3) c) $\displaystyle\int \frac{x \cdot e^{x^2}}{9+e^{2x^2}}\,dx =$

$t = e^{x^2} \rightarrow dt = e^{x^2} \cdot 2x\,dx$

$\displaystyle = \int \frac{1}{9+t^2} \cdot \frac{dt}{2} = \frac{1}{2}\int \frac{dt}{3^2+t^2} =$

$\displaystyle = \frac{\arctan\left(\frac{t}{3}\right)}{6} + c = \frac{\arctan\left(\frac{e^{x^2}}{3}\right)}{6} + c$

d) $\displaystyle\int \frac{\log(\log(x))}{x}\,dx =$

$t = \log(x) \rightarrow dt = \frac{1}{x}\,dx$

$\displaystyle = \int \log(t)\,dt = t\log(t) - \int t \cdot d(\log(t)) =$

$$= t \cdot \log(t) - \int \cancel{t} \cdot \frac{1}{\cancel{t}}\, dt =$$

$$= t \cdot \lg(t) - \int dt = t \cdot \lg(t) - t + c =$$

$$= \lg(x) \cdot \lg(\lg(x)) - \log(x) + c$$

e) $\displaystyle\int x \log(1+x)\, dx =$

$$= \int \log(1+x)\, d\left(\frac{x^2}{2}\right) =$$

$$= \log(x) \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot d(\lg(1+x))\, dx =$$

$$= \lg(x) \cdot \frac{x^2}{2} - \frac{1}{2} \int \frac{x^2}{1+x}\, dx =$$

$$= \lg(x) \cdot \frac{x^2}{2} - \frac{1}{2} \int \frac{x^2+1-1}{1+x}\, dx =$$

$$= \log(x)\cdot\frac{x^2}{2} - \frac{1}{2}\int\frac{(x^2-1)}{1+x}\,dx - \frac{1}{2}\int\frac{1}{1+x}\,dx =$$

$$= \log(x)\cdot\frac{x^2}{2} - \frac{1}{2}\int\frac{\cancel{(x+1)}(x-1)}{\cancel{1+x}}\,dx - \frac{1}{2}\cdot\log|1+x| + c$$

$$= \log(x)\cdot\frac{x^2}{2} - \frac{1}{2}\cdot\left(\frac{x^2}{2} - x\right) - \frac{\log|1+x|}{2} + c$$

$$= \log(x)\cdot\frac{x^2}{2} - \frac{x^2}{4} - \frac{x}{2} - \log\frac{|1+x|}{2} + c$$

f) $$\int\frac{e^{\tan(x)}(\sin(x)+\cos(x))}{\cos^3(x)}\,dx =$$

$$= \int\frac{e^{\tan(x)}\left(\cancelto{\tan(x)}{\sin}(x) + \cancelto{1}{\cos}(x)\right)}{\cos^2 x}\cdot\frac{1}{\cancel{\cos}(x)}\,dx =$$

$$= \int\frac{e^{\tan(x)}(\tan(x)+1)}{\cos^2(x)}\,dx =$$

$$(\tan(x))' =$$

$t = \tan(x)$, $\boxed{dt = \frac{dx}{\cos^2(x)}}$ $= \frac{1}{\cos^2 x}$

$$= \int e^t (t+1)\, dt = \int (t+1) \cdot d(e^t) =$$

$$= e^t \cdot (t+1) - \int e^t \cdot dt =$$

$$= e^t \cdot (t+1) - e^t + c =$$

$$= e^t (t+1-1) + c = e^t \cdot t + c$$

$$= e^{\tan(x)} \cdot \tan(x) + c$$

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4) d) $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2(x) \cdot \cos^3(x)\, dx =$

$$\int_0^{\frac{\pi}{2}} \sin^2(x) \cdot (1 - \sin^2(x)) \cdot \cos(x)\, dx =$$

$$t = \sin(x) \quad , \quad dt = \cos(x)\, dx$$

$$1 \leftarrow \sin\left(\tfrac{\pi}{2}\right)$$

$$\int_0^1 t^2 \cdot (1 - t^2) \cdot dt =$$

$$= \int_0^1 t^2 - t^4 \, dt = \left[\frac{t^3}{3} - \frac{t^5}{5} \right]_0^1 =$$

$$= \left(\frac{(1)^3}{3} - \frac{(1)^5}{5} \right) - \left(\frac{(0)^3}{3} - \frac{(0)^5}{5} \right) = \frac{2}{15}$$

b)

$$\int_0^{\frac{1}{2}} \frac{\sqrt{1-x} + 6}{\sqrt{1-x^2}} \, dx =$$

$$= \int_0^{\frac{1}{2}} \left(\frac{\sqrt{1-x}}{\sqrt{1-x^2}} + \frac{6}{\sqrt{1-x^2}} \right) dx =$$

$$\int_0^{\frac{1}{2}} \quad \int^{\frac{1}{2}}$$

$$= \int_0 \frac{\cancel{\sqrt{1-x}}}{\sqrt{(\cancel{1-x})(1+x)}}\, dx + 6 \int_0 \frac{1}{\sqrt{1-x^2}}\, dx =$$

