$$\int \frac{x e^{x^2}}{9+e^{2x^2}}\, dx$$

si $t = e^{x^2}$, $dt = 2e^{x^2}\, dx$

$$\frac{1}{2}\int \frac{1}{9+t^2}\, dt = \frac{1}{2}\,\frac{\arctan\left(\frac{t}{3}\right)}{6} + c =$$

$$= \frac{1}{2}\,\frac{\arctan\left(\frac{e^{x^2}}{3}\right)}{6} + c$$

$$\int \frac{\log(\log(x))}{x}\, dx$$

sia $t = \log(x)$ $\Rightarrow$ $dt = \frac{1}{x}\, dx$

$e^t = x$ $\qquad$ $t\, dt = dx$

$$\int \log(t) = d(t) =$$

integrazione per parti

$$t \cdot \log(t) - \int \frac{t}{t}\, dt = t \cdot \log(t) - t + c =$$

$$= \log(x) \cdot \log(\log(x)) - \log(x) + c$$

$$\int x \log(1+x)\, dx =$$

$$= \int \log(1+x)\, d\left(\frac{x^2}{2}\right) =$$

$$= \frac{x^2}{2} \log(1+x) - \int \frac{x^2}{2} \cdot \frac{1}{1+x}\, dx =$$

$$= \frac{x^2}{2} \log(1+x) - \frac{1}{2} \int \frac{x^2}{1+x}\, dx$$

$$= \frac{x^2}{2} \log(1+x) - \frac{1}{2} \int \frac{x^2+1-1}{1+x}\, dx =$$

$$= \frac{x^2}{2}\log(1+x) - \frac{1}{2}\int \frac{x-1}{1+x}\,dx + \frac{1}{2}\int -\frac{1}{1+x}\,dx$$

$$= \frac{x^2}{2}\log(1+x) - \frac{1}{2}\int \frac{(x-1)\cancel{(x+1)}}{\cancel{1+x}}\,dx - \frac{1}{2}\ln(1+x) =$$

$$= \frac{x^2}{2}\log(1+x) - \frac{1}{2}\int x\,dx + \frac{1}{2}\int dx - \frac{1}{2}\ln(1+x) =$$

$$= \frac{x^2}{2}\log(1+x) - \frac{1}{2}\cdot\frac{x^2}{2} + \frac{1}{2}x - \frac{1}{2}\ln(1+x) + c =$$

$$= \frac{x^2}{2}\log(1+x) - \frac{x^2}{4} + \frac{x}{2} - \frac{1}{2}\ln(1+x) + c$$

$$\int \frac{e^{\tan(x)}(\sin(x)+\cos(x))}{\cos^3(x)}\,dx$$

Sia $t = \tan(x)$, $dt = \frac{dx}{\cos^2(x)}$

Allora

$$\int \frac{e^{\tan(x)}(\sin(x)+\cos(x))}{\cos^3} =$$

$$\int \frac{\quad}{\cos^2(x)}$$

$$= \int \frac{e^{\tan(x)} \left(\frac{\sin(x) + \cos(x)}{\cos(x)} \right)}{\cos^2(x)} \, dx =$$

$$= \int \frac{e^{\tan(x)} \left(\tan(x) + 1 \right)}{\cos^2(x)} \, dx =$$

$$= \int e^{t} (t+1) \, dt =$$

INTEGRAZIONE PER PARTI

$$e^{t}(t+1) - \int \frac{e^{t}}{(t+1)'} \, dt =$$

$$= e^{t}(t+1) - \int e^{t} \, dt =$$

$$= e^t(t+1) - e^t + c =$$

$$= e^t(t+\cancel{1}-\cancel{1}) + c = e^{\tan(x)}\tan(x) + c$$

$$\int_0^{\frac{\pi}{2}} \sin^2(x)\cos^3(x)\,dx =$$

$$= \int_0^{\frac{\pi}{2}} \sin^2(x)\cos^2(x)\cos(x)\,dx =$$

$$= \int_0^{\frac{\pi}{2}} \sin^2(x)\left(1-\sin^2(x)\right)\cos(x) =$$

Si $t = \sin(x)$, $dt = \cos(x)\,dt$

$$= \int_0^1 t^2(1-t^2)\,dt =$$

$$= \int_0^1 t^2 - t^4\,dt = \left[\frac{t^3}{3} - \frac{t^5}{5}\right]_0^1 =$$

$$\int_0 \left(= \left(\frac{1}{3} - \frac{1}{5} \right) \right)_0 =$$

$$= \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

$$\int_0^{\frac{1}{2}} \frac{\sqrt{1-x} + 6}{\sqrt{1-x^2}} \, dx =$$

$$= \int_0^{\frac{1}{2}} \left(\frac{\sqrt{1-x}}{\sqrt{1-x^2}} + \frac{6}{\sqrt{1-x^2}} \right) dx =$$

$$= \int_0^{\frac{1}{2}} \left(\frac{\cancel{\sqrt{1-x}}}{\cancel{\sqrt{1-x}} \cdot \sqrt{1+x}} + \frac{6}{\sqrt{1-x^2}} \right) dx =$$

$$= \int_0^{\frac{1}{2}} \left((x+1)^{-\frac{1}{2}} + \frac{6}{\sqrt{1-x^2}} \right) dx =$$

$$= \left[\frac{(x+1)^{\frac{1}{2}}}{\frac{1}{2}} + 6 \arcsin(x) \right]_0^{\frac{1}{2}} =$$

$$= \left[2\sqrt{x+1} + 6 \arcsin(x) \right]_0^{\frac{1}{2}} =$$

$$= \left(2\sqrt{\frac{3}{2}} + \cancel{6} \frac{\pi}{\cancel{6}} \right) - \left(2 + 6 \cdot 0 \right) =$$

$$= 2\sqrt{\frac{3}{2}} + \pi - 2$$

$$\int_1^e \frac{1}{x(3+\log(x))^2} \, dx$$

sia $t = \log(x)$, $dt = \frac{1}{x} \, dx$

$$\int_0^1 \frac{1}{(3+t)^2} \, dt =$$

$$= \int_0^1 (3+t)^{-2} \, dt =$$

$s = 3+t \quad , \quad ds = dt$

$$= \int_3^4 s^{-2}\, ds =$$

$$= \left[-\frac{1}{s} \right]_3^4 = -\frac{1}{4} + \frac{1}{3} = \frac{-3+4}{12} = \frac{1}{12}$$

$$\int_0^1 (1+2x^2)\, e^{2x}\, dx$$

INTEGRANDO PER PARTI

$$\int_0^1 (1+2x^2)\, e^{2x}\, dx =$$

$$= \int_0^1 (1+2x^2)\, d\left(\frac{e^{2x}}{2}\right) =$$

$$= \left[(1+2x^2)\,\frac{e^{2x}}{2}\right]_0 - \frac{1}{2}\int_0 e^{2x}\, d(1+2x^2) =$$

