1)

a) $\lim_{n \to \infty} \frac{n^n}{4^{n\log(n)}} =$

$$\frac{n^n}{4^{n\log(n)}} = \frac{4^{n\log_4(n)}}{4^{n\log(n)}} = 4^{n\log_4(n) - n\log(n)}$$

Per $n \to \infty$

CAMBIO BASE

$$n\boxed{\log_4(n)} - n\log(n) = n\,\frac{\ln(n)}{\ln(4)} - n\ln(n) =$$

$$= n\ln(n)\underbrace{\left(\overbrace{\frac{1}{\ln(4)}}^{>1} - 1\right)}_{<0} \to -\infty$$

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$$\lim_{n \to \infty} \frac{n^n}{4^{n\ln(n)}} = 4^{-\infty} = \frac{1}{4^{\infty}} = 0$$

c) $\lim_{n \to \infty} n\left(6^{\frac{1}{n}} - 3^{\frac{1}{n}}\right) = \ln(2)$

Per $n \to \infty$

$$n\left(6^{\frac{1}{n}} - 3^{\frac{1}{n}}\right) = 3^{\frac{1}{n}} \cdot n\left(2^{\frac{1}{n}} - 1\right) =$$

$$= \underbrace{3^{\frac{1}{n}}}_{\to 1} \frac{\overbrace{\left(2^{\frac{1}{n}} - 1\right)}^{\to \ln(2)}}{\frac{1}{n}} \to \ln(2)$$

$$\to \lim_{x \to 0} \frac{a^x - 1}{x} = \ln(a)$$

d) $$\lim_{n \to \infty} \frac{e^{n \log(n+3)}}{e^{n \log(n+1)}}$$

Per $n \to \infty$

$$\frac{e^{n \log(n+3)}}{e^{n \log(n+1)}} = \frac{(n+3)^n}{(n+1)^n} = \frac{\left(\frac{n}{n} + \frac{3}{n}\right)^n}{\left(\frac{n}{n} + \frac{1}{n}\right)^n} =$$

$$= \frac{\left(1 + \frac{3}{n}\right)^n}{\left(1 + \frac{1}{n}\right)^n} \to \frac{e^3}{e^1} = e^2$$

3) a) $f(x) = \cos\left(\frac{\pi}{x}\right)\left(3 + \sqrt{x}\right)$

$$f'(x) = \left(\cos\left(\frac{\pi}{x}\right)\right)' \cdot (3+\sqrt{x}) + \cos\left(\frac{\pi}{x}\right) \cdot (3+\sqrt{x})' =$$

$$= -\sin\left(\frac{\pi}{x}\right) \cdot (3+\sqrt{x}) + \cos\left(\frac{\pi}{x}\right) \cdot \left(\frac{1}{2\sqrt{x}}\right)$$

$$\sqrt{x} = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}}$$

b) $f(x) = x \ln(1+x^2) \cdot \tan(2x)$

$$f'(x) = \left[\ln(1+x^2) + x \cdot \frac{1}{1+x^2} \cdot 2x\right] \cdot \tan(2x) +$$

$$+ x \ln(1+x^2) \cdot \left(\frac{\sin(2x)}{\cos(2x)}\right)' \cdot 2 =$$

$$= \ln(1+x^2) \cdot \tan(2x) + \frac{2x^2}{1+x^2} \tan(2x) + x \ln(1+x^2) \cdot$$

$$\cdot \left(\underbrace{\frac{\cos(2x) \cdot \cos(2x) - \sin(2x) \cdot (-\sin(2x))}{\cos^2(2x)}}_{\frac{\cos^2(2x) + \sin^2(2x)}{\cos^2(2x)}}\right) =$$

$$= \frac{1}{\cos^2(x)}$$

$$= \ln(1+x^2) \cdot \tan(2x) + \frac{2x^2}{1+x^2} \tan(2x) + x \ln(1+x^2) \cdot \frac{2}{\cos^2(2x)}$$

c) $f(x) = \dfrac{x \sin(x)}{1+\cos(x)}$

$$f'(x) = \frac{\left[(\sin(x) + x \cdot \cos(x)) \cdot (1+\cos(x))\right] - \left[(x \sin(x)) \cdot (-\sin(x))\right]}{(1+\cos(x))^2}$$

$$= \frac{\left[(\sin(x) + x \cdot \cos(x)) \cdot (1+\cos(x))\right] + x \cdot \sin^2(x)}{(1+\cos(x))^2}$$

