

CS4533 Lectures 11-12

Slides/Notes

Shading and Illumination; Compositing (Notes, Ch 14, Notes)

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*** Continued on Shading and Illumination:**

- First we reviewed the “Overall Formula” of the Phong Reflection Model (shown in the next 2 slides) as presented last time, which is implemented in the sample program “Handout: rotate-cube-shading.cpp”.
- Discussed the sample program “Handout: rotate-cube-shading.cpp” (complete sample program has been posted at <https://cse.engineering.nyu.edu/cs653/Rotate-Cube-Shading.tar.gz>)
- Some screenshots of the sample program with annotations are then shown next.
- Showed a demo of the sample program “Demo-Rotate-Cube-Shading”.
- Then we finished up the last 2 pages (p.14 & p.15) of the last lecture notes/slides “Lecture-9-10.pdf” on Normal Matrix.

*** New topic: Composition Techniques.**

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Overall formula:

$$I = K_a \cdot L_a \cdot \text{global} + \sum_{\text{light } i} (\text{Attenuation})_i \cdot [K_a \cdot L_a + K_d \cdot L_d \cdot \max\{\ell \cdot n, 0\} + [\text{if } \ell \cdot n \geq 0] \cdot K_s \cdot L_s \cdot (\max\{\ell \cdot n, 0\})^\alpha]_i$$

Note: component-wise multiplications:
 $K_a \cdot L_a, K_d \cdot L_d, K_s \cdot L_s$
 They are attenuated differently

(1) If light i is a distant (directional) light, then

① $(\text{Attenuation})_i = 1$

② vector ℓ (from pt p to light source i) = $-(\text{distant light direction } L)$ $\ell = -L$

(2) If light i is a point source, then

① $(\text{Attenuation})_i = \frac{1}{a + b \cdot d + c \cdot d^2}$ where d = distance from pt p to the light source
 a, b, c : constant, linear, quadratic attenuations

is $\ell = -L$ (Also, use this ℓ to compute $h = \text{normalize}(\ell + v)$)

replaced with $(n \cdot h)$

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(3) If light i is a spotlight then

① $(\text{Attenuation})_i = \frac{1}{a + b \cdot d + c \cdot d^2} \cdot (\text{spotlight-attenuation})_i$ a, b, c, d are as in (2) point source.

② (spotlight-attenuation) $_i$ = ?

θ : spotlight cut-off angle $\theta \in [0, 90^\circ]$

(a) If $\phi > \theta$ then contribution = 0

In the range $[0, 90^\circ]$
 $\phi > \theta \Leftrightarrow \cos \phi < \cos \theta$
 $\Leftrightarrow (L_s \cdot S) < \cos \theta$
 $\Leftrightarrow L_s \cdot (-\ell) < \cos \theta$

(b) Else
 $(\text{spotlight-attenuation})_i = (\cos \phi)^e = [L_s \cdot (-\ell)]^e$ e : spotlight exponent

