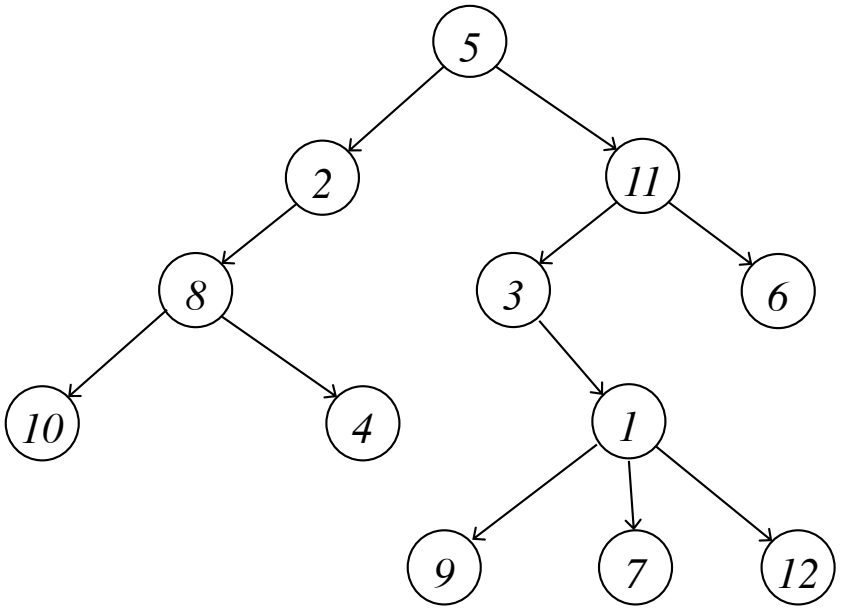
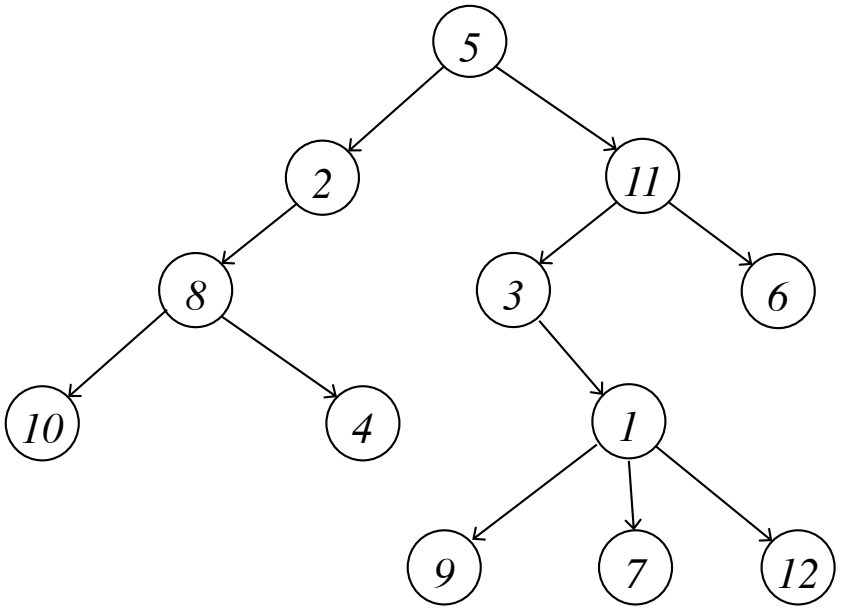


Definitions

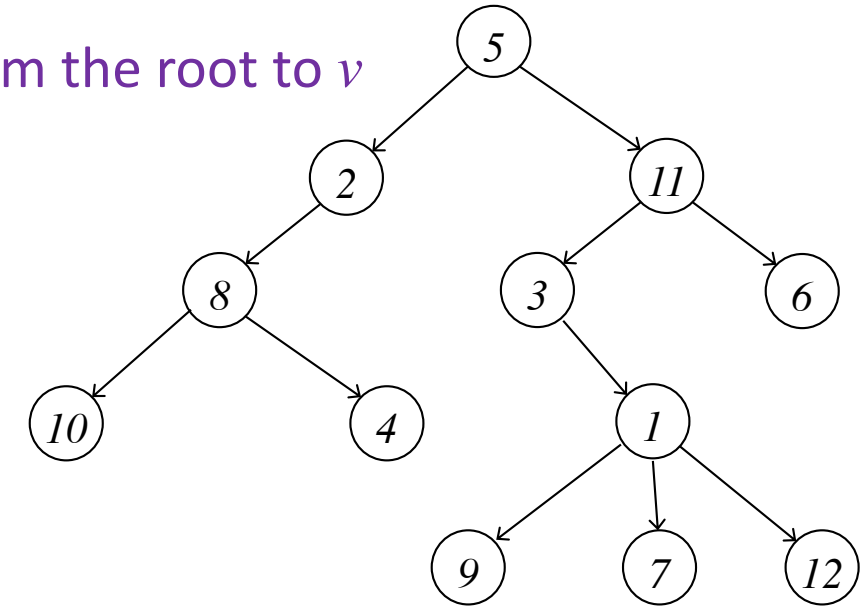
Depth of a node



Definitions

Depth of a node

depth(v) is the length of the path from the root to *v*



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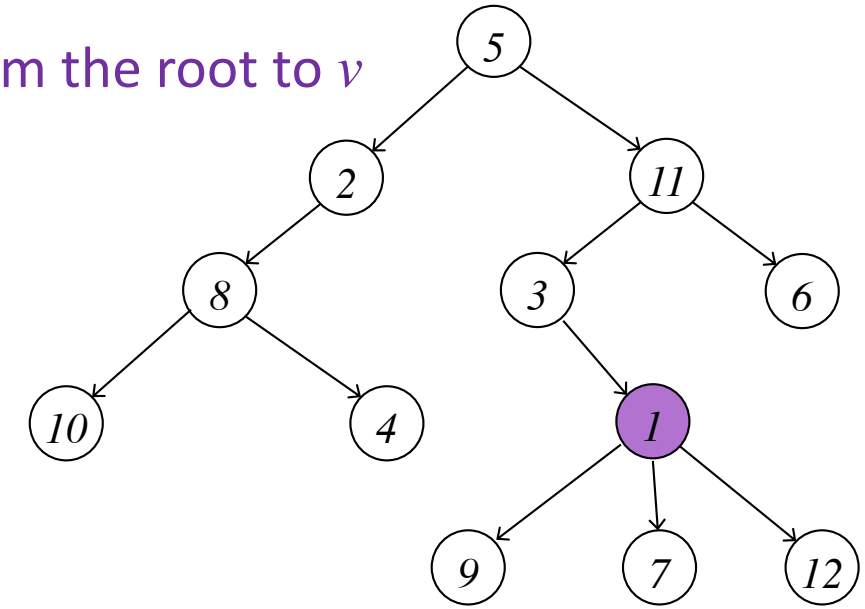
Example: *depth(1) = ?*