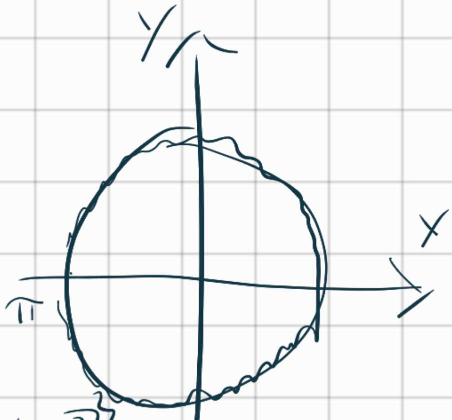
d) $f(x) = \dfrac{\arcsin(x)}{\arccos(x)}$

$$f'(x) = \frac{\frac{1}{\sqrt{1-x^2}} \cdot \arccos(x) - \left(\arcsin \cdot \left(-\frac{1}{\sqrt{1-x^2}}\right)\right)}{\arccos^2(x)} =$$

arccos(x) arcsin arccos(x) + arcsin(x)

$$= \frac{\dfrac{\ldots}{\sqrt{1-x^2}} + \dfrac{\ldots}{\sqrt{1-x^2}}}{\arccos^2(x)} = \frac{\dfrac{\ldots}{\sqrt{1-x^2}}}{\arccos^2(x)}$$

4) a) $f(x) = x + \sin(x)$




$f'(x) = 1 + \cos(x) \geq 0$

$\cos(x) \geq -1$

$$f'(x) = \begin{cases} > 0 & \text{se } x \in \mathbb{R} \setminus \{\pi + 2k\pi : k \in \mathbb{Z}\} \\ = 0 & \text{se } x \in \{\pi + 2k\pi : k \in \mathbb{Z}\} \\ < 0 & \text{mai} \end{cases}$$

b) $f(x) = x^{\frac{1}{x}}$

$$f'(x) = \left(e^{\frac{1}{x}\ln(x)}\right)' = e^{\frac{1}{x}\ln(x)} \cdot \left(\frac{1}{x}\ln(x)\right)' =$$

$$= e^{\frac{1}{x}\ln(x)} \cdot \left(\frac{1}{x} \cdot \frac{1}{x} + \ln(x) \cdot \left(\frac{1}{x}\right)'\right) =$$

$$= e^{\frac{1}{x}\ln(x)} \cdot \left(\frac{1}{x^2} + \ln(x)\cdot\left(-\frac{1}{x^2}\right)\right) =$$

$$= x^{\frac{1}{x}} \cdot \left(\frac{1}{x^2} - \frac{\ln(x)}{x^2}\right) = x^{\frac{1}{x}} \cdot \frac{1-\ln(x)}{x^2} =$$

$$= \frac{x^{\frac{1}{x}}\cdot(1-\ln(x))}{x^2}$$

$$\begin{cases} x \geq 0 \\ x \neq 0 \end{cases} \Rightarrow x > 0$$

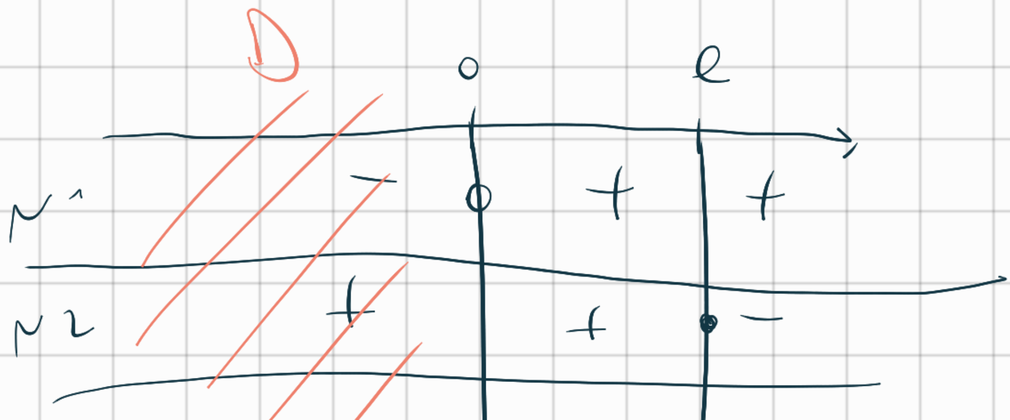
Pl.2 $x \in D = (0, +\infty)$

$f'(x) \geq 0$

N1: $x^{\frac{1}{x}} > 0 \Rightarrow e^{\frac{1}{x}\ln(x)} \geq 0 \Rightarrow x > 0$

