$$\lim_{n\to+\infty} \frac{10^n \cdot n! - 5^n \cdot n^n}{n^5 + n^{n-n}} =$$

$$= \lim_{n\to+\infty} \frac{\overbrace{\frac{10^n}{n^n}}^{\to 0} \cdot \overbrace{\frac{n!}{n^n}}^{\to 0} - \overbrace{\frac{5^n}{n^n}}^{\to 0} \cdot \overbrace{\frac{\cancel{n^n}}{\cancel{n^n}}}^{1}}{\underbrace{\frac{n^5}{n^{n-n}}}_{\to 0} + \overbrace{\frac{\cancel{n^n}}{\cancel{n^n}}}^{1}} \to \frac{0}{1} = 0$$

$$\lim_{n\to+\infty} \frac{(2^n-1)(2-\cos(n!))}{\log(3^n-1)}$$

DATO CHE: $\overbrace{2-\cos(n!)}^{-1 \le -\cos(n!) \le 1} \geq 1$

$$\lim_{n\to+\infty} \frac{(2^n-1)(2-\cos(n!))}{\log(3^n-1)} =$$

$$\geq \lim_{n\to+\infty} \frac{(2^n-1)}{\log\left(3^n\left(1-\frac{1}{3^n}\right)\right)} =$$

$$= \lim_{n\to+\infty} \frac{(2^n-1)}{n\log(3)+\log\left(1-\frac{1}{3^n}\right)} =$$

$$= \lim_{n\to+\infty} \frac{\overbrace{2^n}^{\to+\infty}\overbrace{\left(1-\frac{1}{2^n}\right)}^{\to 1}}{n\left(\lg(2)+\underbrace{\frac{\log\left(1-\frac{1}{2^n}\right)}{n}}_{\to 0}\right)} = +\infty$$

$$\lim_{n\to+\infty} \frac{(5-n)^5+n^5}{(4-n)^4+n^4}$$

SVILUPPO LE POTENZE

$$\frac{\cancel{-n^5}+25n^4+P(n)+\cancel{n^5}}{n^4+q(n)+n^4} = \frac{\frac{25\cancel{n^4}}{\cancel{n^4}}+\overbrace{\frac{P(n)}{n^4}}^{\to 0}}{\frac{2\cancel{n^4}}{\cancel{n^4}}+\underbrace{\frac{q(n)}{n^4}}_{\to 0}} \to \frac{25}{2}$$

POLINOMI DI 3° GRADO

$$\lim_{n\to+\infty} \frac{1}{n}\left(\frac{4^n-n}{2^n+n}-\frac{4^n+n}{2^n-n}\right) =$$

$$= \lim_{n\to+\infty} \frac{1}{n}\left(\frac{(4^n-n)(2^n-n)-(4^n+n)(2^n+n)}{(2^n+n)(2^n-n)}\right) =$$

$$= \lim \frac{1}{n}\left(\frac{(2^{2n}-n)(2^n-n)-(2^{2n}+n)(2^n+n)}{\quad}\right) =$$

$$\lim_{n\to\infty} n\left(\frac{\quad}{(2^{2n}-n^2)}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n}\left(\frac{\cancel{2^{2n}} - 2^{2n}\cdot n - 2^n\cdot n + \cancel{n^2} - \cancel{2^{2n}} - 2^{2n}\cdot n - 2^n\cdot n - \cancel{n^2}}{(2^{2n}-n^2)}\right) =$$

$$= \lim_{n\to\infty} \frac{1}{n}\left(\frac{2^{2n}(-1-1)\cdot n + 2^n(-1\cdot 1)n}{2^{2n}-n^2}\right) =$$

$$= \lim_{n\to\infty} \frac{1}{n}\left(\frac{-2^{n+1}\cdot n - 2^{n+1}\cdot n}{2^{2n}-n^2}\right) =$$

$$= \lim_{n\to\infty} -\frac{2^{n+1}\cdot \cancel{n} - 2^{n+1}\cdot \cancel{n}}{\cancel{n}(2^{2n}-n^2)} =$$

$$= \lim_{n\to\infty} -\frac{2^{2n+1} - 2^{n+1}}{(2^{2n}-n^2)} =$$

$$= \lim_{n\to\infty} -\frac{2^{\cancel{2n}+1}\left(1 - \overbrace{\frac{1}{2^n}}^{\to 0}\right)}{\cancel{2^{2n}}\left(1 - \underbrace{\frac{n^2}{2^{2n}}}_{\to 0}\right)} = -2$$

---

f) $\lim_{n\to\infty} \frac{\log(n^2+3)}{\quad}$

$$\log(n^3+2)$$

Pl. n $\quad n \to \infty$

$$\frac{\log(n^2+3)}{\log(n^3+2)} \sim \frac{\log(n^2)}{\log(n^3)} = \frac{2\cancel{\log(n)}}{3\cancel{\log(n)}} \to \frac{2}{3}$$

oppure:

$$\frac{\log(n^2+3)}{\log(n^3+n)} = \frac{\log\left(n^2\left(1+\frac{3}{n^2}\right)\right)}{\log\left(n^3\left(1+\frac{2}{n^2}\right)\right)} = \frac{2\log(n)+\log\left(1+\frac{3}{n^2}\right)}{3\log(n)+\log\left(1+\frac{2}{n^2}\right)} =$$

$$= \frac{2+\overbrace{\frac{\log\left(1+\frac{3}{n^2}\right)}{\log(n)}}^{\to 0}}{3+\underbrace{\frac{\log\left(1+\frac{2}{n^2}\right)}{\log(n)}}_{\to 0}} \to \frac{2}{3}$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{2^n+3^n}{n^3+n^2}} = \lim_{n\to\infty} \sqrt[n]{\frac{3^n\left(\frac{2^n}{3^n}+1\right)}{n^3\left(\frac{1}{n}+1\right)}} =$$

$$= \lim 3\sqrt[n]{\left(\frac{2}{3}\right)^n+1} = \lim 3\cdot\overbrace{\sqrt[n]{\left(\frac{2}{3}\right)^n+1}}^{\to 1} \to 3$$