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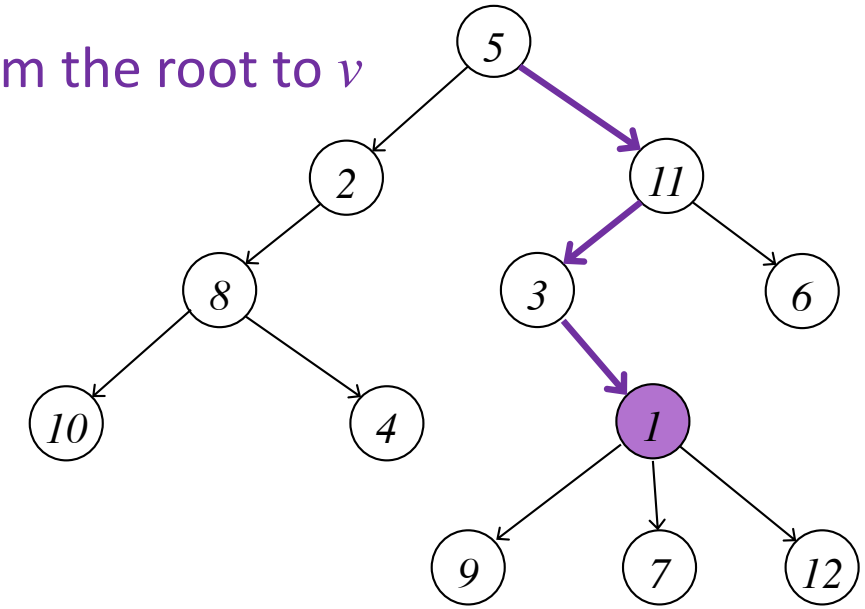

Definitions

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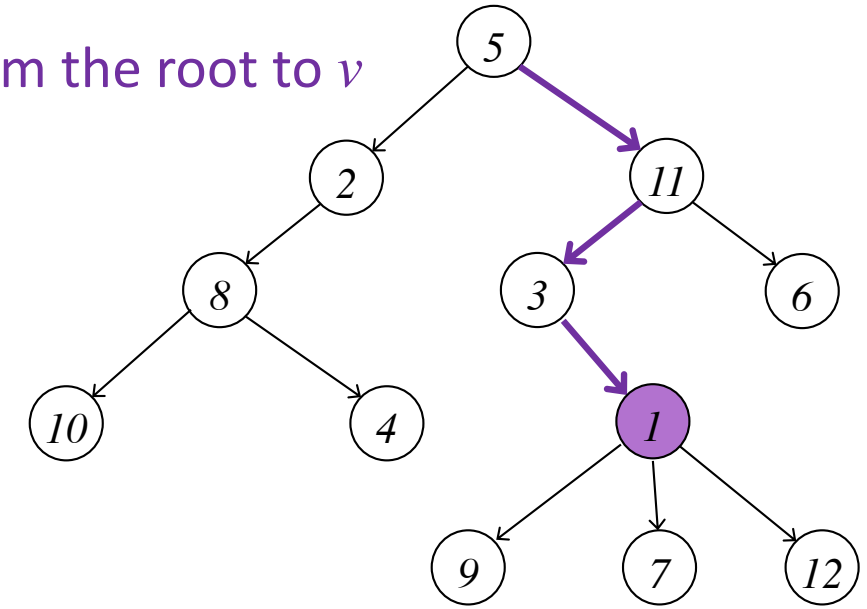

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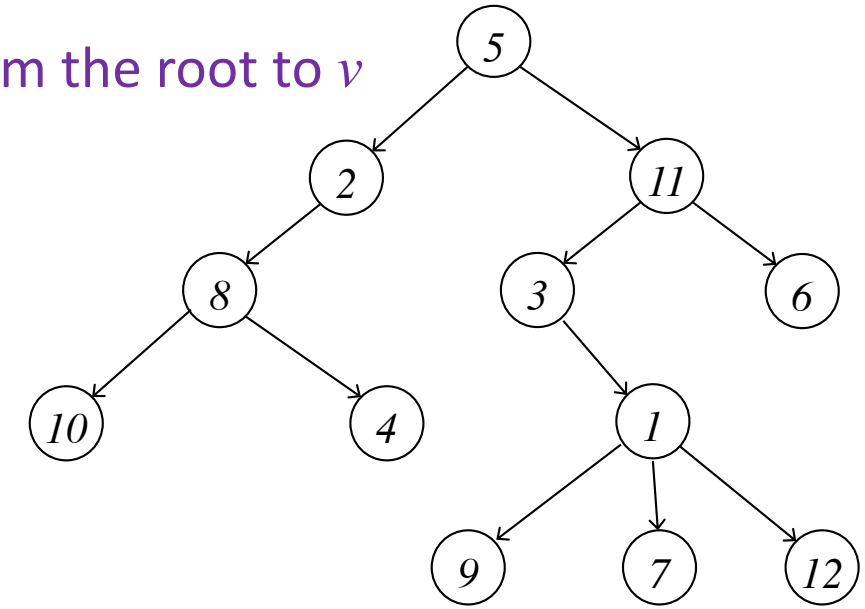