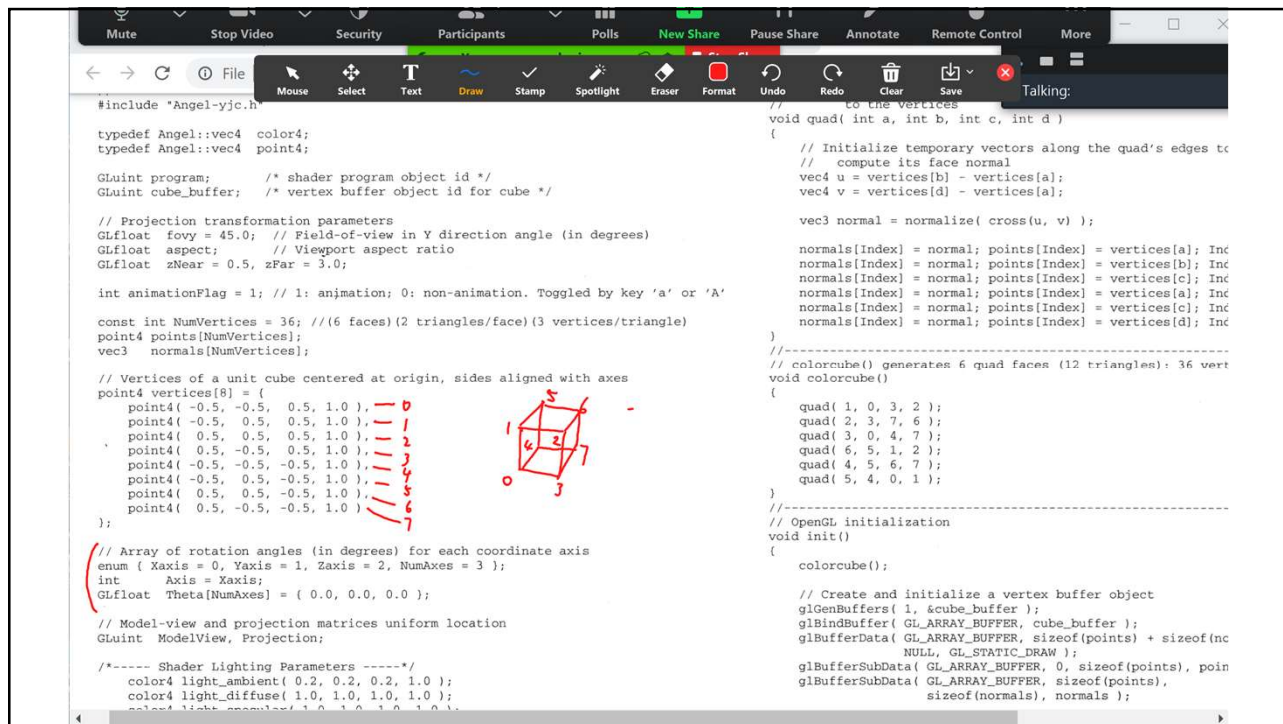
Combining (a), (b): $(\text{spotlight-attenuation})_i = [\text{if } L_s \cdot (-\ell) \geq \cos \theta] \cdot [L_s \cdot (-\ell)]^e$

Let $L_s = \text{normalize}(L_s)$

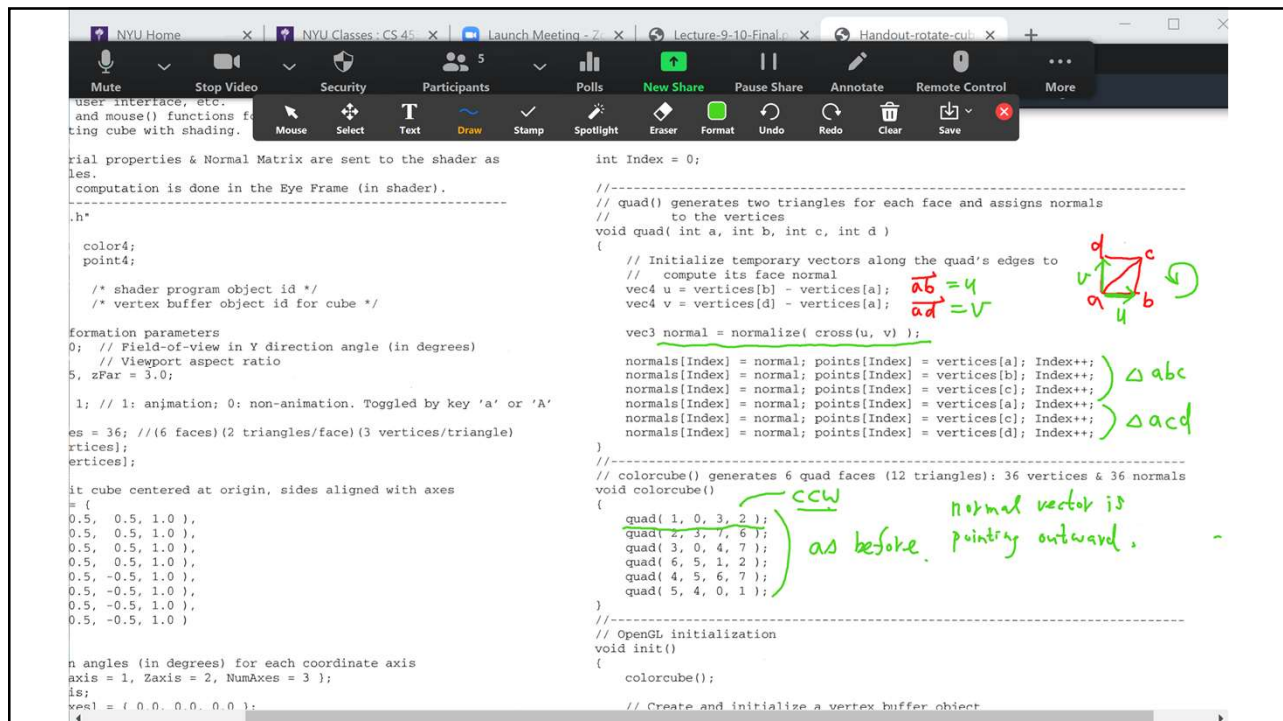
In particular $d = |\vec{P}P_s|$
 $\ell = \text{normalize}(\vec{P}P_s)$
 $= -S$
 $\therefore S = -\ell$

