HW 6

2.

a) The set of x such that its output y has a higher probability of being 1 than 0.

$$P(y=0|X) = 1 - P(y=1|X) = 1 - \frac{1}{1+e^{-z}} = \frac{e^{-z}}{1+e^{-z}}$$

$$P(y=1|X) > P(y=0|X)$$

$$\frac{1}{\cancel{1+e^{-z}}} > \frac{e^{-z}}{\cancel{1+e^{-z}}}$$

$$1 > e^{-z}$$

$$\ln(1) > -z$$

$\beta = [1, 2, 3]^T$

$$0 < z = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

b) The set of x such that its output y has more than 80% probbility of being 1.

$$P(y=1|x) > 0.8$$

$$\frac{1}{1+e^{-z}} > 0.8$$

$$1 > 0.8 + 0.8e^{-z}$$

$$\frac{0.2}{0.8} > e^{-z}$$

$$\ln\left(\frac{1}{4}\right) > -z$$

$$-\ln\left(\frac{1}{4}\right) < z = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \qquad \beta = [1, 2, 3]^T$$

c) The set of feature x_1 such that the othe feature $x_2 = 0.5$ and their output y has more than 80% probability being 1.

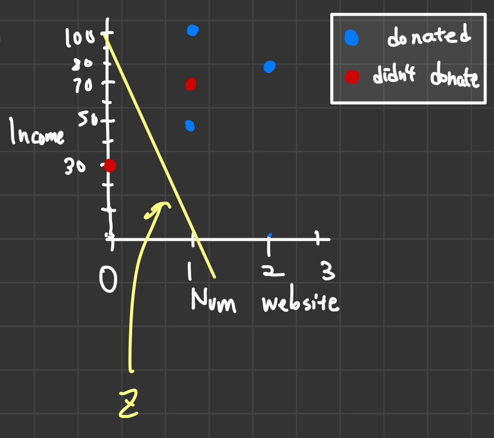
$$-\ln\left(\frac{1}{4}\right) < z = 1 + 2x_1 + 0.5 \times 3$$

$$\frac{-\ln\left(\frac{1}{4}\right) - 1 - 1.5}{2} < x_1$$

$$x_1 > \frac{-\ln\left(\frac{1}{4}\right) - 2.5}{2}$$

3.

a)



b)

$$\begin{cases} 1 & \text{if } z_i > 0 \\ 0 & \text{if } z_i < 0 \end{cases}$$

$$\text{income} + \frac{100}{1}(\text{num\_web}) < 100$$

$$-100 + \text{income} + \frac{100}{1}(\text{num\_web}) < 0$$

$$z_i = W^T x_i + b$$

$$b = -100$$

$$W = [1, 100]$$

c)

$i=0$ $\quad x_{01} = 30 \quad X_{02} = 0 \quad y_0 = 0$

$$-z_0 = w^T X_0 + b$$

$$= \begin{bmatrix} 1 \\ 100 \end{bmatrix} [30, 0] - 100$$

$$P(y_0 = 1 | x_0) = \frac{1}{1+e^{-20}} = \frac{1}{1+e^{70}} \approx 0$$

$$= -70$$

$i=1$

$x_{11} = 50 \quad x_{12} = 1 \quad y_0 = 1$

$$z_0 = \begin{bmatrix} 1 \\ 100 \end{bmatrix} [50, 1] - 100 = 50$$

$$P(y_1 = 1 | x_1) = \frac{1}{1+e^{-50}} \approx 1$$

$i=2$ $\quad x_{21} = 70 \quad x_{22} = 1 \quad y_0 = 0$

$$z_0 = \begin{bmatrix} 1 \\ 100 \end{bmatrix} [70, 1] - 100 = 70$$

$$P(y_2 = 1 | X_2) = \frac{1}{1+e^{-70}} \approx 1$$

$i=3$

$x_{21} = 80 \quad x_{32} = 2 \quad y_0 = 1$

$$z_0 = \begin{bmatrix} 1 \\ 100 \end{bmatrix} [80, 2] - 100 = 180$$

$$P(y_3 = 1 | x_3) = \frac{1}{1+e^{-180}} \approx 1$$

$i=4$

$x_{41} = 100 \qquad x_{42} = 1 \qquad y_0 = 1$

$$z_0 = \begin{bmatrix} 1 \\ 100 \end{bmatrix} [100, 1] - 100 = 100$$

$$P(y_4 = 1 \mid y_4) = \frac{1}{1+e^{-100}} \approx 1$$

Sample $i=2$, the error sample is the least likely.

d)

No, the new parameters will not change $\hat{y}$ in (b) because it is a hard decision classifer

Yes, the new parameter will change likelihood in (c) because it is a soft decision classifer

and will change $z_0 \to \alpha z_0$

4.

$$P(Y=1|X) = \frac{1}{1+e^{-z}} \qquad z = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

$$z = -6 + 0.05X_1 + X_2$$

a) $P(Y=1 | X_1 = 40, X_2 = 3.5)$

$$\frac{1}{1+e^{-z_0}} \qquad z_0 = -6 + 0.05(40) + 3.5 = -0.5$$

$$\frac{1}{1+e^{-(-0.5)}} \approx 0.3775 \approx 38\%$$

b)

$$P(Y=1 | X_1 = a, X_2 = 3.5) = \frac{1}{1+e^{-z_0}} = 0.5$$

find a

$$\frac{1}{1+e^{-z_0}} = 0.5 \qquad 1 = 0.5(1+e^{-z_0})$$

$$1 = 0.5 + 0.5e^{-z_0} \qquad \cancel{0.5} = \cancel{0.5}e^{-z_0}$$

$$1 = e^{-z_0} \qquad \ln(1) = -z_0$$

$$z_0 = 0 = -6 + 0.05(X_1) + 3.5$$

$$0.05X_1 = 2.5$$

$$X_1 = 50 \text{ hours.}$$