3) a) $$\lim_{x\to\sqrt{3}} \frac{\pi - 3\arcsin\left(\frac{x}{2}\right)}{\pi - 3\arctan(x)} = \frac{0}{0}$$

$$= \frac{\pi - \cancel{3}\cdot\frac{\pi}{\cancel{3}}}{\pi - \cancel{3}\cdot\frac{\pi}{\cancel{3}}} = \frac{0}{0}$$

$$\lim_{x\to\sqrt{3}} \frac{-3\cdot\frac{1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}}{-3\cdot\frac{1}{1+x^2}} = \frac{\frac{-3}{2\sqrt{1-\frac{3}{4}}}}{\frac{-3}{1+3}} =$$

$$= \frac{\cancel{-3}}{\frac{\cancel{-3}}{4}} = 4$$

b) $$\lim_{x\to+\infty} \frac{\pi - 2\arctan(5x)}{e^{-\frac{1}{x}} - \sqrt{1-\frac{1}{x}}} =$$

$$= \frac{\pi - 2\cdot\frac{\pi}{2}}{} = \frac{0}{0}$$

$1 - 1$

$\frac{0}{0}$ H

$$= \lim_{x \to +\infty} \frac{-2 \cdot \frac{5}{1+(5x)^2}}{\left(e^{-\frac{2}{x}} \cdot \frac{2}{x^2}\right) - \left(\frac{1}{2\sqrt{1-\frac{1}{x}}} \cdot \frac{1}{x^2}\right)} =$$

$$= \lim_{x \to +\infty} \frac{\frac{-10}{1+25x^2}}{\frac{2 \cdot e^{-\frac{2}{x}}}{x^2} - \frac{1}{2x^2\sqrt{1-\frac{1}{x}}}} =$$

$$= \lim_{x \to +\infty} \frac{\frac{-10}{1+25x^2}}{\frac{1}{x^2}\left(2e^{-\frac{2}{x}} - \frac{1}{2\sqrt{1-\frac{1}{x}}}\right)} =$$

$$= \lim_{x \to +\infty} \frac{-10x^2}{1+25x^2} \cdot \frac{1}{2e^{-\frac{2}{x}} - \frac{1}{2\sqrt{1-\frac{1}{x}}}} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^2}(-10)}{\cancel{x^2}(\underbrace{\tfrac{1}{x^2}}_{\to 0}+25)} \cdot \frac{1}{\underbrace{\boxed{\frac{2}{e^{\frac{2}{x}}}}}_{\to 2} - \frac{1}{2\underbrace{\sqrt{1-\frac{1}{x}}}_{\to 1}}} =$$

$$= -\frac{10}{25} \cdot \frac{1}{2-\frac{1}{2}} = -\frac{2}{5} \cdot \frac{2}{3} = -\frac{4}{15}$$

c) $$\lim_{x\to 2^+} \frac{\cos\left(\frac{\pi}{x}\right)}{\left(\arccos\left(\frac{2}{x}\right)\right)^2} =$$

$$= \frac{\cos\left(\frac{\pi}{2}\right)}{\left(\arccos(1)\right)^2} = \frac{0}{0^2} = \frac{0}{0}$$

$$\overset{\frac{0}{0}}{\underset{H}{=}} \lim_{x\to 2^+} \frac{\frac{-\sin\left(\frac{\pi}{x}\right)\cdot \pi}{\cancel{x^2}}}{2\cdot \arccos\left(\frac{2}{x}\right)\cdot\left(-\frac{1}{\sqrt{1-\left(\frac{2}{x}\right)^2}}\right)\cdot -\frac{2}{\cancel{x^2}}} = \dots$$

$$= \lim_{x\to 2^+} \frac{\frac{\sin\left(\frac{\pi}{x}\right)\cdot\pi}{\cancel{x^2}}}{\frac{+4\cdot \arccos\left(\frac{2}{x}\right)}{+\sqrt{1-\frac{4}{\cancel{x^2}}}}} =$$