$n \to \infty$ $\quad \sqrt[n]{n^2}\,\sqrt[n]{\frac{1}{n}+1}$ $\quad n \to \infty$ $\quad \underbrace{e^{\underbrace{\frac{\log(n)}{n}}_{\to 0}}}_{\to 1} \cdot \underbrace{\sqrt[n]{\underbrace{\frac{1}{n}}_{\to 0}+1}}_{\to 1} = 1$

$$\lim_{n \to \infty} \sqrt[n]{(n+1)^n - n^n}$$

Pir $\quad n \to \infty$

$$\lim_{n \to \infty} \sqrt[n]{n^n \cdot \left(\frac{(n+1)^n}{n^n} - \frac{\cancel{n^n}}{\cancel{n^n}}\right)} = \lim_{n \to \infty} n \cdot \sqrt[n]{\left(\frac{n+1}{n}\right)^n - 1} =$$

$$= \lim_{n \to \infty} n \cdot \sqrt[n]{\overbrace{\underbrace{\left(1+\frac{1}{n}\right)^n}_{\geq 2}}^{\to 1} - 1} = +\infty$$

$$= \sum_{k=0}^{n} \binom{n}{k} \frac{1}{n^k} = 1 + 1 + \underbrace{\sum_{k=2}^{n} \binom{n}{k} \frac{1}{n^k}}_{\geq 0} \geq 2$$

$$\lim_{n \to \infty} \frac{(2n+3)! - (2n)!}{n^2\left((2n+1)! - (2n)!\right)}$$

Pir $\quad n \to \infty$

$$\frac{(2n+3)! - (2n)!}{n^2\left((2n+1)! - (2n)!\right)} = \frac{\dfrac{(2n+3)(2n+2)(2n+1)\cancel{(2n)!} - \cancel{(2n)!}}{\cancel{(2n)!}}}{\dfrac{n^2(2n+1)\cancel{(2n)!} - \cancel{(2n)!}\,n^2}{2\cancel{n!}}} =$$

$$= \frac{(2n+3)(2n+2)(2n+1) - 1}{n^2\left((2n+\cancel{1}) - \cancel{1}\right)} =$$

$$= \frac{n\left(2+\frac{3}{n}\right)\cdot n\left(2+\frac{2}{n}\right)n\cdot\left(2+\frac{1}{n}\right) - 1}{2n^3} =$$

$$= \frac{\cancel{n^3}\left(\left(2+\overbrace{\frac{3}{n}}^{\to 0}\right)\left(2+\overbrace{\frac{2}{n}}^{\to 0}\right)\left(2+\overbrace{\frac{1}{n}}^{\to 0}\right) - \overbrace{\frac{1}{n^3}}^{\to 0}\right)}{2\cancel{n^3}} \to \frac{(2\cdot 2\cdot 2)}{2} = 4$$

$$\lim_{n\to\infty} \frac{n^n}{4^n\cdot n!}$$

APPLICANDO IL TEOREMA DEL RAPPORTO

$$\frac{n^n}{4^n\cdot n!} = \frac{(n+1)^{(n+1)}}{4^{(n+1)}\cdot(n+1)!}\cdot\frac{4^n\cdot n!}{n^n} =$$

$$= \frac{(n+1)^n \cdot \cancel{(n+1)}}{\cancel{4^n} \cdot 4 \cancel{(n+1)} \cdot \cancel{n!}} \cdot \frac{\cancel{4^n} \cdot \cancel{n!}}{n^n} = \frac{1}{4} \cdot \frac{(n+1)^n}{n^n} =$$

$$= \frac{1}{4} \cdot \left(\frac{n+1}{n}\right)^n = \frac{1}{4} \cdot \overbrace{\left(1+\frac{1}{n}\right)^n}^{\to e} \to \frac{e}{4} < 1$$

QUINDI PER IL CRITERIO DEL RAPPORTO

$$\lim_{n \to \infty} \frac{n^n}{4^n \cdot n!} = 0$$

$$\lim_{n \to \infty} \frac{(2n)!}{n! \cdot n^n}$$

APPLICO IL CRITERIO DEL RAPPORTO

$$\frac{(2n)!}{n! \cdot n^n} = \frac{(2n+2)!}{(n+1)!\,(n+1)^{n+1}} \cdot \frac{n! \cdot n^n}{(2n)!} =$$

$$= \frac{(2n+2)(2n+1)\cancel{(2n)!}}{(n+1)\cancel{(n)!}\,(n+1)^n(n+1)} \cdot \frac{\cancel{n!} \cdot n^n}{\cancel{(2n)!}} =$$

$$= \frac{2\cancel{(n+1)}(2n+1)}{\cancel{(n+1)}} \cdot \frac{n^n}{\ldots} =$$

$$(\cancel{n+1})\,(n+1)^n\,(n+1)$$

$$= \frac{2(2n+1)\cdot n^n}{(n+1)^n\cdot(n+1)} = \frac{4n+2}{n+1}\cdot\left(\frac{n}{n+1}\right)^n =$$

$$= \frac{\cancel{n}\left(4+\overbrace{\frac{2}{n}}^{\to 0}\right)}{\cancel{n}\left(1+\underbrace{\frac{1}{n}}_{\to 0}\right)}\cdot\overbrace{\left(1+\frac{1}{n}\right)^{n}}^{\to \frac{1}{e}} \to \frac{4}{e}$$

