

1) STUDIA CON VERSI SISTEMA SOLO UNO,
(IN TAL CASO TRNOVA(E) IL SISTEMA

$$\left\{ \begin{array}{l} x_1 - 2x_2 + x_3 = 1 \\ x_1 - x_2 + 4x_3 = 7 \\ x_1 - 2x_2 + 2x_3 = ? \end{array} \right.$$

$$\left\{ \begin{array}{l} x_1 - 2x_2 + x_3 = 1 \\ x_1 - x_2 + 4x_3 = 7 \\ x_3 = 2 \end{array} \right.$$

$$\left\{ \begin{array}{l} x_1 - 2x_2 + x_3 = 1 \\ x_2 + 3x_3 = 6 \\ x_3 = 2 \end{array} \right.$$

$$\left\{ \begin{array}{l} x_1 - 2x_2 + 2 = 1 \\ x_2 + 6 = 6 \\ x_3 = 2 \end{array} \right.$$

$$\left\{ \begin{array}{l} x_1 - 2x_2 = -1 \\ x_2 = 0 \\ x_3 = 2 \end{array} \right.$$

$$\left\{ \begin{array}{l} x_1 - 0 = -1 \\ x_2 = 0 \\ x_3 = 2 \end{array} \right. \rightarrow \left\{ \begin{array}{l} x_1 = -1 \\ x_2 = 0 \\ x_3 = 2 \end{array} \right.$$

PROV:

$$\left\{ \begin{array}{l} (-1) - 2(0) + (2) = -1 \\ (-1) - (0) + 4(2) = 7 \\ (-1) - 2(0) + 2(2) = 3 \end{array} \right.$$

$$\left\{ \begin{array}{l} 1 = 1 \\ 7 = 7 \\ 3 = 3 \end{array} \right. \quad \underline{\text{OK}}$$

$$\left. \begin{array}{l} (1) \\ (2) \end{array} \right\} \begin{array}{l} 5x_1 + 3x_2 - 2x_3 = 1 \\ x_2 - 2x_3 = -2 \\ x_2 + 2x_3 = 1 \end{array}$$

$$\left| \begin{array}{ccc|c} 5 & 3 & -2 & 1 \\ 0 & 1 & -2 & -2 \\ 0 & 1 & 2 & 1 \end{array} \right| \xrightarrow{\text{Row operations}} \left| \begin{array}{ccc|c} 5 & 3 & -2 & 1 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 4 & 3 \end{array} \right|$$

$$\left. \begin{array}{l} 5x_1 + 3x_2 - 2x_3 = 1 \\ x_2 - 2x_3 = -2 \\ 9x_3 = 3 \end{array} \right\}$$

$$\left. \begin{array}{l} 5x_1 + 3x_2 - 2x_3 = 1 \\ x_2 - 2\left(\frac{3}{4}\right) = -2 \\ x_3 = \frac{3}{4} \end{array} \right\}$$

$$\left. \begin{array}{l} 5x_1 + 3x_2 - 2x_3 = 1 \\ x_2 - \frac{3}{2} = -2 \end{array} \right\}$$

$$\left. \begin{array}{l} 5x_1 + 3\left(-\frac{1}{2}\right) - 2\left(\frac{3}{4}\right) = 1 \\ x_2 = -1 \end{array} \right\}$$

$$x_3 = \frac{3}{4}$$

$$x_3 = -\frac{3}{4}$$

$$\left\{ \begin{array}{l} 5x_1 - \frac{3}{2} - \frac{3}{2} = 1 \end{array} \right.$$

$$x_2 = -\frac{3}{2}$$

$$x_3 = \frac{3}{4}$$

$$\left\{ \begin{array}{l} 5x_1 = 4 \end{array} \right.$$

$$x_2 = -\frac{1}{2}$$

$$x_3 = \frac{3}{4}$$

$$\left\{ \begin{array}{l} x_1 = \frac{4}{5} \\ x_2 = -\frac{1}{2} \end{array} \right.$$

$$x_3 = \frac{3}{4}$$

(1)

