$$\sum_{k=1}^{\infty} \frac{1}{k^{1+\frac{1}{k}}}$$

Per $k \to \infty$

$$\frac{1}{k^{1+\frac{1}{k}}} = \frac{1}{k \cdot \sqrt[k]{k}} \sim \frac{1}{k}$$

Quindi per confronto asintotico, data

che

$$\sum_{k=1}^{\infty} \frac{1}{k} = +\infty \text{ divergente}$$

anche la serie data diverge

$$\sum_{k=1}^{\infty} \frac{\sin\left(\frac{1}{k}\right)}{\lg(k^2+k+1)}$$

Per $k \to \infty$

$$\frac{\left(\frac{1}{k}\right)}{} \qquad \frac{1}{} \qquad \frac{1}{}$$

$$\frac{\sin\left(\frac{1}{k}\right)}{\log(k^2+k+1)} \sim \frac{\frac{1}{k}}{\log(k^2)} = \frac{1}{k \cdot 2\log(k)}$$

Dato che

$$\sum_{k=1}^{\infty} \frac{1}{k 2 \log(k)} = +\infty$$

Dato che $\alpha = \beta = 1$

Anche la serie di partenza diverge

$$\sum_{k=1}^{\infty} \frac{\sqrt{k^2+1} - k}{e^{-k} + \log^2(k)}$$

Per $k \to +\infty$

$$\frac{\sqrt{k^2+1} - k}{e^{-k} + \log^2(k)} = \frac{\sqrt{k^2\left(1+\frac{1}{k^2}\right)} - k}{\log^2(k)\left(\frac{e^{-k}}{\log^2(k)} + 1\right)} \sim \frac{k\left(\sqrt{1+\frac{1}{k^2}} - 1\right)}{\log^2(k)} \sim$$

$$\frac{\cancel{k} \cdot \frac{1}{2k^{\cancel{2}}}}{\log^2(k)} = \frac{1}{2k \log^2(k)}$$

PER IL CONFRONTO ASINTOTICO, DATO CHE:

$$\sum_{k=1}^{\infty} \frac{1}{2k \ln^2(k)} \quad \text{CONVERGE} \quad (\alpha = 1,\ \beta > 1)$$

ANCHE LA SERIE DATA CONVERGE

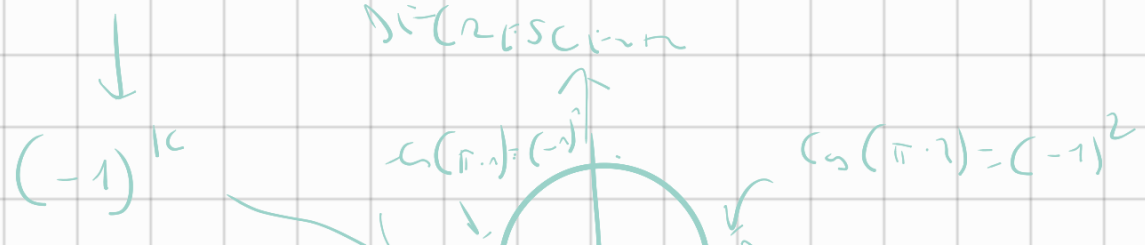
$$\sum_{k=1}^{\infty} k^k e^{-k^2}$$

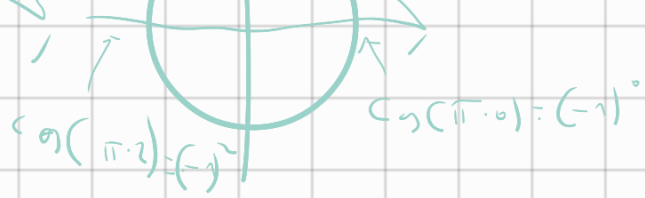
PER $k \to \infty$

$$k^k e^{-k^2} = \frac{k^k}{e^{k^2}} \sim \sqrt[k]{\frac{k^k}{e^{k^2}}} \sim \frac{k}{e^k} \to 0 < 1$$