$$-2\cdot\frac{4}{x^2}\cdot\frac{1}{2\sqrt{1-\frac{4}{x^2}}} = \frac{4}{x^2\sqrt{1-\frac{4}{x^2}}}$$

$$= \lim_{x\to 2^+} \frac{\sin\left(\frac{\pi}{x}\right)\cdot \pi \cdot \sqrt{1-\frac{4}{x^2}}}{4\arccos\left(\frac{2}{x}\right)} = \frac{0}{0}$$

$$\underset{H}{=} \lim_{x\to 2^+} \frac{\pi\left(\frac{\pi\cdot -\cos\left(\frac{\pi}{x}\right)}{x^2}\cdot\sqrt{1-\frac{4}{x^2}} + \sin\left(\frac{\pi}{x}\right)\cdot\frac{4}{x^3\sqrt{1-\frac{4}{x^2}}}\right)}{4\cdot -\frac{1}{\dots}\cdot -2\cdot\frac{1}{x^2}} =$$

$\sqrt{1-\left(\frac{2}{x}\right)^2} \qquad x^2$

$$= \lim_{x \to 2^+} \frac{\pi \left(\frac{\pi \cdot \cos\left(\frac{\pi}{x}\right)}{x^2} \cdot \sqrt{1-\frac{4}{x^2}} + \sin\left(\frac{\pi}{x}\right) \cdot \frac{4}{x^3 \sqrt{1-\frac{4}{x^2}}} \right)}{\frac{8}{\sqrt{1-\frac{4}{x^2}} \cdot x^2}} =$$

$$= \frac{\qquad}{\frac{8}{\qquad}}$$

d) $$\lim_{x \to 1} \left(\frac{e}{e^x - e} - \frac{1}{\ln(x)} \right) =$$

$$= \lim_{x \to 1} \frac{e \cdot \ln(x) - e^x + e}{(e^x - e) \cdot \ln(x)} =$$

$$= \frac{0 - 0}{0 \cdot 0} = \frac{0}{0}$$

$\frac{0}{0} \quad e \quad e^x$

$$\overset{H}{=} \frac{\frac{\cdot}{x} - e}{(e^x - e)' \cdot \lg(x) + (e^x - e) \cdot \frac{1}{x}} =$$

$$= \lim_{x \to 1} \frac{\frac{e - e^x \cdot x}{\cancel{x}}}{\cancel{x} \cdot e^x \cdot \lg(x) + \frac{e^x - e}{\cancel{x}}} =$$

$$\lim_{x \to 1} \frac{e - e^x \cdot x}{e^x \cdot x \cdot \log(x) + e^x - e} = \frac{e - e \cdot 1}{e \cdot 1 \cdot 0 + e - e} = \frac{0}{0}$$

$$\overset{\frac{0}{0}}{\underset{H}{=}} \lim_{x \to 1} \frac{0 - e^x \cdot 1 - e^x \cdot x}{(e^x \cdot x \cdot \lg(x))' + e^x - 0} =$$

$$= \lim_{x \to 1} \frac{-e^x - x \cdot e^x}{(e^x \cdot x)' \cdot \lg(x) + e^x \cdot \cancel{x} \cdot \frac{1}{\cancel{x}} + e^x} =$$

$$= \lim_{x \to 1} \frac{-e^x - x \cdot e^x}{(e^x \cdot x + e^x) \cdot \lg(x) + e^x + e^x} =$$

$$= \lim \frac{-e^x - e^x}{\ldots} =$$

$$= \lim_{x \to 1} e^x \cdot x \cdot \log(x) + e^x \log(x) + 2e^x$$

$$= \lim_{x\to 1} \frac{\cancel{e^x} \cdot (-2)}{\cancel{e^x}\left(x\log(x) + \log(x) + 2\right)} =$$

$$= \lim_{x \to 1} \frac{-2}{x\log(x) + \log(x) + 2} = \frac{-2}{1 \cdot 0 + 0 + 2} = -1$$