$$2x_2 - 4x_3 + x_4 = 1$$

$$x_1 - 3x_2 - x_3 + x_4 = 0$$

$$x_1 - x_2 + 5x_3 - 2x_4 = -1$$

$$2x_1 - 2x_2 - x_3 + 2x_4 = 0$$

0	2	-4	1	1	X	1	-3	-1	1	0
1	-3	-1	1	0	X	0	2	-4	1	1
1	-1	4	-2	-1	.	1	-1	4	-2	-1
2	-2	-1	2	0		2	-2	-1	2	0

$$\left| \begin{array}{ccccc} 1 & -3 & -1 & 1 & 0 \\ 0 & 2 & -4 & 1 & 1 \\ 0 & 2 & 5 & -3 & -1 \\ 0 & 4 & 0 & 1 & 0 \end{array} \right| \xrightarrow{\quad} \left| \begin{array}{ccccc} 1 & -3 & -1 & 1 & 0 \\ 0 & 2 & -4 & 1 & 1 \\ 0 & 0 & 9 & 2 & -2 \\ 0 & 0 & 8 & -1 & 0 \end{array} \right| \xrightarrow{\quad} \left| \begin{array}{ccccc} 1 & -3 & -1 & 1 & 0 \\ 0 & 2 & -4 & 1 & 1 \\ 0 & 0 & 9 & 2 & -2 \\ 0 & 0 & -\frac{25}{9} & 0 & 0 \end{array} \right|$$

$$\cdot \frac{16}{9} \cancel{-9} = \frac{-25}{9}$$

$$\left| \begin{array}{ccccc} 1 & -3 & -1 & 1 & 0 \\ 0 & 2 & -4 & 1 & 1 \\ 0 & 0 & 9 & 2 & -2 \\ 0 & 0 & 0 & -\frac{25}{9} & 0 \end{array} \right|$$

$$\left. \begin{array}{l} x_1 - 3x_2 - x_3 + x_4 = 0 \\ 2x_2 - 4x_3 + x_4 = 1 \\ 9x_3 + 2x_4 = -2 \\ -\frac{25}{9}x_4 = 0 \end{array} \right\} \quad \left. \begin{array}{l} x_1 - 3x_2 - \left(-\frac{2}{9}\right) = 0 \\ 2x_2 - 4\left(-\frac{2}{9}\right) = 1 \\ x_3 = -\frac{2}{9} \\ x_4 = 0 \end{array} \right\}$$

$$\left. \begin{array}{l} x_1 - 3x_2 = -\frac{2}{9} \\ 2x_2 = \frac{1}{9} \end{array} \right\} \quad \left. \begin{array}{l} x_1 - 3x_2 = -\frac{2}{9} \\ x_2 = \frac{1}{18} \end{array} \right\}$$

$$x_3 = -\frac{1}{3}$$

$$x_4 = 0$$

$$x_3 = -\frac{1}{3}$$

$$x_4 = 0$$

$$\left\{ \begin{array}{l} x_1 - 3\left(\frac{1}{18}\right) = -\frac{2}{3} \\ x_2 = -\frac{1}{18} \\ x_3 = -\frac{2}{9} \\ x_4 = 0 \end{array} \right. \quad \left\{ \begin{array}{l} x_1 = -\frac{2}{3} + \frac{1}{6} = -\frac{4+3}{18} = -\frac{1}{18} \\ x_2 = -\frac{1}{18} \\ x_3 = -\frac{2}{9} \\ x_4 = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} x_1 = -\frac{1}{18} \\ x_2 = \frac{1}{18} \\ x_3 = -\frac{2}{9} \\ x_4 = 0 \end{array} \right.$$

⑧
⑨

$$\begin{array}{c|ccccc|ccccc} & 1 & -1 & 2 & -2 & & 1 & -1 & 2 & -2 \\ \textcircled{1} & 2 & -2 & 1 & -1 & & 0 & 0 & -3 & 3 \\ \textcircled{2} & 0 & 1 & 2 & 3 & & 0 & & & \end{array} \rightarrow \begin{array}{c|ccccc|ccccc} & 1 & -1 & 2 & -2 & & 1 & -1 & 2 & -2 \\ & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 \\ & 0 & & & & & 0 & 0 & 0 & 0 \end{array}$$

3 | 2 1 0 | 0

$$P_1 = 1$$

$$P_2 = 0$$

DATO CHIE $P_2 = 0$ LA MATRICE E'