אינטגרציה פי חלקים

$$= 3\cdot\frac{e^2}{2} - 1\cdot\frac{1}{2} - \frac{1}{\cancel{2}}\int_0^1 \cancel{4}x\cdot d\left(\frac{e^{2x}}{2}\right) =$$

$$= \frac{3e^2}{2} - \frac{1}{2} - 2\cdot\left[x\cdot\frac{e^{2x}}{2}\right]_0^1 + \int_0^1 e^{2x}\, dx =$$

$$= \frac{3e^2}{2} - \frac{1}{2} - \cancel{2}\cdot\frac{e^2}{\cancel{2}} + \left[\frac{e^{2x}}{2}\right]_0^1 =$$

$$= \frac{3e^2}{2} - \frac{1}{2} - e^2 + \frac{e^2}{2} - \frac{1}{2} =$$

$$= \frac{3e^2 - 1 - 2e^2 + e^2 - 1}{2} = \frac{2e^2 - 2}{2} = e^2 - 1$$

$$\int^{\pi} |x|\left(-\cos(x) - 2\sin(x)\right)\cdot dx =$$

$$\int_{-\pi}$$

$$= \underbrace{\int_{-\pi}^{\pi} |x| \cos(x)\,dx}_{\text{PARI}} - 2\underbrace{\int_{-\pi}^{\pi} |x| \sin(x)\,dx}_{\text{DISPARI}} =$$

$$= 2\int_{0}^{\pi} x \cos(x)\,dx - 2 \cdot 0 =$$

INTEGRAZIONE PER PARTI

$$= 2\int_{0}^{\pi} x \, D(\sin(x)) =$$

$$= 2\left[x \cdot \sin(x)\right]_{0}^{\pi} - 2\int_{0}^{\pi} \sin(x)\,dx =$$

$$= 2\left(\pi \cdot 0 - 0 \cdot 0\right) + 2\left[\cos(x)\right]_{0}^{\pi} =$$

$$= 2\left(-1-(+1)\right) = 2\cdot(-2) = -4$$

$$\int_0^2 \log(4+x^2)\,dx$$

INTEGRO PER PARTI

$$\int_0^2 \log(4+x^2)\,dx = \left[x\log(4+x^2)\right]_0^2 - \int_0^2 x\,d\left(\log(4+x^2)\right) =$$

$$= 2\log(8) - \int_0^2 \frac{x}{4+x^2}\cdot 2x\,dx =$$

$$= 2\log(8) - 2\int_0^2 \frac{x^2}{4+x^2}\,dx =$$

$$= 2\log(8) - 2\int_0^2 \frac{x^2+4-4}{4+x^2}\,dx =$$

$$\log(8) - \int_0^2 \cancel{\frac{x^2}{4}} \quad \int_0^2 4$$

$$= 2\log(8) - 2\int_0 \frac{x+2}{x^2+4}\,dx + 2\int_0 \frac{}{x^2+4}\,dx =$$

$$= 2\log(8) - 2\left[x\right]_0^2 + 2\left[2\arctan\left(\frac{x}{2}\right)\right]_0^2 =$$

$$= 2\log(8) - 4 + \cancel{2}\left(2\cdot\frac{\pi}{\cancel{4}\,\cancel{2}} - 2\cdot 0\right) =$$

$$= 2\log(8) - 4 + \pi = 2\log(2^3) - 4 + \pi =$$

$$= 6\log(2) - 4 + \pi$$

$$\int_0^{\frac{\pi}{6}} \frac{1}{\cos(x)}\,dx$$

$$\int_0^{\frac{\pi}{6}} \frac{1}{\cos(x)}\,dx = \int_0^{\frac{\pi}{6}} \frac{1}{\cos(x)}\cdot\frac{\cos(x)}{\cos(x)}\,dx =$$

$$= \int_0^{\frac{\pi}{6}} \frac{-\cos(x)}{\cos^2(x)}\, dx = \int_0^{\frac{\pi}{6}} \frac{\cos(x)}{1-\sin^2(x)}\, dx =$$

sia $t = \sin(x)$, $dt = \cos(x)\, dx$

quindi

$$= \int_0^{\frac{1}{2}} \frac{1}{1-t^2}\, dt =$$

$$= \int_0^{\frac{1}{2}} \frac{1}{(1-t)(1+t)}\, dt$$

$$\frac{1}{\cancel{(1-t)(1+t)}} = \frac{A}{1-t} + \frac{B}{1+t} = \frac{A(t+1)+B(1-t)}{\cancel{(1-t)(1+t)}}$$

$$1 = At + A + B - Bt$$

$$\begin{cases} A+B=1 \\ A-B=0 \end{cases} \rightarrow \begin{cases} 2A=1 \\ A=B \end{cases} \rightarrow \begin{cases} A=\frac{1}{2} \\ B=\frac{1}{2} \end{cases}$$

$$\int_0^{\frac{1}{2}} \frac{1}{(1-t^2)}\,dx = \int_0^{\frac{1}{2}} \left(\frac{1}{2(1-t)} + \frac{1}{2(1+t)}\right) dt =$$

$$= \frac{1}{2}\int_0^{\frac{1}{2}} \frac{1}{1-t}\,dt + \frac{1}{2}\int_0^{\frac{1}{2}} \frac{1}{1+t}\,dt =$$

$$= \frac{1}{2}\left[-\ln(1-t)\right]_0^{\frac{1}{2}} + \frac{1}{2}\left[\ln(1+t)\right]_0^{\frac{1}{2}} =$$