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4) POLINOMI DI TAYLOR

a) $f(x) = \arcsin(x)$ in $x_0 = 0$

ordine $n = 3$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} = (1-x^2)^{-\frac{1}{2}}$$

$$f''(x) = \left((1-x^2)^{-\frac{1}{2}}\right)' = -\frac{1}{\cancel{2}} \cdot (1-x^2)^{-\frac{3}{2}} \cdot -\cancel{2}x =$$

$$= x \cdot (1-x^2)^{-\frac{3}{2}}$$

$$f'''(x) = (1-x^2)^{-\frac{3}{2}} + \left(x \cdot -\frac{3}{\cancel{2}} \cdot (1-x^2)^{-\frac{5}{2}} \cdot -\cancel{2}x\right) =$$

$$= (1-x^2)^{-\frac{3}{2}} + 3x^2 \cdot (1-x^2)^{-\frac{5}{2}}$$

Quindi

$$f(0) = \arcsin(0) = 0$$

$$f'(0) = (1-0)^{-\frac{1}{2}} = \frac{1}{\sqrt{1}} = 1$$

$$f''(0) = 0 \cdot (1-0)^{-\frac{3}{2}} = 0$$

$$f'''(0) = (1-0)^{-\frac{3}{2}} + 3(0) \cdot (1-0)^{-\frac{5}{2}} = 1$$

Quindi

$$\sum_{k=0}^{3} \frac{f^{k}(0)}{k!} \cdot x^{k} = \cancel{\frac{0}{0} x^{0}} + \frac{1}{1!} \cdot x^{1} + \cancel{\frac{0}{2!} \cdot x^{2}} + \frac{1}{3!} \cdot x^{3}$$

Quindi

$$T_3(x) = \sum_{k=0}^{3} \frac{f^{k}(0)}{k!} \cdot x^{k} = x + \frac{x^3}{6}$$

b) $x_0 = 0$

$$f(x) = \frac{1}{\sqrt{1+4x}} = (1+4x)^{-\frac{1}{2}}$$

$n = 3$

$$f'(x) = -\frac{1}{2} \cdot (1+4x)^{-\frac{3}{2}} \cdot 4 = -2 \cdot (1+4x)^{-\frac{3}{2}}$$

$$f''(x) = -2 \cdot -\frac{3}{2} (1+4x)^{-\frac{5}{2}} \cdot 4 = 12 (1+4x)^{-\frac{5}{2}}$$

$$f'''(x) = \cancel{12}^{6} \cdot \left(-\frac{5}{\cancel{2}}\right) \cdot (1+4x)^{-\frac{7}{2}} \cdot 4 = -120 (1+4x)^{-\frac{7}{2}}$$

Quindi

$$f(0) = \frac{1}{\sqrt{1+0}} = 1$$

$$f'(0) = -2 \cdot (1+0)^{-\frac{3}{2}} = -2$$

$$f''(0) = 12 (1+0)^{-\frac{5}{2}} = 12$$

$$f'''(0) = -120 (1+0)^{-\frac{7}{2}} = -120$$

Quindi

$$T = \sum^{3} f^{(k)}(0)\, x^{k}$$

$$T_3(x) = \sum_{k=0} \frac{f^{(k)} \cdot x^k}{k!} =$$

$$= \frac{1 \cdot x^0}{0!} + \frac{-2 \cdot x^1}{1!} + \frac{\overset{6}{\cancel{12}} \cdot x^2}{\cancel{2!}} + \frac{-\overset{20}{\cancel{120}}\, x^3}{\cancel{3!}} =$$

$$= 1 - 2x + 6x^2 - 20x^3$$

c) $$f(x) = \log\left(\frac{x}{2-x}\right) = \lg(x) - \lg(2-x)$$

$n = 4$

$x_0 = 1$

$$f'(x) = \frac{1}{x} - \frac{1}{2-x} \cdot (-1) = \frac{1}{x} + \frac{1}{2-x}$$

$$f''(x) = -\frac{1}{x^2} + (-1) \cdot \left(-\frac{1}{(2-x)^2}\right) = -\frac{1}{2} + \frac{1}{(2-x)^2}$$

