# The use of Monte Carlo simulation in evaluating the elevator round trip time under up-peak traffic conditions and conventional group control

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# The Use of Monte Carlo Simulation in Evaluating the Elevator Round Trip Time under Up-peak Traffic Conditions

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**Abstract**

The design of vertical transportation systems still relies on the evaluation of the round trip of the elevators during the up peak (incoming) traffic conditions in a building. The evaluation of the round trip time for anything other than the most straightforward case becomes very complicated and requires the use of advanced special case formulae. These formulae become even more complicated when a combination of the special cases exist within the building being designed. The most two prominent examples of these special cases are the case of multiple entrances to the building (rather than a single entrance) and the case where the top speed is not attained within one floor journey (or even two or three floor journeys).

The use of the Monte Carlo simulation is presented in this paper as a simple and practical means to calculate the round trip time for an elevator during the up peak (incoming) traffic conditions, under a combination of any or all of the special conditions such as: multiple entrances, top speed not attained within one or more floor journeys, unequal floor heights and unequal floor populations.

Analytical methods are used to show that the Monte Carlo simulation produces the same results for real life cases of multiple entrances and where the top speed is not attained in a one floor journey. The structure and architecture of the Monte Carlo simulation tool used is discussed in detail. The practical details that are used to ensure the speed of the tool in producing an answer are also discussed.

**Keywords:** Monte Carlo simulation, elevator, lift, round trip time, interval, up peak traffic, basement, entrance, sub-entrance, highest reversal floor, probable number of stops.

## 1. INTRODUCTION

The design of vertical transportation systems in a building involves the selection of the suitable number, speed and capacity of elevators that achieves the required performance parameter. The main two methods currently used are calculation and simulation.

Calculation is a deterministic method that works from first principles to calculate the time it would take one elevator to do a complete cycle within the

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building during up-peak (incoming) traffic conditions. The round trip time is the time taken by an elevator to pick up passengers from the ground floor (also referred to as the lobby or main terminal) and delivering them to their destinations and then returning back to the ground floor. By dividing the round trip time by the number of elevators in the group, the interval is obtained and this is used as the performance parameter for the design. The interval is the time between elevator arrivals in the ground floor. The interval is effectively an elevator centric (rather than a passenger centric) measure of performance.

Simulation is a probabilistic method that carries out a time-slice based simulation in the building by generating passengers and ‘processing’ them within the elevator system until they are delivered to their destinations. The outcome of the simulation is generally the passenger waiting time, passenger travelling and overall journey time. These parameters are effectively passenger centric parameters.

Nevertheless, the interval is still widely quoted as a performance parameter. It is still invariably used as the acceptance criterion for the performance for vertical transportation systems in office buildings between the developer and the client. For these reasons there is still a need to calculate the interval. Calculating the interval relies on the accurate calculation of the round trip time.

The classical method for calculating the round trip is simple for the basic straightforward case, but tends to get very complicated as the building configuration becomes more complicated and as the elevator parameters deviate from the simple conditions. The classical method can deal with unequal populations, by amending the formula for the highest reversal floor ($H$) and the probable number of stops ($S$). Specifically, the classical method of calculating the round trip time tends to get complicated in any of the following situations:

1. The classical method assumes that the passenger arrivals occur from one entrance. Where multiple entrances exist the basic formula needs to be adjusted.

2. The classical method assumes that the top elevator speed is attained within a one floor journey. Where the speed is too high to be achieved in a one floor journey, this introduces an error.

3. The classical method assumes equal floor heights.

4. The classical method assumes equal floor populations.

The classical method becomes extremely complicated and intractable when more than one (or even all) of the above conditions are combined. This papers presents an alternative method of calculating the round trip time when any or all of these special cases exist. This alternative method is based on the use of the Monte Carlo simulation tool.

The formula used in the classical method for calculating the round trip time is first examined. The use of the Monte Carlo simulation in the calculation of the round trip time is introduced. The method is then compared to the two most important special cases: the case of multiple entrances (e.g., basements) and the case where the top speed is not attained in a one floor journey. It is shown how the Monte Carlo simulation method provides the same answer to these two special cases when compared to the complicated calculation methods required in these two special cases. Numerical examples are given for both cases. More details are then given regarding the practical implementation of the Monte Carlo simulation method for calculating the round trip time within Matlab and how the response time of the method can be kept to a minimum value to make it practical to be used as an everyday calculation method.

## 2. THE CLASSICAL METHOD OF CALCULATING OF THE RTT

The traditional method used in the design of vertical transportation systems is the calculation of the round trip time for an elevator during the up-peak traffic. This assumes that all traffic is entering the building (i.e., incoming) and that the elevator picks up the passengers from the main entrance and delivers them to the upper levels. The round trip time ($\tau$) is the cycle time taken by the elevator to pick up the passengers from the main entrance, deliver them to the upper levels and then return back to the main entrance.

The value of the round trip time ($\tau$) for an elevator during up peak conditions can be calculated as follows [1], [2]:

$$\tau = 2 \cdot H \cdot \left( \frac{d_f}{v} \right) + (S+1) \cdot \left( t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao} \right) + P(t_{pi} + t_{po}) \qquad (1)$$

where:

$\tau$ is the round trip time in s

$H$ is the highest reversal floor (where floors are numbered 0, 1, 2….$N$

$S$ is the probable number of stops (not including the stop at the ground floor)

$d_f$ is the typical height of one floor in metres

$v$ is the top rated speed in metres per second

$t_f$ is the time taken to complete a one floor journey in seconds assuming that the elevator attains the top speed $v$

$P$ is the number of passengers in the car when it leaves the ground floor

$t_{do}$ is the door opening time in seconds

$t_{dc}$ is the door closing time in seconds

$t_{sd}$ is the motor start delay in seconds

$t_{ao}$ is the door advance opening time in seconds (where the door starts opening before the car comes to a complete standstill)

$t_{pi}$ is the passenger boarding time in seconds

$t_{po}$ is the passenger alighting time in seconds

The probable number of stops $S$ for equal floor populations can be calculated as follows [1] and [2]:

$$S = N \cdot \left( 1 - \left( 1 - \frac{1}{N} \right)^{P} \right) \quad (2)$$

…and as follow for unequal floor populations (where $U_i$ is the population of floor *i* and *U* is the total building population) [1], [2]:

$$S = N - \sum_{i=1}^{N} \left( 1 - \frac{U_i}{U} \right)^{P} \quad (3)$$

The highest reversal can be calculated as follows for equal floor populations [1] and [2]:

$$H = N - \sum_{i=1}^{N-1} \left( \frac{i}{N} \right)^{P} \quad (4)$$

…and as follows for unequal populations (where $U_i$ is the population of floor *i* and *U* is the total building population) [1], [2]:

$$H = N - \sum_{j=1}^{N-1} \left( \sum_{i=1}^{j} \frac{U_i}{U} \right)^{P} \quad (5)$$

The formula for the round trip time implicitly makes a number of assumptions:

1. The floor heights are equal.

2. The top speed is attained in one floor to floor journey.

3. All passenger arrivals are from the main entrance. It is assumed that this is the only entrance (i.e., no basement or other entrances).

The effect of the first assumption can be accounted for by taking the average of the height of all floors without major loss in accuracy.

However, the second assumption is very significant especially at high rated speeds. For speeds above 2.5 m/s and typical floor heights of around 4 m, the error can be become very significant. This issue is addressed by the work in [3] that amends the round trip time equation to account for the fact that the top speed is not attained one floor jump.

The third assumption is also very significant especially with large numbers of basements having a high percentage of the arrivals.

