$$f(x) = \frac{1}{\sin(2x)}$$

$n = 4$

$x_0 = \frac{\pi}{4}$

Ponendo $t = x - \frac{\pi}{4}$ si ha

$$f(x) = \frac{1}{\sin\left(2\left(t + \frac{\pi}{4}\right)\right)} = \frac{1}{\sin\left(2t + \frac{\pi}{2}\right)} = \frac{1}{\cos(2t)}$$

$$\frac{1}{\cos(2t)} = \frac{1}{1 - \frac{(2t)^2}{2!} + \frac{(2t)^4}{4!} + o(t^5)} =$$

$$= \frac{1}{1 - \frac{\cancel{4}^{2} t^2}{\cancel{2}} + \frac{\cancel{16}^{2} t^4}{\cancel{24}_{3}} + o(t^5)} =$$

$$= \left(1 - 2t^2 + \frac{2}{3}t^4 + o(t^5)\right)^{-1}$$

$$(1+s)^{-1} = 1 - s + s^2 + o(s^2)$$

$$= 1 - \left(-2t^2 + \frac{2}{3}t^4 + o(t^5)\right) + \left(-2t^2 + \frac{2}{3}t^4 + o(t^5)\right)^2 + o(t^4) =$$

$$= 1 + 2t^2 - \frac{2}{3}t^4 + 4t^4 + o(t^4) =$$

$$= 1 + 2t^2 + \frac{-2+12}{3}t^4 + o(t^4) =$$

$$= 1 + 2t^2 + \frac{10}{3}t^4 + o(t^4)$$

QUINDI IL POLINOMIO DI TAYLOR DI ORDINE $n=4$ IN CENTRO $x_0 = \frac{\pi}{4}$ DI f È

$$T_4(x) = 1 + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{10}{3}\left(x - \frac{\pi}{4}\right)^4$$

$$f(x) = (\cos(x))^x \qquad n = 6, \quad x_0 = 0$$

PER $x \to 0$

$$x \lg(\cos(x))$$

$$f(x) = e^{\ln\left(1-\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)\right)} =$$

$$= e^{x \ln\left(1+\left(-\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)\right)\right)} =$$

$$= e^{x\left(\left(-\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)\right)-\frac{1}{2}\left(-\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)\right)^2\right)} =$$

$$= e^{x\left(-\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)-\frac{1}{2}\left(\frac{x^4}{4}+o(x^4)\right)\right)}$$

$$= e^{x\left(-\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)-\frac{x^4}{8}+o(x^4)\right)} =$$

$$= e^{-\frac{x^3}{2}+\frac{x^5}{24}-\frac{x^5}{8}+o(x^5)} = e^{-\frac{x^3}{2}+\frac{(5-3)x^5}{24}+o(x^5)}$$

$$= e^{-\frac{x^3}{2}+\frac{1}{12}x^5+o(x^5)} =$$

$$= 1+\left(-\frac{x^3}{2}+\frac{1}{12}x^5+o(x^5)\right)+\frac{1}{2}\left(-\frac{x^3}{2}+\frac{1}{12}x^5+o(x^5)\right)^2 =$$

$$= 1 - \frac{x^3}{2} + \frac{1}{12}x^5 + o(x^5) + \frac{1}{2}\left(\frac{x^6}{4} + o(x^6)\right) =$$

$$= 1 - \frac{x^3}{2} + \frac{1}{12}x^5 + \frac{1}{8}x^6 + o(x^6)$$

Quindi il polinomio di TAYLOR di ordine $n=6$ in centro $x_0 = 0$ di f è

$$T_6(x) = 1 - \frac{x^3}{2} + \frac{x^5}{12} + \frac{x^6}{8}$$

$$f(x) = \frac{1}{1+\lg(x)} - \frac{4}{x^{2x} + \sin(x^2 - x)} \qquad n = 2 \qquad x_0 = 1$$

Pongo $t = x - 1$

$$f(x) = \frac{1}{1+\lg(t+1)} - \frac{4}{e^{(2t+1)\lg(t+1)} + \sin((t+1)^2 - t - 1)} =$$

$$= \frac{1}{1+4\lg(t+1)} - \frac{4}{e^{(2t+1)\lg(t+1)} + \sin(t^2+1-2t+t+1)} =$$

$$= \frac{1}{1+4\lg(t+1)} - \frac{4}{e^{(2t+1)\lg(t+1)} + \sin(t^2+t)}$$

Or,

$$\frac{1}{1+4\lg(t+1)} = \frac{1}{1+4\left(t - \frac{t^2}{2!} + o(t^2)\right)} =$$

$$= \frac{1}{1+4t-2t^2+o(t^2)} = \left(1+4t-2t^2+o(t^2)\right)^{-1} =$$

$$= 1 - \left(4t - 2t^2 + o(t^2)\right) + \left(4t + o(t^2)\right)^2 =$$

$$= 1 - 4t + 2t^2 + o(t^2) + 16t^2 + o(t^2) =$$

$$= 1 - 4t + 18t^2 + o(t^2)$$

$\bar{t}$

$$\frac{4}{e^{(2t+2)\log(t+1)} + \sin(t^2+t)} =$$

$$= \frac{4}{e^{(2t+2)\left(t - \frac{t^2}{2} + o(t^2)\right)} + t + t^2 + o(t^2)} =$$