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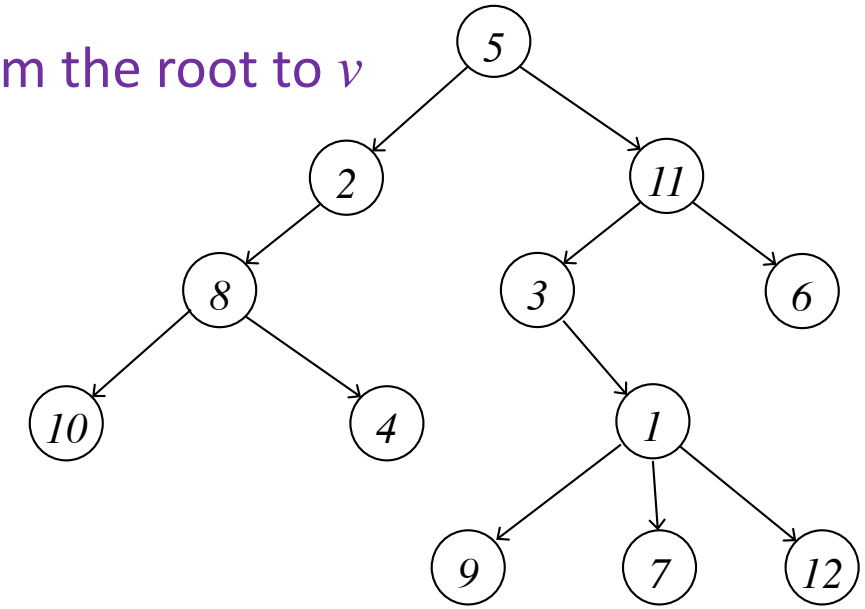

Level 0

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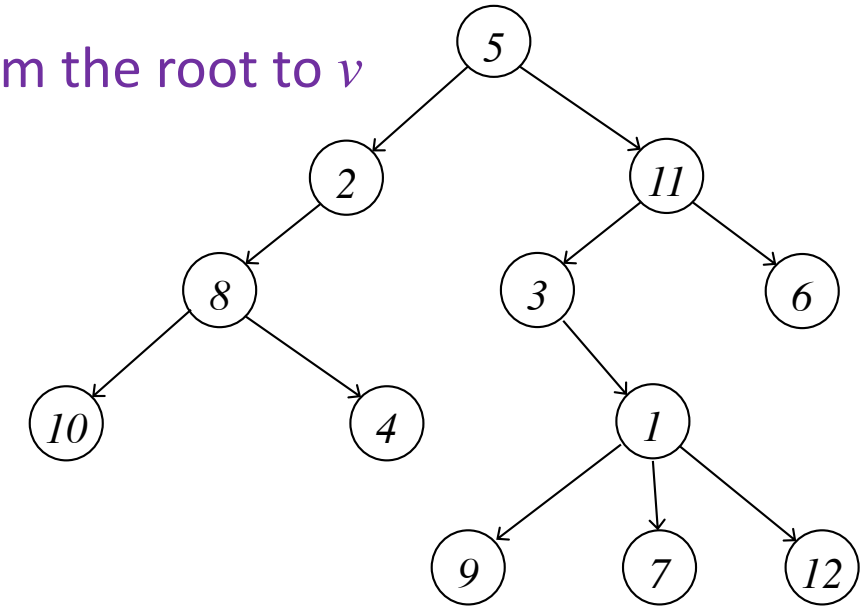

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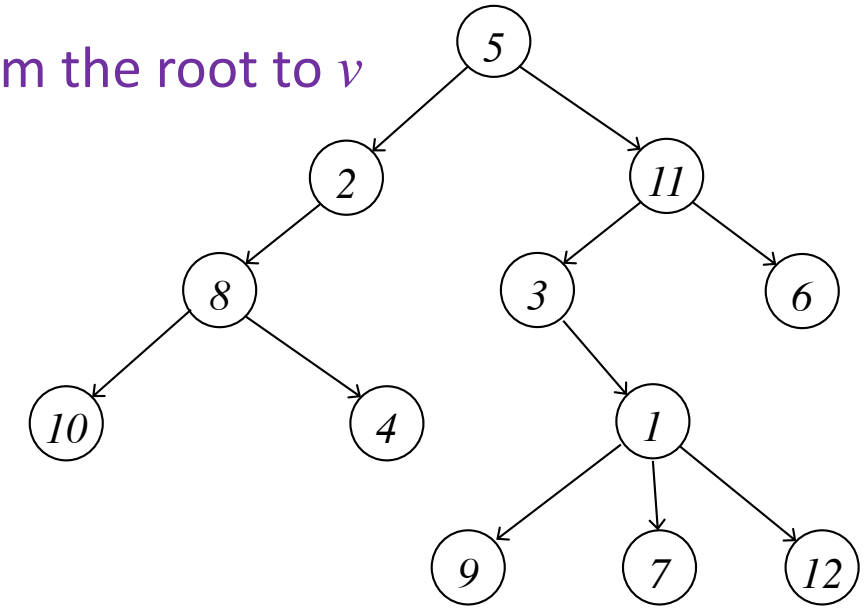

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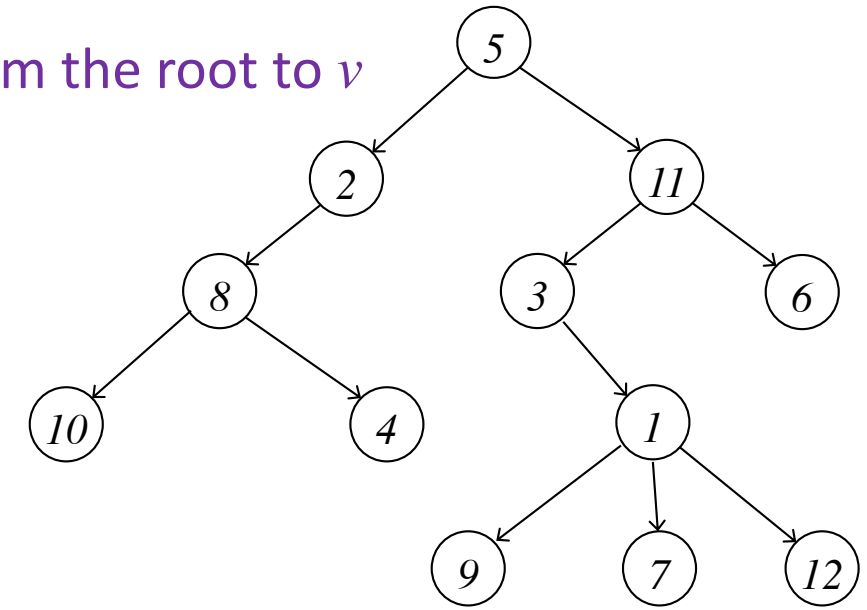