$$\lim_{n\to\infty} n^2 \ln\left(\cos\left(\frac{1}{n}\right)\right)$$

für $n\to\infty$

$$n^2 \ln\left(\cos\left(\frac{1}{n}\right)\right) = n^2 \ln\left(1+\left(\cos\left(\frac{1}{n}\right)-1\right)\right) =$$

$$= \frac{n^2 \ln\left(1+\left(\cos\left(\frac{1}{n}\right)+1\right)\right)}{\cos\left(\frac{1}{n}\right)-1}\cdot\cos\left(\frac{1}{n}\right)-1 =$$

$$= \underbrace{\frac{\ln\left(1+\left(\cos\left(\frac{1}{n}\right)+1\right)\right)}{\cos\left(\frac{1}{n}\right)-1}}_{\to 1}\cdot\underbrace{\frac{\cos\left(\frac{1}{n}\right)-1}{\frac{1}{n^2}}}_{\to -\frac{1}{2}}$$

$$\lim_{n\to+\infty} \frac{n^n}{4^{n\log(n)}}$$

per $n \to +\infty$

$$\frac{n^n}{4^{n\log n}} = \frac{4^{n\log_4(n)}}{4^{n\log(n)}} = 4^{n\log_4(n) - n\ln(n)}$$

ora

$$n\log_4(n) - n\ln(n) = n\frac{\ln(n)}{\log_4(n)} - n\ln(n) =$$

$$= n\ln(n)\underbrace{\left(\frac{1}{\log_4(n)} - 1\right)}_{\leq 0} \longrightarrow -\infty$$

quindi

$$\lim_{n\to+\infty} \frac{n^n}{4^{n\ln(n)}} = 0$$

$$\lim_{n\to+\infty} \frac{(2n)!}{n^{2n}}$$

APPLICO IL CRITERIO DEL RAPPORTO

$$\frac{(2n)!}{\ } = \frac{(2n+2)!}{\ } \cdot \frac{n^{2n}}{\ } =$$

$$\frac{\quad}{n^{2n}} \cdot \frac{\quad}{(n+1)^{(2n+1)}} \cdot \frac{\quad}{(2n)!}$$

$$= \frac{2(\cancel{n+1})(2n+1)(\cancel{2n})!}{(n+1)^{2n}\cdot(n+1)^{\cancel{2}}} \cdot \frac{n^{2n}}{(\cancel{2n})!} =$$

$$= \frac{2(2n+1)}{n+1} \cdot \frac{n^{2n}}{(n+1)^{2n}} = \frac{4n+2}{n+1} \cdot \left(\frac{n}{n+1}\right)^{2n} =$$

$$= \frac{\cancel{n}\left(4+\overbrace{\frac{2}{n}}^{\to 0}\right)}{\cancel{n}\left(1+\underbrace{\frac{1}{n}}_{\to 0}\right)} \cdot \overbrace{\left(\frac{n+1}{n}\right)^{-2n}}^{\to \frac{1}{e^2}} = \frac{4}{e^2} < 1$$

QUINDI PER IL CRITERIO DEL RAPPORTO

$$\lim_{n\to\infty} \frac{(2n)!}{n^{2n}} = 0$$

$$\lim_{n\to\infty} n\left(6^{\frac{1}{n}} - 3^{\frac{1}{n}}\right)$$

PER $n \to \infty$

$$n\left(6^{\frac{1}{n}} - 3^{\frac{1}{n}}\right) = n \cdot 3^{\frac{1}{n}}\left(2^{\frac{1}{n}} - 1\right) =$$

$$\frac{1}{n}\left(2^{\frac{1}{n}} - 1\right) \qquad \lim_{n}$$

$$= \underbrace{3}_{\to 1} \frac{\left(\dots\right)}{\underbrace{\frac{1}{n}}_{h(n)}} \to \ln(2)$$

$$\downarrow$$

$$a > 0 \qquad \lim_{x \to 0} \frac{a^x - 1}{x} = \ln(a)$$

$$\lim_{n \to \infty} \frac{e^{n \ln(n+3)}}{e^{n \ln(n+1)}}$$

Pour $n \to \infty$

$$\frac{e^{n \ln(n+3)}}{e^{n \ln(n+1)}} = \frac{(n+3)^n}{(n+1)^n} = \frac{\cancel{n^n} \overbrace{\left(1 + \frac{3}{n}\right)^n}^{\to e^3}}{\cancel{n^n} \underbrace{\left(1 + \frac{1}{n}\right)^n}_{e}} \to \frac{e^3}{e} = e^2$$

$$\lim_{n \to \infty} n \left(\sqrt{\frac{n-1}{n+5}} - 1 \right)$$

Pour $n \to \infty$

$$n \left(\sqrt{\frac{n-1}{n+5}} - 1 \right) = n \left(\sqrt{1 + \frac{5}{n+5}} - 1 \right) = \frac{5n}{n+5} \frac{\left(\sqrt{1 + \frac{5}{n+5}} - 1 \right)}{\frac{5}{n+5}}$$

$$= \frac{5 \cancel{n}}{\cancel{n}\left(1 + \frac{5}{n}\right)} \cdot \frac{\overbrace{\left(1 + \frac{5}{n+5}\right)^{\frac{1}{2}} - 1}^{x}}{\frac{5}{n+5}} \to 5 \cdot \frac{1}{1} = 5$$

$$\frac{(1+x)^a - 1}{x} \to a$$

$$\lim_{n \to \infty} \left(\sqrt{n} - \sqrt{n - \sqrt{n}} \right)$$

Pi n $\quad n \to \infty$

$$\sqrt{n} - \sqrt{n - \sqrt{n}} = \sqrt{n}\left(1 - \sqrt{1 - \frac{1}{\sqrt{n}}}\right) = -\frac{\left(\sqrt{1 - \frac{1}{n}} - 1\right)}{\frac{1}{\sqrt{n}}} =$$