$$= \frac{1}{2}\left[-\ln(1-t) + \ln(1+t)\right]_0^{\frac{1}{2}} =$$

$$= \frac{1}{2}\left[\ln\left(\frac{1+t}{1-t}\right)\right]_0^{\frac{1}{2}} =$$

$$= \frac{1}{2}\left(\ln\left(\frac{\frac{3}{\cancel{2}}}{\frac{1}{\cancel{2}}}\right)\right) = \frac{1}{2}\ln(3)$$

$$\int \frac{x^4+1}{(x-1)^2(x+1)}\,dx$$

$$\frac{x^4+1}{(x-1)^2(x+1)} = \frac{x^4+1}{(x^2-2x+1)(x+1)} = \frac{x^4+1}{x^3-\cancel{2x^2}+\cancel{x}+\cancel{x^2}-\cancel{2x}+1} =$$

$$= \frac{x^4+1}{x^3-x^2-x+1}$$

DIVIDO

$$\begin{array}{rrrrr|l}
x^4 & & & & 1 & x^3-x^2-x+1 \\ \cline{6-6}
-x^4 & +x^3 & +x^2 & -x & & x+1 \\ \cline{1-4}
// & +x^3 & +x^2 & -x & & \\
 & -x^3 & +x^2 & +x & -1 & \\ \cline{2-5}
 & & 2x^2 & // & // & \\
\end{array}$$

QUINDI

$$\int \frac{x^4+1}{\quad} \,dx =$$

$$\int (x-1)^2(x-1)$$

$$= \int (x+1)\left(\frac{2x^2}{(x-1)^2(x-1)}\right) dx$$

$$\frac{2x^2}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

$$B = \lim_{x \to 1} \frac{2x^2}{x+1} = 1$$

$$C = \lim_{x \to -1} \frac{2x^2}{(x-1)^2} = \frac{2}{4} = \frac{1}{2}$$

così

$$\frac{2x^2}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{1}{(x-1)^2} + \frac{\frac{1}{2}}{x+1}$$

sostituendo a $x = 0$

$$0 = -\frac{A}{1} + \frac{1}{2} - \frac{1}{2}$$

$$0 = \frac{-2A+3}{2} \Rightarrow 2A = 3 \Rightarrow A = \frac{3}{2}$$

Quindi

$$\int \frac{x^4+1}{(x-1)^2(x-1)}\,dx =$$

$$= \int \left(x+1 + \frac{\frac{3}{2}}{x-1} + \frac{1}{(x-1)^2} + \frac{\frac{1}{2}}{x+1} \right) dx =$$

$$= \frac{x^2}{2} + x + \frac{3}{2}\ln|x-1| - \frac{1}{x-1} + \frac{1}{2}\ln|x+1| + c$$

$$\int \frac{e^x+1}{e^{2x}+1}\,dx$$

sia $t = e^x$ $\quad dt = e^x\,dx \Rightarrow \frac{1}{t}\,dt = dx$

Allora

$$\int \frac{e^x+1}{e^{2x}+1}\,dx = \int \frac{t+1}{t^2+1}\cdot\frac{1}{t}\,dt =$$

$$= \int \frac{t+1}{t(t^2+1)}\,dt$$

ora

$$\frac{t+1}{t(\cancel{t^2}+1)} = \frac{A}{t} + \frac{B\cdot t+C}{t^2+1} = \frac{At^2+A+Bt^2+Ct}{t(t^2+1)}$$

$$t+1 = At^2+A+Bt^2+Ct$$

$$t+1 = (A+B)t^2+Ct+A$$

$$\begin{cases} A+B=0 \\ C=1 \\ A=1 \end{cases} \qquad \begin{cases} B=-1 \\ C=1 \\ A=1 \end{cases}$$

Quindi

$$\int \ldots$$

$$\int \frac{t+1}{t(t^2+1)}\,dt = \int \left(\frac{1}{t} + \frac{1}{t^2+1} - \frac{t}{t^2+1}\right) dt =$$

$$= \ln|t| + \arctan(t) - \int \frac{t}{t^2+1}\,dt$$

sia $s = t^2+1 \rightarrow ds = 2t\,dt$

$$\frac{ds}{2s} = dt$$

$$= \ln|t| + \arctan(t) - \int \frac{1}{2(s)}\,dt =$$

$$= \ln|t| + \arctan(t) - \frac{1}{2}\ln(s) + c =$$

$$= \ln|t| + \arctan(t) - \frac{1}{2}\ln(t^2+1) + c =$$

$$= \ln|e^x| + \arctan(e^x) - \frac{1}{2}\ln(e^{2x}+1) + c =$$

$$= x + \arctan(e^x) - \frac{1}{2}\ln(1+e^{2x}) + c$$

$$\int \frac{1}{\ldots}$$

$$\int \frac{1}{\sqrt{x}\,(x-9)}\,dx$$

Si $t = \sqrt{x} \rightarrow dt = \frac{1}{2\sqrt{x}}\,dx \rightarrow 2 \cdot t\,dt = dx$

quindi

$$\int \frac{1}{\sqrt{x}\,(x-9)}\,dx = \int \frac{1}{\cancel{t}(t^2-9)}\,2\cancel{t}\,dt =$$

$$= 2\int \frac{1}{t^2-3^2}\,dt = 2\int \frac{1}{(t-3)(t+3)}\,dt$$

$$\frac{1}{(t-3)(t+3)} = \frac{A}{t-3} + \frac{B}{t+3} = \frac{A(t+3)+B(t-3)}{(t-3)(t+3)}$$

quindi

$$At + 3A + Bt - 3B = 1$$

$$(A+B)t + (3A-3B) = 1$$

$$\begin{cases} A+B = 0 \\ \end{cases} \qquad \begin{cases} A = -B \\ \end{cases} \qquad \begin{cases} B = -\frac{1}{6} \\ \end{cases}$$

$$\begin{cases} \\ 3A - 3B = 1 \end{cases} \qquad \begin{cases} \\ 3A + 3A = 1 \end{cases} \rightarrow \begin{cases} \\ A = \frac{1}{6} \end{cases}$$