$$f'''(x) = \frac{2}{x^3} + (-2) \cdot \left(-\frac{1}{(2-x)^3}\right) = \frac{2}{x^3} + \frac{2}{(2-x)^3}$$

$$f''''(x) = \frac{-6}{x^4} + \frac{6}{(2-x)^4}$$

$(2-1)$

QUINDI

$$f(1) = \log(1) - \log(1) = 0$$

$$f'(1) = 1 + 1 = 2$$

$$f''(1) = -1 + 1 = 0$$

$$f'''(1) = 2 + 2 = 4$$

$$f''''(1) = -6 + 6 = 0$$

QUINDI PER $x_0 = 1$ E $n = 4$

$$T_4(x) = \sum_{k=0}^{4} \frac{f^{(k)}(1)}{k!}(x-1)^k =$$

$$= \cancel{\frac{0 \cdot (x-1)^0}{0!}} + \frac{2 \cdot (x-1)^1}{1!} + \cancel{\frac{0 \cdot (x-1)^2}{2!}} + \frac{\cancel{4}^{2} \cdot (x-1)^3}{\cancel{3!}} +$$

$$+ \cancel{\frac{0 \cdot (x-1)^4}{4!}} =$$

$$= 0 \cdot (x-1) + 2 \cdot (x-1)^2$$

$$2\cdot(x-1)+\frac{\;}{3}(x-1)$$

d) $$f(x)=\frac{1}{2-e^{-x}}=\left(2-e^{-x}\right)^{-1} \qquad x_0=1$$

$$n=3$$

$$f'(x)=-\left(2-e^{-x}\right)^{-2}\cdot\left(-e^{-x}\right)\cdot(-1)=$$

$$=-\frac{e^{-x}}{\left(2-e^{-x}\right)^{2}}=-e^{-x}\cdot\left(2-e^{-x}\right)^{-2}$$

$$f''(x)=-\left[-1\cdot e^{-x}\cdot\frac{1}{\left(2-e^{-x}\right)^{2}}+e^{-x}\cdot(-2)\cdot\left(2-e^{-x}\right)^{-3}\cdot e^{-x}\right]$$

$$=\frac{e^{-x}}{\left(2-e^{-x}\right)^{2}}+\frac{2e^{-2x}}{\left(2-e^{-x}\right)^{3}}=e^{-x}\cdot\left(2-e^{-x}\right)^{-2}+2e^{-2x}\cdot\left(2-e^{-x}\right)^{-3}$$

$$f'''(x)=-e^{-x}\cdot\left(2-e^{-x}\right)^{-2}+e^{-x}\cdot\left(-2\left(2-e^{-x}\right)^{-3}\cdot e^{-x}\right)+$$

$$+-4e^{-2x}\cdot\left(2-e^{-x}\right)^{-3}+2e^{-2x}\cdot\left(-3\left(2-e^{-x}\right)^{-4}\cdot e^{-x}\right)$$

$$= \frac{-e^{-x}}{(2-e^{-x})^2} - \frac{2e^{-2x}}{(2-e^{-x})^3} - \frac{4e^{-2x}}{(2-e^{-x})^3} - \frac{6e^{-3x}}{(2-e^{-x})^4} =$$

$$= \frac{-e^{-x}}{(2-e^{-x})^2} + \frac{-2e^{-2x} - 4e^{-2x}}{(2-e^{-x})^3} - \frac{6e^{-3x}}{(2-e^{-x})^4} =$$

$$= \frac{-e^{-x}}{(2-e^{-x})^2} - \frac{6e^{-2x}}{(2-e^{-x})^3} - \frac{6e^{-3x}}{(2-e^{-x})^4}$$

quindi

$$f(0) = \frac{1}{2-e^{-0}} = 1$$

$$f'(0) = -\frac{e^{0}}{(2-e^{0})^2} = -\frac{1}{1} = -1$$

$$f''(0) = \frac{e^{0}}{(2-1)^2} + \frac{2 \cdot e^{0}}{(2-1)^3} = \frac{1}{1} + \frac{2}{1} = 3$$