$$= \frac{\sqrt{1 - \frac{1}{\sqrt{n}}} - 1}{-\frac{1}{\sqrt{n}}} = \frac{\left(1 - \frac{1}{\sqrt{n}}\right)^{\frac{1}{2}} - 1}{-\frac{1}{2}} \to \frac{1}{2}$$

$$\lim_{n \to \infty} \left(1 - \frac{2}{\sqrt{n}}\right)^n$$

Pi n $\quad n \to \infty$

$$\left(1 - \frac{2}{\sqrt{n}}\right)^n = e^{n \ln\left(1 - \frac{2}{\sqrt{n}}\right)} =$$

$$= e^{n \cdot \frac{\ln\left(1-\frac{2}{\sqrt{n}}\right)}{-\frac{2}{\sqrt{n}}} \cdot \left(-\frac{2}{\sqrt{n}}\right)} \to 0$$

$$\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^{\sqrt{n}}$$

per $n \to \infty$

$$\left(1+\frac{2}{n}\right)^{\sqrt{n}} = e^{\sqrt{n}\,\ln\left(1+\frac{2}{n}\right)} =$$

$$= e^{\sqrt{n}\,\frac{\ln\left(1+\frac{2}{n}\right)}{\frac{2}{n}} \cdot \frac{2}{n}} \to 1$$

$$\lim_{n\to\infty} \left(1+\frac{1}{2^{n-1}}\right)^{2^{n+1}}$$

Più $n \to \infty$

$$2^{n-1} \cdot 2^{2} = 2^{n-1+2} = \boxed{2^{n+1}}$$

$$\left(1+\frac{1}{2^{n-1}}\right)^{\boxed{2^{n+1}}} = \left(\left(1+\frac{1}{2^{n-1}}\right)^{\boxed{2^{n-1}}}\right)^{\boxed{2^{2}}} = \left(\left(1+\frac{1}{2^{n-1}}\right)^{2^{n-1}}\right)^{4} \to e^{4}$$

$$\lim_{n \to \infty} \left(1 + \frac{3}{\log(n^2)}\right)^{\log(\sqrt{n})}$$

Pir $n \to \infty$

$$\left(1 + \frac{3}{\log(n^2)}\right)^{\log(\sqrt{n})} = \left(1 + \frac{3}{2\log(n)}\right)^{\frac{1}{2}\log(n)} =$$

$$= \left(\left(1 + \frac{\frac{3}{2}}{\log(n)}\right)^{\log(n)}\right)^{\frac{1}{2}} \to \left(e^{\frac{3}{2}}\right)^{\frac{1}{2}} = e^{\frac{3}{4}}$$

$$\lim_{n \to \infty} \left(\frac{2n^2 + 5n + 4}{2n^2 - 3n + 6}\right)^{3n+2}$$

Pir $n \to \infty$

$$\left(\frac{2n^2 + 5n + 4}{2n^2 - 3n + 6}\right)^{3n+2} = \exp\left((3n+2) \cdot \ln\left(\frac{2n^2 + 5n + 4}{2n^2 - 3n + 6}\right)\right) =$$

$$= \exp\left((3n+2) \frac{\ln\left(1+\frac{8n-2}{2n^2-3n+6}\right)}{\frac{8n-2}{2n^2-3n+6}} \cdot \frac{8n-2}{2n^2-3n+6} \right) =$$

$$= \exp\left(\frac{(8n-2)(3n+2)}{2n^2-3n+6} \cdot \frac{\ln\left(1+\frac{8n-2}{2n^2-3n+6}\right)}{\frac{8n-2}{2n^2-3n+6}} \right) =$$

$$= \exp\left(\frac{24n^2+10n-4}{2n^2-3n+6} \cdot \underbrace{\frac{\ln\left(1+\frac{8n-2}{2n^2-3n+6}\right)}{\frac{8n-2}{2n^2-3n+6}}}_{\to 1} \right) =$$

$$= \exp\left(\underbrace{\frac{\cancel{n^2}\left(\overset{\to 12}{\cancel{24}}+\overbrace{\frac{10}{n}-\frac{4}{n^2}}^{\to 0}\right)}{\cancel{n^2}\left(\cancel{2}-\underbrace{\frac{3}{n}+\frac{6}{n^2}}_{\to 0}\right)}}_{\to 12} \cdot \underbrace{\frac{\ln\left(1+\frac{8n-2}{2n^2-3n+6}\right)}{\frac{8n-2}{2n^2-3n+6}}}_{\to 1} \right) \to e^{12}$$

$$\lim_{n\to\infty} \left(\frac{3^n+2^{n+1}}{n^{n-1}}\right)^{\left(\frac{3}{2}\right)^n}$$

Pir $n \to \infty$

$$\left(\frac{3^n + 2^{n+1}}{3^n + 2^{n-1}}\right)^{\left(\frac{3}{2}\right)^n} = \frac{\left(3^n + 2^{n+1}\right)^{\left(\frac{3}{2}\right)^n}}{\left(3^n + 2^{n-1}\right)^{\left(\frac{3}{2}\right)^n}} =$$

$$= \frac{3^n\left(1 + \frac{2^n \cdot 2}{3^n}\right)^{\left(\frac{3}{2}\right)^n}}{3^n\left(1 + \frac{2^n \cdot 2^{-1}}{3^n}\right)^{\left(\frac{3}{2}\right)^n}} = \frac{\left(1 + \left(\frac{2}{3}\right)^n \cdot 2\right)^{\left(\frac{3}{2}\right)^n}}{\left(1 + \left(\frac{2}{3}\right)^n \cdot \frac{1}{2}\right)^{\left(\frac{3}{2}\right)^n}} =$$