If $L_s \cdot (-\ell) < \cos \theta$ then $(\text{spotlight-attenuation})_i = 0$

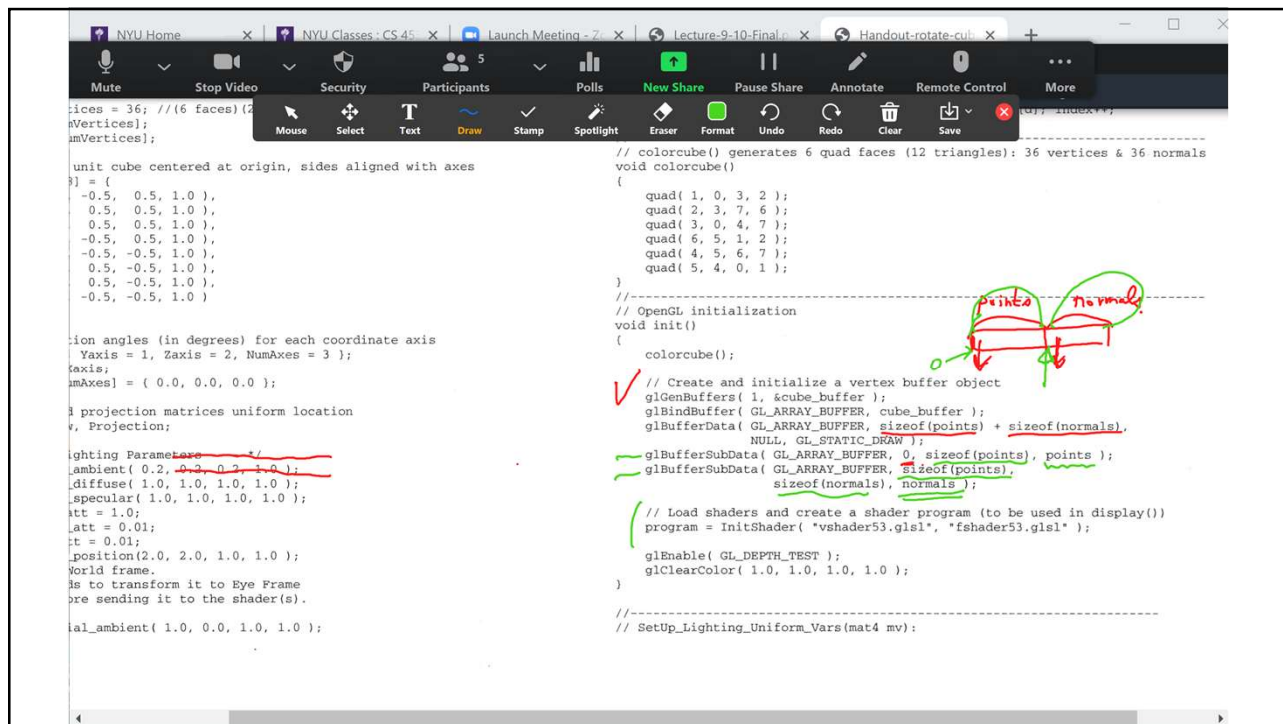
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