$$f'''(0) = -\frac{e^{0}}{(2-1)^2} - \frac{6 \cdot e^{0}}{(2-1)^3} - \frac{6 \cdot e^{0}}{(2-1)^4} =$$

$$= -1 - 6 - 6 = -13$$

QUINDI

$$T_3(x) = \sum_{k=0}^{3} \frac{f^{(k)}(0)}{k!} \cdot x^k = \frac{1}{0!} \cdot x^0 + \frac{-1}{1!} \cdot x^1 + \frac{3}{2!} \cdot x^2 + \frac{-13}{3!} \cdot x^3 =$$

$$= 1 - x + \frac{3}{2} x^2 - \frac{13}{6} x^3$$

e) $f(x) = \tan(x) = \dfrac{\sin(x)}{\cos(x)}$ $\qquad x_0 = \dfrac{\pi}{4}$, $n = 3$

$$f'(x) = \frac{\cos(x) \cdot \cos(x) - (\sin(x) \cdot -\sin(x))}{(\cos(x))^2} =$$

$$= \frac{\cos^2(x) + \sin^2(x)}{(\cos(x))^2} = \frac{1}{(\cos(x))^2}$$

$$f''(x) = -2\,(\cos(x))^{-3} \cdot (-\sin(x)) =$$

$$= 2(\cos(x))^{-3} \cdot \sin(x)$$

$$f'''(x) = 2 \cdot \left[-3(\cos(x))^{-4} \cdot (-\sin(x)) \cdot \sin(x) + \right.$$

$$\left. + (\cos(x))^{\cancel{-3}\,-2} \cdot \cancel{\cos}(x) \right]$$

$$= 6\,\frac{(\sin(x))^2}{(\cos(x))^4} + \frac{2}{(\cos(x))^2}$$

quindi

$$f\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

$$f'\left(\frac{\pi}{4}\right) = \frac{1}{\left(\cos\left(\frac{\pi}{4}\right)\right)^2} = \frac{1}{\frac{2}{4}} = 2$$

$$f''\left(\frac{\pi}{4}\right) = \frac{2}{\left(\cos\left(\frac{\pi}{4}\right)\right)^3} \cdot \sin\left(\frac{\pi}{4}\right) =$$

$$= \frac{2}{(\cos(\frac{\pi}{4}))^2} \cdot \frac{\sin(\frac{\pi}{4})}{\cos(\frac{\pi}{4})} = 4$$

$$f'''(\tfrac{\pi}{4}) = 6 \cdot \left(\tan(\tfrac{\pi}{4})\right)^2 \cdot \frac{1}{(\cos(\frac{\pi}{4}))^2} + \frac{2}{(\cos(\frac{\pi}{4}))^2} =$$

$$= 6 \cdot \frac{\cancel{4}^2}{\cancel{4}} + \frac{\cancel{8}^4}{\cancel{2}} = 16$$

$$T_3(x) = \sum_{k=0}^{3} \frac{f^{(k)}(\frac{\pi}{4})}{k!} \cdot \left(x - \frac{\pi}{4}\right)^k =$$

$$= \frac{1 \cdot (x - \frac{\pi}{4})^0}{0!} + \frac{2 \cdot (x - \frac{\pi}{4})^1}{1!} + \frac{\cancel{4}^2 \cdot (x - \frac{\pi}{4})^2}{\cancel{2!}} + \frac{\cancel{16}^8 \cdot (x - \frac{\pi}{4})^3}{3!} =$$

$$= 1 + 2 \cdot \left(x - \frac{\pi}{4}\right) + 2 \cdot \left(x - \frac{\pi}{4}\right)^2 + 8 \cdot \left(x - \frac{\pi}{4}\right)^3$$