Q1 MD1

$$2\int \frac{1}{(t-3)(t+3)}\,dt = 2\int \left(\frac{1}{6(t-3)} - \frac{1}{6(t+3)}\right) dt =$$

$$= \frac{2}{3 \cdot 6}\int \left(\frac{1}{t-3} - \frac{1}{t+3}\right) dt =$$

$$= \frac{1}{3}\left(\ln|t-3| - \ln|t+3|\right) + c =$$

$$= \frac{1}{3}\left(\ln\left|\frac{t-3}{t+3}\right|\right) + c =$$

$$= \frac{1}{3}\left(\ln\frac{|\sqrt{x}-3|}{\sqrt{x}+3}\right) + c$$

$$\int \log\left(x + \sqrt{1+x^2}\right) dx =$$

אינטגרציה בחלקים

$$= x \cdot \lg\left(x+\sqrt{1+x^2}\right) - \int x \cdot d\left(\lg\left(x+\sqrt{1+x^2}\right)\right) =$$

$$= x \lg\left(x+\sqrt{1+x^2}\right) - \int \frac{x}{x+\sqrt{1+x^2}} \cdot \left(1+\frac{x}{\sqrt{1+x^2}}\right) dx =$$

$$= x \lg\left(x+\sqrt{1+x^2}\right) - \int \frac{x}{\cancel{x+\sqrt{1+x^2}}} \cdot \frac{\cancel{\sqrt{1+x^2}+x}}{\sqrt{1+x^2}} dx =$$

$$= x \lg\left(x+\sqrt{1+x^2}\right) - \int \frac{x}{\sqrt{1+x^2}} dx =$$

שים

$$t = \sqrt{1+x^2}, \quad dt = \frac{x}{\sqrt{1+x^2}} dx$$

$$\left(\left(1+x^2\right)^{\frac{1}{2}}\right)' = \frac{\left(1+x^2\right)^{-\frac{1}{2}}}{2} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$$

אינטגרל

$$= x \lg\left(x+\sqrt{1+x^2}\right) - \int dt =$$

$$= x \log\left(x + \sqrt{1+x^2}\right) - t + c =$$

$$= x \log\left(x + \sqrt{1+x^2}\right) - \sqrt{1+x^2} + c$$

$$\int_0^4 e^{\sqrt{x}}\, dx \qquad \left(x^{\frac{1}{2}}\right)' = \frac{1}{2\sqrt{x}}$$

SIA $t = \sqrt{x}$, $dt = \frac{1}{2\sqrt{x}}\, dx \Rightarrow 2t\, dt = dx$

QUINDI

$$\int_0^4 e^{\sqrt{x}}\, dx = 2\int_0^4 t \cdot e^t\, dt =$$

INTEGRO PER PARTI

$$2\int_0^2 t \cdot d(e^t) = 2\left[e^t \cdot t\right]_0^2 - 2\int_0^2 e^t \cdot dt =$$

$$= 2\left(2 \cdot e^2\right) - 2\left[e^t\right]_0^2 = 4e^2 - 2e^2 + 2 =$$

$$= 2e^2 + 2 = 2\left(e^2 + 1\right)$$

$$\int_1 (|x-2| - x) \log(x)\, dx$$

$$|x-2| = \begin{cases} +2-x & \text{si } 1 \leq x \leq 2 \\ x-2 & \text{si } 2 < x \leq 4 \end{cases}$$

Quindi

$$\int_1^4 (|x-2| - x) \lg(x)\, dx =$$

$$= \int_1^2 ((2-x) - x) \lg(x)\, dx + \int_2^4 ((x-2) - x) \lg(x)\, dx =$$

$$= \int_1^2 (2-2x) \lg(x)\, dx + \int_2^4 (-2) \lg(x)\, dx =$$

$$= \int_1^2 2 \lg(x)\, dx - \int_1^2 2x \lg(x)\, dx - 2\int_2^4 \lg(x)\, dx =$$

$$= 2\int_1^2 \lg(x)\, dx - 2\int_1^2 x \lg(x)\, dx - 2\int_2^4 \lg(x)\, dx =$$

$$= 2\left(\left[x\lg(x)\right]_1^2 - \int \frac{\cancel{x}}{\cancel{x}}\,dx\right) - \cancel{2}\left[\frac{x^2}{\cancel{2}}\cdot\lg(x)\,dx\right]_1^2 + \cancel{2}\int_1^2 \frac{x^{\cancel{2}}}{\cancel{2}}\cdot\frac{1}{x}\,dx -$$

$$-2\left(\left[x\lg(x)\right]_2^4 - \int_2^4 \frac{x}{x}\,dx\right) =$$

$$= 2\left[x\lg(x) - x\right]_1^2 + \left[-x^2\lg(x) + \frac{x^2}{2}\right]_1^2 - 2\left[x\lg(x) - x\right]_2^4 =$$

$$= 2\left(2\ln(2) - 2 - 1\right) + \left((-4)\lg(2) + 2 - \frac{1}{2}\right) -$$

$$-2\left(4\ln(4) - 4 - 2\ln(2) + 2\right) =$$

$$= \cancel{4\ln(2)} - 2 - \cancel{4\ln(2)} + \frac{3}{2} - 8\ln(4) + 8 + 4\ln(2) - 4 =$$

$$= 4\ln(2) - 8\ln(2) + \frac{-4+3+16-8}{2} =$$

$$= 4\ln(2) - 16\ln(2) + \frac{7}{2} = \frac{7}{2} - 12\ln(2)$$

$$\int^{+\infty}$$

$$\int_{-\infty} e^{-|x|} \lg(1+e^x)\, dx =$$

$$= \int_{-\infty}^{0} e^{x} \lg(1+e^x)\, dx + \int_{0}^{+\infty} e^{-x} \log(1+e^x)\, dx$$