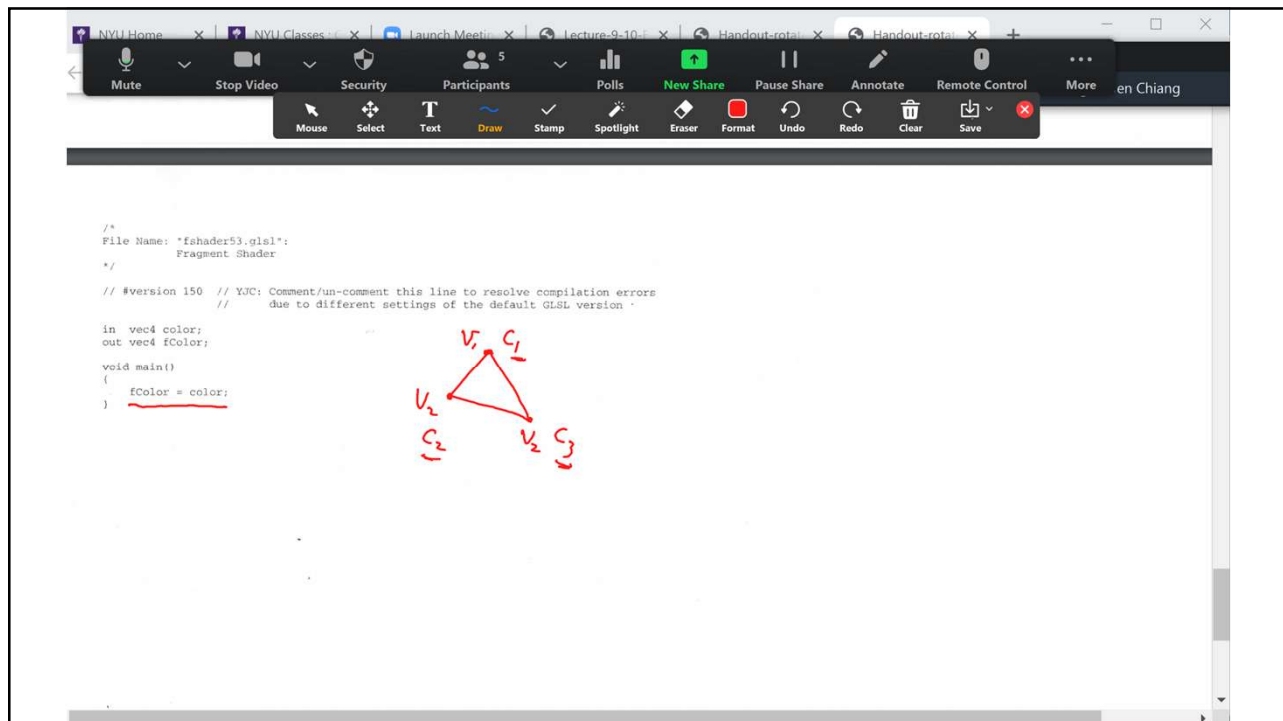
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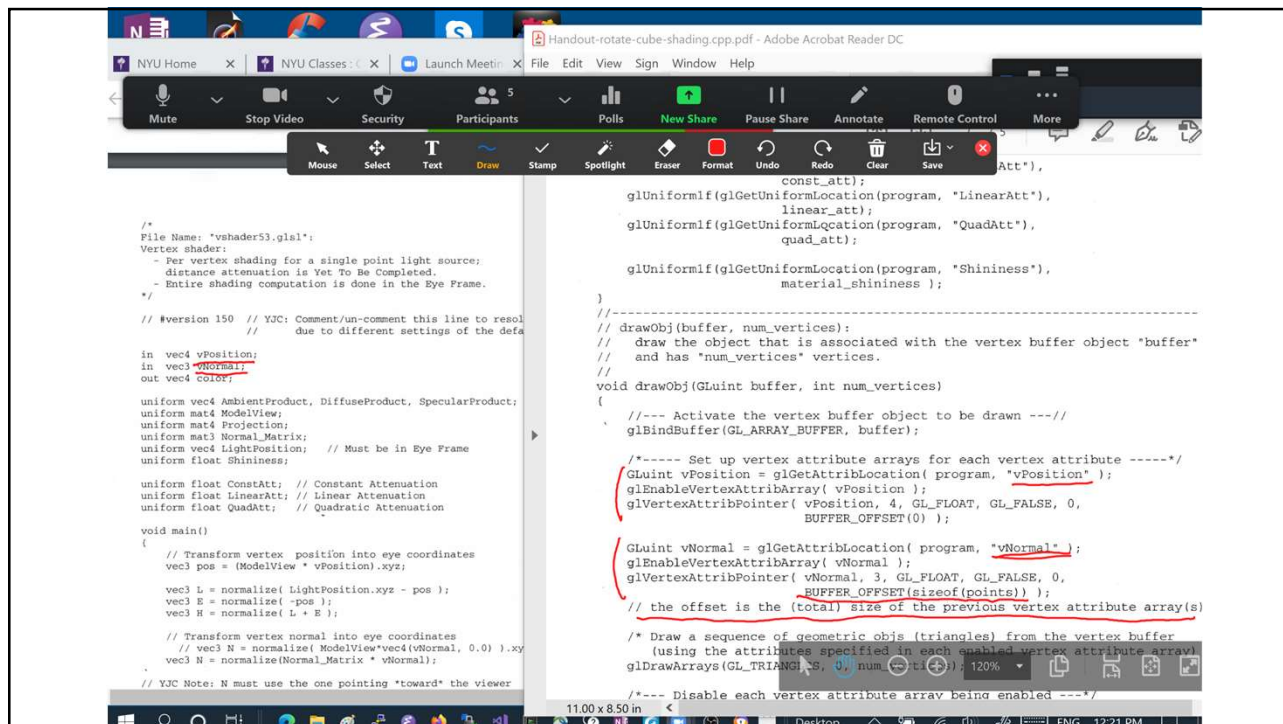