$$= \frac{\left(1 + \frac{2}{\left(\frac{3}{2}\right)^n}\right)^{\left(\frac{3}{2}\right)^n}}{\left(1 + \frac{\frac{1}{2}}{\left(\frac{3}{2}\right)^n}\right)^{\left(\frac{3}{2}\right)^n}} \to \frac{e^2}{e^{\frac{1}{2}}} = e^{\frac{3}{2}}$$

$$\lim_{x \to 0} \frac{(1+2x^2)^{\pi} - 1}{(1+x)^3 + (1-x)^3 - 2}$$

Pir $x \to 0$

$$\frac{(1+2x^2)^{\pi}-1}{(1+x)^3+(1-x)^3-2} = \frac{(1+2x^2)^{\pi}-1}{\cancel{1}+\cancel{3x}+3x^2+\cancel{1}-\cancel{3x}+3x^2-2} =$$

$$= \frac{(1+2x^2)^{\pi}}{6x^2} = \frac{1}{3}\cdot\overbrace{\frac{(1+2x^2)^{\pi}}{2x^2}}^{\to\pi} \to \frac{\pi}{3}$$

$$\lim_{x\to 0} \frac{9^x-4^x}{\operatorname{tg}(2x)}$$

per $x\to 0$

$$\frac{9^x-4^x}{\operatorname{tg}(2x)} = (9^x-4^x)\cdot\frac{1}{\operatorname{tg}(x)}\cdot\frac{2x}{2x} =$$

$$= \frac{9^x-4^x}{x}\cdot\frac{1}{2}\cdot\frac{2x}{\operatorname{tg}(2x)} =$$

$$= \left(\overbrace{\frac{9^x-1}{x}}^{\log(9)} - \overbrace{\frac{4^x-1}{x}}^{\log(4)}\right)\cdot\frac{1}{2}\cdot\overbrace{\frac{2x}{\operatorname{tg}(2x)}}^{\to 1} \to \frac{\log(9)-\log(4)}{2} =$$

$$= \frac{1}{2} \log\left(\frac{9}{4}\right) = \log\left(\sqrt{\frac{9}{4}}\right) = \log\left(\frac{3}{2}\right)$$

$$\lim_{x \to 0} \frac{\log(1+2x)}{\sin(3^x - 1)}$$

VOLCIO ACCIUGLIA

$2x$ e $3^x - 1$

$$\frac{\sin(x)}{x} \to 1$$

$$\lim_{x \to 0}$$

$$\frac{\log(1+2x)}{\sin(3^x-1)} = \underbrace{\frac{\log(1+2x)}{2x}}_{\to 1} \cdot 2 \cdot \underbrace{\frac{x}{3^x - 1}}_{\to \frac{1}{\log(3)}} \cdot \underbrace{\frac{3^x - 1}{\sin(3^x-1)}}_{\to 1} \to$$

$$\to \frac{2}{\log(3)}$$

$$\lim_{x \to 0^+} \frac{\sin(2^x - 1)}{\log(1+2x^2)}$$

VOLCIO $2x^2$ e $\frac{1}{2^x - 1}$

$$\lim_{x \to 0^+}$$

$$\frac{\sin(2^x - 1)}{\log(1+2x^2)} = \underbrace{\sin(2^x-1) \cdot \frac{2x^2}{2x^2}}_{\to 1} \cdot \underbrace{\frac{2^x - 1}{2^x - 1}}_{\to 1} \cdot \underbrace{\frac{1}{\log(1+2x^2)}}_{\log(2)} =$$

$$= \frac{\sin(2^x-1)}{2^x-1} \cdot \frac{2x^2}{\lg(2x^2)} \cdot \underbrace{\frac{1}{2x}}_{\to +\infty} \cdot \frac{2^x-1}{x} \to +\infty$$

$$\lim_{x \to 0} \frac{2\arctan(2x^3) - x^2}{x^2(\cos(3\sqrt{x}) - 1)}$$

voglio $\frac{1}{2x^3}$

$(3\sqrt{x})^2 = 9x$

Per $x \to 0$

$$\frac{2\arctan(2x^3) - x^2}{x^2(\cos(3\sqrt{x}) - 1)} = \frac{x^2\left(\frac{2\arctan(2x^3)}{x^2} - 1\right)\cdot\frac{2}{2}}{-x^2(1-\cos(3\sqrt{x}))\cdot\frac{9}{9}} =$$

$$= \frac{\cancel{x^2}\left(2\frac{\arctan(2x^3)}{x^2} - 1\right)\cdot\frac{2}{2}}{-\cancel{x^2}\left(\frac{1-\cos(3\sqrt{x})}{x}\right)\frac{9}{9}} = \frac{4\overbrace{\frac{\arctan(2x^3)}{2x}}^{\to 1} - 1}{-9\underbrace{\frac{1-\cos(3\sqrt{x})}{9x}}_{\frac{1}{2}}} \to$$

$$\to \frac{4-1}{-9\cdot\frac{1}{2}} = -\frac{6}{9} = -\frac{2}{3}$$

$$\lim_{n \to \infty} n\left(\sqrt[3]{8 + \sin\left(\frac{2}{n}\right)} - 2\right)$$

Per $x \to \infty$

$$n\left(\sqrt[3]{8+\sin\left(\frac{2}{n}\right)}-2\right)=$$

VOLGLIO $\frac{\sin\left(\frac{2}{n}\right)}{8}$; AFFINCHÉ $\frac{1}{\frac{\sin\left(\frac{2}{n}\right)}{8}}$

$$=n\left(\sqrt[3]{8\left(1+\frac{\sin\left(\frac{2}{n}\right)}{8}\right)}-2\right)=$$

$$=n\left(2\sqrt[3]{1+\frac{\sin\left(\frac{2}{n}\right)}{8}}-2\right)=2n\left(\left(1+\frac{\sin\left(\frac{2}{n}\right)}{8}\right)^{\frac{1}{3}}-1\right)=$$