It is important to note that the work in this paper is restricted to the special case of up-peak (incoming) traffic only. The design of vertical transportation systems is still mainly based on the calculation of the round trip time during the morning up-peak (incoming) traffic conditions.

## 3. MONTE CARLO SIMULATION AS A CALCULATION TOOL

The block diagram of the Monte Carlo simulation tool for the calculation of the round trip time of the elevator during up peak conditions is shown in Figure 1. It comprises the following main blocks:

1. The random scenario parameter generator.

2. The kinematics calculator.

3. The round trip time calculator.

In addition a master module controls the three blocks above to run each block and produce the final result and interface with the user. These three blocks above are explained in more detail below.

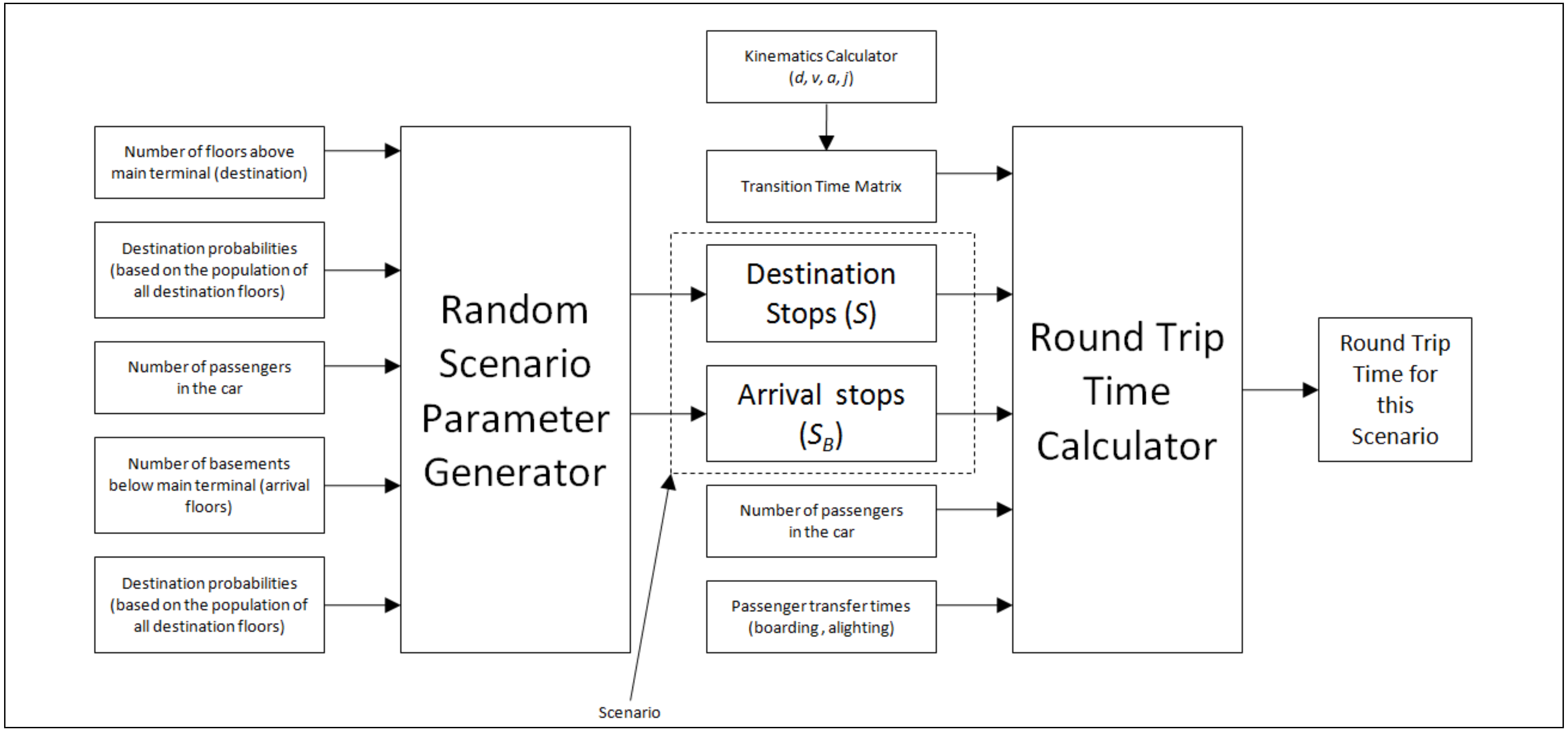


**Figure 1: Block diagram of the Monte Carlo Simulation round trip time tool.**

Under up peak (incoming traffic) conditions, it is assumed that passengers arrive at the arrival floors, denoted as *NB* to *B1* as well as *G* (the main terminal or the ground floor). They make their destination selections and then alight at their respective floors. This is shown in a diagrammatic format in Figure 2 below.

