Trovare una formula chiusa per

$$\sum_{i=0}^{m} \sum_{j=1}^{m} (i+j)$$

Stiamo sommando tutti i numeri della seguente tabella

$i \backslash j$	1	2	3	...	$m-2$	$m-1$	m
0	1	2	3	...	$m-2$	$m-1$	m
1	2	3	4	...	$m-1$	m	$m+1$
2	3	4	5	...	m	$m+1$	$m+2$
3	4	5	6	...	$m+1$	$m+2$	$m+3$
$\vdots$	...	...	...	...	...	...	...
$m-2$	$m-1$	m	$m+1$	...	$m+m-4$	$m+m-3$	$m+m-2$
$m-1$	m	$m+1$	$m+2$	...	$m+m-3$	$m+m-2$	$m+m-1$
m	$m+1$	$m+2$	$m+3$	...	$m+m-2$	$m+m-1$	$m+m$

Quindi la somma è

$$= \sum_{i=0}^{n} \left((1+2+3 \dots + n) + i \cdot n \right) =$$

$$= \sum_{i=0}^{n} \left(\binom{n+1}{n} + i \cdot n \right) =$$

$$= \sum_{i=0}^{n} \binom{n+1}{n} + \sum_{i=0}^{n} (i \cdot n) =$$

$$= \underbrace{\binom{n+1}{2} + \dots + \binom{n+1}{2}}_{\substack{n+1 \\ \text{VOLTE}}} + n \cdot \sum_{i=0}^{n} i =$$

$$= (n+1) \cdot \binom{n+1}{2} + n \cdot \binom{n+1}{2}$$

CONCLUDENDO

$$\sum_{i=0}^{n} \sum_{j=1}^{n} (i+j) = (2n+1) \binom{n+1}{2}$$

TROVARE UNA FORMULA CHIUSA PER

$n \quad n-i$

$$\sum_{i=0} \sum_{j=0} (n-j)$$

Stiamo sommando tutti i numeri della seguente tabella

$i \backslash j$	0	1	2	....	$n-2$	$n-1$	n
0	n	$n-1$	$n-2$	...	..	..	0
1	n	$n-1$	$n-2$	...	..	1	//
2	n	$n-1$	$n-2$	...	2	//	//
3	n	$n-1$	$n-2$	...	//	//	//
...	...	...	...	...	//	//	//
$n-2$	n	$n-1$	$n-2$	...	//	//	//
$n-1$	n	$n-1$	//	...	//	//	//
n	n	//	//	...	//	//	//

Quindi la somma è

$$= (n+1)n + (n-1)\cdot n + (n-2)(n-1) + (n-2)(n-3) + \dots$$

$$\dots + 3\cdot 2 + 2\cdot 1 + 1\cdot 0$$

$$= \sum_{i=1}^{n+1} i(i-1) = \sum_{i=1}^{n+1} i^2 - i$$