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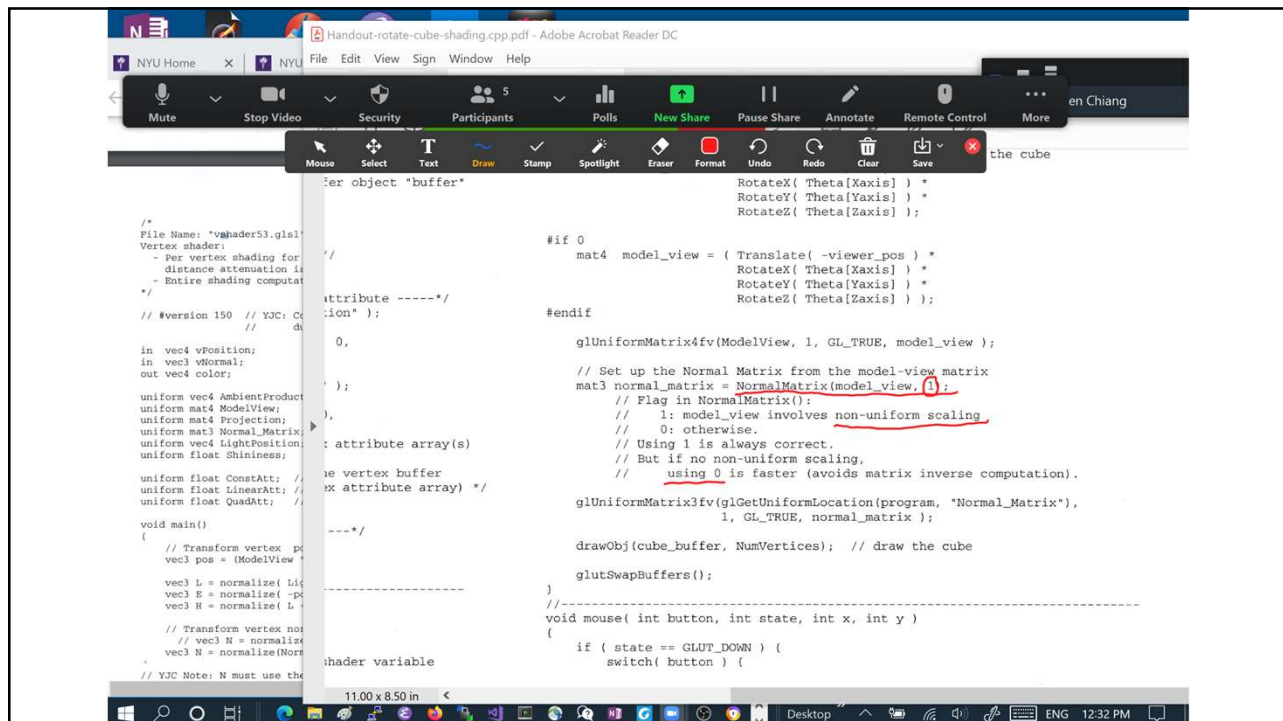
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* Normal Matrix