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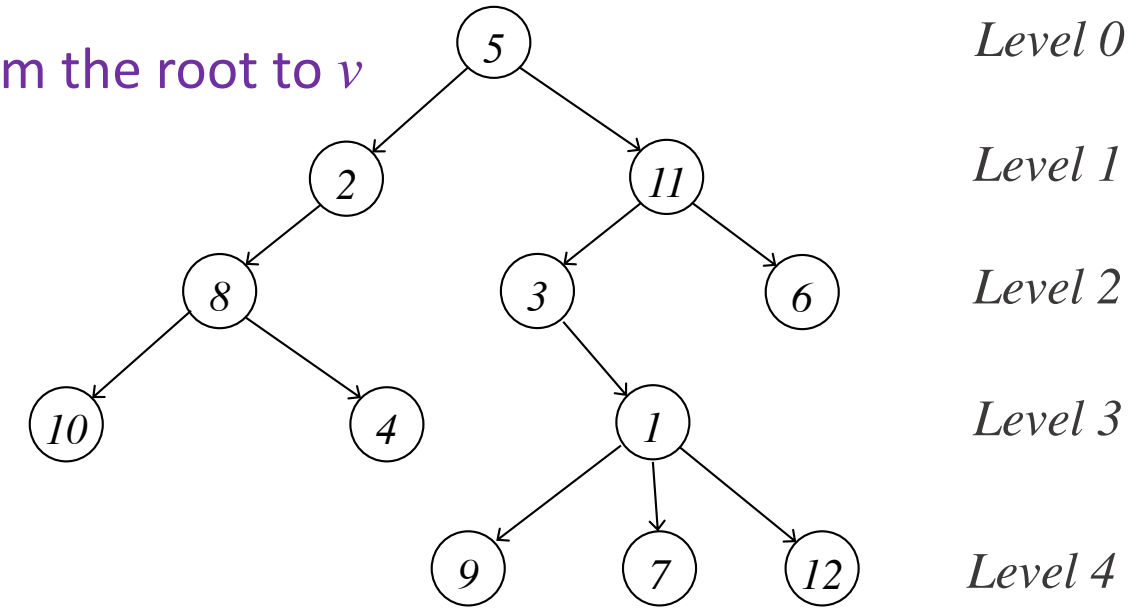

Height of a tree:

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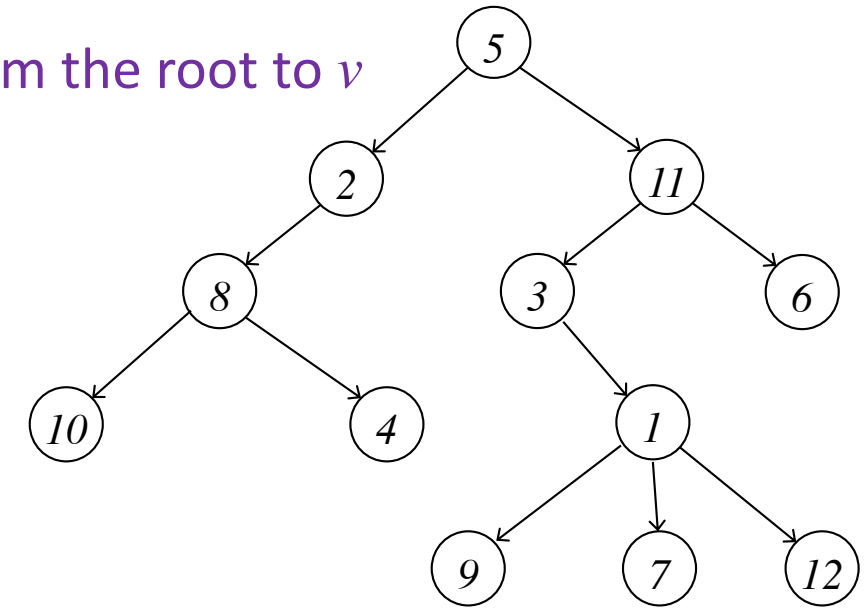

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height(T) is the length of a longest path in *T*.

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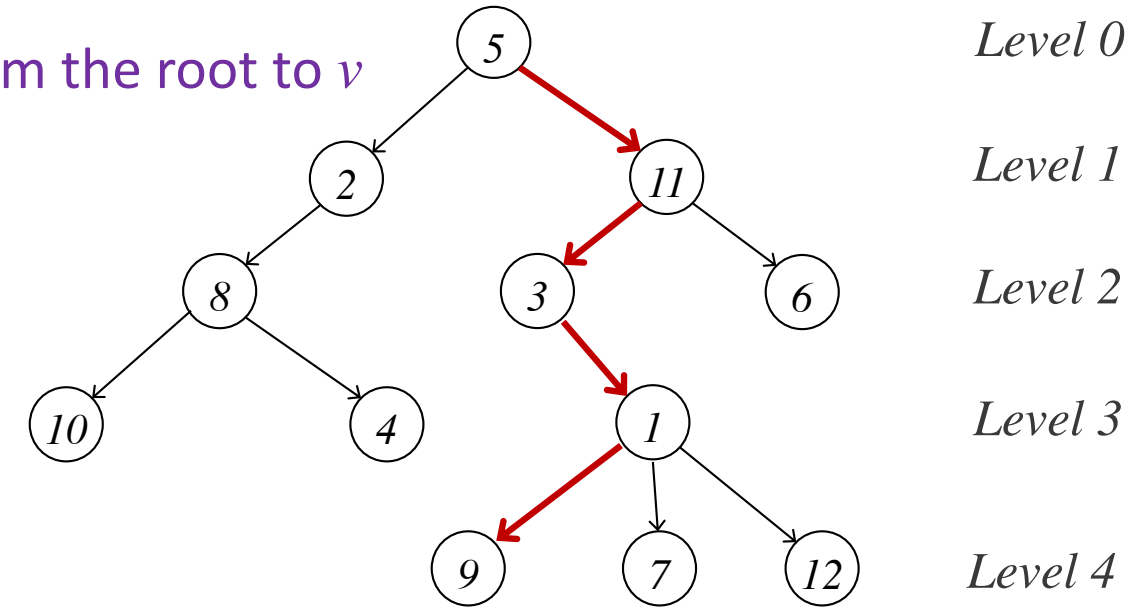

