HW 4

a) Linear, no undermodeling, $\beta = (1, 2, 0)$

b) non linear, no undermodeling, $\beta = (a_0, a_1, b_0, b_1)$
$= (2, 0, 2, 3)$

c)

$$f_0(x) = (x_1 - x_2)^2 = x_1^2 - 2x_1x_2 + x_2^2$$

linear, undermodeling (no x_1x_2)

2.

$$\hat{y} = \sum_{j=0}^{d} \beta_j e^{-\frac{j}{d} u}$$

for model order $d = 1$

$$\hat{y} = \beta_0 + \beta_1 e^{-u}$$

for model order $d = 2$

$$\hat{y} = \beta_0 + \beta_1 e^{-\frac{1}{2} u} + \beta_2 e^{-u}$$

code:

```
nsamp = len(u)

ntr = nsamp // 2
nts = nsamp - ntr

xtr = u[:ntr]
ytr = y[:ntr]

xts = u[ntr:]
yts = u[ntr:]

model = LinearRegression()
RSS_tst = []
for d in dtest:
    Xtr = transform(xtr, d)  # Xtr.shape = (ntr, d+1)
    Xts = transform(xts, d)
    model.fit(Xtr, ytr)
    yhat = model.predict(Xts)

    RSS = np.mean((yhat - y)**2)
    RSS_tst.append(RSS)
```

```
imin =  np.argmin(RSS_tst) + 1   # argmin returns indizes
print(imin)

# Takes a vector X and some degree d and return a matrix X.
def transform(x , d):
    d_array = np.arange(d+2)  #  0 ~ d+1
    X =  np.exp( -j * x[:, None]/ d[None, :])
    return X
```

5.

a)

X_1 = cancer volume

X_2 = patient's age

X_3 = cancer type

$\phi_1(X) = X_1$

$\phi_2(X) = X_2$

X_3	$\phi_3(X)$	$\phi_4(X)$
TYPE I	X_1	0
TYPE II	0	X_1

Model 1:

$$\hat{y} = \beta_0 + \beta_1 \phi_1(X)$$

Model 2:

$$\hat{y} = \beta_0 + \beta_1 \phi_1(X) + \beta_2 \phi_2(X)$$

Model 3:

$$\hat{y} = \beta_0 + \beta_1 \phi_2(X) + \beta_2 \phi_3(X) + \beta_3 \phi_4(X)$$

b)

Model 1: 2
Model 2: 3
Model 3: 4

Model 3 most complex. But model 2 and 3 have same number of features.

c)

Model 1:

$$A = \begin{pmatrix} 1 & 0.7 \\ 1 & 1.3 \\ 1 & 1.6 \\ \vdots & \vdots \end{pmatrix}$$

Model 2:

$$A = \begin{pmatrix} 1 & 0.7 & 55 \\ 1 & 1.3 & 65 \\ 1 & 1.6 & 70 \\ \vdots & \vdots & \vdots \end{pmatrix}$$

Model 3 :

$$A = \begin{pmatrix} 1 & 55 & 0.7 & 0 \\ 1 & 65 & 0 & 1.3 \\ 1 & 70 & 0 & 1.6 \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

d)

$$SE[p_0] = \frac{\sigma[p_0]}{\sqrt{K-1}} = \frac{0.05}{\sqrt{10-1}} = 0.01667$$

$$S_{tgt} = \bar{S}[p_0] + SE[p_0] = 0.70 + 0.01667 = 0.71667$$

$$\hat{p} = \min\{p \mid \bar{S}[p] \leq S_{tgt}\} = 3$$

Model 3.