SIRGOLARO, POI CHI' SONO TUTTI I PIVI FAMANDO
DIVISI DA 0

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$$\left| \begin{array}{ccccc|ccccc} 1 & 0 & 3 & 7 & 4 & 1 & 0 & 3 & 7 & 4 \\ -1 & 1 & 0 & 2 & -1 & 0 & 1 & 3 & 9 & 3 \\ 2 & 2 & 0 & 2 & 2 & 0 & 2 & -6 & -10 & -6 \\ 3 & 1 & 4 & -1 & 7 & 0 & -1 & -5 & -20 & -5 \\ 0 & 1 & 3 & 0 & 3 & 0 & 1 & 0 & 0 & 0 \end{array} \right| \xrightarrow{\quad} \left| \begin{array}{ccccc|ccccc} 1 & 0 & 3 & 7 & 4 & 1 & 0 & 3 & 7 & 4 \\ 0 & 1 & 3 & 7 & 4 & 0 & 2 & -6 & -10 & -6 \\ 0 & -1 & -5 & -20 & -5 & 0 & -1 & 0 & 0 & 0 \end{array} \right| \xrightarrow{\quad}$$

$$P_1 = -1$$

$$P_2 = -1$$

$$\left| \begin{array}{ccccc|ccccc} 1 & 0 & 3 & 7 & 4 & 1 & 0 & 3 & 7 & 4 \\ 0 & 1 & 3 & 7 & 4 & 0 & 1 & 3 & 9 & 3 \\ 0 & 0 & -12 & -28 & -12 & 0 & 0 & -12 & -28 & -12 \end{array} \right| \xrightarrow{\quad} \left| \begin{array}{ccccc|ccccc} 1 & 0 & 3 & 7 & 4 & 1 & 0 & 3 & 7 & 4 \\ 0 & 1 & 3 & 9 & 3 & 0 & 1 & 3 & 9 & 3 \\ 0 & 0 & -12 & -28 & -12 & 0 & 0 & -12 & -28 & -12 \end{array} \right|$$

$$\left| \begin{array}{ccccc} 0 & 0 & -8 & -29 & -8 \\ 0 & 6 & -3 & -9 & -3 \end{array} \right| \quad \left| \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right| \quad \left| \begin{array}{ccccc} 0 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 2 & 0 \end{array} \right|$$

$e_1 = -12$

$e_4 = -\frac{3}{3}$

$e_5 = 0$

DATO $c + \lambda e_s = 0$ UNA MATRICE DI SURROGAZI

15) $P_1 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \quad P_2 = \begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix}$

r: $\overrightarrow{OP'} = \overrightarrow{OP_0} + l \overrightarrow{OQ'}$

$P_0 = P_1$

$\overrightarrow{OQ'} = \overrightarrow{OP'_1} - \overrightarrow{OP_1} = \begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 2 \end{pmatrix}$

r: $\begin{cases} x = 2 - 4l \\ y = 6l \\ z = 1 + 2l \end{cases}$

$\tilde{\Pi}: \begin{cases} x = 1 + t + s \\ y = 1 + t - s \\ z = 1 + 4t + 3s \end{cases}$

r 0 π

$$\begin{cases} x = -4l - 1 - t - s \\ y = 6l - 1 - t + s \\ z = +2l \end{cases} \quad \left\{ \begin{array}{l} -4l - t - s = -1 \\ 6l - t + s = 1 \\ 2l = -4t - 3s \end{array} \right. \quad \left\{ \begin{array}{l} -4l - t - s = -1 \\ 6l - t + s = 1 \\ 2l - 4t - 3s = 0 \end{array} \right.$$

$$\left| \begin{array}{ccc|c} -4 & -1 & -1 & -1 \\ 6 & -1 & 1 & 1 \\ 2 & -4 & -3 & 0 \end{array} \right| \xrightarrow{\text{Row 1} \leftrightarrow \text{Row 3}} \left| \begin{array}{ccc|c} 2 & -4 & -3 & 0 \\ 6 & -1 & 1 & 1 \\ -4 & -1 & -1 & -1 \end{array} \right| \xrightarrow{\text{Row 2} \rightarrow}$$

$$\Rightarrow \left| \begin{array}{ccc|c} 2 & -4 & -3 & 0 \\ 0 & 11 & 10 & 1 \\ 0 & -9 & -7 & -1 \end{array} \right| \xrightarrow{\text{Row 3} \rightarrow} \left| \begin{array}{ccc|c} 2 & -4 & -3 & 0 \\ 0 & 11 & 10 & 1 \\ 0 & 0 & 0 & \frac{13}{11} - \frac{2}{11} \end{array} \right|$$