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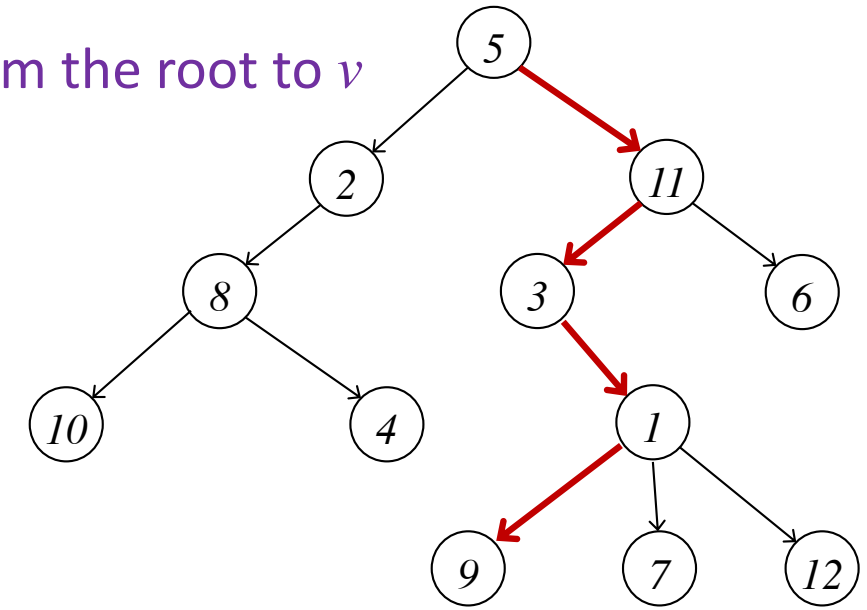

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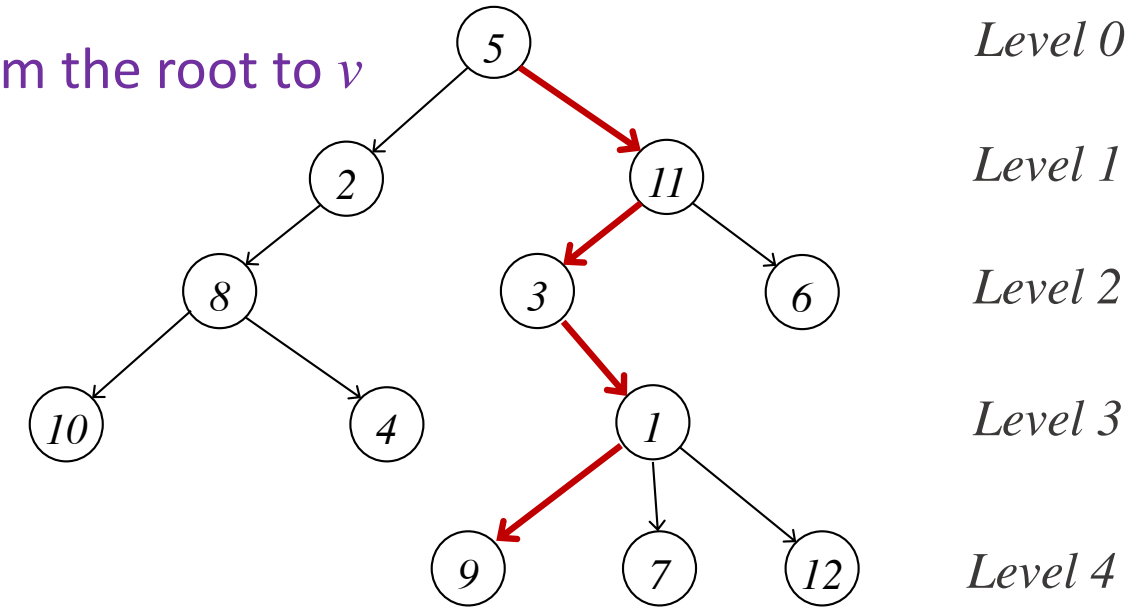

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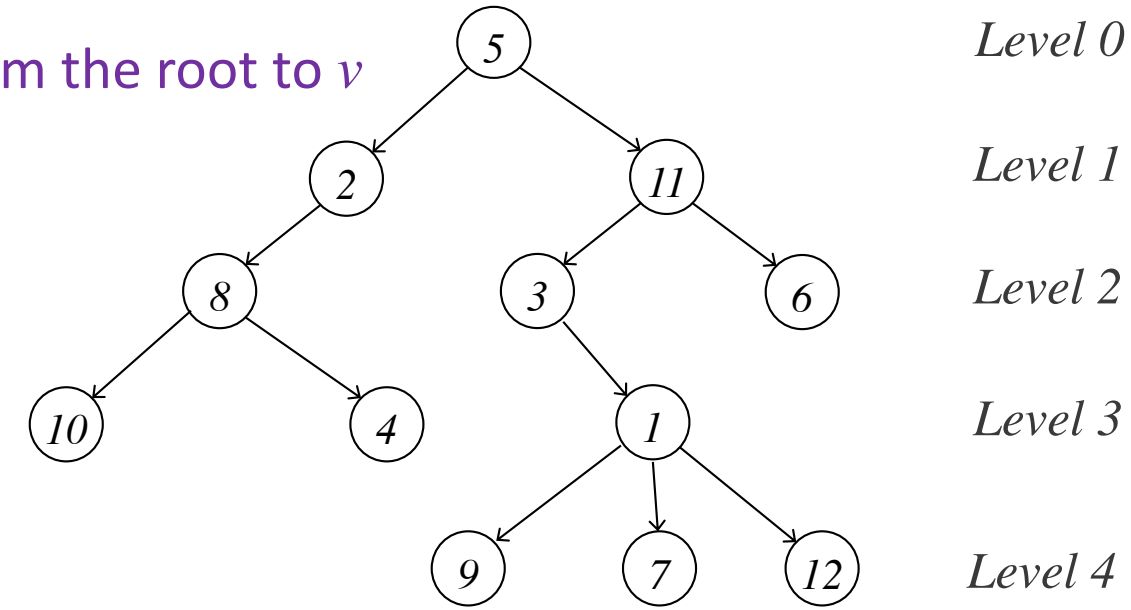

