# HW 7

Q 2, 4, 5.

2.

$$J(w,b) = \sum_{i=1}^{N} (\log(y_i) - \log(\hat{y}_i))^2 \qquad \hat{y}_i = \sum_{j=1}^{p} x_{ij} w_j + b$$

a)

$$\frac{\partial J}{w_j} = \sum_{i=1}^{N} \frac{\partial J}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial w_j} = 2 \sum_{i=1}^{N} \left[\log(y_i - \log(\hat{y}_i))\right] \frac{x_{ij}}{y_i}$$

$$\frac{\partial J}{\partial b} = \sum_{i=1}^{N} \frac{\partial J}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial b} = 2 \sum_{i=1}^{N} \left[\log(y_i - \log(y_i))\right]\left(\frac{1}{\hat{y}_i}\right)$$

b)

```
def Jeval(w,b, X, y):
    yhat = X.dot(w)+b
    J = np.sum((np.log(y) - np.log(yhat))** 2)

    y_error = np.log(yhat) - np.log(y)

    b = 2* yerr/yhat
    Jgrad_w = X.T.dot(b )

    Jgrad_b = np.sum(b)

    return J, Jgrad_w, Jgrad_w
```

4.

$$J(a,b) = \sum_{i=1}^{N} \log(1+e^{z_i}) - y_i z_i \qquad z_i = \sum_{j=1}^{d} a_j e^{-(x_i-b_j)^2/2}$$

a)

$$\frac{\partial J}{\partial a_j} = \sum_{i=1}^{N} \frac{\partial J}{\partial J_i}\frac{\partial J}{\partial z_i}\frac{\partial z_i}{\partial a_j} = \sum_{i=1}^{N}\left(\frac{e^{z_i}}{e^{z_i}+1} - y_i\right)\left(e^{-(x_i-b_j)^2/2}\right)$$

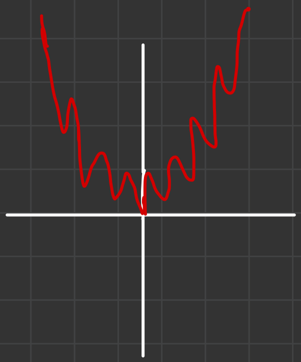
$$\frac{\partial J}{\partial b_j} = \sum_{i=1}^{N} \frac{\partial J}{\partial J_i}\frac{\partial J}{\partial z_i}\frac{\partial z_i}{\partial b_j} = \sum_{i=1}^{N}\left(\frac{e^{z_i}}{e^{z_i}+1} - y_i\right)\left(a_j(b_j-x_i)\right)\left(e^{-(x_i-b_j)^2/2}\right)$$

b)

```
def Jeval(a, b, X, y):

    c = x[:, None] - b[None:]

    d = np.exp(-c**2/2)

    z = d.dot(a)

    J = np.sum(np.log(1+np.exp(z)) - z*y)

    J_Ji = 1/(1+np.exp(-z)) - y

    Jgrad_a = d.T.dot(J_Ji)
    Jgrad_b = -np.sum(a[None:] * c * d, axis=0)

    return J, Jgrad_a, Jgrad_b
```

5.

a.



$f(x) = \frac{1}{4}x^2 + 1 - \cos(2\pi x)$

b

$f'(x) = \frac{x}{2} + 2\pi \sin(x)$

$x_{k+1} = x_k + a_k f'(x_k)$

c $x = 0$

d $x$ between $1.6 - 2.5$ will converge to local minimal $x = 2$.