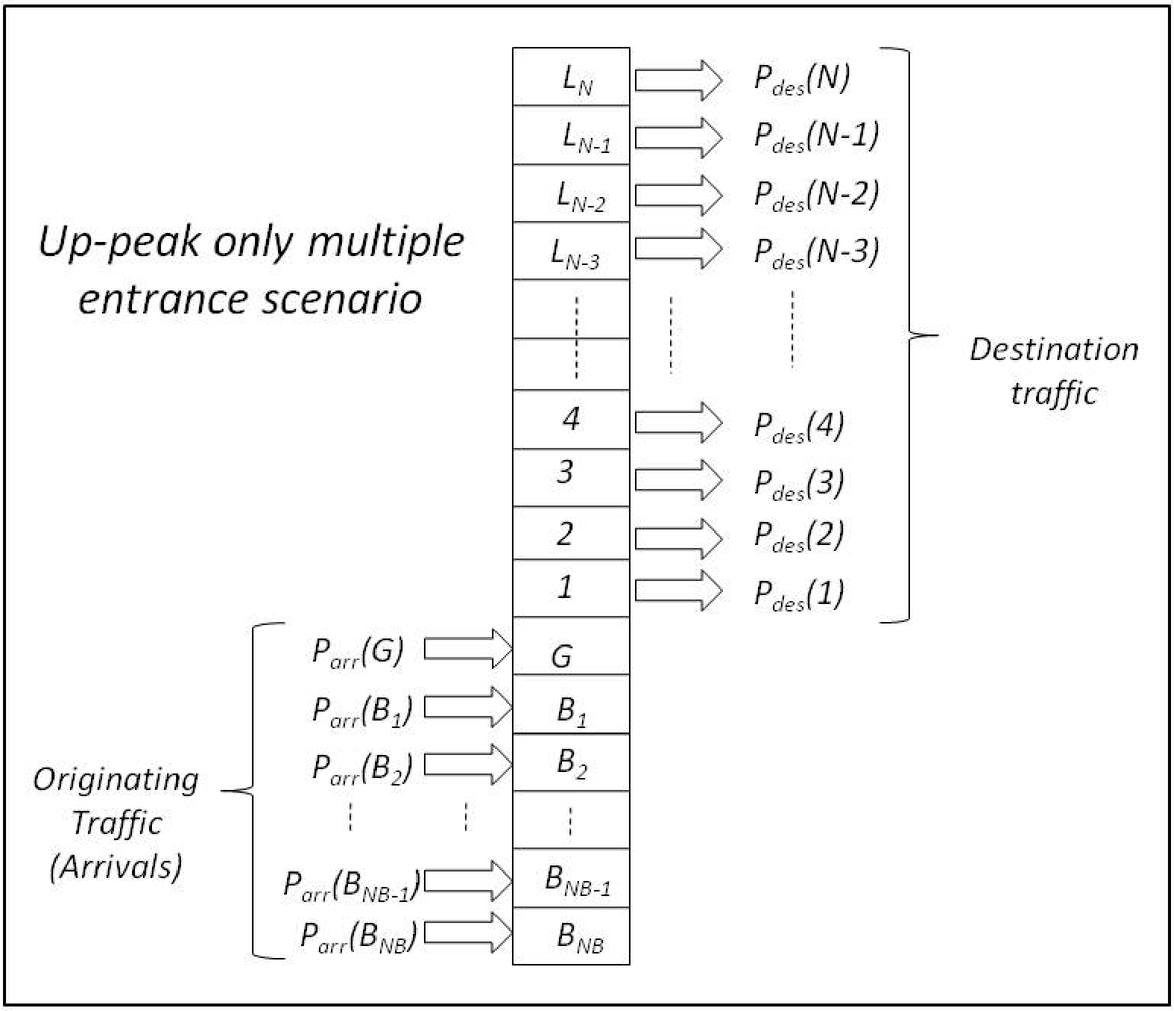

Figure 2: Demand and performance model of an elevator system.

### 3.1 Random scenario parameter generator (RSPG)

The purpose of this block is to generate the stops that the elevator will stop at the arrival floors (e.g., ground and basement floors) and the stops at the destination floors (floors above the main terminal). These stops represent the stops for one round trip time scenario only. In order to generate these stops in the arrival floors and destination floors, the following five pieces of information are needed:

a) The number of passengers in the car, denoted as *P*. This is defined as the number of passengers in the car as it leaves the highest arrival floor heading for the destination floors. Alternatively, it can be defined as the total number of passengers arriving from the arrival floors and the total number of passengers heading for the destination floors. This number does not necessarily need to be a whole number (integer). When expressed as a non-integer (e.g., 8.4) it represents an average number over a number of journeys. As will be discussed later, the Monte Carlo simulation tool can deal with this case as well.

b) The number of arrival floors (e.g., basements and ground floor). This is usually denoted as *NB*.

c) The probability of passenger arrivals from the arrival floors. This is denoted as $P_{arr}(i)$ which stands for the probability of passenger arrival from the arrival floor *i* where *i* runs from 1 to *NB*.

d) The number of destination floors (floors above the main terminal). This is usually denoted as *N*.

e) The probability of passengers heading to the destination floors. This is denoted as $P_{dest}(i)$ which stands for the probability of passenger arrival from the arrival floor *i* where *i* runs from 1 to *N*.

The probabilities of arrival and destination are then used to randomly generate the arrival and destination floor based on the number of passengers in the car. The number of stops in the arrival floors is denoted as $S_{NB}$ and the number of stops in the destination floors is denoted as $S_N$. These stops will vary from one scenario to the next, but the average of the values from all scenarios will converge to the value that is given by the formulae (2) and (3) for the probable number of stops.

It is worth noting that for the destination floors the following holds true:

$$1 \leq S_N \leq N \qquad (6)$$

where
$S_N$ is the number of stops in one round trip at the destination floors
*N* is the number of destination floors

In a similar way, the following applies to the arrival floors:

$$1 \leq S_{NB} \leq NB \qquad (7)$$

where
$S_{NB}$ is the number of stops in one round trip at the arrival floors
*NB* is the number of arrival floors

### 3.2 The kinematic calculator (KC)

The kinematic calculator calculates the time taken by the elevator to travel from any floor to another floor. This produces a two dimensional matrix that shows the time required to travel from the floor i to the floor *j*. The starting floor *i* can be any of the arrival or destination floors (*NB* to *N*) and the destination floor *j* can be any of the arrival or destination floors (*NB* to *N*). A diagram of such a matrix is shown in Table 1.

The following can be noted from the matrix:

1. As expected the diagonal values are all zero, which is as expected as the time to go from a floor to the same floor is zero.

$$t_{ij} = 0 \quad \text{where } i = j \tag{8}$$

2. The upper triangle of the matrix is identical to the lower triangle of the matrix. And this is expected as the time to travel form a floor *i* to floor *j* is equal to the time to travel from time floor *j* to floor *i*.

$$t_{ij} = t_{ji} \quad \text{for any value of } i \text{ and } j \tag{9}$$

So it is only necessary to calculate the upper triangle and equate it to the lower triangle.

**Table 1: Transition times matrix.**

| | NB | NB-1 | NB-2 | ... | ... | B1 | G | 1 | 2 | ... | ... | ... | N-2 | N-1 | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **NB** | 0 | $t_{NB-1\ NB}$ | $t_{NB-2\ NB}$ | ... | ... | $t_{B1\ NB}$ | $t_{G\ NB}$ | $t_{1\ NB}$ | $t_{2\ NB}$ | ... | ... | ... | $t_{N-2\ NB}$ | $t_{N-1\ NB}$ | $t_{N\ NB}$ |
| **NB-1** | $t_{NB\ NB-1}$ | 0 | $t_{NB-2\ NB-1}$ | ... | ... | $t_{B1\ NB-1}$ | $t_{G\ NB-1}$ | $t_{1\ NB-1}$ | $t_{2\ NB-1}$ | ... | ... | ... | $t_{N-2\ NB-1}$ | $t_{N-1\ NB-1}$ | $t_{N\ NB-1}$ |
| **NB-2** | $t_{NB\ NB-2}$ | $t_{NB-1\ NB-2}$ | 0 | ... | ... | $t_{B1\ NB-2}$ | $t_{G\ NB-2}$ | $t_{1\ NB-2}$ | $t_{2\ NB-2}$ | ... | ... | ... | $t_{N-2\ NB-2}$ | $t_{N-1\ NB-2}$ | $t_{N\ NB-2}$ |
| **...** | ... | ... | ... | 0 | $t_{ij}$ | ... | ... | ... | ... | $t_{ij}$ | $t_{ij}$ | $t_{ij}$ | ... | ... | ... |
| **...** | ... | ... | ... | $t_{ij}$ | 0 | ... | ... | ... | ... | $t_{ij}$ | $t_{ij}$ | $t_{ij}$ | ... | ... | ... |
| **B1** | $t_{NB\ B1}$ | $t_{NB-1\ B1}$ | $t_{NB-2\ B1}$ | ... | ... | 0 | $t_{G\ B1}$ | $t_{1\ B1}$ | | ... | ... | ... | $t_{N-2\ B1}$ | $t_{N-1\ B1}$ | $t_{N\ B1}$ |
| **G** | $t_{NB\ G}$ | $t_{NB-1\ G}$ | $t_{NB-2\ G}$ | ... | ... | $t_{B1\ G}$ | 0 | $t_{1\ G}$ | $t_{2\ G}$ | ... | ... | ... | $t_{N-2\ G}$ | $t_{N-1\ G}$ | $t_{N\ G}$ |
| **1** | $t_{NB\ 1}$ | $t_{NB-1\ 1}$ | $t_{NB-2\ 1}$ | ... | ... | $t_{B1\ 1}$ | $t_{G\ 1}$ | 0 | $t_{2\ 1}$ | ... | ... | ... | $t_{N-2\ 1}$ | $t_{N-1\ 1}$ | $t_{N\ 1}$ |
| **2** | $t_{NB\ 2}$ | $t_{NB-1\ 2}$ | $t_{NB-2\ 2}$ | ... | ... | $t_{B1\ 2}$ | $t_{G\ 2}$ | $t_{1\ 2}$ | 0 | ... | ... | ... | $t_{N-2\ 2}$ | $t_{N-1\ 2}$ | $t_{N\ 2}$ |
| **...** | ... | ... | ... | $t_{ij}$ | $t_{ij}$ | ... | ... | ... | ... | 0 | $t_{ij}$ | $t_{ij}$ | ... | ... | ... |
| **...** | ... | ... | ... | $t_{ij}$ | $t_{ij}$ | ... | ... | ... | ... | $t_{ij}$ | 0 | $t_{ij}$ | ... | ... | ... |
| **...** | ... | ... | ... | $t_{ij}$ | $t_{ij}$ | ... | ... | ... | ... | $t_{ij}$ | $t_{ij}$ | 0 | ... | ... | ... |
| **N-2** | $t_{NB\ N-2}$ | $t_{NB-1\ N-2}$ | $t_{NB-2\ N-2}$ | ... | ... | $t_{B1\ N-2}$ | $t_{G\ N-2}$ | $t_{1\ N-2}$ | $t_{2\ N-2}$ | ... | ... | ... | 0 | $t_{N-1\ N-2}$ | $t_{N\ N-2}$ |
| **N-1** | $t_{NB\ N-1}$ | $t_{NB-1\ N-1}$ | $t_{NB-2\ N-1}$ | ... | ... | $t_{B1\ N-1}$ | $t_{G\ N-1}$ | $t_{1\ N-1}$ | $t_{2\ N-1}$ | ... | ... | ... | $t_{N-2\ N-1}$ | 0 | $t_{N\ N-1}$ |
| **N** | $t_{NB\ N}$ | $t_{NB-1\ N}$ | $t_{NB-2\ N}$ | ... | ... | $t_{B1\ N}$ | $t_{G\ N}$ | $t_{1\ N}$ | $t_{2\ N}$ | ... | ... | ... | $t_{N-2\ N}$ | $t_{N-1\ N}$ | 0 |