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Definitions

Definitions

Binary Tree

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A tree T is a *binary tree*, if the number of children of each node in T is ≤ 2

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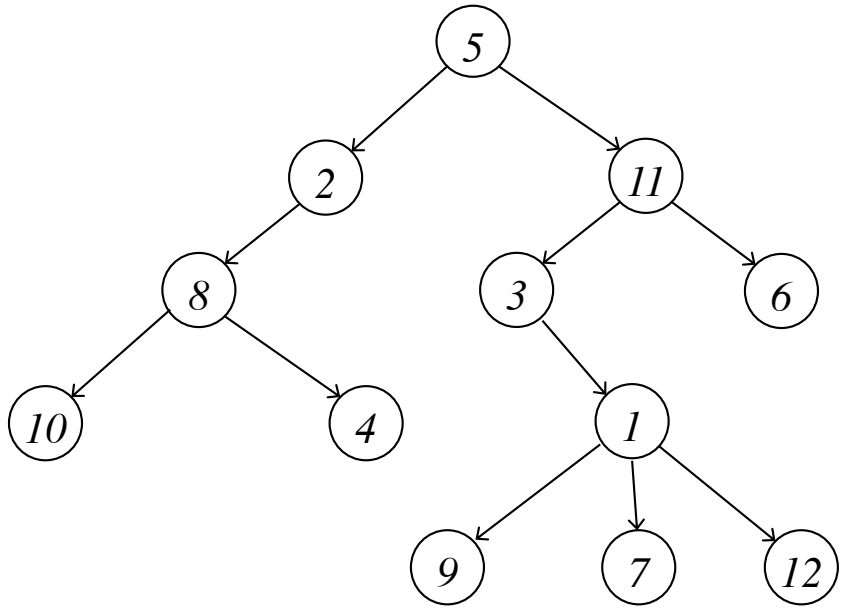

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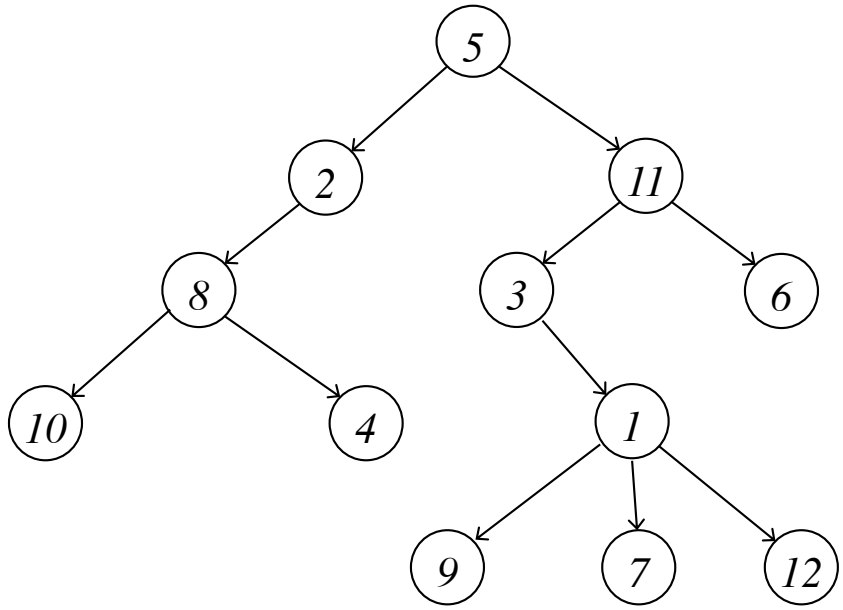

Not a Binary Tree

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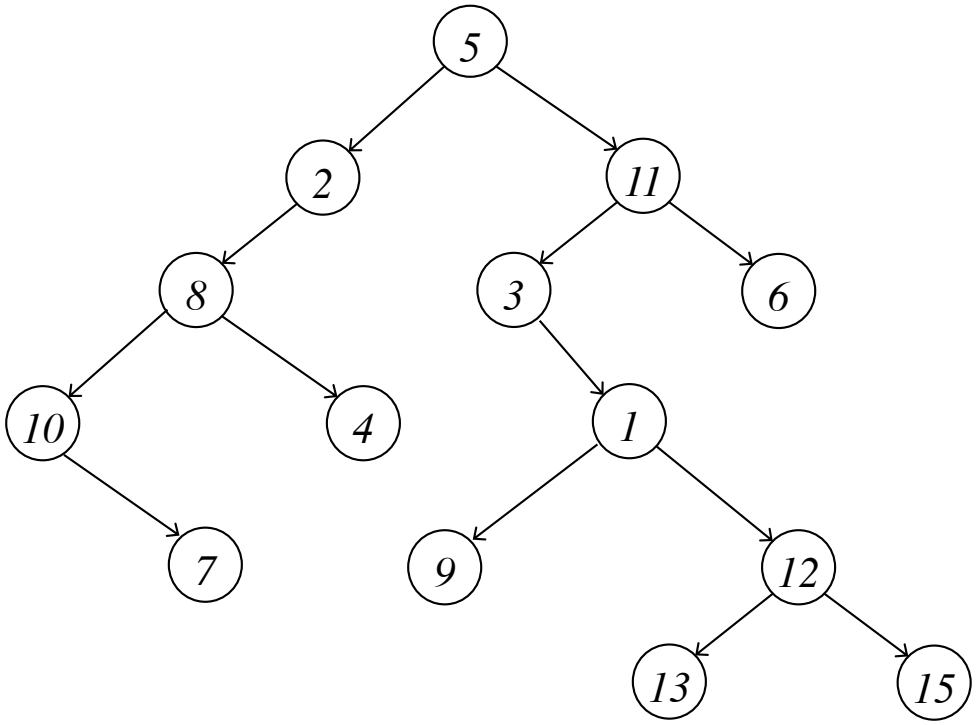