$$= \int_0^{\frac{1}{2}} (1+x)^{-\frac{1}{2}}\, dx + 6\left[\arcsin(x)\right]_0^{\frac{1}{2}} =$$

$$= \left[\frac{(1+x)^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^{\frac{1}{2}} + 6\cdot\left(\arcsin\left(\tfrac{1}{2}\right) - \arcsin(0)\right) =$$

$$= 2\left(\sqrt{1+\tfrac{1}{2}} - 1\right) + 6\cdot\left(\frac{\pi}{6} - 0\right) =$$

$$= 2\cdot\sqrt{\tfrac{3}{2}} - 2 + \pi =$$

$$= \sqrt{6} - 2 + \pi$$

c) $\displaystyle\int_1^e \frac{1}{x(3+\log))^2}\, dx$

$t = \log(x)$, $dt = \frac{1}{x}\, dx$

$1 \to \ell_\gamma(e)$

$$\int_0^1 \frac{1}{(3+t)^2}\,dt = \int_0^1 (3+t)^{-2}\,dt =$$

$0 \to \ell_\gamma(1)$

$$= \left[-(3+t)^{-1}\right]_0^1 = -\left(\frac{1}{3+1}\right) + \left(\frac{1}{3}\right) =$$

$$= -\frac{1}{4} + \frac{1}{3} = \frac{1}{12}$$

d) $$\int_0^1 (1+2x^2)e^{2x}\,dx =$$

$$= \int_0^1 \left(e^{2x} + 2x^2 e^{2x}\right)dx =$$

$$= \int_0^1 e^{2x}\,dx + 2\int_0^1 x^2 e^{2x}\,dx =$$

$$= \left[\frac{e^{2x}}{2}\right]_0 + 2\left(\left[x^2 \cdot \frac{e^{2x}}{2}\right]_0 - \int_0 \frac{e^{2x}}{\cancel{2}} \cdot \cancel{2}x \, dx\right) =$$

$$= \left[\frac{e^{2x}}{2}\right]_0^1 + 2\left(\left[\frac{e^{2x}}{2} \cdot x^2\right]_0^1 - \left(\left[x \cdot \frac{e^{2x}}{2}\right]_0^1 - \int_0^1 \frac{e^{2x}}{2} \, dx\right)\right) =$$

$$= \left[\frac{e^{2x}}{2}\right]_0^1 + 2\left(\left[\frac{e^{2x}}{2} \cdot x^2\right]_0^1 - \left(\left[x \cdot \frac{e^{2x}}{2}\right]_0^1 - \int_0^1 \frac{e^{2x}}{2} \, dx\right)\right) =$$

$$= \left[\frac{e^{2x}}{2}\right]_0^1 + \cancel{2}\left(\left[\frac{e^{2x}}{\cancel{2}} \cdot x^2\right]_0^1 - \left(\left[x \cdot \frac{e^{2x}}{\cancel{2}}\right]_0^1 - \left[\frac{e^{2x}}{4}\right]_0^1\right)\right) =$$

$$= \left[\frac{e^{2x}}{2}\right]_0^1 + \left[e^{2x} \cdot x^2\right]_0^1 - \left[x \cdot e^{2x}\right]_0^1 + \left[\frac{e^{2x}}{2}\right]_0^1 =$$

$$= \cancel{2}\left[\frac{e^{2x}}{\cancel{2}}\right]_0^1 + \left[e^{2x} \cdot x^2\right]_0^1 - \left[x \cdot e^{2x}\right]_0^1 =$$

$$= \cancel{e^2} - 1 + e^2 - \cancel{e^2} = e^2 - 1$$

e) $\int^{\pi}$

e) $\int_{-\pi} |x| \left(\cos(x) - 2\sin(x)\right) dx =$

PARI DISPARI

$$= \int_{-\pi}^{\pi} |x| \cdot \cos(x)\, dx - 2\int_{-\pi}^{\pi} |x| \sin(x)\, dx =$$

$$2 \cdot \int_{0}^{\pi} x \cdot \cos(x)\, dx - \cancel{2 \cdot 0} =$$

$$= 2\left(\left[x \sin(x) \right]_0^{\pi} - \int_0^{\pi} \sin(x)\, dx \right) =$$

$$= 2 \cdot \left(\left[x \sin(x) \right]_0^{\pi} + \left[\cos(x) \right]_0^{\pi} \right) =$$

$$= 2 \cdot \pi \cdot 0 + 2 \cdot (-1 - 1) = -4$$

f) $\int_{0}^{\pi} \left(\sin^2(x) + \cos^2(x)\right) dx =$

DISPARI NELL' INTERVALLO $[0, \pi]$

$$\int_0^{\pi} \sin^2(x)\,dx + \int_0^{\pi} \cos^3(x)\,dx =$$

$$= \int_0^{\pi} \sin^2(x)\,dx + 0 =$$

$$= \int_0^{\pi} \frac{(1 - \cos(2x))}{2}\,dx =$$

$$= \frac{1}{2} \left(\int_0^{\pi} 1\,dx - \int_0^{\pi} \cos(2x)\,dx \right) =$$

$$= \frac{1}{2} \left(\left[x \right]_0^{\pi} - \left[\frac{\sin(2x)}{2} \right]_0^{\pi} \right) =$$