The inputs to the kinematics calculator are as follows:

a) The floor to floor distances for all floors.

b) The top speed, the top acceleration and the top jerk ($v_{max}$, $a_{max}$ and $j_{max}$).

The transition time between two floors separated by a distance *d*, with a top speed of *v*, top acceleration *a,* and top jerk *j* can be calculated as follows under the three different conditions [4]:

$$\text{If } d \geq \left( \frac{a^2 v + v^2 j}{aj} \right) \text{ then } t = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} \tag{10}$$

$$\text{If } \frac{2a^3}{j^2} \leq d < \left( \frac{a^2 v + v^2 j}{aj} \right) \text{ then } t = \frac{a}{j} + \sqrt{\frac{4d}{a} + \left( \frac{a}{j} \right)^2} \tag{11}$$

$$\text{If } d < \frac{2a^3}{j^2} \text{ then } t = \left( \frac{32d}{j} \right)^{\frac{1}{3}} \tag{12}$$

**3.3 Round trip time calculator**

The round trip time calculator uses the outputs of the random scenario parameter generator and the kinematic calculator to find the value of the round trip time. The random scenario parameter generator produces the position of all stops in the arrival floors and the destination floors.  The kinematic calculator produces the transition matrix that gives the times required to travel between the various floors.

The round trip time is made up of the following items (Figure 3):

a) The time taken to travel upwards between all the stops (arrival stops and destination stops).  This is taken from the transition matrix, where it is used as a lookup table.

b) The time taken to express back from the top most stop to the lowest most stop in the next scenario.  This time is also taken from the transition matrix, where it is used as a lookup table.

c) The time spent opening and closing the doors at each stop.

d) The time taken by each passenger to alight and board.

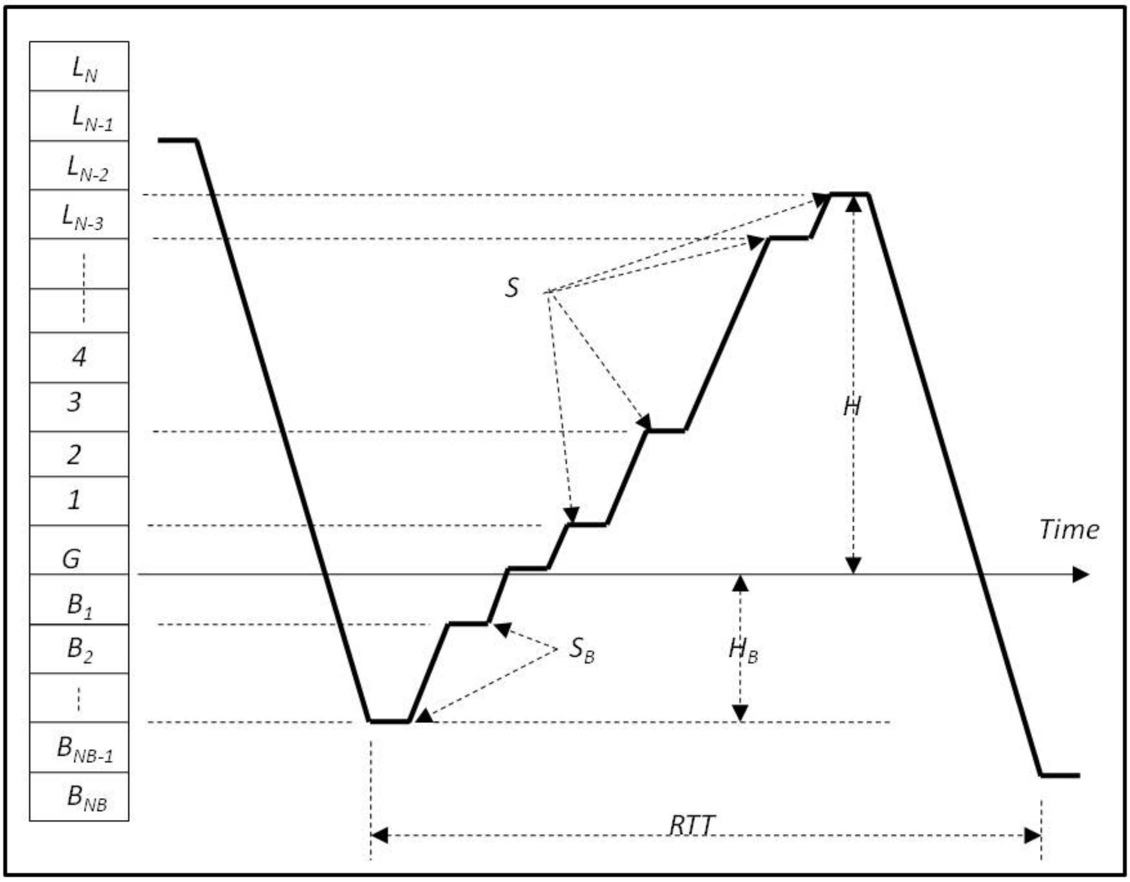


Figure 3: Round trip time timeline where the elevator goes down to the basement.

### 3.4 Scenario running module

This module is the overall controller that runs the scenarios.

It first runs the kinematics calculator at the start in order to generate the transition matrix. The advantage of doing this is to save time during the running of the scenarios later. Once the transition matrix is populated, it takes much less time to find the time to travel between any two floors by looking up the value from the matrix rather than doing the calculation.

It then starts running the scenarios. A large number of scenarios is required (e.g., 10 000 or 100 000). The user can control the number of required scenarios. The higher the number of scenarios, the better the confidence of the final obtained value for the round trip time.

The running of the software is shown in the flowchart in Figure 4. The next two sections show how the results from the Monte Carlo simulation calculator are verified against classical advanced calculation methods for two special cases: the case of multiple entrances (e.g. basements) and the case where the top elevator speed is not attained in a one floor journey.