SIA $t = e^x$, $x = \log(t)$, $\frac{dt}{t} = dx$ ci

$$\int_{-\infty}^{0} e^{x} \lg(1+e^x)\, dx + \int_{0}^{+\infty} e^{-x} \log(1+e^x)\, dx =$$

$$= \int_{0}^{1} \frac{\cancel{t} \lg(1+t)\, dt}{\cancel{t}} + \int_{1}^{+\infty} \frac{t^{-1} \lg(1+t)\, dt}{t} =$$

$$= \int^{1} \lg(1+t)\, dt + \int^{+\infty} \lg(1+t)\, dt =$$

$$\int_0 \qquad \int_1 \frac{\qquad}{t^2}$$

$$s = 1+t \quad , \quad ds = dt$$

$$\int_1^2 \ln(s)\, ds = \left[s \ln(s)\right]_1^2 - \int_1^2 \frac{s}{s}\, ds =$$

$$= \left[s \ln(s) - s\right]_1^2 = 2\ln(2) - 2 - 0 + 1 =$$

$$= 2\ln(2) - 1$$

$$= 2\ln(2) - 1 + \int_1^{+\infty} \log(1+t)\, d\left(-\frac{1}{t}\right) =$$

$$= 2\ln(2) - 1 + \left[\log(1+t)\cdot\left(-\frac{1}{t}\right)\right]_1^{+\infty} + \int_1^{+\infty} \frac{1}{t}\cdot\frac{1}{(t+1)}\, dt =$$

$$= 2\ln(2) - 1 - \left[\log(1+t)\left(-\frac{1}{t}\right)\right]_1^{+\infty} + \int_1^{+\infty} \frac{1}{t(t+1)}\, dt$$

$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} = \frac{At + A + Bt}{t(t+1)}$$

$$\begin{cases} A = 1 \\ A + B = 0 \end{cases} \qquad \begin{cases} A = 1 \\ B = -1 \end{cases}$$

$$= 2\ln(2) - 1 + \left[-\frac{\log(1+t)}{t}\right]_1^{+\infty} + \int_1^{+\infty}\left(\frac{1}{t} - \frac{1}{t+1}\right)dt =$$

$$= 2\ln(2) - 1 + \left[-\frac{\log(1+t)}{t}\right]_1^{+\infty} + \left[\ln(t) - \ln(t+1)\right]_1^{+\infty} =$$

$$= 2\ln(2) - 1 + \left[-\frac{\log(1+t)}{t}\right]_1^{+\infty} + \left[\ln\left(\frac{t}{t+1}\right)\right]_1^{+\infty} =$$

$$\left(\lim_{t\to+\infty} -\frac{\log(1+t)}{t}\right) - \left(-\frac{\log(1+1)}{1}\right) =$$

$$= 0 + \log(2)$$

$$\left(\lim_{t\to+\infty} \ln\left(\frac{t}{t+1}\right)\right) - \ln\left(\frac{1}{2}\right) =$$

$$= \lim_{t\to+\infty} \underbrace{\ln\left(\overbrace{\frac{\cancel{t}(1)}{\cancel{t}\left(1+\underbrace{\frac{1}{t}}_{\to 0}\right)}}^{\to 1}\right)}_{\to 0} + \ln(2) =$$

$$= \ln(2)$$

$$= 2\ln(2) - 1 + \ln(2) + \ln(2) =$$

$$= 4\ln(2) - 1$$

$$\int_{\lg(2)}^{+\infty} \frac{1}{\sqrt{e^{2x}-1}}\, dx$$

SIA $t = \sqrt{e^{2x}-1}$, ALLORA

$$\ln(1+t^2) = \ln\left(e^{2x} - \cancel{1} + \cancel{1}\right) = 2x$$

$$\underbrace{\log(1+t^2)}_{\text{DERIVATA}} = 2\log(t) \cdot \underbrace{1+t^2}_{\text{DERIVATA}}$$

$$\frac{2t}{1+t^2}\,dt = 2\,dx$$

ALLORA

$$\int_{\log(2)}^{+\infty} \frac{1}{\sqrt{e^x-1}} = \frac{1}{\cancel{2}}\int_{\sqrt{3}}^{+\infty} \frac{1}{\cancel{t}} \cdot \frac{\cancel{2}\cancel{t}}{1+t^2}\,dt =$$

$$= \int_{\sqrt{3}}^{+\infty} \frac{1}{1+t^2}\,dt = \Big[\arctan(t)\Big]_{\sqrt{3}}^{+\infty} =$$

$$= \frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi - 2\pi}{6} = \frac{1}{3}\pi$$

$$\int_0^1 x \log\left(\frac{x}{2-x}\right) dx =$$

INTEGRANDO PER PARTI

$$= \left[x^2 \cdot \log\left(\frac{x}{2-x}\right)\right]_0^1 - \int_0^1 x^2\, d\left(\log\left(\frac{x}{2-x}\right)\right) =$$

$$= \left(\frac{\quad}{2} \quad \int_0^{\quad} \quad \int_0^{2} \quad \left(\quad \right)\right)$$

DERIVATA

$$= \frac{1}{2} \cdot 0 - 0 - \int_0^1 \frac{x^{\cancel{2}}}{\cancel{2}} \cdot \frac{2-x}{x} \cdot \frac{x}{(2-x)^2} \, dx =$$

$$= -\int_0^1 \frac{x\,\cancel{(2-x)}}{(2-x)^{\cancel{2}}} \, dx =$$

$$= +\int_0^1 \frac{-x \overset{+2}{-2}}{2-x} \, dx =$$

$$= \int_0^1 \frac{\cancel{-x+2}}{\cancel{2-x}} \, dx + \int_0^1 \frac{-2}{2-x} \, dx =$$

$$= \int_0^1 dx - \int_0^1 \frac{2}{2-x} \, dx = \left[x\right]_0^1 + 2\int_0^1 \frac{1}{x-2} \, dx =$$

$$= 1 + \left[\log|x-2|\right]_0^1 = 1 + 2\left(\log|-1| - \log|-2|\right) =$$

$$= 1 + 2\left(\cancel{\log(1)} - \log(2)\right) = 1 - 2\log(2)$$

$$\int_0^1 \frac{\log(x+3)}{\sqrt{x}}\,dx =$$

INTEGRO PER PARTI

$$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}$$

$$= 2\int_0^1 \log(x+3)\cdot d(\sqrt{x}) =$$

$$= 2\left[\sqrt{x}\,\log(x+3)\right]_0^1 - 2\int_0^1 \sqrt{x}\cdot d\left(\log(x+3)\right) =$$