SORPRAI AVERE $\frac{2}{n}$

$$=\lim\frac{\left(\left(1+\frac{\sin\left(\frac{2}{n}\right)}{8}\right)^{\frac{1}{3}}-1\right)}{\frac{\sin\left(\frac{2}{n}\right)}{8}}\cdot\frac{\sin\left(\frac{2}{n}\right)}{8}=$$

$$=\frac{\left(\left(1+\frac{\sin\left(\frac{2}{n}\right)}{8}\right)^{\frac{1}{3}}-1\right)}{\frac{\sin\left(\frac{2}{n}\right)}{8}}\cdot\frac{\sin\left(\frac{2}{n}\right)}{\frac{2}{n}\cdot 2}=$$

$\frac{x}{n}$

VOGLIO SEMPLIFICARE

$$=\frac{1}{2}\cdot\frac{\overbrace{\left(\left(1+\frac{\sin\left(\frac{2}{n}\right)}{8}\right)^{\frac{1}{3}}-1\right)}^{\frac{1}{3}}}{\frac{\sin\left(\frac{2}{n}\right)}{8}}\cdot\overbrace{\frac{\sin\left(\frac{2}{n}\right)}{\frac{2}{n}}}^{\to 1}\Rightarrow$$

$$\Rightarrow\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$$

$$\lim\ \log(2)-\log(x)$$

$x \to 2$ $\quad \dfrac{\log(2) - \log(x)}{\sqrt{2} - \sqrt{x}}$

Più $x \to 2$

$$\frac{\log(2) - \log(x)}{\sqrt{2} - \sqrt{x}} = \frac{\log\left(\frac{2}{x}\right)}{\sqrt{2} - \sqrt{x}} =$$

$$= \frac{\log\left(1 + \left(\frac{2}{x} - 1\right)\right)}{\sqrt{2} - \sqrt{x}} \cdot \frac{\frac{2}{x} - 1}{\frac{2}{x} - 1} =$$

vuoto ottenuto

$$= \frac{\log\left(1 + \left(\frac{2}{x} - 1\right)\right)}{\frac{2}{x} - 1} \cdot \frac{\frac{2}{x} - 1}{\sqrt{2} - \sqrt{x}} \cdot \frac{\sqrt{2} + \sqrt{x}}{\sqrt{2} + \sqrt{x}} =$$

$$= \frac{\log\left(1 + \left(\frac{2}{x} - 1\right)\right)}{\frac{2}{x} - 1} \cdot \frac{\frac{2}{x} - 1}{2 - x} \cdot \sqrt{2} + \sqrt{x} =$$

$$= \underbrace{\frac{\log\left(1 + \left(\frac{2}{x} - 1\right)\right)}{\frac{2}{x} - 1}}_{\to 1} \cdot \underbrace{\frac{\frac{2 - x}{x}}{2 - x}}_{\to \frac{1}{x}} \cdot \underbrace{\sqrt{2} + \sqrt{x}}_{\to 2\sqrt{2}} \to \sqrt{2}$$

$\lim \quad 3\log\left(1 + \frac{1}{x}\right) + 4\log(1 + 2^x)$

$$\lim_{x\to+\infty} \frac{\quad}{\sqrt{1+x^2}+x}$$

Pl'n $x\to+\infty$

$$\frac{3\log\left(1+\frac{1}{x}\right)+4\log(1+2^x)}{\sqrt{1+x^2}+x} = \frac{3\log\left(1+\frac{1}{x}\right)+4\log\left(2^x(1+2^{-x})\right)}{x\left(\sqrt{\frac{1+x^2}{x^2}}+1\right)} =$$

$$= \frac{3\log\left(1+\frac{1}{x}\right)+4\left(\log(2^x)+\log(1+2^{-x})\right)}{x\left(\sqrt{\frac{1}{x^2}+1}+1\right)} =$$

$$= \frac{3\log\left(1+\frac{1}{x}\right)+4x\log(2)+4\log(1+2^{-x})}{x\left(\sqrt{\frac{1}{x^2}+1}+1\right)} =$$

$$= \frac{\cancel{x}\left(3\cdot\overbrace{\frac{\log\left(1+\frac{1}{x}\right)}{x}}^{\to 0}+4\log(2)+\overbrace{\frac{4\log(1+2^{-x})}{x}}^{\to 0}\right)}{x\left(\underbrace{\sqrt{\frac{1}{x^2}+1}}_{\to 1}+1\right)} \to$$

$$\frac{4\ln(2)}{2} = 2\ln(2)$$

$$\lim_{x\to\infty} \frac{3\log\left(1+\frac{1}{x}\right)+4\log(1+2^x)}{\sqrt{1+x^2}+x}$$

pour $x \to -\infty$

$$\frac{3\log\left(1+\frac{1}{x}\right)+4\log(1+2^x)}{\sqrt{1+x^2}+x} = \frac{3\log\left(1+\frac{1}{x}\right)+4\log(1+2^x)}{-x\left(\sqrt{1+\frac{1}{x^2}}-1\right)} =$$

$$= -\frac{3\,\log\left(1+\frac{1}{x}\right)+4\,2^x\,\frac{\log(1+2^x)}{2^x}}{x\left(\sqrt{1+\frac{1}{x^2}}-1\right)} \cdot \frac{x}{x} =$$