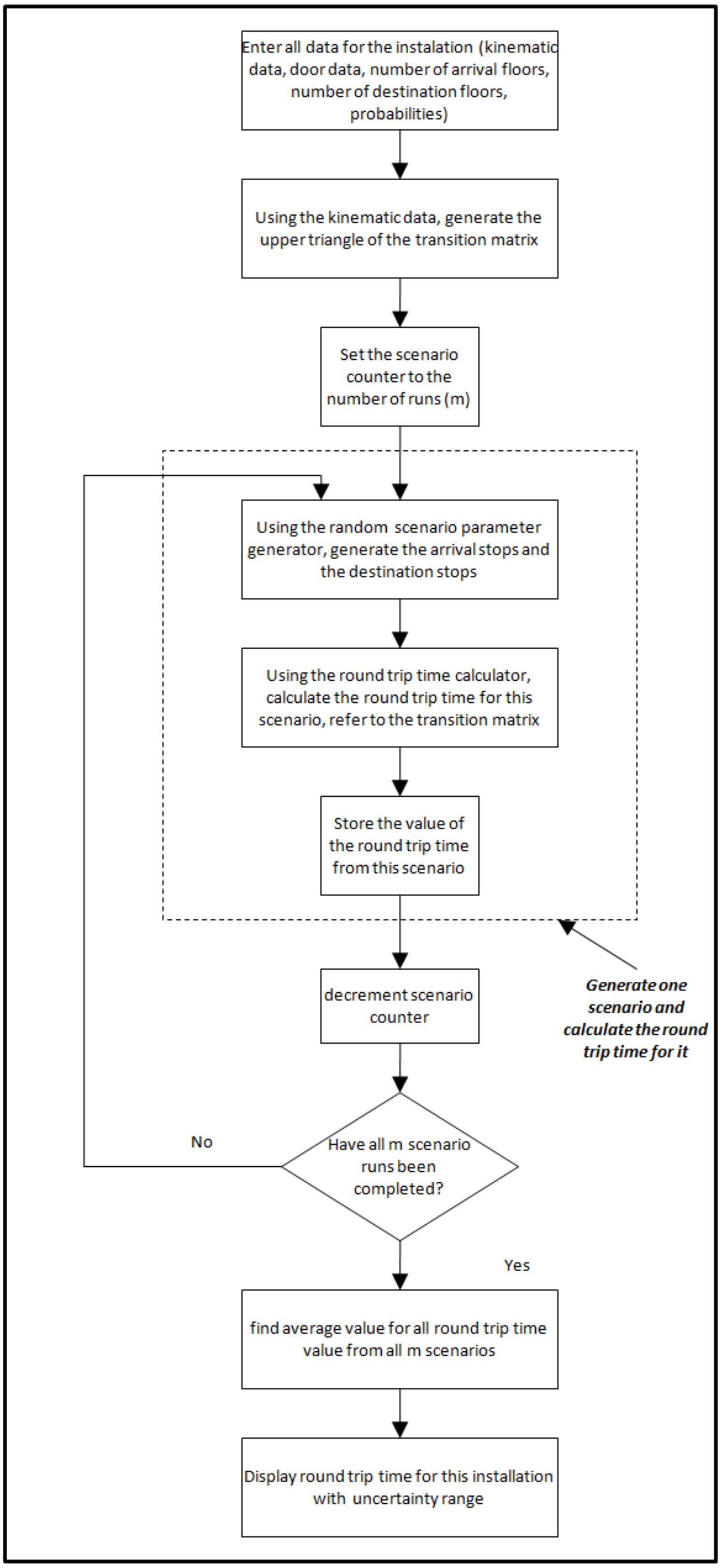


**Figure 4: Flowchart for the Monte Carlo round trip time calculator.**

## 4. MULTIPLE ENTRANCE ARRIVALS

The first special case that is dealt with is the case of the multiple entrances. This is encountered in buildings that have car parks beneath the main terminal. Passengers arrive in the car park entrances as well as the main terminal (Ground floor). The mathematical analysis is introduced in the next sub-section. A numerical example is then given in the following sub-section that compares the results from the Monte Carlo simulation calculator with one of the classical calculation methods for multiple entrances.

### 4.1 Mathematical Analysis

Multiple entrance scenarios are encountered in practice where a number of car parks exist below the main terminal (the ground floor). Passengers arrive in the car parks (having parked their cars) and access the elevator system from their respective car park floors. A percentage of passengers arrive in the main terminal (Ground floor). The basic round trip equation assumes one single entrance and thus cannot be used in this case, because the elevator will travel down to the basement floors, pick up passengers from there and then travel upwards and pick up passengers from the ground floor and then deliver them to the upper destination floors as usual.

The $H_BS_B$ method is one of two methods developed in [5] to deal with the case of multiple entrances in the calculation of the round trip time. It uses the concept of highest reversal floor ($H$) and the probable number of stops ($S$) that has been traditionally used in the upper destination floors in the classical round trip time equation and finds their equivalent in the arrival floors ($H_B$ and $S_B$). The classical round trip time formula for the single arrival entrance case is then developed to cover the various entrance floors by including $H_B$ and $S_B$ in the equation.

### 4.2 Example Case Study

In order to illustrate the use of the Monte Carlo simulation method in the case of multiple entrances, a case study is shown below. It uses the so-called $H_BS_B$ methods mentioned in the last sub-section.

An example from an office building is used to illustrate the use of the Monte Carlo simulation in the case of a building with multiple entrances. The parameters for a typical office building are shown below.

a. Number of floors above ground is 14 floors.
b. Car capacity is 21 person (1600 kg).
c. Floor height is 4.5 m (finished floor level to finished floor level).
d. Basement floor height is 4.5 m.
e. Top value of speed, $v$, is 1.6 $m \cdot s^{-1}$.
f. Top value of acceleration, $a$, is 1 $m \cdot s^{-2}$.
g. Top value of jerk, $j$, is 1 $m \cdot s^{-3}$.
h. Passenger transfer time out of the car is 1.2 s.
i. Passenger transfer time into the car is 1.2 s.

j. Door opening time is 2 s.
k. Door closing time is 3 s.
l. Advanced door opening is 0.5 s.
m. Start delay is 1 s.
n. Total building population of 1100 persons.
o. Equal floor populations.

The first step is to calculate the round trip time without the basement ($\tau_G$). The equation for the round trip time where the only entrance is the ground floor ($\tau_G$) can be written as follows (adapted from (1) with the subscript *G* added):

$$\tau_G = 2 \cdot H \cdot \left(\frac{d_f}{v}\right) + (S+1) \cdot \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P\left(t_{pi} + t_{po}\right) \qquad (13)$$

Where:
$\tau_G$ is the round trip time in s
*H* is the highest reversal floor (where floors are numbered 0, 1, 2….*N*
S is the probably number of stops
$d_f$ is the typical height of one floor in m
*v* is the top rated speed in m/s
$t_f$ is the time taken to complete a one floor journey in s assuming that the elevator attains the top speed *v*
*P* is the number of passengers in the car when it leaves the ground floor
$t_{do}$ is the door opening time in s
$t_{dc}$ is the door closing time in s
$t_{sd}$ is the motor start delay in s
$t_{ao}$ is the door advance opening time in s (where the door starts opening before the car comes to a complete standstill)
$t_{pi}$ is the passenger boarding time in s
$t_{po}$ is the passenger alighting time in s