f) $f(x) = \log(\cos(x))$ $\qquad x_0 = 0$

$n = 4$

$$f'(x) = \frac{1}{\quad} \cdot (-\sin(x)) = -\tan(x)$$

$-\cos(x)$

$$f''(x) = \left(-\frac{\sin(x)}{-\cos(x)}\right)' = -\left[\frac{-\cos(x)\cdot -\cos(x) - \sin(x)\cdot(\sin(x))}{(-\cos(x))^2}\right]$$

$$= -\frac{1}{(-\cos(x))^2}$$

$$f'''(x) = -2\,(-\cos(x))^{-3} \cdot -\sin(x) \cdot$$

$$= \frac{-2\cdot\sin(x)}{(-\cos(x))^3}$$

$$f^{(4)}(x) = -2\left[\cancel{-\cos(x)}\cdot \cancel{-\cos}(x)^{-\cancel{3}\,2} + (\sin(x)\cdot(-3)(-\cos(x))^{-4}(\sin(x))\right]$$

$$= \frac{-2}{(-\cos(x))^{-2}} + \frac{6(\sin(x))^2}{(-\cos(x))^{-4}}$$

Quindi:

$$f(0) = \log(-\cos(0)) = \log(1) = 0$$

$$f'(0) = -\tan(0) = 0$$

$$f''(0) = -\frac{1}{(\cos(0))^{-2}} = -\frac{1}{1} = -1$$

$$f'''(0) = \frac{-2 \cdot \sin(0)}{(\cos(0))^{-3}} = \frac{-2 \cdot 0}{1^{3}} = 0$$

$$f^{(4)}(x) = \frac{-2}{(\cos(x))^{-2}} + \frac{6 \cdot (\sin(0))^2}{(\cos(x))^{-4}}$$

$$= -2 + \frac{0}{1} = -2$$

Quiz 1

$$T_4(x) = \sum_{k=0}^{4} \frac{f^{(k)}(0)}{k!}(x)^k = \frac{-1 \cdot x^2}{2!} + \frac{-2 \cdot x^4}{4 \cdot 3 \cdot 2}$$

$$= -\frac{x^2}{2} - \frac{x^4}{12}$$

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5) a)

5) a)

$$\lim_{x \to 0} \frac{\arcsin(x) - \sin(x)}{x \cdot \ln(\cos(x))}$$

Per $x \to 0$ si usa T.a e TF.

$$\frac{\left(x + \frac{x^3}{6} + o(x^3)\right) - \left(x - \frac{x^3}{6} + o(x^3)\right)}{x \cdot \left(-\frac{x^2}{2} + o(x^4)\right)} =$$

$$= \frac{\cancel{x} + \frac{x^3}{6} + o(x^3) - \cancel{x} + \frac{x^3}{6} + o(x^3)}{-\frac{x^3}{2} + o(x^3)} =$$

$$= \frac{\frac{\cancel{x}^3}{3} + o(x^3)}{-\frac{\cancel{x}^2}{2} + o(x^3)} \to -\frac{2}{3}$$

b) $$\lim_{x \to \frac{\pi}{4}} \left(\tan(x)\right)^{\frac{1}{4x - \pi}}$$

per $x \to \frac{\pi}{4}$, $t = x - \frac{\pi}{4} \to 0$, t per

4 . e

$$\left(\tan(x)\right)^{\frac{1}{4x-\pi}} = \left(1 + 2\cdot\left(x - \frac{\pi}{4}\right) + o\left(\cancel{x} - \frac{\pi}{4}\right)\right)^{\frac{1}{4x-\pi}} =$$

$$= \left(1 + 2\cdot t + o(t)\right)^{\frac{1}{4t}} =$$

$$= e^{\frac{1}{4t}\log\left(1+2t+o(t)\right)} =$$

$$= e^{\frac{1}{4t}\cdot\left(2t+o(t)\right)} = e^{\frac{\cancel{2}\cancel{t}}{2\cancel{4}\cancel{t}} + \frac{o(t)}{\cancel{t}}} \to e^{\frac{1}{2}} = \sqrt{e}$$