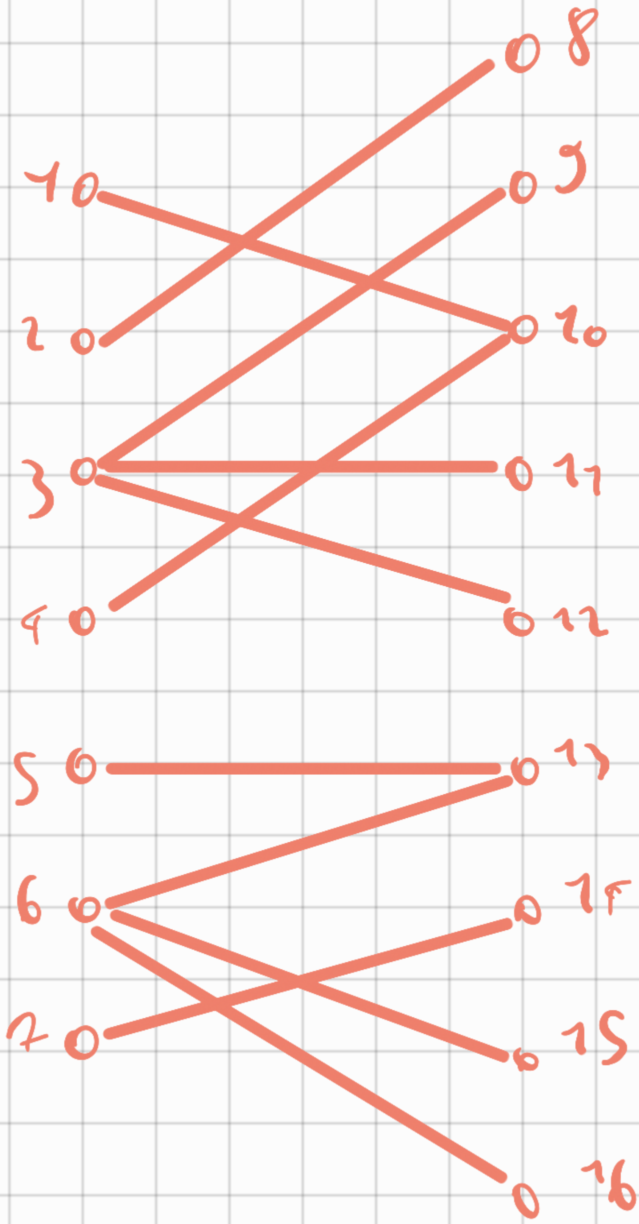
S_{12}

$G =$ 

ESISTE UN ACCOPPIAMENTO DI V_1 IN V_2?

UN ACCOPPIAMENTO DA V_1 IN V_2 NON ESISTE PERCHÉ, SE

$$S \stackrel{def}{=} \{1, 4\} \Rightarrow N_G(S) = \{10\} \text{ E}$$

$$|S| = 2 \not\leq 1 = |N_G(S)|$$

S_{13} $G = (V, E)$ DOVE

$$V \stackrel{def}{=} \binom{[5]}{2} \cup \binom{[5]}{3}$$

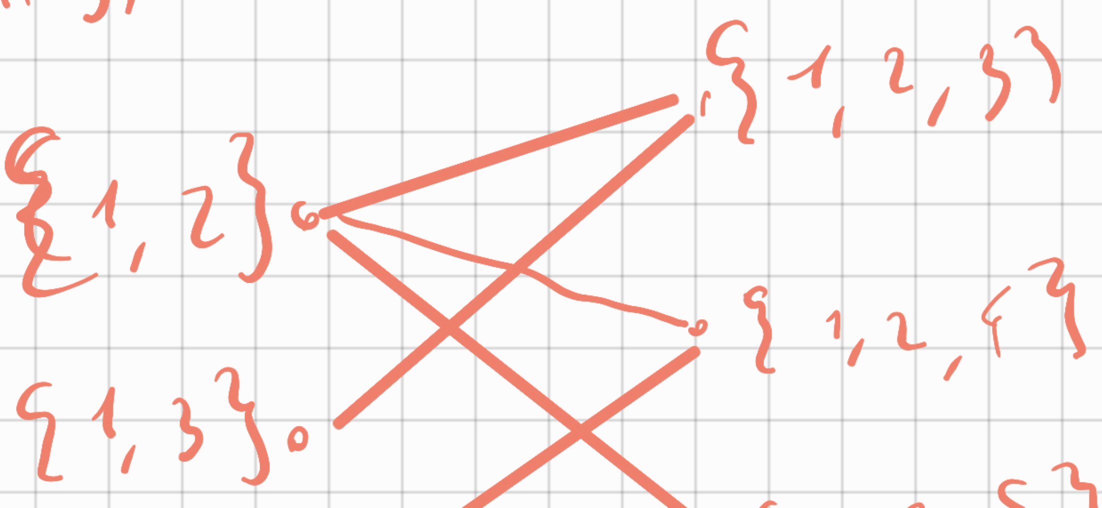
$\binom{S}{k} \stackrel{def}{=} \{A \subseteq S : |A| = k\}$), E se