N2: $1 - \ln(x) \geq 0 \Rightarrow \ln(x) \leq 1 \Rightarrow x \leq e$

D: $x^2 > 0 \Rightarrow x > 0$



| | | | | | |
|---|---|---|---|---|---|
| $D$ | | $-$ | $0$ | $+$ | $+$ |
| $\frac{N}{D}$ | | $+$ | $0$ | $+$ | $-$ |

$$f'(x) = \begin{cases} >0 & \text{si } x \in (0, e) \\ =0 & \text{si } x = e \\ <0 & \text{si } x \in (e, +\infty) \end{cases}$$

c) $f(x) = \dfrac{1 - |2x+1|}{x^2+1}$ $\qquad x^2 = -1 \rightarrow x \nexists \sqrt{-1}$

$D = x \in \mathbb{R}$

• $2x+1 > 0 \rightarrow x > -\frac{1}{2}$

$$f'(x) = \left(\frac{1-2x-1}{x^2+1}\right)' = \left(\frac{-2x}{x^2+1}\right)' = \frac{-2\cdot(x^2+1) - (-2x \cdot 2x)}{(x^2+1)^2} =$$

$$= \frac{-2x^2 - 2 + 4x^2}{(x^2+1)^2} = \frac{2x^2-2}{(x^2+1)^2} = \frac{2(x^2-1)}{(x^2+1)^2}$$

$$2x+1 < 0 \rightarrow x < -\frac{1}{2}$$

$$f'(x) \left(\frac{1+2x+1}{x^2+1}\right)' = \left(\frac{2x+2}{x^2+1}\right)' = \frac{2\cdot(x^2+1)-((2x+2)\cdot 2x)}{(x^2+1)^2} =$$

$$= \frac{2x^2+2-4x^2-4x}{(x^2+1)^2} = \frac{-2(x^2-2x+1)}{(x^2+1)}$$

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$$f'(x) = \begin{cases} \dfrac{2(x^2-1)}{(x^2+1)^2} & \text{SE } x > -\dfrac{1}{2} \\[2ex] \dfrac{-2(x^2-2x+1)}{(x^2+1)^2} & \text{SE } x < -\dfrac{1}{2} \end{cases}$$

$f(x)$ NON DERIVABILE IN $x = -\frac{1}{2}$

STUDIO DEL SEGNO

$$f'(x) \leq 0 \rightarrow -\frac{1}{2} < \frac{2(x^2-1)}{(x^2+1)^2} < 0$$

d) $f(x) = e^{-x^2}(x^4 - 3x^2 + 1)$

$$f'(x) = -2x \cdot e^{-x^2}(x^4 - 3x^2 + 1) + e^{-x^2} \cdot (4x^3 - 6x) =$$

$$= e^{-x^2}\left(-2x \cdot (x^4 - 3x^2 + 1) + (4x^3 - 6x)\right)$$

$$= e^{-x^2}\left(-2x^5 + 6x^3 - 2x + 4x^3 - 6x\right) =$$

$$= e^{-x^2}\left(-2x^5 + 10x^3 - 8x\right) =$$

$$= -2e^{-x^2} \cdot x\left(x^4 - 5x^2 + 4\right) =$$

$$= -2e^{-x^2} \cdot x(x+2)(x-2)(x+1)(x-1)$$

$f(x) \geq 0 \Rightarrow -2e^{-x^2} \cdot x(x+2)(x-2)(x+1)(x-1) \geq 0$

$N1: e^{-x^2} \geq 0 \Rightarrow \forall x \in \mathbb{R}$

$N2: x \geq 0$

$N3: x+2 \geq 0 \rightarrow x \geq 2$

$N4: x \geq -2$

$N5: x \geq 1$

$N6: x \geq -1$

$N7: -2 \geq 0$ nAl

| | | -2 | | -1 | | 0 | | 1 | | 2 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| N7: | − | | − | | − | | − | | − | | − |
| N1: | + | | + | | + | | + | | + | | + |
| N2: | − | | − | | − | • | + | | + | | + |
| N3: | − | | − | | − | | − | | − | • | + |
| N4: | − | • | + | | + | | + | | + | | + |
| N5: | − | | − | • | + | | + | | + | | + |
| N6: | − | | − | | − | | − | • | + | | + |
| | + | • | − | | + | • | − | • | + | • | − |