QUINDI LA SERIE CONVERGE PER IL CRITERIO DELLA RADICE

$$\sum_{k=0}^{\infty} \underbrace{\cos(\pi k)}_{(-1)^k} \underbrace{\arctan\left(\frac{1}{k}\right)}_{\text{DECRESCENTE}}$$





Dunque per il criterio di Leibniz la serie è convergente

$$\sum_{k=1}^{\infty} \left(\sqrt{k^3+1} - \sqrt{k^3-1}\right)$$

per $k \to +\infty$

$$\sqrt{k^3+1} - \sqrt{k^3-1} = k^{\frac{3}{2}}\left(\sqrt{1+\frac{1}{k^3}} - \sqrt{1-\frac{1}{k^3}}\right) =$$

$$= k^{\frac{3}{2}}\left(1 + \frac{1}{2k^3} + o\left(\frac{1}{k^3}\right) - \left(\cancel{1} - \frac{1}{2k^3} + o\left(\frac{1}{k^3}\right)\right)\right) =$$

$$= k^{\frac{3}{2}}\left(\frac{1}{k^3} + o\left(\frac{1}{k^3}\right)\right) = k^{\frac{3}{2}-3} + o\left(\frac{1}{k^3}\right) =$$

$$= k^{-\frac{3}{2}} + o\left(\frac{1}{k^3}\right) \sim \frac{1}{k^{\frac{3}{2}}}$$

Così per il confronto asintotico, dato che

$$\sum_{k=1}^{\infty} \frac{1}{k^{\frac{3}{2}}} \quad \text{convergente} \quad \left(\frac{3}{2} < 1\right)$$

ANCHE LA SERIE DATA CONVERGE

$$\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

SIA $a_k = \dfrac{(k!)^2}{(2k)!}$, ALLORA

$$\frac{(k!)^2}{(2k)!} = \frac{((k+1)!)^2}{(2(k+1))!} \cdot \frac{(2k)!}{(k!)^2} = \frac{(k+1)^2 \cancel{(k!)^2}}{(2k+2)!} \cdot \frac{(2k)!}{\cancel{(k!)^2}} =$$

$$= \frac{(k+1)^2 \cancel{(2k)!}}{(2k+2)(2k+1)\cancel{(2k)!}} = \frac{(k+1)^2}{(2k+2)(2k+1)} = \frac{(k+1)^{\cancel{2}}}{2\cancel{(k+1)}(2k+1)} =$$

$$= \frac{k+1}{2(2k+1)} = \frac{k+1}{4k+2} = \frac{\cancel{k}\left(1+\frac{1}{k}\right)}{4\cancel{k}\left(1+\frac{1}{k}\right)} \to \frac{1}{4} < 1$$

QUINDI PER IL CRITERIO DEL RAPPORTO LA

SERIE CONVERGE

$$\sum_{k=1}^{\infty} \frac{1}{2^{\log(k)}}$$

SI HA CHE

$$2^{\log(k)} = e^{\log(k)\cdot\log(2)} = k^{\log(2)}$$

QUINDI

$$\frac{1}{2^{\log(k)}} = \frac{1}{k^{\log(2)}}$$ CON DATO CHE $\log(2) < 1$

LA SERIE DIVERGE A $+\infty$

$$\sum_{k=1}^{\infty} \frac{\log(k^2+3k) - 2\log(k)}{(\log(\sqrt{k}+1))^{3\alpha}}$$

PER $k \to \infty$

$$\frac{\log(k^2+3k) - 2\log(k)}{(\log(\sqrt{k}+1))^{3\alpha}} = \frac{\log\left(\frac{k^2+3k}{k^2}\right)}{\left(\frac{1}{2}\log(k)\cdot\log(2)\right)^{3\alpha}} \sim$$

$$\frac{3}{k} \quad \sim C$$

$$\frac{k}{\frac{1}{2}\lg(k)^{3\alpha}} = \frac{k}{k\lg^{3\alpha}(k)}$$

COSÌ PER CONFRONTO ASINTOTICO, LA SERIE

CONVERGE SE E SOLTANTO SE $3\alpha > 1$ $(\gamma = 1, \beta = 3\alpha)$ OSSIA

SE $\alpha > \frac{1}{3}$

$$\sum_{k=1}^{\infty} \frac{(4^k + k)\alpha^{2k}}{3^k - 1}$$

PER $k \to \infty$

$$\frac{(4^k + k)\alpha^{2k}}{3^k - 1} \sim \frac{4^k \alpha^{2k} +}{3^k} = \left(\frac{4\alpha^2}{3}\right)^k \sim \sqrt[k]{\left(\frac{4\alpha^2}{3}\right)^k}$$