$$= \frac{1}{2} \left(\pi - \cancel{0} + \cancel{0} \right) = \frac{\pi}{2}$$

g) $\int_0^{2} \log(4 + x^2)\,dx =$

INTEGRAZIONE PER PARTI

2

$$= \left[x \log(4+x^2)\right]_0^2 - \int_0^2 x \cdot \frac{2x}{4+x^2}\,dx =$$

$$= 2\cdot\log(8) - 2\int_0^2 \frac{x^2}{4+x^2}\,dx =$$

$$= 2\log(8) - 2\int_0^2 \frac{x^2+4-4}{4+x^2}\,dx =$$

$$= 2\log(8) - 2\int_0^2 \left(\frac{\cancel{x^2+4}}{\cancel{x^2+4}} - \frac{4}{4+x^2}\right)dx =$$

$$= 2\log(8) - 2\left(\int_0^2 dx - 4\int_0^2 \frac{1}{4+x^2}\,dx\right) =$$

$$= 2\log(8) - 2\left(\left[x\right]_0^2 - \cancel{4}\int_0^2 \frac{1}{\cancel{4}\left(1+\frac{x^2}{4}\right)}\,dx =$$

$$u = \frac{x}{2} \Rightarrow du = \frac{1}{2}\,dx \Rightarrow dx = 2\,du$$

$$2\left(\left[x\right]^2 \quad \int^2 \frac{1}{\;}\,du\right) =$$

$$= 2\log(8) - 2\left(\left[x\right]_0 - 2\int_0 \frac{}{x^2+1}\,dx\right) =$$

$$= 2\log(8) - 2\left(\left[x\right]_0^2 - 2\left[\arctan(x)\right]_0^2\right) =$$

$$= 2\log(8) - 2\left(\left[x\right]_0^2 - 2\left[\arctan\left(\frac{x}{2}\right)\right]_0^2\right) =$$

$$= 2\log(8) - 2\left(2 - \cancel{2}\left(\frac{\pi}{\cancel{4}}\right)\right) =$$

$$= 2\log(8) - 4 - \pi$$

h) $$\int_0^{\frac{\pi}{6}} \frac{1}{\cos(x)}\,dx =$$

$\cos^2(x) + \sin^2(x) = 1$

$\cos^2(x) = 1 - \sin^2(x)$

$\frac{\cos^2(x)}{\cos(x)} = \frac{1-\sin^2(x)}{\cos(x)}$

$$= \int_0^{\frac{\pi}{6}} \left(\frac{1}{1-\sin^2(x)} \cdot \frac{\cos(x)}{\sin(x)}\right)dx =$$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos(x)}{1-\sin^2(x)}\,dx$$

$t = \sin(x)$, $dt = \cos(x)\,dx$

$$\int_0^{\frac{1}{2}} \frac{1}{1-t^2}\,dt =$$

$$= \int_0^{\frac{1}{2}} \left(\frac{A}{(1-t)} + \frac{B}{1+t} \right) dt$$

$$1 = A(1+t) + B(1-t) = A + At + B - Bt$$

$$= A + B + (A-B)t$$

$$\begin{cases} A + B = 1 \\ A - B = 0 \end{cases} \rightarrow \begin{matrix} A = \frac{1}{2} \\ B = \frac{1}{2} \end{matrix}$$

quindi

$$\int_0^{\frac{1}{2}} \frac{1}{1-t^2}\,dt = \frac{1}{2} \int_0^{\frac{1}{2}} \left(\frac{1}{1-t} + \frac{1}{1+t} \right) dt =$$

$$= \frac{1}{2}\left(\int_0^{\frac{1}{2}} \frac{1}{1-t}\,dt + \int_0^{\frac{1}{2}} \frac{1}{1+t}\,dt\right) =$$

$$= \frac{1}{2}\left(-\int_0^{\frac{1}{2}} \frac{-1}{1-t}\,dx + \left[\log|1+t|\,dt\right]_0^{\frac{1}{2}}\right) =$$

$$= \frac{1}{2}\left(-\left[\log|1-t|\right]_0^{\frac{1}{2}} + \left[\log|1+t|\right]_0^{\frac{1}{2}}\right) =$$

$$= \frac{1}{2}\left(-\log\left(\frac{1}{2}\right) + \log\left(\frac{3}{2}\right) + 0\right) =$$

$$= \frac{1}{2}\left(\log\left(\frac{\frac{3}{2}}{\frac{1}{2}}\right)\right) =$$

$$= \frac{\log\left(\frac{3 \cdot 2}{2}\right)}{2} = \frac{\log(3)}{2}$$