A check needs to be carried out to ensure that the elevator will attain top speed in a one floor journey, as follows [4]:

$$d_f = 4.5 \geq \left(\frac{a^2 v + v^2 j}{aj}\right) = \left(\frac{1 \cdot 1.6 + 1.6^2 \cdot 1}{1}\right) = 4.16 \text{ m} \qquad (14)$$

So the top speed of 1.6 m/s will be attained in a one floor jump. In this case, the time taken to complete a one floor journey, $t_f$ will be [4]:

$$t_f = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 5.4\,s \qquad (15)$$

The value of *P* based on 80% car rated capacity is 16.8 passengers (on average). Moving on to calculate *H* and *S* as follows (assuming equal floor population), using (2) for *S*:

$$S = N \cdot \left(1 - \left(1 - \frac{1}{N}\right)^{P}\right) = 14 \cdot \left(1 - \left(1 - \frac{1}{14}\right)^{16.8}\right) = 9.97 \tag{16}$$

…and using (4) for *H*:

$$H = N - \sum_{i=1}^{N-1}\left(\frac{i}{N}\right)^{P} = 14 - \sum_{i=1}^{13}\left(\frac{i}{14}\right)^{16.8} = 14 - 0.288 - 0.075 - 0.0174...... = 13.62 \tag{17}$$

Substituting in the round trip time equation (13) gives:

$$\begin{aligned}\tau_G &= 2 \cdot H \cdot \left(\frac{d_f}{v}\right) + (S+1) \cdot \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P\left(t_{pi} + t_{po}\right) \\ &= 2 \cdot 13.62 \cdot \left(\frac{4.5}{1.6}\right) + (9.97+1) \cdot \left(5.4 - \frac{4.5}{1.6} + 2 + 3 + 1 - 0.5\right) + 16.8 \cdot (1.2 + 1.2) \\ &= 76.5 + 89 + 40.3 = 205.77\ s\end{aligned} \tag{18}$$

The next step is to include the multiple entrances (basements). The $H_B$ $S_B$ method will be used to amend the round trip time to account for the multiple entrances as follows. There are three basements so *NB* is 3. $P_B$ is the number of passengers arriving from the basement in each elevator round trip journey. It can be calculated as follows:

$$P_B = P \cdot \sum_{i=1}^{NB} P_{arr}(B_i) = 16.8 \cdot (0.05 + 0.05 + 0.05) = 2.52 \tag{19}$$

So on average the elevator will pick up 2.52 passengers from the basements, when it does go to the basement in a round trip journey. $S_B$ *and* $H_B$ can be found as follows:

$$S_B = N_B \cdot \left(1 - \left(1 - \frac{1}{N_B}\right)^{P_B}\right) = 3 \cdot \left(1 - \left(1 - \frac{1}{3}\right)^{2.52}\right) = 1.92 \tag{20}$$

$$H_B = N_B - \sum_{i=1}^{N_B-1}\left(\frac{i}{N_B}\right)^{P_B} = 3 - \sum_{i=1}^{2}\left(\frac{i}{3}\right)^{2.52} = 3 - 0.063 - 0.36 = 2.577 \tag{21}$$

Thus the extra term ($\tau_{GB}$) becomes:

$$\begin{aligned}
&\Delta\tau_{GB} = 2 \cdot H_B \cdot \left(\frac{d_{fB}}{v}\right) + \left(S_B\right) \cdot \left(t_f - \frac{d_{fB}}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) = \\
&2 \cdot 2.577 \cdot \left(\frac{4.5}{1.6}\right) + (1.92) \cdot \left(5.4 - \frac{4.5}{1.6} + 2 + 3 + 1 - 0.5\right) = \\
&14.5 + 15.53 = 30\ s
\end{aligned} \tag{22}$$

The probability of the elevator going to the basement in any one round trip journey can be found as follows:

$$\begin{aligned}
P_{stop}(B) &= \left(1 - \prod_{i=1}^{NB} \left(1 - P_{arr}(B_i)\right)^P\right) \\
&= 1 - \left((0.95)^{16.8}\right) \cdot \left((0.95)^{16.8}\right) \cdot \left((0.95)^{16.8}\right) \\
&= 0.9246
\end{aligned} \tag{23}$$

Substituting the result for the difference gives the value of the round trip time where the elevator serves the ground and the basements as follows:

$$\tau_{GB} = \tau_G + P(basement) \cdot \Delta\tau_{GB} = 205.77 + 0.9246 \cdot 30 = 233.5\ \text{s} \tag{24}$$

Using the Monte Carlo simulation for this building gives a value for the round trip time of 233.0974± 0.308 s (with a certainty of 95.4%) based on 10000 runs. This is an error of less than 0.25%.

## 5. TOP SPEED NOT ATTAINED

The second main source of complication in the use of the classical round trip time calculation is the case where the top speed is not attained in one journey (and in many cases this could be two or three floor journeys). This is now becoming more prevalent as elevator speeds are increased in order to improve performance. There are two main reasons why rated elevator speeds are increasing:

1. Higher performance is now expected and delivered from elevator systems. This can be clearly demonstrated by the following performance parameter. It was usually acceptable to find the rated speed required form an elevator in an office building by dividing the total travel of the elevator by 30 (a rough method of stating that the elevator has to travel between terminal floor in 30 seconds). Nowadays it has become customary to find the rated speed of the elevator by dividing by 20 (a rough method of stating that the elevator has to travel between terminal floor in 20 seconds).

2. As building become higher, increasing in many cases beyond 20 floors above the main terminal, it becomes necessary to split them into two zones (and more zones where the number of floors exceeds 40 floors above the main terminal). In these cases, the distance that has to be travelled by the upper zone elevators increases dramatically and their speed has to be increased accordingly. This point becomes more pronounced in cases where the elevator serves basement floor with a significant percentage arrivals and has a high top speed due to the fact that it is serving an upper zone in a high rise building. For example in a building with 40 floors and 3 basements, it would not be unusual for a group of elevators to have a rated speed of 5 m/s and serve the upper zone of the building (e.g., floors 21 to 40).

This fact makes it even more critical to address the case where the top elevator speed is not attained in one floor journey. This is one of the main sources of inaccuracy from the use of the classical round trip time equation (shown in (1)).

For example, an elevator that has a rated speed of 4 m/s and serves 20 floor above ground at a typical floor to floor height of (assuming a top acceleration of 1 $m \cdot s^{-2}$ and a top jerk of 1 $m \cdot s^{-3}$) will only get up to top speed in a 5 floor journey.

An overview is given of the mathematical analytical method for calculating the round trip time for the case where the top speed is not attained in a one floor journey. A numerical example is then presented and the results compared between the analytical method and the output from the Monte Carlo simulation method.