QUINDI LA SERIE CONVERGE, PER IL CRITERIO DELLA RADICE E PER IL CONFRONTO ASINTOTICO,

SOLO E SOLTANTO SE $\left|\frac{4\alpha^2}{3}\right| < 1$ DA CUI $|\alpha| < \frac{\sqrt{3}}{2}$

$$\sum_{k=1}^{\infty} (-1)^k \frac{\sqrt{k} + (-1)^k k^{\alpha}}{k^{\alpha}\sqrt{k}} =$$

$$= \sum^{\infty} \frac{(-1)^k \sqrt{k} + (-1)^{2k} k^{\alpha}}{k^{\alpha}\sqrt{k}} =$$

$$k=1 \qquad k \quad \sqrt{k}$$

$$= \sum_{k=1}^{\infty} \left(\frac{(-1)^k \cancel{\sqrt{k}}}{k^a \cancel{\sqrt{k}}} + \frac{(-1)^{2k} \cancel{k^a}}{\cancel{k^a} \sqrt{k}} \right) =$$

$$= \sum_{k=1}^{\infty} \frac{(-1)^k}{k^a} + \sum_{k=1}^{\infty} \frac{(-1)^{2k}}{\sqrt{k}} = L + \infty = \infty$$

DOVE

$\sum_{k=1}^{\infty} \frac{(-1)^k}{k^a}$ È CONVERGENTE PER IL CRITERIO DI LEIBNIZ

MENTRE

$$\sum_{k=1}^{\infty} \frac{(-1)^{2k}}{\sqrt{k}} = +\infty$$

$$\sum_{k=1}^{\infty} \frac{k^a + k^2}{k^{2a} + k^4}$$

PER $k \to \infty$

$$k^a + k^2 \sim \begin{cases} k^a & \text{SE } a > 2 \\ 2k^2 & \text{SE } a = 2 \\ k^2 & \text{SE } 0 < a < 2 \end{cases}$$

ci

$$k^{2a}+k^3 \sim \begin{cases} k^{2a} & \text{se } 2a>3 \to a>\frac{3}{2} \\ 2k^3 & \text{se } 2a=3 \to a=\frac{3}{2} \\ k^3 & \text{se } 0<2a<3 \to 0<a<\frac{3}{2} \end{cases}$$

Quindi

$$\frac{k^a+k^2}{k^{2a}+k^3} \sim \begin{cases} \dfrac{k^{\cancel{a}}}{k^{\cancel{2a}}} = \dfrac{1}{k^a} & \text{se } a>2 \\ \dfrac{2k^2}{k^{2a}} = \dfrac{2}{k^{2a-2}} & \text{se } a=2 \ \left(a>\frac{3}{2}\right) \\ \dfrac{k^2}{2k^3} = \dfrac{1}{2k} & \text{se } a=\frac{3}{2} \ (0<a<2) \\ \dfrac{k^2}{k^3} = \dfrac{1}{k} & \text{se } 0<a<\frac{3}{2} \ (a<2) \\ \dfrac{k^2}{k^{2a}} = \dfrac{1}{k^{2a-2}} & \text{se } \frac{3}{2}<a<2 \end{cases}$$

Allora per il confronto asintotico convergenza solo se e soltanto se $a>\frac{3}{2}$ (cioè $2a-2>1$)

$$\sum^{\infty} \left(\frac{\cos^2\left(\frac{1}{k}\right)-\sin^2\left(\frac{2}{k}\right)}{\ldots}\right)^{k^3}$$