$$= -\frac{3\cdot\overbrace{\frac{\log\left(1+\frac{1}{x}\right)}{\frac{1}{x}}}^{\to 1}+4\,\overbrace{x\,2^x}^{\to 0}\,\overbrace{\frac{\log(1+2^x)}{2^x}}^{\to 1}}{\underbrace{\frac{\sqrt{1+\frac{1}{x^2}}-1}{x^2}}_{\to \frac{1}{2}}} \to$$

$$\to -\frac{3\cdot 1+4\cdot 0\cdot 1}{\frac{1}{2}} = -6$$

$$\lim_{x\to 0} \frac{x\cos(x)-x^2}{\sqrt{1+x^4}-1}$$

$\sqrt{1+3x^4} - 1$

Pin $x \to 0$

$\arcsin(x) = x + \frac{x^3}{6} + o(x^3)$

E

$\sqrt{1+3x^4} = (1+3x^4)^{\frac{1}{2}} = 1 + \frac{1}{2}(3x^4) + o(x^4)$

Quindi

$$\frac{x \arcsin(x) - x^2}{\sqrt{1+3x^4} - 1} = \frac{x\left(x + \frac{x^3}{6} + o(x^3)\right) - x^2}{\cancel{1} + \frac{3x^4}{2} + o(x^4) - \cancel{1}} =$$

$$= \frac{\cancel{x^2} + \frac{x^4}{6} + o(x^4) - \cancel{x^2}}{\frac{3x^4}{2} + o(x^4)} = \frac{\frac{x^4}{6} + o(x^4)}{\frac{3x^4}{2} + o(x^4)} =$$

$$= \frac{\cancel{\frac{1}{2}x^4} + o(x^4)}{\cancel{\frac{18}{9}x^4} + o(x^4)} = \frac{1}{9} + o(1) \to \frac{1}{9}$$

$$\lim_{x \to 1} \frac{x^x - x}{\log^3(x)} - \frac{1}{\log(x)}$$

Pin $x \to 1$ $f = 1$ $x \to 0$ Vedi

$t = x - 1 \to 0$, donc :

$$x^x = \exp\left((1+t)\ln(1+t)\right) =$$

$$= \exp\left((1+t)\left(t - \frac{t^2}{2} + \frac{t^3}{3} + o(t^3)\right)\right) =$$

$$= \exp\left(t - \frac{t^2}{2} + \frac{t^3}{3} + t^2 - \frac{t^3}{2} + \frac{t^4}{3} + o(t^4)\right) =$$

$$= \exp\left(t + \frac{t^2}{2} + \frac{2t^2 - 3t^3}{6} + \frac{t^4}{3} + o(t^4)\right) =$$

$$= \exp\left(t + \frac{t^2}{2} - \frac{t^3}{6} + \frac{t^4}{3} + o(t^4)\right) =$$

$$= 1 + \left(t + \frac{t^2}{2} - \frac{t^3}{6} + \cancel{t^4} + o(t^3)\right) + \frac{1}{2}\left(t + \frac{t^2}{2} + o(t^2)\right)^2 + \frac{1}{6}\left(t + o(t)\right)^3$$

$$= 1 + t + \frac{t^2}{2} - \frac{t^3}{6} + \cancel{t^4} + o(t^4) + \frac{t^2}{2} + \cancel{t^3} + \frac{t^3}{2} + o(t^3) + \frac{t^3}{6} + o(t^3)$$

$$= 1 + t + t^2 + \frac{\cancel{-1+3}\,1}{\cancel{6}\,2} t^3 + \quad + o(t^3) =$$

$$= 1 + t + t^2 + \frac{t^3}{2} + o(t^3)$$

i

$$\log(x) = \log(1+t) = t - \frac{t^2}{2} + o(t^2)$$

ausl

$$\frac{x^x - x}{\log^3(x)} - \frac{1}{\log(x)} = \frac{x^x - x - \log^2(x)}{\log^3(x)} =$$

$$= \frac{\cancel{1} + \cancel{t} + t^2 + \frac{t^3}{2} + o(t^3) - \cancel{t} - \cancel{1} - \left(t - \frac{t^2}{2} + o(t^2)\right)^2}{\left(t - t^2 + o(t^2)\right)^3} =$$

$$= \frac{\cancel{t^2} + \frac{t^3}{2} + o(t^3) - \left(\cancel{t^2} - t^3 + o(t^3)\right)}{t^3 + o(t^3)} =$$

$$= \frac{3\frac{t^3}{2} + o(t^3)}{t^3 + o(t^3)} = \frac{3\cancel{t^3} + o(t^3)}{2\cancel{t^3} + o(t^3)} \to \frac{3}{2}$$

$$\lim_{x \to +\infty} \frac{x^2 \cos\left(\frac{5}{x}\right) - (x^2 - x - 3)e^{\frac{1}{x}}}{\left(1 - \frac{1}{x}\right)^x}$$

Pen $x \to +\infty$

$$\left(1 - \frac{1}{x}\right)^x \to e^{-1}$$

t si $t = \frac{1}{x} \to 0^+$

$$x^2 \cos\left(\frac{5}{x}\right) - (x^2 - x - 3)e^{\frac{1}{x}} =$$

$$= \frac{1}{t^2} \cos(5t) - \left(\frac{1}{t^2} - \frac{1}{t} - 3\right) e^t =$$

$$= \frac{1}{t^2}\left(1 - \frac{(5t)^2}{2!} + o(t)\right) - \left(\frac{1 - t - 3t^2}{t^2}\right)\left(1 + t + \frac{t^2}{2} + o(t)\right)$$

$$= \frac{1}{t^2}\left(1 - \frac{25t^2}{2} + o(t^2)\right) - (1 - t - 3t^2)\left(1 + t + \frac{t^2}{2} + o(t^2)\right)$$

$$= \frac{1}{t} - \frac{25}{2} + o(t) - \left(1 + \cancel{t} + \frac{t^2}{2} - \cancel{t} - t^2 - 3t^2 + o(t^2)\right)$$