* Typically we perform shading computation in the (eye frame) (i.e. the right-handed eye frame where the eye/camera is at the origin looking at the -z direction. This is the frame obtained by applying LookAt() to the world frame).

Let \vec{T} be the tangent at pt p being shaded.

\vec{n} normal

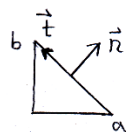
\vec{n}, \vec{T} are in the model frame

M the model-view matrix

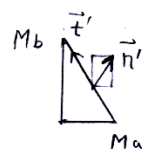
i.e. Mp puts p in the eye frame
in x, y, z dimensions are different

(1) Suppose M involves (non-uniform scaling) (i.e. scaling factors

e.g. $S(1, 2)$
in 2D:



$S(1, 2)$



$$\vec{T}' = M\vec{b} - M\vec{a} = M(\vec{b} - \vec{a})$$

$= M\vec{T}$ is the tangent after transformation

i.e. We can still apply M to \vec{T} to obtain the new tangent \vec{T}' correctly.

But applying M to \vec{n} does NOT give the correct normal vector.

(since \vec{n}' is NOT perpendicular to \vec{T}')

(2) Deriving the correct matrix for normal vector: the (normal matrix)

$$\text{Let } \vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$$

$$\vec{n} \cdot \vec{T} = 0$$

The dot product can be expressed as matrix multiplication:

$$\vec{n} \cdot \vec{T} = \begin{bmatrix} \vec{n} \end{bmatrix}^T \begin{bmatrix} \vec{T} \end{bmatrix} = (\vec{n})^T \vec{T} \quad (*) \quad (\vec{n})^T: \text{transpose of } \vec{n} = \begin{bmatrix} \end{bmatrix}$$

