HW   q 2, 3, 5, 6

2.

a) $\hat{y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2$

b) $\hat{y} = A\beta$

$$A = \begin{pmatrix} 1 & X_{11} & X_{12} \\ \vdots & \vdots & \vdots \\ 1 & X_{n1} & X_{n2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$$

$$\beta = (A^T A)^{-1} A^T y = \begin{pmatrix} 0.75 \\ 2.5 \\ 3.5 \end{pmatrix}$$

$$\hat{y} = 0.75 + 2.5X_1 + 3.5X_2$$

3.

a) $\hat{y} = (a_1 x_1 + a_2 x_2) e^{-x_1} \cdot e^{-x_2}$

$$= a_1 x_1 e^{-x_1} e^{-x_2} + a_2 x_2 e^{-x_1} e^{-x_2}$$

$$\beta = (a_1, a_2) \qquad \phi_1(X) = x_1 e^{-x_1} e^{-x_2}$$

$$\phi_2(X) = x_2 e^{-x} e^{-x_2}$$

$$\hat{y} = \underset{\underset{a_1}{\parallel}}{\beta_1} \phi_1(x) + \underset{\underset{a_2}{\parallel}}{\beta_2} \phi_2(x) = [a_1, a_2] \begin{bmatrix} x_1 e^{-x_1} e^{-x_2} \\ x_2 e^{-x} e^{-x_2} \end{bmatrix}$$

b) $\hat{y} = \begin{cases} a_1 + a_2 X & \text{if } x < 1 \\ a_3 + a_4 X & \text{if } x \geq 1 \end{cases}$

$$\beta = (a_1, a_2, a_3, a_4)$$

$$\phi_1(x) = \begin{cases} 1 \text{ if } x < 1 \\ 0 \text{ else} \end{cases} \qquad \phi_2(x) = \begin{cases} x \text{ if } x < 1 \\ 0 \quad \text{else} \end{cases}$$

$$\phi_3(x) = \begin{cases} 1 \text{ if } x \geq 1 \\ 0 \quad \text{else} \end{cases} \qquad \phi_4(x) = \begin{cases} x \text{ if } x \geq 1 \\ 0 \quad \text{else} \end{cases}$$

$$\hat{y} = [a_1, a_2, a_3, a_4] \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \\ \phi_3(x) \\ \phi_4(x) \end{bmatrix}$$

c)

$$\hat{y} = (1+a_1x_1)e^{-x_2+a_2} = (1+a_1x_1)e^{-x_2}e^{a_2}$$

$$= e^{-x_2}e^{a_2} + a_1x_1e^{-x_2}e^{a_2}$$

$$\hat{y} = \beta_1\phi_1(x) + \beta_2\phi_2(x)$$

$$\beta = (\beta_1, \beta_2) = (e^{a_2}, a_1e^{a_2})$$

$$\phi(x) = (\phi_1(x), \phi_2(x)) = (e^{a_2}, x_1e^{-x_2})$$

$$\hat{y} = \beta^T\phi(x) = [e^{a_2}, a_1e^{a_2}]\begin{bmatrix} e^{a_2} \\ x_1e^{-x_2} \end{bmatrix}$$

$$\beta_1 = e^{a_2} \qquad a_2 = \ln(\beta_1)$$

$$\beta_2 = a_1e^{a_2}$$

$$a_1 = \frac{\beta_2}{e^{a_2}} = \frac{\beta_2}{\beta_1}$$

5.

a) If Ω_l is given.

$$x \approx A\beta$$

$$\overset{1 \times 2l}{\beta^T = (a_1, b_1, a_2, b_2, \ldots\ldots, a_l, b_l)}$$

$$\phi_1(\Omega) = \cos(\Omega_1(k))$$

$$\phi_2(\Omega) = \sin(\Omega_1(k))$$

$$\vdots$$

$$\phi_{2l-1}(\Omega) = \cos(\Omega_l(k))$$

$$\phi_{2l}(\Omega) = \sin(\Omega_l(k))$$

$$A = (\phi_1(\Omega), \phi_2(\Omega), \ldots\ldots, \phi_{2l-1}(\Omega), \phi_{2l}(\Omega))$$

$$N \times 2l$$

$$= \begin{pmatrix} \cos(\Omega_1(0)) & \sin(\Omega_1(0)) & \cdots & \cos(\Omega_l(0)) & \sin(\Omega_l(0)) \\ \cos(\Omega_1(1)) & \sin(\Omega_1(1)) & \cdots & \cos(\Omega_l(1)) & \sin(\Omega_l(1)) \\ \vdots & \vdots & & \vdots & \vdots \\ \cos(\Omega_1(N-1)) & \sin(\Omega_1(N-1)) & \cdots & \cos(\Omega_l(N-1)) & \sin(\Omega_l(N-1)) \end{pmatrix}$$

$$\underset{N \times 1}{X} \approx \underset{N \times 2l}{A}\ \underset{2l \times 1}{\beta}$$

$\beta_1 = a_1$ ... $\beta_{2l-1} = a_l$

$\beta_2 = b_1$ ... $\beta_{2l} = b_l$

6.

a)

```
yhat = beta[0] * X[:, 0] + beta[1] * X[:,1] + beta[2] * X[:,1] * X[:,2]
```

b)

```
yhat = np.sum( alpha[None, :] * np.exp(-beta[None, :] * x[:, None]), axis = 1)
```

c)

```
x[:, None ,:]   # shape  n, 1, d

y[ None ,:, :]  # shape  1, m, d
```

```
dist = np.sum((X[:, None,:] - y[None, :,:]) **2 , axis = 2)
```