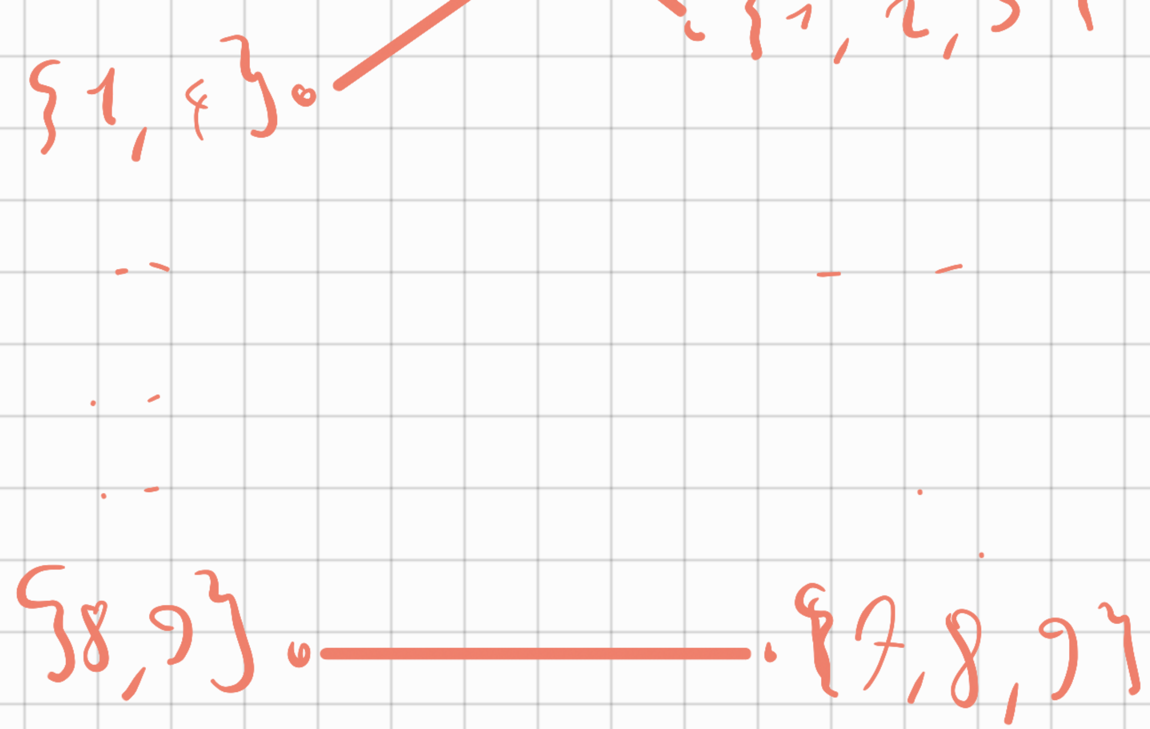
$X, Y \in V$ allora

$$\{X, Y\} \in E$$
$$\Updownarrow$$
$$X \subsetneq Y \text{ o } Y \subsetneq X$$

(Per es: $\{\{1,3\},\{1,3,4\}\} \in E$, $\{\{1,3\},\{2,3,4\}\} \notin E$)

Quindi





Esiste un accoppiamento di V_1, V_2?

Beh

$$|V_1| = \left|\binom{[9]}{2}\right| = \binom{9}{2} = \frac{9\cdot 8}{2} = 45$$

$$|V_2| = \left|\binom{[9]}{3}\right| = \binom{9}{3} = \frac{9\cdot 8\cdot 7}{3\cdot 2} = 84$$

– Teorema di Hall? Forse no

– Costretto nei gradi da V_1 in V_2?

Sia $x \in V_1 \Rightarrow x = \{a,b\}$ con $a,b \in [9]$

$a \neq b$

: Allora

$$d(X) = |\{Y \in V_2 : \{X, Y\} \in E\}|$$

$$= |\{Y \in \binom{[9]}{3} : X \subseteq Y\}|$$

$$= |\{\{a, d, e\} \in \binom{[9]}{3} : \{a, b\} \subseteq \{a, d, e\}\}|$$

$$= |\{\{a, b, c\} : c \in [9] \setminus \{a, b\}\}| = 7$$

Sia $Y \in \binom{[9]}{3}$, $Y = \{a, b, c\}$, $a, b, c \in [9]$

Allora

$$d(Y) = |\{X \in \binom{[9]}{2} : X \subseteq Y\}|$$

$$= |\{X \in [9] : |X| = 2,\ X \subseteq \{a, b, c\}\}|$$

$$= |\{\{a, b\}, \{a, c\}, \{b, c\}\}| = 3$$

Pertanto

$$d(X) = 7 \neq 3 = d(Y)$$

$d(x) = 7 \geq 3 = d(y)$

$\forall x \in V_1$ e $\forall y \in V_2 \Rightarrow G$ è costruito nei grafi da V_1 in $V_2 \Rightarrow$ sappiamo dalla teoria che $\exists$ un accoppiamento di V_1 in V_2

Sia $G =$