$$= \frac{1}{t} - \frac{25}{2} + o(t) - 1 + \frac{7}{2} t^2 + o(t^2)$$

$$= \frac{2 - 25t - 2t + 7t^3 + o(t^3)}{2t}$$

$$= \frac{1 - 27t + 7t^2 + o(t^2)}{2t}$$

$$\lim_{x \to 0} \left(1 + \sin(x) + \ln(1 - x + x^2)\right)^{\frac{1}{x^2}}$$

Pour $x \to 0$

$$\left(1 + \sin(x) + \ln(1 + (-x + x^2))\right)^{\frac{1}{x^2}} =$$

$$= \left(1 + (\cancel{x} + o(x^2)) + (-\cancel{x} + x^2) - \frac{(-x + x^2)^2}{2} + o(x^2)\right)^{\frac{1}{x^2}} =$$

$$= \left(1 + x^2 - \frac{x^2}{2} + o(x^2)\right)^{\frac{1}{x^2}} =$$

$$= \left(1 + \frac{x^2}{2} + o(x^2)\right)^{\frac{1}{x^2}} =$$

$$= \exp\left(\frac{1}{x^2} \cdot \ln\left(1 + \left(\frac{x^2}{2} + o(x^2)\right)\right)\right) =$$

$$= \exp\left(\frac{1}{\cancel{x^2}} \cdot \left(\frac{\cancel{x^2}}{2} + o(x^2)\right)\right) =$$

$$= \exp\left(\frac{1}{2} + o(1)\right) \to \sqrt{e}$$

$$\lim_{x \to \frac{\pi}{4}} \left(\frac{2}{\tan(x) - 1} - \frac{1}{\sqrt{2}\sin(x) - 1}\right)$$

Sia $t = x - \frac{\pi}{4}$, allora

$$\sqrt{2}\sin(x) = \sqrt{2}\sin\left(t + \frac{\pi}{4}\right) = \sin(t) + \cos(t) =$$

$$= \left(t + o(t^2)\right) + \left(1 - \frac{t^2}{2} + o(t^2)\right) =$$

$$= 1 + t - \frac{t^2}{2} + o(t^2)$$

è

$$\tan(x) = \tan\left(t + \frac{\pi}{4}\right) = \frac{1+\tan(t)}{1-\tan(t)} =$$

$$= \frac{1+t+o(t^2)}{1-t+o(t^2)} = \left(1+t+o(t^2)\right)\left(1-t+o(t^2)\right)^{-1} =$$

$$= \left(1+t+o(t^2)\right)\left(1+t+t^2+o(t^2)\right) =$$

$$= 1+t+t^2+t+t^2+o(t^2) =$$

$$= 1+2t+2t^2+o(t^2)$$

Così per $t \geq 0$

$$\frac{2}{\tan(x)-1} - \frac{1}{\sqrt{2}\sin(x)-1} =$$

$$= \frac{2}{2t+2t^2+o(t^2)} - \frac{1}{t-\frac{t^2}{2}+o(t^2)} =$$

$$= \frac{\cancel{2t}-t^2 - \cancel{2t}-2t^2+o(t^2)}{(2t+2t^2+o(t^2))(t-\frac{t^2}{2}+o(t^2))} =$$

$$= \frac{-3t^2 + o(t^2)}{2t^2 + o(t^2)} \Rightarrow -\frac{3}{2}$$

$$\lim_{x \to 0} \frac{x^2\left(\cos(2\sin(x)) - \cos(2x)\right)}{x^5 + x^3 - \sin(x^3) - x^5 e^{-x}}$$

Pour $x \to 0$

- $\sin(x) = x - \frac{x^3}{3!} + o(x^3)$

- $\cos(2\sin(x)) = 1 - \frac{\left(2x - \frac{x^3}{3!} + o(x^3)\right)^2}{2} + \frac{1}{24}\left(2x - \frac{x^3}{3} + o(x^3)\right)^4$

$= 1 - \frac{4x^2 - \frac{4x^4}{3}}{2} + o(x^4) + \frac{1}{24}\left(16x^4 + o(x^4)\right) + o(x^4) =$

$= 1 - 2x^2 + \frac{4x^4}{3} + o(x^4)$

- $\cos(2x) = 1 - \frac{4x^2}{2} + \frac{16x^4}{24} + o(x^4)$

- $\sin(x^3) = x^3 - \frac{(x^3)^3}{6} + o(x^9) = x^3 + o(x^6)$

- $e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + o(x^4)$

$$\frac{x^2\left(\cos(2\sin(x)) - \cos(2x)\right)}{x^5 + x^2 - \sin(x^2) - x^5 e^{-x}} =$$

$$= \frac{x^2\left(\cancel{1} - \cancel{2x^2} + \frac{4x^4}{3} + o(x^4) - \cancel{1} + \cancel{\frac{4x^2}{2}} - \frac{\cancel{16}^{\,2}x^4}{\cancel{24}_{\,3}} + o(x^4)\right)}{\cancel{x^5} + \cancel{x^2} - \cancel{x^2} + \frac{x^6}{6} + o(x^6) - x^5\left(\cancel{1} - x + o(x)\right)} =$$

$$= \frac{x^2\left(\left(\frac{4}{3} - \frac{2}{3}\right)x^4 + o(x^4)\right)}{\left(\frac{1}{6} + 1\right)x^6 + o(x^6)} = \frac{\frac{2}{3}x^6 + o(x^6)}{\frac{7}{6} + o(x^6)} =$$

$$= \frac{\cancel{4}^{\,2}\,\cancel{x^6} + o(x^6)}{\cancel{7}_{\,7}\,\cancel{x^6} + o(x^6)} \longrightarrow \frac{4}{7}$$