### 5.1 Mathematical Analysis

The comprehensive method for dealing with the fact that the elevator does not attain its top speed is to find the probability of each of the possible journeys of differing lengths. Thus the probable number (expected number) of one floor journeys is calculated, the expected number of two floor journeys, as well as 3, 4, 5 and up to the number of floors *N*. This is done using the piece of work developed in [3]. The formula is reproduced below using different notation:

$$J_r\Big|_{1\le r\le N} = \left(\sum_{i=1}^{N-(r-1)}\left(\left(1-\sum_{k=i}^{i+(r-2)}P_k\right)^P-\left(1-\sum_{k=i}^{i+(r-1)}P_k\right)^P\right)\right)-\left(\sum_{i=1}^{N-(r-1)}\left(\left(1-\sum_{k=i}^{i+(r-1)}P_k\right)^P-\left(1-\sum_{k=i}^{i+r}P_k\right)^P\right)\right) \quad (25)$$

Where:
$J_r$ is the expected number of journeys of length *r* floors in any round trip
$P_k$ is the percentage of the population on the $k^{th}$ floor
*r* is the journey length in the number of floors
*N* is the number of floors above the main terminal

For the special case of equal floor populations, the probability of stopping at any floor *k* becomes for any value of *k*:

$$P_k = \frac{1}{N} \tag{26}$$

Hence the expression in (25) above reduces to:

$$J_r\big|_{1\le r\le N} = (N-r+1)\cdot\left(\left(1-\frac{r-1}{N}\right)^P - \left(1-\frac{r}{N}\right)^P\right) - (N-r)\cdot\left(\left(1-\frac{r}{N}\right)^P - \left(1-\frac{r+1}{N}\right)^P\right) \tag{27}$$

It must be noted that the probable number of journeys of length r is also equal to the probable number of stops from journeys of length *r*. This is expressed as shown below:

$$J_r = S_r \tag{28}$$

Where:
$J_r$ is the expected number of journeys of length *r* floors in any round trip
$S_r$ is the expected number of stops from journeys of length *r* floors in any round trip

The round trip time can be taken as the sum of the time taken to travel in the up direction, the time taken to travel in the down direction as well as the time taken for passenger to board and alight (passenger transfer time):

$$\tau = \tau_U + \tau_D + \tau_P \tag{29}$$

Each of these three components is detailed below. The time taken to travel in the up direction is shown below:

$$\tau_U = \sum_{i=1}^{r}(S_r \cdot T_r) + (S\cdot(t_{do} + t_{dc} + t_{sd} - t_{ao})) \tag{30}$$

where:
$S_r$ is the expected number of stops from journeys of length *r* floors in any round trip
$S$ is the probable number of stops at the destination floors
$T_r$ is the time taken to travel a journey of length *r* floors in seconds
$t_{do}$ is the door opening time in seconds
$t_{dc}$ is the door closing time in seconds
$t_{sd}$ is the motor start delay in seconds
$t_{ao}$ is the advanced door opening time in seconds

The first part of the formula in (30) above is the time taken travelling in the up direction. The second part is the time taken stopping at each floor. The sum of all stops from different journey length is equal to the probable number of stops:

$$S = \sum_{r=1}^{N} S_r \tag{31}$$

The time taken travelling in the down direction is:

$$\tau_D = \left(t_{dH} + \left(t_{do} + t_{dc} + t_{sd} - t_{ao}\right)\right) \tag{32}$$

where
$t_{dH}$ is the time taken to travel between the main terminal and the highest reversal floor (*H*) in seconds
$t_{do}$ is the door opening time in seconds
$t_{dc}$ is the door closing time in seconds
$t_{sd}$ is the motor start delay in seconds
$t_{ao}$ is the advanced door opening time in seconds

The first part of the formula in (32) is the time taken to travel from the highest reversal floor back to the main terminal. The second part of the formula in (32) is the time for the door opening during the stop back at the main terminal as soon as the elevator arrives back at the main terminal.

The time taken to allow passengers to transfer is:

$$\tau_P = \left(P \cdot \left(t_{pi} + t_{po}\right)\right) \tag{33}$$

Where:
*P* is the number of passengers in the car
$t_{pi}$ is the time taken by each passenger to board the elevator in seconds
$t_{po}$ is the time taken by each passenger to alight from the elevator in seconds

Substituting these three parts in the formula in (29) above gives:

$$\tau = \sum_{i=1}^{r} \left(S_r \cdot T_r\right) + \left(S \cdot \left(t_{do} + t_{dc} + t_{sd} - t_{ao}\right)\right) + \left(t_{dH} + \left(t_{do} + t_{dc} + t_{sd} - t_{ao}\right)\right) + \left(P \cdot \left(t_{pi} + t_{po}\right)\right) \tag{34}$$

Rearranging gives the round trip time formula for the case where the top speed is not attained in one floor journey:

$$\tau = t_{dH} + \left(\sum_{i=1}^{r} \left(S_r \cdot T_r\right)\right) + \left(S + 1\right) \cdot \left(t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P \cdot \left(t_{pi} + t_{po}\right) \tag{35}$$

A numerical example is given next to show that the Monte Carlo simulator can deal with this case comparing its output with the result from the formula in (35).

### 5.2 Approximate method

An approximate method for dealing with the cases where the top speed is not attained has been presented in [2] and [6]. It basically assumes that the average journey length is found by dividing the total distance travelled between the highest reversal floor and the main terminal by the number of stops.

$$d_{ave} = \frac{dH}{S} \qquad (36)$$

This provides the average length of each journey. The time that this takes is then calculated using one of the relevant formulae from (10), (11) and (12). This time is then substituted for the value of $t_f$ in the classical formula (1). $d_{ave}$ is then used in place of $d_f$ in the classical formula (1). All other variables in the equation remain the same.

### 5.3 Case Study

Let us take a building with 20 floors above ground, with a floor to floor distance of 4 m (assuming equal floor to floor distances for all floors). The total travel distance is 80 m. It would not be unreasonable to select a top speed of 4 m·s$^{-1}$ (using the industry rule of thumb of dividing the total travel by 20). The kinematic parameters are given as follows:

$v_{max}$= 4 m·s$^{-1}$
$a_{max}$=1 m·s$^{-2}$
$j_{max}$=1m·s$^{-3}$

The effective passenger number $P$ is assumed to be 10.4 passengers (based on 80% of the car carrying capacity of 13 passengers).

The door times are given as follows:

$t_{do}$ is equal to 2 seconds
$t_{dc}$ is equal to 3 seconds
$t_{sd}$ is equal to 0.5 seconds
$t_{ao}$ is equal to 0 seconds

The passenger transfer time is assumed to be 1 seconds for boarding time and alighting time.

Calculating the round trip time using the classical equation requires the calculation of the highest reversal floor using formula (4) and the calculation of the probable number of stops ($S$) using formula (2). These are shown below:

$H$= 18.7
$S$= 8.3

Substituting into equation (1) gives the value the round trip time using the classical method as 147.39 seconds.

The approximate method is then used to calculate the round trip time. As the floor to floor distances are equal, the value of $dH$ is:

$$dH = H \cdot d_f = 18.3 \cdot 4 = 74.8 \text{ m} \tag{37}$$

Where:
$d_f$ is the floor to floor distance in m

The time taken to travel this distance is given by using the relevant formula from (10), (11) or (12), as follows:

$$d \geq \left( \frac{a^2 v + v^2 j}{aj} \right) \tag{38}$$

then:

$$t_{dH} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{74.8}{4} + \frac{4}{1} + \frac{1}{1} = 23.7 \text{ s} \tag{39}$$

The average length of each stop journey is:

$$d_{ave} = \frac{dH}{S} = \frac{74.8}{8.3} = 9.012 \text{ m} \tag{40}$$

The time taken to travel this distance can be found, which gives:

$$t_{df}\text{=7.086 s} \tag{41}$$

Substituting the results above into the round trip time equation gives the value:

$$\tau = 153.98 \text{ s} \tag{42}$$

In order to find the exact value of the round trip time using the method summarised in formula (35), the expected number of journeys at each of the possible journey lengths (i.e., one floor jump, two floor jump….etc) is calculated. These are denoted as $S_r$, where the letter $S$ is chosen to denote the number of stops associated with each journey length. Note that the subscript $r$ denotes the length of the journey in floors. This calculation is shown in detail in Table 2. This gives the following value:

$$\sum_{i=1}^{r}\left(S_r \cdot T_r\right)=56.505\ \text{s} \tag{43}$$

The average round trip time is then calculated using the formula (35) giving a value of the round trip time of 151.98 s.