$f'(x) = \begin{cases} > 0 & \text{si } x \in (-\infty, -2) \cup (-1, 0) \cup (1, 2) \\ = 0 & \text{si } x \in \{-2, -1, 0, 1, 2\} \end{cases}$

$<0$ si $x \in (-2,-1) \cup (0,1) \cup (2,+\infty)$

e) $f(x) = x \log|x|$

$|x| > 0 \Rightarrow x \neq 0$

$f'(x) = \log|x| + x \cdot \frac{1}{|x|} = \overbrace{\log|x|}^{} + 1$

$D = \mathbb{R} \setminus \{0\}$

$f'(x) \geq 0 \Rightarrow \log|x| + 1 \geq 0$

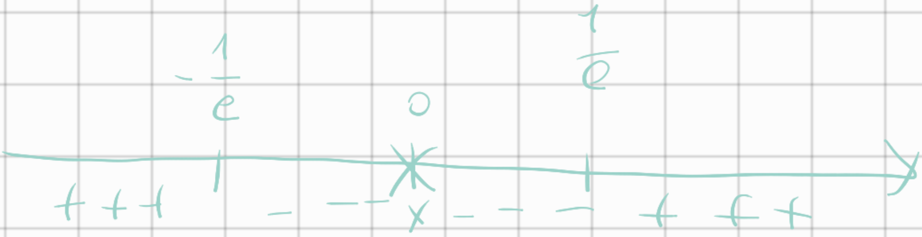
$\log|x| \geq -1 \Rightarrow e^{\log|x|} \geq e^{-1} \Rightarrow |x| \geq \frac{1}{e}$

$x \geq \frac{1}{e} \cup -x \geq \frac{1}{e}$

$x \geq \frac{1}{e} \cup x \leq -\frac{1}{e}$

$$f'(x) = \begin{cases} >0 & \text{si } x \in \left(-\infty, -\frac{1}{e}\right) \cup \left(\frac{1}{e}, +\infty\right) \\ =0 & \text{si } x \in \left\{-\frac{1}{e}, \frac{1}{e}\right\} \\ <0 & \text{si } x \in \left(\ \ \frac{1}{}\ \ \right) \cup \left(0\ \ \frac{1}{}\ \ \right) \end{cases}$$

$\leq 0$ se $x \in \left(-\frac{1}{e}, 0\right) \cup \left(0, \frac{1}{e}\right)$




f) $f(x) = \sqrt{x} - 4 \log(\sqrt{x} + 1)$

$$f'(x) = \frac{1}{2\sqrt{x}} - 4 \cdot \frac{1}{\sqrt{x}+1} \cdot \frac{1}{2\sqrt{x}} =$$

$$\frac{\sqrt{x}+1 - 4}{2\sqrt{x} \cdot (\sqrt{x}+1)} = \frac{\sqrt{x} - 3}{2\sqrt{x}(\sqrt{x}+1)}$$

$$\begin{cases} x \geq 0 \\ 2\sqrt{x}(\sqrt{x}+1) \neq 0 \end{cases} \Rightarrow 2x + 2\sqrt{x} \neq 0$$

$$x + \sqrt{x} \neq 0$$

$$x \neq 0$$

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$D = (0, +\infty)$

Pi-R $\quad x \in D = (0, +\infty)$

$$f'(x) \geq 0 \Rightarrow \frac{\sqrt{x} - 3}{2\sqrt{x}(\sqrt{x}+1)} \geq 0$$

N: $\sqrt{x} - 3 \geq 0 \Rightarrow \sqrt{x} \geq 3 \Rightarrow x \geq 9$

D: $2\sqrt{x}(\sqrt{x}+1) > 0 \Rightarrow 2x + 2\sqrt{x} > 0 \Rightarrow x + \sqrt{x} > 0$

$\Rightarrow x > 0$

| | | 0 | | 9 | |
|---|---|---|---|---|---|
| N: | − | | − | o | + |
| D: | − | o | + | | + |
| N/D: | + | o | − | o | + |

$$f'(x) = \begin{cases} > 0 & \text{se } x \in (9, +\infty) \\ = 0 & \text{se } x = 9 \\ < 0 & \text{se } x \in (0, 9) \end{cases}$$