$k=1$ $\left(\quad -\cos\left(\frac{1}{k}\right) \quad \right)$

Pir $k \to \infty$ $\left(\frac{1}{k} \to 0 \right)$

$$\frac{\cos^2\left(\frac{1}{k}\right) - \sin^2\left(\frac{2}{k}\right)}{-\cos\left(\frac{3}{k}\right)} = \frac{\left(1 - \frac{1}{2k^2} + o\left(\frac{1}{k^2}\right)\right)^2 - \left(\frac{2}{k} + o\left(\frac{1}{k^2}\right)\right)^3}{1 - \frac{9}{2k^2} + o\left(\frac{1}{k^2}\right)} =$$

$$= \left(\left(1 - \frac{2}{2k^2} + o\left(\frac{1}{k^2}\right)\right) - \left(\frac{4}{k^2} + o\left(\frac{1}{k^2}\right)\right)\right)\left(1 + \frac{9}{2k^2} + o\left(\frac{1}{k^2}\right)\right) =$$

$$= \left(1 + \left(\frac{\overbrace{-2-8}^{-10}}{2k^2}\right) + o\left(\frac{1}{k^2}\right)\right)\left(1 + \frac{9}{2k^2} + o\left(\frac{1}{k^2}\right)\right) =$$

$$= \left(1 - \frac{5}{k^2} + o\left(\frac{1}{k^2}\right)\right)\left(1 + \frac{9}{2k^2} + o\left(\frac{1}{k^2}\right)\right) =$$

$$= 1 + \frac{9}{2k^2} - \frac{5}{k^2} + o\left(\frac{1}{k^2}\right) =$$

$$= 1 + \frac{9-10}{2k^2} + o\left(\frac{1}{k^2}\right) = 1 + \frac{1}{2k^2} + o\left(\frac{1}{k^2}\right)$$

QUINDI

$$\sqrt[k]{a_k} = \sqrt[k]{\left(\frac{-\cos^2\left(\frac{1}{k}\right) - \sin^2\left(\frac{2}{k}\right)}{-\cos\left(\frac{3}{k}\right)}\right)^{k^2}} =$$

$$= \left(1 - \frac{1}{2k^2} + o\left(\frac{1}{k^2}\right)\right)^{k^2} \to e^{-\frac{1}{2}}$$

PER IL CRITERIO DELLA RADICE,

DATO CHE $e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} < 1$, LA SERIE

DATA È CONVERGENTE

$$\sum_{k=3}^{\infty}\left(\exp\left(\frac{k^2+3k}{k^2+1}\right) - \exp\left(\frac{k}{k-2}\right)\right)$$

PER $k \to \infty$

$$e^{\frac{k^2+3k}{k^2+1}} - e^{\frac{k}{k-2}} = e\left(e^{\frac{k^2+3k}{k^2+1}-1} - e^{\frac{k}{k-2}-1}\right) =$$

$$\left(\frac{\cancel{k^2}+3k-\cancel{k^2}-1}{\qquad} \qquad \frac{\cancel{k}-\cancel{k}+2}{\qquad}\right)$$

$$= e\left(e^{\frac{k-1}{k^2+1}} - e^{\frac{k-1}{k-2}}\right) =$$

$$= e\left(e^{\overbrace{\frac{3k-1}{k^2+1}}^{\to 0}} - e^{\overbrace{\frac{2}{k-2}}^{\to 0}}\right) =$$

$$= e\left(\cancel{1} + \frac{3k-1}{k^2+1} + o\left(\frac{1}{k}\right) - \left(\cancel{1} + \frac{2}{k-2} + o\left(\frac{1}{k}\right)\right)\right) =$$

$$= e\left(\frac{3k-1}{k^2+1} - \frac{2}{k-2} + o\left(\frac{1}{k}\right)\right) \sim$$

$$\sim e \cdot \left(\frac{3\cancel{k}}{k^{\cancel{2}}} - \frac{2}{k}\right) = e \cdot \left(\frac{3}{k} - \frac{2}{k}\right) =$$

$$= e \cdot \left(\frac{1}{k}\right) = \frac{e}{k}$$

PER IL CRITERIO DEL CONFRONTO ASINTOTICO, DATO CHE LA SERIE $\sum_{k=1}^{\infty} \frac{1}{k}$ DIVERGE,

ARGITZ LA SERIE PRIMA DI SINCRONO