$$= 2\left(1\log(4)\right) - 2\int_0^1 \frac{\sqrt{x}}{x+3}\,dx =$$

SIA $t=\sqrt{x}$, $x=t^2$, $dx = 2t\,dt$

$$= 2\log(2^2) - 2\int_0^1 \frac{t}{t^2+3}\cdot 2t\,dt =$$

$$= 4\log(2) - 4\int_0^1 \frac{t^2+3-3}{t^2+3}\,dt =$$

$$= 4\log(2) - 4\left(\int_0^1 dt - 3\int_0^1 \frac{}{t^2+3}\,dt\right) =$$

$$= 4\log(2) - 4\left(\left[x\right]_0^1 - 3\left[\frac{\arctan\left(\frac{t}{\sqrt{3}}\right)}{\sqrt{3}}\right]_0^1\right) =$$

$$= 4\log(2) - 4\left(1 - 3\left(\frac{\arctan\left(\frac{1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan(0)}{\sqrt{3}}\right)\right) =$$

$$= 4\log(2) - 4\left(1 - 3\left(\frac{\frac{\pi}{6}}{\sqrt{3}}\right) + 0\right) =$$

$$= 4\log(2) - 4 + \left(\frac{2\pi}{\sqrt{3}}\right)$$

$$\int_0^{+\infty} \frac{x+5}{(x+3)(x^2+3x+2)}\,dx =$$

$$= \int_0^{+\infty} \frac{x+5}{(x+3)(x+1)(x+2)}\,dx$$

Or,

$$\frac{x+5}{(x+3)(x+1)(x+2)} = \frac{A}{x+3} + \frac{B}{x+1} + \frac{C}{x+2} =$$

$$A = \lim_{x \to -3} \frac{x+5}{(x+1)(x+2)} = \frac{2}{(-2)(-1)} = 1$$

$$B = \lim_{x \to -1} \frac{x+5}{(x+3)(x+2)} = \frac{4}{2 \cdot 1} = 2$$

$$C = \lim_{x \to -2} \frac{x+5}{(x+3)(x+1)} = \frac{3}{1 \cdot (-1)} = -3$$

Quindi

$$\int_0^{+\infty} \frac{x+5}{(x+3)(x+1)(x+2)} \, dx =$$

$$= \int_0^{+\infty} \left(\frac{1}{x+3} + \frac{2}{x+1} - \frac{3}{x+2} \right) dx =$$

$$= \left[\log|x+3| + 2\log|x+1| - 3\log|x+2| \right]_0 =$$

$$= \left[\log \left| \frac{(x+3)(x+1)^2}{(x+2)^3} \right| \right]_0^{+\infty} =$$

$$= \left(\lim_{x \to +\infty} \log \left(\frac{(x+3)(x+1)^2}{(x+2)^3} \right) \right) - \log\left(\frac{3}{8}\right) =$$

$$= \left(\lim_{x \to +\infty} \log \left(\frac{(x+3)(x^2+1+2x)}{x^3+8+6x^2+12x} \right) \right) - \log\left(\frac{3}{8}\right) =$$

$$= \left(\lim_{x \to +\infty} \log \left(\frac{x^3+9x+3x^2+3}{x^3+8+6x^2+12x} \right) \right) + \log\left(\frac{8}{3}\right) =$$

$$= \left(\lim_{x \to +\infty} \log \left(\frac{\cancel{x^3}\overbrace{\left(1+\frac{9}{x^2}+\frac{3}{x}+\frac{3}{x^3}\right)}^{\to 0}}{\cancel{x^3}\underbrace{\left(1+\frac{8}{x^3}+\frac{6}{x}+\frac{12}{x^2}\right)}_{\to 1}} \right) \right) + \log\left(\frac{8}{3}\right) =$$

$$= \log(1) + \log\left(\frac{8}{3}\right) = \log\left(\frac{8}{3}\right)$$

$$\int_0^{+\infty} \frac{1}{\sqrt{x}+x^2}\,dx$$

sia $t=\sqrt{x}$, $t^2=x$, $dx=2t\,dt$

allora

$$\int_0^{+\infty} \frac{1}{\sqrt{x}+x^2}\,dx = \int_0^{+\infty} \frac{1}{t+t^4}\cdot 2t\,dt =$$

$$= 2\int_0^{+\infty} \frac{1}{\cancel{t}(1+t^3)}\,\cancel{t}\,dt = 2\int_0^{+\infty} \frac{1}{t^3+1}\,dt =$$

$$= 2\int_0^{+\infty} \frac{1}{(t+1)(t^2-t+1)}\,dt$$

ora

$$\frac{1}{(t+1)(t^2-t+1)} = \frac{A}{t+1} + \frac{Bt+C}{t^2-t+1} \rightarrow$$

$$\rightarrow At^2 - At + A + Bt^2 + Bt + Ct + C = 1$$

$$(A+B)t^2 + (-A+B+C)t + (A+C) = 1$$

$$\begin{cases} A+B=0 \\ -A+B+C=0 \\ A+C=1 \end{cases} \qquad \begin{cases} A=-B \\ B+B+C=0 \\ -B+C=1 \end{cases}$$

$$\begin{cases} A=-B \\ B=-\frac{C}{2} \\ \frac{C}{2}+C=1 \end{cases} \qquad \begin{cases} A=\frac{1}{3} \\ B=-\frac{1}{3} \\ C=\frac{2}{3} \end{cases}$$

Quindi

$$2\int_0^{+\infty} \frac{1}{(t+1)(t^2-t+1)}\,dt =$$

$$\int_0^{+\infty} \left(\frac{1}{\ldots} \quad \frac{-t+2}{\ldots} \right) dt =$$

$$= 2 \cdot \frac{1}{3} \int_0 \left(\frac{\quad}{(t+1)} + \frac{\quad}{(t^2-t+1)} \right)$$

$$= \frac{2}{3} \left(\left[\ln|t+1| \right]_0^{+\infty} - \int_0^{+\infty} \frac{t-2}{t^2-t+1} \, dt \right)$$

Si $s = t - \frac{1}{2}$

$$\int \frac{t-2}{t^2-t+1} \, dt = \int \frac{s + \frac{1}{2} - 2}{\left(s+\frac{1}{2}\right)^2 - s - \frac{1}{2} + 1} \, ds =$$

$$= \int \frac{s - \frac{3}{2}}{s^2 + \cancel{s} + \frac{1}{4} - \cancel{s} - \frac{1}{2} + 1} \, ds = \int \frac{s - \frac{3}{2}}{s^2 + \frac{3}{4}} \, ds =$$

$$= \int \frac{s}{s^2 + \frac{3}{4}} \, ds - \frac{3}{2} \int \frac{1}{s^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \, ds =$$