From (*) we have

$$0 = (\vec{n})^T \vec{T} = (\vec{n})^T M^{-1} (M\vec{T})$$

$$\text{But } M = \begin{bmatrix} l & T \\ 0 & 1 \end{bmatrix}$$

and the 4th component of \vec{T} is 0 \Rightarrow We can ignore the 4th column of M , i.e. $\begin{bmatrix} T_x \\ T_y \\ T_z \\ 1 \end{bmatrix}$ and can be ignored

\Rightarrow Then the 4th row of the remaining columns are 0

\therefore In (*) we can use l to replace M :

$$((\vec{n})^T l^{-1}) (l \vec{T}) = 0$$

$$= \text{transformed tangent } \vec{T}'$$

$(\vec{x})^T$ where \vec{x} is the transformed normal, in the form of (*):

$$\therefore (\vec{x})^T = (\vec{n})^T l^{-1} \Rightarrow \vec{x} = [(\vec{n})^T l^{-1}]^T = (l^{-1})^T (\vec{n})$$

cf. In (*): $(\vec{n})^T \vec{T} = 0$

Here: $(\vec{x})^T \vec{T}' = 0$

\Rightarrow Desired normal \vec{x} is obtained by $N \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$ where the 3x3 matrix N (normal matrix) is $(l^{-1})^T$

Simplification:

- (3) If M only involves translations, rotations, uniform scaling, and LookAt() then: translations have no effect on l LookAt() has translation and rotation $\Rightarrow l$ only involves rotations and uniform scaling
- But uniform scaling has no effect after we normalize the transformed normal vector

$\Rightarrow l \equiv R$. But $R^{-1} = R^t$

i $(l^{-1})^t \equiv (R^{-1})^t = (R^t)^t = R \equiv l$

ie ① We can use (l) to replace $(l^{-1})^t$

ie ② We can use the model-view matrix M (4×4) to apply to normal $\vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$

4 components \rightarrow

① \equiv ②

Screenshot for Elaboration:

in 2D.

is. We can still apply M to \vec{T} to obtain the new tangent \vec{T}' correctly. But applying M to \vec{n} does NOT give the correct normal vector. (since \vec{n}' is NOT perpendicular to \vec{T}')

(2) Deriving the correct matrix for normal vector: the (normal matrix)

Let $\vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$. $\vec{n} \cdot \vec{T} = 0$. The dot product can be expressed as matrix multiplication:

$$\vec{n} \cdot \vec{T} = \begin{bmatrix} n_x & n_y & n_z & 0 \end{bmatrix} \begin{bmatrix} T_x \\ T_y \\ T_z \\ T_w \end{bmatrix} = 0$$

From (*) we have $0 = (\vec{n})^t \vec{T} = (\vec{n})^t M^{-1} (M \vec{T})$. But $M = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}$ and the 4th component of \vec{T} is 0 \Rightarrow We can ignore the 4th column of M . is $\begin{bmatrix} T_x \\ T_y \\ T_z \\ 1 \end{bmatrix}$ and can be ignored.

\Rightarrow Then the 4th row of the remaining columns are 0

\therefore In (*) we can use L to replace M :

$$(\vec{n})^t L^{-1} L \vec{T} = 0$$

\Rightarrow transformed tangent \vec{T}

$(\vec{x})^t$ where \vec{x} is the transformed normal, in the form of (*):

$$\vec{x} = (\vec{n})^t L^{-1} = (L^{-1})^t \vec{n}$$

\Rightarrow Desired normal \vec{x} is obtained by $N \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$ where the 3×3 matrix N (normal matrix) is $(L^{-1})^t$

cf. In (*): $(\vec{n})^t \vec{T} = 0$
Here: $(\vec{x})^t \vec{T} = 0$ \leftarrow same

New Topic: Compositing Techniques

Recall:

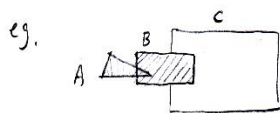
* Fragments : generated by the rasterization of geometric primitives (polygons, etc.)
Each fragment corresponds to a single pixel

* Compositing Techniques : compositing, α -blending

* How do we model transparent objects? — the alpha channel

RGBA (RGB α) color : (r, g, b, α)
 α → opacity : $\begin{matrix} 1 & \text{opaque} \\ \updownarrow & \\ 0 & \text{transparent} \end{matrix}$
 (transparency = $1 - \alpha$)

* α value controls how the RGB values are written to the frame buffer.