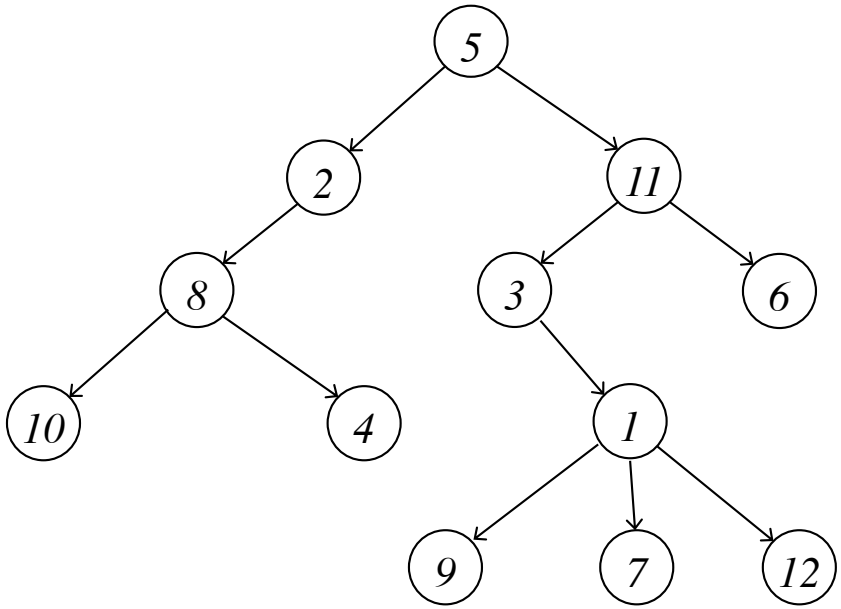

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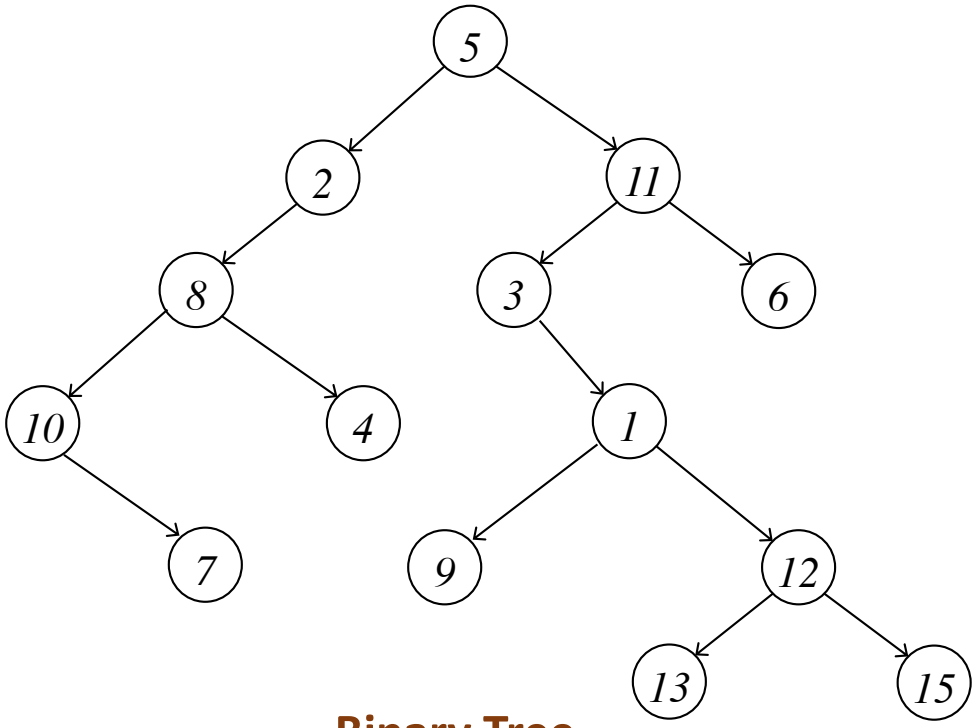

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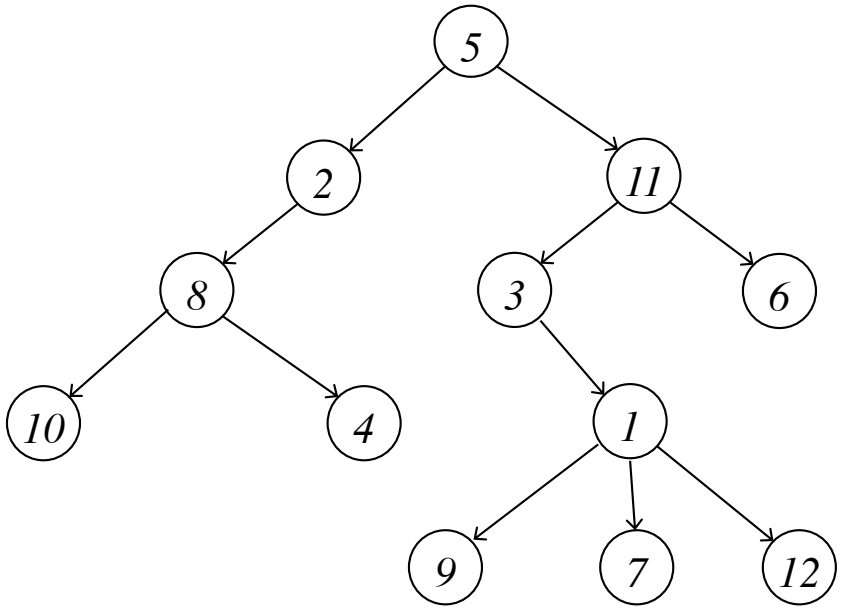


