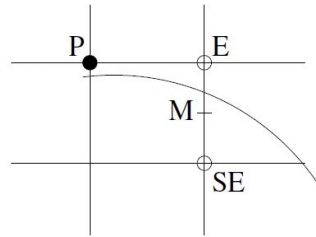
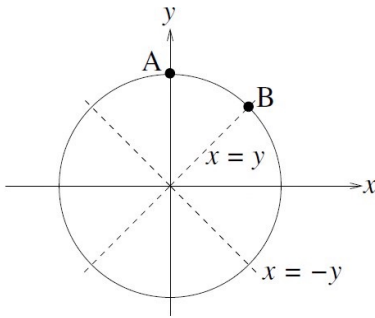


HW Write up

a)

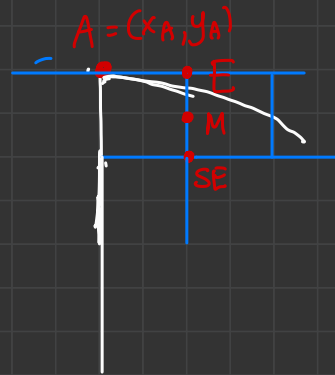


$$F(x, y) = x^2 + y^2 - r^2$$

$$(x_A, y_A) = (0, r) \quad M = (x_A + 1, y_A - \frac{1}{2})$$

$$E = (x_A + 1, y_A)$$

$$SE = (x_A + 1, y_A - 1)$$



Objective : Find D where

if $D < 0$, M is inside circle, choose SE

if $D \geq 0$, M is outside circle, choose E

$$F(E) = (x_A + 1)^2 + y_A^2 - r^2 \leftarrow \text{always } > 0$$

$$F(SE) = (x_A + 1)^2 + (y_A - 1)^2 - r^2 \leftarrow \text{always } < 0$$

$D = F(E) + F(SE)$ if $D < 0$, M in circle, because radius is further away from $F(SE)$

if $D \geq 0$, M on or out of circle, because radius is further away from $F(E)$

$$D = (x_A + 1)^2 + y_A^2 - r^2 + (x_A + 1)^2 + (y_A - 1)^2 - r^2$$

$$= 2(x_A + 1)^2 + y_A^2 + (y_A - 1)^2 - 2r^2$$

$$(x_A, y_A) = (0, r) \text{ since center is origin. } (0, 0)$$

$$= 2(0 + 1)^2 + r^2 + (r - 1)^2 - 2r^2$$

$$= 2 + r^2 + r^2 - 2r + 1 - 2r^2 = 3 - 2r$$

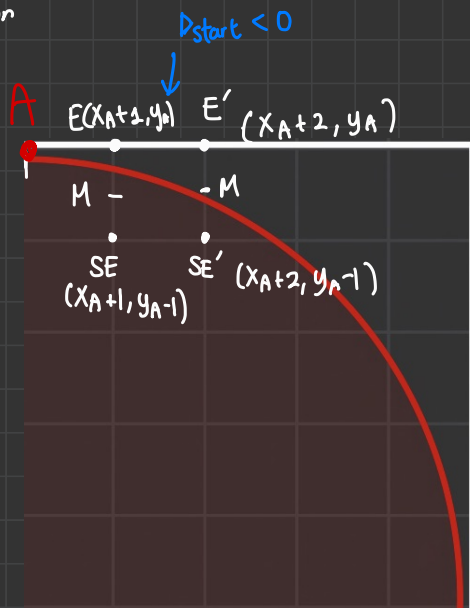
$$D_{\text{start}} = 3r - 2 \quad \text{if } D_{\text{start}} < 0 \text{ pick E}$$

$$D_{\text{start}} \geq 0 \text{ pick SE}$$

This also means that for any circle with radius greater or equal to $\frac{2}{3}$, the first option to choose Δ will always be E.
from point A

Incremental Computation

if $D_{\text{old}} < 0$



$$E' = (x_A + 2, y_A) \quad \text{only increment } x$$

$$SE' = (x_A + 2, y_A - 1)$$

$$D_{\text{new}} = F(E') + F(SE') = (x_A + 2)^2 + y_A^2 - r^2 + (x_A + 2)^2 + (y_A - 1)^2 - r^2$$

$$= 2(x_A^2 + 4x_A + 4) + y_A^2 + (y_A - 1)^2 - 2r^2$$

$$= 2x_A^2 + 8x_A + 8 + y_A^2 + (y_A - 1)^2 - 2r^2$$

$$= (2x_A^2 + 4x_A + 2) + y_A^2 + (y_A - 1)^2 - 2r^2 + 4x_A + 6$$

$$\downarrow$$

$$2(x_A + 1)^2$$

$$= \underbrace{2(x_A + 1)^2 + y_A^2 + (y_A - 1)^2 - 2r^2}_{D_{\text{old}}} + 4x_A + 6$$

$$D_{\text{new}} = D_{\text{old}} + 4x + 6$$

if $D_{old} \geq 0$

$$E' = (x_A + 2, y_A - 1) \quad \text{increment } x$$

$$SE' = (x_A + 2, y_A - 2) \quad \text{decrement } y$$

$$D_{new} = F(E') + F(SE') = (x_A + 2)^2 + (y_A - 1)^2 - r^2 + (x_A + 2)^2 + (y_A - 2)^2 - r^2$$

$$= 2(x_A^2 + 4x_A + 4) + (y_A - 1)^2 - 2r^2 + y_A^2 - 4y_A + 4$$

$$= 2x_A^2 + 8x_A + 8 + (y_A - 1)^2 - 2r^2 + y_A^2 - 4y_A + 4$$

$$= (2x_A^2 + 4x_A + 2) + (y_A - 1)^2 - 2r^2 + y_A^2 - 4y_A + 4 + 4x_A + 6$$

$$= \underbrace{2(x_A + 1)^2 + y_A^2 + (y_A - 1)^2 - 2r^2}_{D_{old}} - 4y_A + 4 + 4x_A + 6$$

$$D_{new} = D_{old} + 4(x - y) + 10$$

Summary:

$$D_{start} = 3 - 2r$$

$$D_{new} = D_{old} + 4x + 6 \quad \text{if } D_{old} < 0$$

$$D_{new} = D_{old} + 4(x - y) + 10 \quad \text{if } D_{old} \leq 0$$