Si $u = s^2 + \frac{3}{4} \rightarrow du = 2s \, ds$

$$= \frac{1}{2} \int \frac{1}{u} \, du - \frac{\cancel{3}}{\cancel{4}} \left(\frac{\cancel{2}\sqrt{3} \arctan\left(\frac{2\sqrt{3}\, s}{3}\right)}{\cancel{3}} \right) + C =$$

$$= \frac{1}{2}\log(u) - \sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(t-\frac{1}{2}\right)}{3}\right) + c =$$

$$= \frac{1}{2}\log\left(t^2+\frac{3}{4}\right) - \sqrt{3}\arctan\left(\frac{2\sqrt{3}t-\sqrt{3}}{3}\right) + c =$$

$$= \frac{1}{2}\log\left(\left(t-\frac{1}{2}\right)^2+\frac{3}{4}\right) - \sqrt{3}\arctan\left(\frac{\cancel{\sqrt{3}}(2t-1)}{\cancel{3}\sqrt{3}}\right) + c =$$

$$= \frac{1}{2}\log\left(t^2-t+\frac{1}{4}+\frac{3}{4}\right) - \sqrt{3}\arctan\left(\frac{2t-1}{\sqrt{3}}\right) + c =$$

$$= \frac{1}{2}\log\left(t^2-t+1\right) - \sqrt{3}\arctan\left(\frac{2t-1}{\sqrt{3}}\right) + c$$

QUINDI

$$\frac{2}{3}\left(\left[\ln|t+1|\right]_0^{+\infty} - \int_0^{+\infty}\frac{t-2}{t^2-t+1}\,dt\right) =$$

$$= \frac{2}{3}\left[\ln(t+1) - \frac{1}{2}\log(t^2-t+1) + \sqrt{3}\arctan\left(\frac{2t-1}{\sqrt{3}}\right)\right]_0^{+\infty} =$$

$$= \frac{2}{3}\left[\ln\left(\frac{t+1}{\sqrt{t^2-t+1}}\right) + \sqrt{3}\arctan\left(\frac{2t-1}{\sqrt{3}}\right)\right]_0^{+\infty} =$$

$$= \frac{2}{3}\left[\ln\left(\frac{t\left(1+\frac{1}{t}\right)}{\sqrt{t^2\left(1-\frac{1}{t}+\frac{1}{t^2}\right)}}\right) + \sqrt{3}\arctan\left(\frac{2t-1}{\sqrt{3}}\right)\right]_0^{+\infty} =$$

$$\frac{2}{3}\left[\ln\left(\frac{t\left(1+\frac{1}{t}\right)}{t\sqrt{1-\frac{1}{t}+\frac{1}{t^2}}}\right)+\sqrt{3}\arctan\left(\frac{2t-1}{\sqrt{3}}\right)\right]_0^{+\infty}=$$

$$\lim_{t\to+\infty}\left(\ln\left(\overbrace{\frac{1+\frac{1}{t}}{\sqrt{1-\frac{1}{t}+\frac{1}{t^2}}}}^{\to 1}\right)+\sqrt{3}\overbrace{\arctan\left(\frac{2t-1}{\sqrt{3}}\right)}^{\to\frac{\pi}{2}}\right)=$$

$$=\ln(1)+\sqrt{3}\cdot\frac{\pi}{2}=\frac{\sqrt{3}\pi}{2}$$

$$=\frac{2}{3}\left(\frac{\sqrt{3}\pi}{2}-\sqrt{3}\arctan\left(-\frac{1}{\sqrt{3}}\right)\right)=$$

$$=\frac{2}{3}\left(\frac{\sqrt{3}\pi}{2}-\sqrt{3}\cdot\left(-\frac{\pi}{6}\right)\right)=$$

$$=\frac{2}{3}\left(\frac{\sqrt{3}\pi}{2}+\frac{\sqrt{3}\pi}{6}\right)=\frac{3}{2}\left(\frac{3\sqrt{3}\pi+\sqrt{3}\pi}{6}\right)=$$

$$=\frac{\cancel{4}}{3}\left(\frac{\cancel{4}^{2}\sqrt{3}\pi}{\cancel{6}\,3}\right)=\frac{4\sqrt{3}\pi}{9}=\frac{4\cancel{\sqrt{3}}\pi}{\cancel{\sqrt{3}}\cdot\sqrt{3}\cdot 3}=\frac{4\pi}{3\sqrt{3}}$$

$$\int_0^{+\infty}\frac{1}{\quad}\,dx$$

$$\int_0 \sqrt{e^{4x}-1}$$

sia $t = \sqrt{e^{4x}-1} \rightarrow t^2 = e^{4x}-1 \rightarrow$

$\rightarrow \log(t^2+1) = \log(e^{4x}-\cancel{1}+\cancel{1}) \rightarrow$

$\rightarrow \log(t^2+1) = 4x \rightarrow \frac{\log(t^2+1)}{4} = x$

$$dx = \frac{t}{2(t^2+1)}\,dt \qquad \text{e}$$

$$\int_0^{+\infty} \frac{1}{\sqrt{e^{4x}-1}}\,dx = \int_0^{+\infty} \frac{1}{\cancel{t}} \cdot \frac{\cancel{t}}{2(t^2+1)}\,dt =$$

$$= \int_0^{+\infty} \frac{1}{2(t^2+1)}\,dt = \frac{1}{2}\int_0^{+\infty} \frac{1}{t^2+1}\,dt =$$

$$= \frac{1}{2}\Big[\arctan(t)\Big]_0^{+\infty} = \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4}$$

$$\int_0^1 \frac{\arccos(x)}{\sqrt{1-x}}\,dx$$

INTEGRAZIONE PER PARTI

$$\int_0^1 \frac{\arccos(x)}{\sqrt{1-x}}\,dx = \int_0^1 \arccos(x)\, d\left(-2\sqrt{1-x}\right) =$$

$$= \left[\arccos(x)\cdot\left(-2\sqrt{1-x}\right)\right]_0^1 - \int_0^1 \left(-2\sqrt{1-x}\right)\cdot\left(\left(\arccos(x)\right)'\right)$$

$$= -\frac{\pi}{2}\cdot -2 + 2\int_0^1 \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx =$$

$$= \pi + 2\int_0^1 \frac{\sqrt{1-x}}{\sqrt{(1-x)(1+x)}}\,dx =$$

$$= \pi + 2\int_0^1 \frac{\cancel{\sqrt{1-x}}}{\cancel{\sqrt{1-x}}\sqrt{1+x}}\,dx = \pi + 2\int_0^1 \frac{1}{\sqrt{1+x}}\,dx =$$

$$= \pi + 2\left[2\sqrt{1+x}\right]_0^1 = \pi + 2\left(2\sqrt{2} - 2\right) =$$

$$= \pi + 4\sqrt{2} - 4$$

$$\int_0^{\frac{\pi}{4}} \sin(2x)\log^2(\sin^2(x))\,dx$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