Not a Binary Tree

Definitions

Definitions

Definitions

Full (Proper) Binary Tree

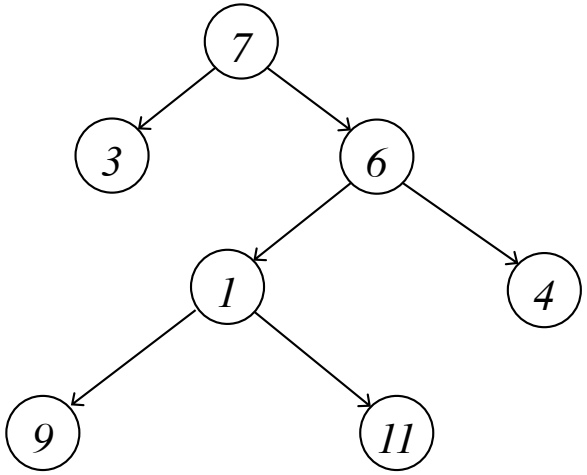
Definitions

Full (Proper) Binary Tree

Complete Binary Tree

Definitions

Full (Proper) Binary Tree

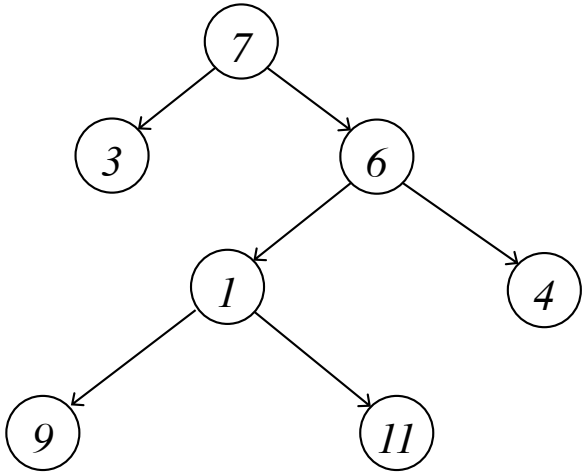


Complete Binary Tree

Definitions

Full (Proper) Binary Tree

A binary tree T is a *full binary tree*, if the number of children of each node in T is either 2 or 0



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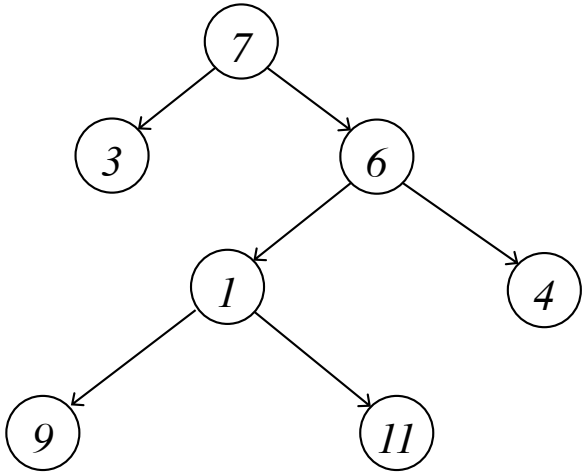
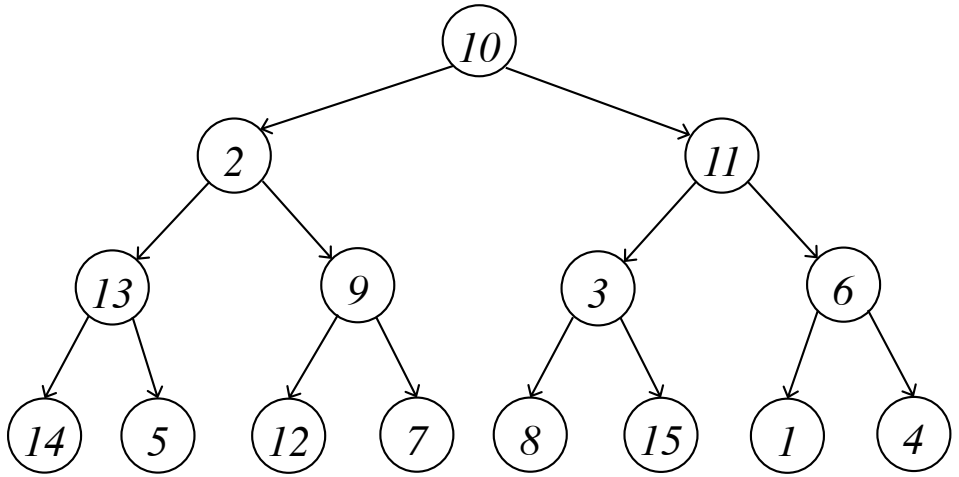


Complete Binary Tree



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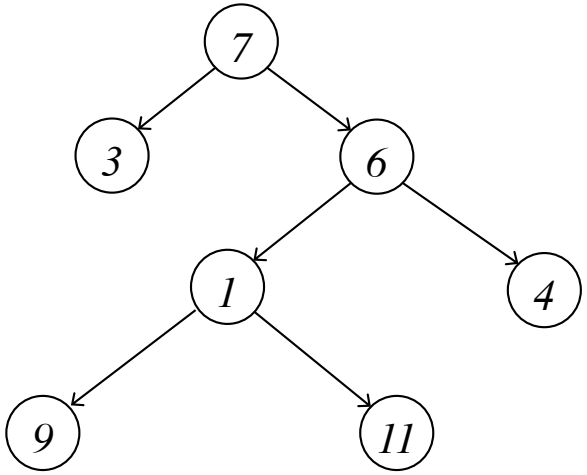
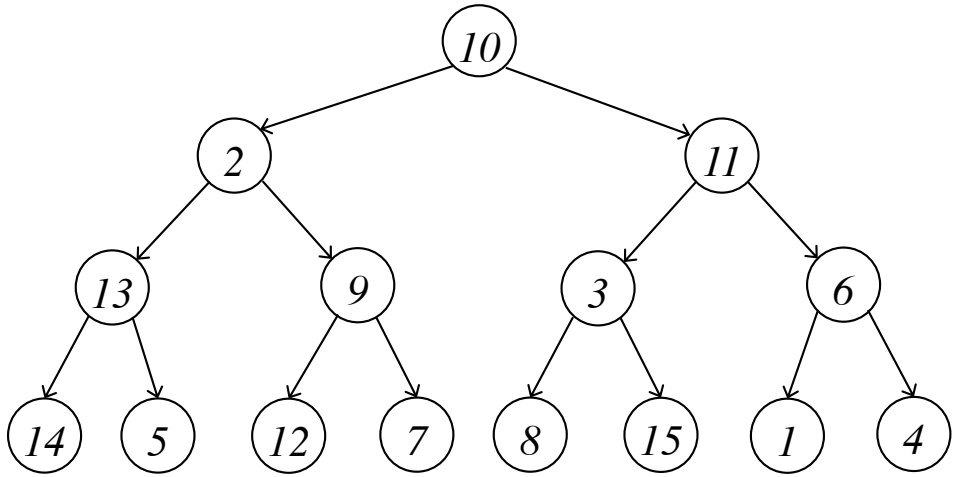
Full (Proper) Binary Tree

A binary tree T is a *full binary tree*, if the number of children of each node in T is either 2 or 0



Complete Binary Tree

A binary tree T is a *complete binary tree*, if all the levels of T contain all possible nodes