Using the Monte Carlo simulation method to calculate the round trip time gives a value of the round trip time of 151.94 s, which is very near to the exact method shown above.

**Table 2: The calculation of the total upward travelling time for the case where the top speed is not attained.**

| *r* | *(n-r+1)* | *(1-((r-1)/N))^P* | *(1-((r)/N))^P* | *n-r* | *(1-r/N)^P* | *((1-(r+1/n))^P)* | $S_r$ | $T_r$ *(s)* | $S_r.T_r$ *(s)* |
|---|---|---|---|---|---|---|---|---|---|
| **1** | 20 | 1.0000 | 0.5866 | 19 | 0.5866 | 0.3343 | 3.475 | 5.123 | 17.803 |
| **2** | 19 | 0.5866 | 0.3343 | 18 | 0.3343 | 0.1845 | 2.097 | 6.745 | 14.143 |
| **3** | 18 | 0.3343 | 0.1845 | 17 | 0.1845 | 0.0982 | 1.230 | 8.000 | 9.838 |
| **4** | 17 | 0.1845 | 0.0982 | 16 | 0.0982 | 0.0502 | 0.699 | 9.062 | 6.330 |
| **5** | 16 | 0.0982 | 0.0502 | 15 | 0.0502 | 0.0245 | 0.383 | 10.000 | 3.827 |
| **6** | 15 | 0.0502 | 0.0245 | 14 | 0.0245 | 0.0113 | 0.201 | 11.000 | 2.214 |
| **7** | 14 | 0.0245 | 0.0113 | 13 | 0.0113 | 0.0049 | 0.101 | 12.000 | 1.212 |
| **8** | 13 | 0.0113 | 0.0049 | 12 | 0.0049 | 0.0020 | 0.048 | 13.000 | 0.624 |
| **9** | 12 | 0.0049 | 0.0020 | 11 | 0.0020 | 0.0007 | 0.021 | 14.000 | 0.300 |
| **10** | 11 | 0.0020 | 0.0007 | 10 | 0.0007 | 0.0002 | 0.009 | 15.000 | 0.133 |
| **11** | 10 | 0.0007 | 0.0002 | 9 | 0.0002 | 0.0001 | 0.003 | 16.000 | 0.054 |
| **12** | 9 | 0.0002 | 0.0001 | 8 | 0.0001 | 0.0000 | 0.001 | 17.000 | 0.019 |
| **13** | 8 | 0.0001 | 0.0000 | 7 | 0.0000 | 0.0000 | 0.000 | 18.000 | 0.006 |
| **14** | 7 | 0.0000 | 0.0000 | 6 | 0.0000 | 0.0000 | 0.000 | 19.000 | 0.002 |
| **15** | 6 | 0.0000 | 0.0000 | 5 | 0.0000 | 0.0000 | 0.000 | 20.000 | 0.000 |
| **16** | 5 | 0.0000 | 0.0000 | 4 | 0.0000 | 0.0000 | 0.000 | 21.000 | 0.000 |
| **17** | 4 | 0.0000 | 0.0000 | 3 | 0.0000 | 0.0000 | 0.000 | 22.000 | 0.000 |
| **18** | 3 | 0.0000 | 0.0000 | 2 | 0.0000 | 0.0000 | 0.000 | 23.000 | 0.000 |
| **19** | 2 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.000 | 24.000 | 0.000 |
| **20** | 1 | 0.0000 | 0.0000 | 0 | 0.0000 | | | 25.000 | |
| | | | | | | ***S=*** | **8.268** | | **56.505** |

The values obtained from the different methods are summarised in Table 3 below:

**Table 3: Comparison of the four methods of calculating the round trip time for the example case study.**

| Basic method | Approximate method | Exact method | Monte Carlo simulation |
|---|---|---|---|
| 147.39 s | 153.98 s | 151.98 s | 151.94 s |

The special cases that have been analysed in the last two sections dealt with each special case on its own. It is important to note that when all of these special cases are combined in one case study, using conventional calculation methods becomes very complicated (e.g., unequal floor populations, unequal floor heights, speed not attained in one floor journey, multiple entrance arrivals). This further emphasises the power of the Monte Carlo simulation method, as it can deal with a combination of any or all of these special cases easily and without any loss of accuracy.

## 6. PRACTICAL IMPLEMENTIATON OF THE MONTE CARLO SIMULATION

The use of Monte Carlo simulation has become practical only with the availability of powerful computing facilities that are accessible nowadays in desktops and laptops. This makes it feasible to run tens of thousands and even hundreds of thousands of simulation runs in a fraction of a second. Although Monte Carlo is run as a simulation, it is in fact used in most application as a means of calculation replacing what would otherwise be a complex and intractable analytical calculation method.

In this section the means of implementation of the Monte Carlo simulation for calculating the round trip time of an elevator during up peak traffic conditions is discussed. The software was implemented in Matlab, and many of the features within Matlab allowed the software to run 100 000 thousand simulation runs in a fraction of a second.

As discussed earlier, the kinematics calculator creates the kinematics matrix prior to starting the simulations, in order to save time during the simulation runs. This greatly reduces the run time.

The most important part of the algorithm is creating the simulation results. As a large number of runs are performed, creating a loop for the runs is too time-consuming. This is why the algorithm makes use of the powerful matrix capabilities of the MATLAB language to create one matrix which contains all of the runs data.

The aim is to create a $P$ by $m$ array, where $P$ is the number of passengers and $m$ the number of trials. Each column of this matrix corresponds to the chosen floors for one set of passengers (i.e., one simulated run).

$P$ is the number of passengers in the car (strictly speaking the total number of passengers arriving from the arrival floors which is the same as the number of passenger alighting at the destination floors in one round trip time journey). Where $P$ is not an integer number the software can still be run. It splits the number of simulation runs in the same ratio as the distance between the upper integer and the lower integer around $P$. As an example, where the number of passengers is 10.4 (indicating an average number of passengers in the car over a large number of journeys), and assuming that the number of simulation runs is 100 000 runs, then the software carries out 60 000 runs with *P* equal to 10 passengers and 40 000 runs with *P* equal to 11 passengers.

The use of time-saving matrix operations along with suitable indexing and mathematical operations greatly reduce the run time for the algorithm, especially when compared to a typical loop.

## 7. CONCLUSIONS

The design methodology for vertical transportation systems relies on the derivation of the round trip time during the up peak traffic. The simplest case comprises equal floor heights, equal floor population, one entrance floor and a top speed that is attained in a one floor run. Any deviation from these simple conditions complicates the calculation methodology.

The Monte Carlo simulation is used as a tool to arrive at the value of the round trip time during up peak (incoming) traffic conditions, for any or all of the special conditions above with ease of programming and without loss of accuracy. A large number of simulations are completed in a fraction of a second by the use of the powerful processing power available from modern computing facilities and the use of advanced matrix operations in Matlab.

The Monte Carlo simulation method has been compared with the exact analytical methods for the case of multiple entrances and for the case where the tops speed is not attained in a one floor journey, with excellent results of less than 0.5% inaccuracy. The method is also very powerful in dealing with all of the special cases combined together in one building, something that becomes virtually impossible to calculate with analytical exact techniques.

Due to its practicality and speed, the Monte Carlo simulation method as presented here can be used as a viable alternative due to classical analytical methods for calculating the round trip time for elevator under up peak (incoming) traffic conditions. It can thus be used as a tool to arrive that the interval and thus to design the vertical transportation systems in buildings.

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