SIA $t = \sin^2(x)$, ALLORA

$dt = 2\sin(x)\cos(x)\,dx$ E

$$\int_0^{\frac{\pi}{4}} \sin(2x)\log^2(\sin^2(x))\,dx =$$

$$= \int_0^{\frac{1}{2}} \log^2(t)\,dt =$$

INTEGRO PER PARTI

$$= \left[t\log^2(t)\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} t \cdot d(\log^2(t)) =$$

$$= \frac{1}{2}\log^2(2^{-1}) - \int_0^{\frac{1}{2}} 2\,\frac{\log(t)}{\cancel{t}}\,\cancel{t}\,dt =$$

$$= \frac{1}{2}\log^2(2) - 2\int_0^{\frac{1}{2}} \log(t)\,dt =$$

INTEGRO PER PARTI

$$= \frac{1}{2}\log^2(2) - 2\left(\left[t\log(t)\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \cancel{t}\cdot\left(\frac{1}{\cancel{t}}\right)\right) =$$

$$= \frac{\log^2(2)}{2} - 2\left(\frac{1}{2}\log\left(\frac{1}{2}\right) - \frac{1}{2}\right)$$

$$= \frac{\log^2(2)}{2} + \log(2) + 1$$

$$\int_1^{+\infty} \frac{\log(x^2-x)}{x^3}\,dx =$$

INTEGRO PER PARTI

$$= \int_1^{+\infty} \lg(x^2-x)\, d\left(-\frac{1}{2x^2}\right) =$$

$$= \left[-\frac{1}{2x^2}\lg(x^2-x)\right]_1^{+\infty} + \int_1^{+\infty} \frac{1}{2x^2}\, d\left(\lg(x^2-x)\right)$$

$$= \left[-\frac{1}{2x^2}\lg(x^2-x)\right]_1^{+\infty} + \int_1^{+\infty} \frac{1}{2x^2}\cdot\frac{2x-1}{x^2-x}\, dx =$$

$$= \left[-\frac{1}{2x^2}\lg(x^2-x)\right]_1^{+\infty} + \int_1^{+\infty} \frac{\cancel{2}\left(x-\frac{1}{2}\right)}{\cancel{2}x^4-\cancel{2}x^3}\, dx =$$

$$= \left[-\frac{1}{2x^2}\lg(x^2-x)\right]_1^{+\infty} + \int_1^{+\infty} \frac{x-1}{x^3(x-1)}\, dx =$$

oz.

$$\frac{x-\frac{1}{2}}{x^3(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1} =$$

$$D = \lim_{x \to -1} \frac{x - \frac{1}{2}}{x^3} = \frac{1}{2}$$

$$C = \lim_{x \to 0} \frac{2x - \frac{1}{9}}{x - 1} = \frac{1}{2}$$

Por vamos $x = \frac{1}{2}$ obtemos

$$2A + 4B + \frac{\frac{1}{2}}{\frac{1}{8}} + \frac{\frac{1}{2}}{-\frac{1}{2}} = 0$$

$$2A + 4B + 4 - 1 = 0$$

$$2A + 4B = -3$$

E por fim $x = -1$ obtemos

$$-A + B = 0$$

Daí

$$\text{Cui}$$

$$\begin{cases} 2A + 4B = -3 \\ -A + B = 0 \end{cases} \qquad \begin{cases} A = B \\ 6A = -3 \end{cases} \qquad \begin{cases} A = B \\ A = -\frac{1}{2} \end{cases}$$

DONC

$$\left[-\frac{1}{2x^2} \lg(x^2 - x) \right]_1^{+\infty} + \int_1^{+\infty} \frac{x-1}{x^3(x-1)} \, dx =$$

$$= \left[-\frac{1}{2x^2} \lg(x^2 - x) \right]_1^{+\infty} + \int_1^{+\infty} \left(-\frac{1}{2x} - \frac{1}{2x^2} + \frac{1}{2x^3} + \frac{1}{2(x-1)} \right) dx$$

$$= \left[-\frac{1}{2x^2} \lg(x^2 - x) \right]_1^{+\infty} + \left[\frac{\lg|x|}{2} + \frac{1}{2x} - \frac{1}{4x^2} + \frac{1}{2} \lg|x-1| \right]_1^{+\infty}$$

$$= \left[-\frac{1}{2x^2} \lg(x^2 - x) + \frac{1}{2x} - \frac{1}{4x^2} + \frac{1}{2} \lg\left| \frac{x-1}{x} \right| \right]_1^{+\infty} =$$

$$\lim_{x \to +\infty} -\frac{1}{2x^2} \lg(x^2 - x) + \frac{1}{2x} - \frac{1}{4x^2} + \frac{1}{2} \lg\left| \frac{x-1}{x} \right| =$$

$$= \lim_{x \to +\infty} - \frac{\lg\left(x^2\left(1-\frac{1}{x}\right)\right)}{2x^2} + \frac{1}{2x} - \frac{1}{4x^2} + \frac{1}{2}\lg\left|1-\frac{1}{x}\right|$$

$$= 0$$

$$= 0 - \frac{1}{2}\lg(0) + \frac{1}{2} - \frac{1}{4} + \frac{1}{2}\lg(0) =$$

$$= -\cancel{\frac{0}{2}} + \frac{1}{2} - \frac{1}{4} + \cancel{\frac{0}{2}} = \frac{2-1}{4} = \frac{1}{4}$$