3) b) $f(x) = \dfrac{\tan(\frac{x}{2})}{1-\cos(4x)}$

$\begin{cases} \frac{x}{2} \neq \frac{\pi}{2} + k\pi \\ 1-\cos(4x) \neq 0 \end{cases}$ $\begin{cases} x \neq \pi + 2k\pi \\ \cos(4x) \neq 1 \end{cases}$

$\begin{cases} x \neq \pi + 2k\pi \\ x \neq \frac{2k\pi}{4} = \frac{k\pi}{2} \end{cases}$

$\cos(x) = 1$

$x = 0 = 2\pi$

$\cos(4x) = 1$

$x = \frac{2\pi}{4}$

$D = \{x \in \mathbb{R} : x \neq \frac{k\pi}{2} \quad k \in \mathbb{Z}\}$

d) $f(x) = \arccos(x - \sqrt{x^2 - 3x}$

$-1 \leq x - \sqrt{x^2 - 3x} \leq 1$

$\begin{cases} x - \sqrt{x^2-3x} \geq -1 \\ x - \sqrt{x^2-3x} \leq 1 \end{cases}$ $\begin{cases} \sqrt{x^2-3x} \leq 1+x \\ \sqrt{x^2-3x} \geq -1+x \end{cases}$

$$\begin{cases} x^2-3x\geq 0 \\ 1+x\geq 0 \\ x^2-3x\leq (1+x)^2 \end{cases} \qquad \begin{cases} x(x-3)\geq 0 \\ x\geq -1 \\ \cancel{x^2}-3x\leq 1+\cancel{x^2}+2x \end{cases}$$

$$\begin{cases} x\leq 0 \ \vee \ x\geq 3 \\ x\geq -1 \\ -5x\leq 1 \end{cases} \qquad \begin{cases} x\leq 0 \ \vee \ x\geq 3 \\ x\geq -1 \\ x\geq -\frac{1}{5} \end{cases}$$

$$x\in\left[-\frac{1}{5},0\right]$$

$$\begin{cases} x(x-3)\geq 0 \\ -1+x<0 \\ \cancel{x^2-3x\leq 1+x^2-2x} \end{cases} \qquad \begin{cases} x\leq 0 \ \vee \ x\geq 3 \\ x<1 \end{cases}$$

$$x\in(-\infty,0]$$

Quindi

$$D=\left[-\frac{1}{5},0\right]$$

4) a) $f(x) = \dfrac{4x+1}{x-2}$

$D = \mathbb{R} \setminus \{2\}$

$y = \dfrac{4x+1}{x-2}$

$yx - 2y = 4x + 1$

$yx - 2y - 4x = 1$

$x(y-4) = 2y+1$

$x = \dfrac{2y+1}{y-4}$ con $y \neq 4$

Quindi

$f(D) = \mathbb{R} \setminus \{4\}$ è invertibile con inversa

$f^{-1}(x) = \dfrac{1+2x}{x-4}$

b) $f(x) = \frac{x^2+1}{2x}$

$D = \mathbb{R} \setminus \{0\}$

$y = \frac{x^2+1}{2x}$

$2xy = x^2+1$

$2xy - x^2 - 1 = 0$

$x^2 - 2xy + 1 = 0$

$\Updownarrow$

$x_1 = y + \sqrt{y^2-1}$

$x_2 = y - \sqrt{y^2-1}$ con $|y| \geq 1$

$f(D) = (-\infty, -1] \cup [1, +\infty)$

NON È INVERTIBILE POICHÉ x_1 E x_2 SONO DISTINTI E QUINDI f NON È INIETTIVA

c) $f(x) = \dfrac{2}{3+\log\left(\frac{x+1}{x}\right)}$

$$\begin{cases} 3+\log\left(\frac{x+1}{x}\right) \neq 0 \\ x \neq 0 \\ \frac{x+1}{x} > 0 \end{cases} \qquad \begin{cases} \log\left(\frac{x+1}{x}\right) \neq -3 \\ \frac{x+1}{x} > 0 \\ x \neq 0 \end{cases}$$

$$\begin{cases} \frac{x+1}{x} \neq e^{-3} \\ x < -1 \quad \cup \quad x > 0 \end{cases}$$

$\rightarrow \frac{x+1}{x} \neq \frac{1}{e^3}$

$\frac{e^3 x + e^3}{x} \neq 1$

$e^3 x + e^3 \neq x$

$e^3 x - x \neq -e^3$

$x(e^3 - 1) \neq -e^3$

$x \neq -\frac{e^3}{e^3-1}$

$$\begin{cases} x \neq -\frac{e^3}{e^3-1} \end{cases}$$

$x < -1 \lor x > 0$

$D = (-\infty, -1) \setminus \left\{ -\frac{e^3}{e^3 - 1} \right\} \cup (0, +\infty)$

$y = \frac{2}{3 + \log\left(\frac{x+1}{x}\right)}$

$3y + y \log\left(\frac{x+1}{x}\right) = 2$

$y \log\left(\frac{x+1}{x}\right) = -3y + 2$

$\log\left(\frac{x+1}{x}\right) = \frac{-3y+2}{y}$

$\frac{x+1}{x} = e^{\frac{-3y+2}{y}}$

$1 + \frac{1}{x} = e^{\frac{-3y+2}{y}}$

$\frac{1}{x} = e^{\frac{-3y+2}{y}} - 1$

$x = \frac{1}{e^{\frac{-3y+2}{y}} \ldots}$ con $\boxed{y \neq 0}$

$-3y+2$

$$e^{\cdots} - 1 \qquad \cdot\, e^{\frac{\cdots}{y}} \neq 1$$

$$\Updownarrow$$

$$e^{\frac{-3y+2}{y}} \neq e^{0}$$

$$\frac{-3y+2}{y} \neq 0$$

$$-3y \neq -2 \iff \boxed{y \neq \frac{2}{3}}$$

$f(D) = \mathbb{R} \setminus \{0, \frac{2}{3}\}$ e f è

invertibile con inversa

$$f^{-1}(x) = \frac{1}{e^{\frac{-3x+2}{x}} - 1}$$