$$\left\{ \begin{array}{l} 2l - 4t - 3s = 0 \\ 11t + 10s = 1 \\ -\frac{13}{11}s = -\frac{2}{11} \end{array} \right.$$

$$\left(2l - 4t - 3s = 0 \quad (2l - 4t - 3) \left(\frac{2}{11} \right) = 0 \right)$$

$$11t + 10s = 1$$

$$s = -\frac{2}{13} - \frac{11}{13}t = \frac{2}{13}$$

$$11t + 10\left(\frac{2}{13}\right) = 1$$

$$s = \frac{2}{13}$$

$$2l - 4t = \frac{6}{13}$$

$$11t = -\frac{2}{13},$$

$$s = \frac{2}{13}$$

$$2l -$$

$$t =$$

$$-4l - t - s = -1$$

$$6l - t + s = 1$$

$$2l - 4t - 3s = 0$$

$$-4l - t - s = -1$$

$$6l - t + s = 1$$

$$l = \frac{+4t + 3s}{2}$$

$$-\frac{3}{4}\left(\frac{4t + 3s}{2}\right) - t - s = -1 \quad \begin{cases} +5t + 7s = +1 \\ 11t + 10s = 1 \end{cases}$$

$$\frac{3}{4}\left(\frac{4t + 3s}{2}\right) - t + s = 1$$

$$l = \frac{4t + 3s}{2}$$

$$l = \frac{4t + 3s}{2}$$

$$3\left(\frac{-10s}{2}\right) + 7s - t, \quad \frac{-9-13s}{2} = 1$$

$$t = \frac{1 - 10s}{11}$$

$$t = \frac{1 - 10s}{11}$$

$$l = \frac{s \left(\frac{1 - 10s}{11} \right) + 3s}{2}$$

$$l = \frac{4 - 7s}{22}$$

$$s = \left(\frac{2}{11} \right) \cdot \left(\frac{11}{13} \right)$$

$$s = -\frac{2}{13}$$

$$t = \frac{1 - 10s}{11}$$

$$t = 1 - 10 \left(-\frac{2}{13} \right)$$

$$l = \frac{4 - 7s}{22}$$

$$l = 4 - 7 \left(-\frac{2}{13} \right)$$

$$s = -\frac{2}{13}$$

$$s = -\frac{2}{13}$$

$$t = \frac{1 + \frac{20}{13}}{11}$$

$$t = \frac{33}{13 \cdot 11} = \frac{33}{143} = \frac{1}{13}$$

$$l = \frac{8 + \frac{14}{13}}{22}$$

$$l = \frac{66}{11 \cdot 22} = \frac{66}{242} = \frac{33}{143} = \frac{1}{13}$$

$$x = -4l - 1 - t - s$$

$$y = 6l - 1 - t + s$$

$$x = -4t - 3s$$

$$\left\{ \begin{array}{l} x = -4 \left(\frac{1}{13} \right) - 1 - \left(\frac{1}{13} \right) - \left(-\frac{2}{13} \right) \\ y = 6 \left(\frac{1}{13} \right) - 1 - \left(\frac{1}{13} \right) + \left(-\frac{2}{13} \right) \\ z = 2 \left(\frac{1}{13} \right) - 1 - \left(\frac{1}{13} \right) - 3 \left(-\frac{2}{13} \right) \end{array} \right.$$

$$\left\{ \begin{array}{l} x = -\frac{4}{13} - 1 - \frac{1}{13} + \frac{2}{13} \\ y = \frac{6}{13} - 1 - \frac{1}{13} + \frac{2}{13} \\ z = \frac{2}{13} - \frac{4}{13} + \frac{6}{13} \end{array} \right.$$

$$\left\{ \begin{array}{l} x = \frac{-4 - 13 - 1 + 2}{13} \\ y = \frac{6 - 13 - 1 + 2}{13} \\ z = \frac{2 - 4 + 6}{13} \end{array} \right. \quad \left\{ \begin{array}{l} x = -\frac{16}{13} \\ y = -\frac{9}{13} \\ z = \frac{4}{13} \end{array} \right.$$

C_A (C-C-T)A P(T-T(CS)) C_A LC P(A AND NOT C) ELC

PV, T0

$$P = \begin{pmatrix} -\frac{16}{13} \\ -\frac{9}{13} \\ \frac{4}{13} \end{pmatrix}$$

