Sistem Homogen. **1.**

$$x + y + z = 0$$
$$x + 2y + 3z = 0$$
$$x + 4y + 9z = 0$$

$\boxed{\text{Rank matriks}}$ $= ?$

Matriks Elementer

$$Ag = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 4 & 9 & 0 \end{bmatrix}$$

$$E_0 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c} \approx \\ B_2 - B_1 \\ B_3 - B_1 \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 3 & 8 & 0 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}.$$

Eliminasi Gauss

$$\begin{array}{c} \approx \\ \\ B_3 - 3B_2 \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}.$$

dst.

$$\begin{array}{c} \approx \\ \\ \frac{1}{2}B_3 \end{array} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$\begin{array}{c} \approx \\ B_2 - 2B_3 \end{array} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

I

$$\begin{array}{c} B_1 - B_2 \\ \approx \\ \text{Gauss Jordan} \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} x = 0 \\ y = 0 \\ z = 0 \end{array}$$

$x = y = z = 0$ solusi trivial.

2

Bentuk Eselon [baris tereduksi]

1. Jika suatu baris tidak semua elemennya nol, maka bilangan tak nol pertama dlm baris tsb. 1
   yang disebut : $\boxed{\text{1 - utama.}}$
2. Jika ada baris yg semua unsurnya nol, maka ditempatkan pd baris² paling bawah.
3. Tiap 2 baris berurutan yg tidak semua unsurnya nol, maka 1 utama baris yg bawah hrs terletak pd kolom yg dikanan dibanding 1 utama yg di atasnya.
4. Tiap kolom yg memuat 1 utama maka unsur lain pd kolom tsb adalah nol.

Contoh. 1-utama

1-utama

$$E = \begin{bmatrix} 0 & 1 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank$(E) = 2$

baris nol.

Sistem dengan banyak persamaan lebih sedikit dari banyak un-known

$$x_1 + 3x_2 - 2x_3 + 2x_5 = 0$$

$$2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = -1$$

$$5x_3 + 10x_4 + 15x_6 = 5$$

$$2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 = 6$$

$$\bar{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

$$Ag = \left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{array}\right]$$

$$\begin{array}{l} \\ B_2 - 2B_1 \\ \\ B_4 - 2B_1 \end{array} \approx \left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{array}\right]$$

$$\begin{array}{l} \\ \\ B_3 + 5B_2 \\ B_4 + 4B_2 \end{array} \approx \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \end{bmatrix}$$

$$\downarrow$$

1

$$B_3 \rightleftarrows B_4 \approx \begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \end{array} \\ \left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x_2 \quad x_4. \quad x_5$ bebas

Syarat ada solusi

1. Rank (A) = Rank (A_b).

$3 = 3 \Rightarrow$ ada Solusi

Unknow = 6

Rank = 3

Parameter bebas = 3

$x_5 = t$

$x_4 = s$

$x_2 = r$.

solusi SPL :

$x_6 = \frac{1}{3}$

$x_5 = t$

$x_4 = s$

$x_3 = -2x_4 = -2s$

$x_2 = r$

$x_1 = -2t + 2(-2s) - 3.r$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -2t-4s-3r \\ r \\ -2s \\ s \\ t \\ \frac{1}{3} \end{bmatrix}$$

$$= t\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} -4 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + r\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \cancel{0} & 0 \\ \cancel{0} & 0 \\ \cancel{0} & 0 \\ \cancel{0} & 0 \\ \cancel{1} & 0 \\ & \frac{1}{3} \end{bmatrix}$$

Matriks invers.

A = MBS, B = MBS dgn order yg sama.

$A.B = B.A = I \Longrightarrow$ B disb invers dr A, A = invers dr B.

Mendapatkan invers A, (A^{-1}), dgn Gauss Jordan.

$$(A \,\vdots\, I) \xRightarrow{OBE} (I \,\vdots\, A^{-1}).$$

$\Rightarrow$ SPL : $A\bar{x} = \bar{b}$; A^{-1} ada.

$$\Rightarrow A^{-1}(A\bar{x}) = A^{-1}.b.$$

$$\underbrace{A^{-1}A}\bar{x} = A^{-1}b.$$

$$I x = A^{-1}b.$$

$$\boxed{x = A^{-1}b.}$$

solusi dgn metode invers.

Contoh.

$$A_g = \left(\begin{array}{ccc:ccc} \multicolumn{3}{c}{\overbrace{\hphantom{1\ 1\ 1}}^{A}} & \multicolumn{3}{c}{\overbrace{\hphantom{1\ 0\ 0}}^{I}} \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{array}\right) \approx$$

Gauss jordan.

$$E_0 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\approx \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 3 & 8 & -1 & 0 & 1 \end{array}\right] \quad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\approx \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 2 & 2 & -3 & 1 \end{array}\right] \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

$$\underset{B_2 - 2B_3}{\approx} \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -\frac{3}{2} & \frac{1}{2} \end{array}\right] \quad E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$\underset{B_1 - B_2}{\overset{B_1 - B_3}{\approx}} \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 0 & \frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -3 & 4 & -1 \\ 0 & 0 & 1 & 1 & -\frac{3}{2} & \frac{1}{2} \end{array}\right] \quad E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \quad E_6 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_7 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -\frac{5}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -3 & 4 & -1 \\ 0 & 0 & 1 & 1 & -\frac{3}{2} & \frac{1}{2} \end{array}\right]$$

$$\underbrace{}_{I} \qquad \underbrace{\phantom{3\ -\tfrac{5}{2}\ \tfrac{1}{2}}}_{A^{-1}}.$$

$$A^{-1} = \begin{bmatrix} 3 & -\frac{5}{2} & \frac{1}{2} \\ -3 & 4 & -1 \\ 1 & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

?

$$= \frac{1}{2} \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 1 & -3 & 1 \end{bmatrix}$$

?

Cek : $A \cdot A^{-1} = A^{-1} \cdot A = I$

$$\boxed{A^{-1} = E_k E_{k-1} \cdots E_2 E_1 I_n .}$$

$$E_7 E_6 \cdot E_5 \cdot E_4 \cdot E_3 \cdot E_2 \cdot E_1 \cdot I = A^{-1}$$