B is opaque, blocking C in ^{the} overlapped portion.

A is transparent, the portion overlapped with B is blended with the color of B (blending the colors of A & B)

* Many fragments, each coming from a different object, may correspond

to the same pixel \Rightarrow each such fragment contributes ^{to} the color of the pixel.

the final color of the pixel is obtained by blending the fragment colors.

The corresponding objects are blended or composited together.

* When a polygon is processed, pixel-size fragments are computed.

The fragments are assigned colors based on the shading model used.

Regard the fragment as the (source pixel)

the frame-buffer pixel as the (destination pixel)

Previously : z-buffer, opaque : source pixel is closer to viewer \Rightarrow source pixel (replaces) the destination pixel.

destination pixel : \Rightarrow source pixel is (blocked) no action.

Now : blend the source and destination pixels in various ways

color of source pixel: $s = [s_r \ s_g \ s_b \ s_a]$ source blending factor $b = [b_r \ b_g \ b_b \ b_a]$

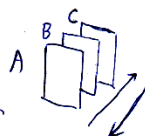
destination: $d = [d_r \ d_g \ d_b \ d_a]$ destination: $c = [c_r \ c_g \ c_b \ c_a]$

$$d' \leftarrow bs + cd$$

compositing: replace d with $d' = [b_r s_r + c_r d_r \ b_g s_g + c_g d_g \ b_b s_b + c_b d_b \ b_a s_a + c_a d_a]$

the resulting r, g, b, a values are clamped to $[0.0, 1.0]$ $\begin{pmatrix} \geq 1 \Rightarrow 1.0 \\ \leq 0 \Rightarrow 0.0 \end{pmatrix}$

Depth Cueing and Fog



"Over" operation: back-to-front.

$$\begin{cases} C_d' = \alpha_s C_s + (1 - \alpha_s) C_d \\ \alpha_d' = \alpha_s + (1 - \alpha_s) \alpha_d \end{cases}$$

transparency: the fraction that the "behind color" survives

* Depth Cueing: create illusion of depth by drawing objects farther from the viewer dimmer

* Fog Effect: extend depth cueing.

create the illusion of partially translucent space (fog) between the object and the viewer, by blending in a (distance-dependent color) as each fragment is processed

f : fog factor, given by the fog equation $f(z)$, ($f = f(z)$)

C_s : fragment color

C_f : fog color

z : distance between a (fragment) being rendered and the (viewer) given in the (eye coordinates)

(** Note: The Handout for the "Over" operation has been posted at NYU Brightspace:

"Handouts -> CS4533_Over-Op-Associativity.pdf")

Resulting color: $C_{s'} = f C_s + (1-f) C_f$ — (*)

fog-mode

fog equation $f(z) = f$

linear fog

$$f = \frac{\text{end} - z}{\text{end} - \text{start}}$$

linear, depth-cueing effect.

exponential fog

$$f = e^{-(\text{density} \cdot z)}$$

exponential

exponential square fog

$$f = e^{-(\text{density} \cdot z)^2}$$

Gaussian

} fog effect

* f specified is clamped to $[0,1]$ and then used in (*) to compute $C_{s'}$.

o From fog equation:
 $z \uparrow \Rightarrow f \downarrow$ (f is clamped to $[0,1]$)
Plugging f into (*)
 $\Rightarrow C_s$ has more weight
 \Rightarrow when object is farther ($z \uparrow$)
we see more of the fog color (C_f)