$$= \frac{4}{e^{2t^2+2t+o(t^2)} + t + t^2 + o(t^2)} =$$

$$= \frac{4}{1 + \left(2t^2+2t+o(t^2)\right) + \frac{1}{2}\left(2t+o(t^2)\right)^2 + t + t^2 + o(t^2)} =$$

$$= \frac{4}{1+2t^2+2t+2t^2+t+t^2+o(t^2)} =$$

$$= \frac{4}{1+3t+4t^2+o(t^2)} =$$

$$= 4\cdot\left(1+3t+4t^2+o(t^2)\right)^{-1}$$

$$= 4\cdot\left(1-\left(3t+4t^2+o(t^2)\right)+\left(3t+o(t)\right)^2+o(t^2)\right)$$

$$= 4\left(1-\left(3t+4t^2+o(t^2)\right)+9t^2+o(t^2)\right) =$$

$$= 4\left(1 - 3t + 5t^2 + o(t^2)\right) =$$

$$= 4 - 12t + 20t^2 + o(t^2)$$

FACCIO LA DIFFERENZA

$$f(x) = 1 - 4t + 18t^2 + o(t^2) - \left(4 - 12t + 20t^2 + o(t^2)\right) =$$

$$= 1 - 4t + 18t^2 - 4 + 12t - 20t^2 + o(t^2) =$$

$$= -3 + 8t - 2t^2 + o(t^2)$$

QUINDI IL POLINOMIO DI TAYLOR DI GRADO $n=2$ CENTRATO $x_0 = 1$ DI f È

$$T_2(x) = -3 + 8(x-1) - 2(x-1)^2$$

$$f(x) = \sqrt{\cos(2x + x^2) + \frac{x \sin(x)}{1+x^2}} \qquad n = 4, \quad x_0 = 0$$

Per $x \to 0$

$$\cos(2x+x^2) = 1 - \frac{(2x+x^2)^2}{2} + \frac{(2x)^4}{24} + o(x^4) =$$

$$= 1 - \frac{4x^2 + x^4 + 4x^3}{2} + \frac{16x^4}{24} + o(x^4) =$$

$$= 1 - 2x^2 - \frac{x^4}{2} - 2x^3 + \frac{2}{3}x^4 + o(x^4) =$$

$$= 1 - 2x^2 - 2x^3 + \frac{x^4}{6} + o(x^4)$$

6

$$\frac{x \sin(x)}{1+x^2} = \frac{x\left(x - \frac{x^3}{6} + o(x^3)\right)}{1+x^2} = \frac{x^2 - \frac{x^4}{6} + o(x^4)}{1+x^2} =$$

$$= \left(x^2 - \frac{x^4}{6} + o(x^4)\right)(1+x^2)^{-1} =$$

$$= \left(x^2 - \frac{x^4}{6} + o(x^4)\right)\left(1 - x^2 + o(x^3)\right) =$$

$$= x^2 - x^4 - \frac{x^4}{6} + o(x^4) = x^2 + \frac{-6-1}{6}x^4 + o(x^4) =$$

$$= x^2 - \frac{7x^4}{6} + o(x^4)$$

$$\cos(2x+x^2) + \frac{x \sin(x)}{1+x^2} =$$

$$= 1 - 2x^2 - 2x^3 + \frac{x^4}{6} + o(x^4) + x^2 - \frac{7x^4}{6} + o(x^4) =$$

$$= 1 - x^2 - 2x^3 - x^4 + o(x^4)$$

ALLORA

$$f(x) = \sqrt{1 + (-x^2 - 2x^3 - x^4) + o(x^4)} =$$

$$= 1 + \frac{1}{2}\left(-x^2 - 2x^3 - x^4 + o(x^4)\right) - \frac{1}{8}\left(-x^2 + o(x^4)\right)^2 =$$

$$= 1 - \frac{x^2}{2} - x^3 - \frac{x^4}{2} + o(x^4) - \frac{x^4}{8} + o(x^4) =$$

$$= 1 - \frac{x^2}{2} - x^3 + \frac{-4x^4 - x^4}{8} + o(x^4) =$$

$$= 1 - \frac{x^2}{2} - x^3 - \frac{5x^4}{8} + o(x^4)$$

Così il polinomio di Taylor di ordine $n=4$ è di centro $x=0$ di f è

$$T_4(x) = 1 - \frac{x^2}{2} - x^3 - \frac{5x^4}{8}.$$

b)

$$f'(x) = \left(1 - \frac{x^2}{2} - x^3 - \frac{5x^4}{8}\right)' =$$

$$= -x - 3x^2 - \frac{\cancel{20}^{5}\, x^3}{\cancel{8}_{2}} = -x - 3x^2 - \frac{5x^3}{2}$$

ora

$$f'(0) = -0 - 3(0) - \frac{5(0)}{2} = 0$$

Quindi $x=0$ è un punto stazionario

ora

$$f''(x) = \left(-x - 3x^2 - \frac{5x^3}{2}\right)' = -1 - 6x - \frac{15x^2}{2}$$

$f''(0) = -1 < 0$

QUINDI f È STRETTAMENTE CONCAVA

NELL'INTORNO $x = 0$. QUINDI

$x = 0$ È UN PUNTO DI MASSIMO RELATIVO