Trovare una formula chiusa per

$$\sum_{k=0}^{n} (k^2+k)$$

Sappiamo dalla teoria che $\exists f \in \mathbb{R}[x]$

tale che $\deg(f(x)) \leq 2+1$ e

$\Rightarrow$ grado di

$$\sum_{k=0}^{n} (k^2+k) = f(n) \qquad \forall n \in \mathbb{N}$$

Quindi $\exists\ a, b, c, d \in \mathbb{R}$ tale che

$$\sum_{k=0}^{n} (k^2+k) = a n^3 + b n^2 + c n + d$$

$\forall n \in \mathbb{N}$. Ma allora

$$\begin{cases} 0 = d & (n=0) \\ 2 = a+b+c+d & (n=1) \\ 6 = 8a+4b+2c+d & (n=2) \\ 20 = 27a+9b+3c+d & (n=3) \end{cases}$$

$$\Rightarrow d=0 \qquad \begin{cases} 2 = a+b+c \\ 6 = 8a+4b+2c \\ 20 = 27a+9b+3c \end{cases}$$

$$\Rightarrow c = 2 - a - b$$

$$\begin{cases} 8 \cdot a + 4b + 2(2 - a - b) = 8 \\ 27 \cdot a + 9b + 3 \cdot (2 - a - b) = 20 \end{cases}$$

$$\begin{cases} 8a + 4b + 4 - 2a - 2b = 8 \\ 27a + 9b + 6 - 3a - 3b = 20 \end{cases}$$

$$\begin{cases} \frac{6a}{2} + \frac{2b}{2} = \frac{4}{2} \\ \frac{24a}{2} + \frac{6b}{2} = \frac{14}{2} \end{cases} \Rightarrow \begin{cases} 3a + b = 2 \\ 12a + 3b = 7 \end{cases}$$

$$\Rightarrow b = 2 - 3a$$

$$12 \cdot a + 3(2 - 3a) = 7$$

$$\Rightarrow 12a + 6 - 9a = 7$$

$$3a = 1 \Rightarrow a = \frac{1}{3}$$

$$\Rightarrow b = 2 - 3 \cdot \left(\frac{1}{3}\right)$$

$$b = 1$$

$$\Rightarrow c = 2 - \frac{1}{3} - 1$$

$$c = \frac{6-1-3}{3}$$

$$c = \frac{2}{3}$$

Concludendo

$$\sum_{k=0}^{n} (k^2+k) = \frac{n^3}{3} + n^2 + \frac{2}{3} \cdot n$$

Trovare un'espressione asintotica chiusa per

$$\sum_{k=1}^{m} (k^2 - 6k - 4)$$

Sappiamo dalla teoria che $\exists\, P(x) \in \mathbb{R}[x]$

tale che $\deg(P) \leq 2+1$ e

$$\sum^{n} (k^2 - 6k - 4) = P(x) \quad \forall n \in \mathbb{N}$$

$K=1$

QUINDI $\exists\, a, b, c, d \in \mathbb{R}$ TALI CHE:

$$f(x) = an^3 + bn^2 + cn + d$$

$\forall n \in \mathbb{N}$. MA ALLORA

$$\begin{cases} 0 = d & (n=0) \\ -9 = a+b+c+d & (n=1) \\ -21 = 8a+4b+2c+d & (n=2) \\ -34 = 27a+9b+3c+d & (n=3) \end{cases}$$

$$\Rightarrow d = 0$$

$$\begin{cases} a+b+c = -9 \\ 8a+4b+2c = -21 \\ 27a+9b+3c = -34 \end{cases}$$

$$\Rightarrow c = -a-b-9$$

$$\Rightarrow \begin{cases} 8a+4b+2(-a-b-5) = -2 \\ 24a+9b+3(-a-b-5) = -34 \end{cases}$$

$$\begin{cases} 6a+2b = -3 \\ 24a+6b = -7 \end{cases}$$

$$\Rightarrow \frac{2b}{2} = \frac{-3-6a}{2}$$

$$b = \frac{-3-6a}{2}$$

$$\Rightarrow 24a + \cancel{6}^{3}\left(\frac{-3-6a}{\cancel{2}}\right) = -7$$

$$24a - 9 - 18a = -7$$

$$6a = 2$$

$$a = \frac{1}{3}$$

$$\Rightarrow b = \frac{-3-\cancel{6}^{2}\left(\frac{1}{\cancel{3}}\right)}{} = -\frac{5}{}$$

$$\Rightarrow c = -\left(\frac{1}{3}\right) - \left(-\frac{5}{2}\right) - 9$$

$$c = \frac{-2+15-54}{6} = -\frac{41}{6}$$

Concludendo

$$\sum_{i=1}^{n} (k^2 - 6k - 4) = \frac{n^3}{3} - \frac{5}{2}n^2 - \frac{41}{6}n$$