Institut Teknologi Sepuluh Nopember

Alternating Current

NURRISMA PUSPITASARI

Chapter 6

Alternating Current

Trancient Induction

- RL series circuit in DC Sources
- RL series circuit in DC Sources

Part 1

AC Sources

- Resistors in an AC Circuit
- Inductors in an AC Circuit
- Capacitors in an AC Circuit
- Power in an AC Circuit
- Resonance in a Series RLC Circuit
- The Effective Current and Votlage

Part 2

Induksi Transien/Peralihan

Current growth in an *R-L* circuit

We can learn a great deal about inductor behavior by analyzing the circuit of **Fig.1**. A circuit that includes both a resistor and an inductor, and possibly a source of emf, is called an ***R-L* circuit.**

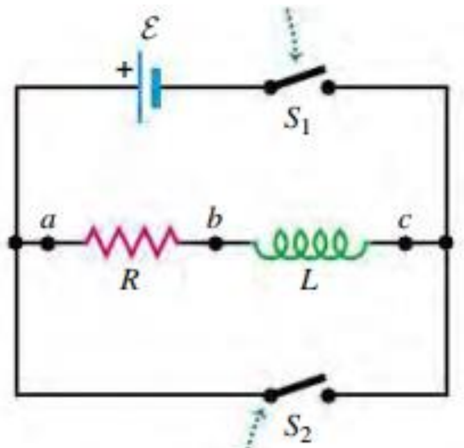
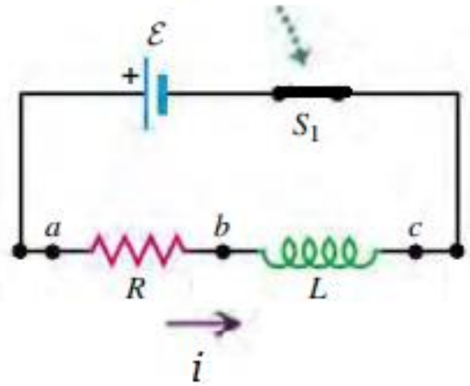


Fig.1 The *R-L* circuit

By closing switch S_1, we can connect the *R-L* combination to a source with constant emf $\boldsymbol{\varepsilon}$. (We assume that the source has zero internal resistance, so the terminal voltage equals $\boldsymbol{\varepsilon}$.)

switch S_1 is closed at t = 0



We apply Kirchhoff's loop rule

$$\boldsymbol{\varepsilon} - \mathrm{iR} - \mathrm{L}\frac{di}{dt} = 0$$

$$\boldsymbol{\varepsilon} = \mathrm{iR} + \mathrm{L}\frac{di}{dt}$$

$$\boldsymbol{\varepsilon} = \mathrm{iR} + \mathrm{L}\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{1}{L}(\boldsymbol{\varepsilon} - iR)$$

$$\int_0^{i(t)} \frac{di}{(\boldsymbol{\varepsilon} - iR)} = \int_0^t \frac{dt}{L}$$

$$\int_0^{i(t)} \frac{di}{(\boldsymbol{\varepsilon} - iR)} = \int_0^t \frac{dt}{L}$$

misal $\mathrm{u} = \boldsymbol{\varepsilon} - iR$

$$\mathrm{du} = -Rdi$$

$$di = -\frac{du}{R}$$

$$\int_0^{i(t)} -\frac{du}{Ru} = \int_0^t \frac{dt}{L}$$

$$-\frac{1}{R}\int_0^{i(t)} \frac{du}{u} = \int_0^t \frac{dt}{L}$$

$$-\frac{1}{R}\ln[u]_0^i = \frac{1}{L}t$$

$$-\frac{1}{R}\ln[\boldsymbol{\varepsilon} - iR\,]_0^i = \frac{1}{L}t$$

$$-\frac{1}{R}(\ln[\boldsymbol{\varepsilon} - iR] - \ln\boldsymbol{\varepsilon}) = \frac{1}{L}t$$

$$-\frac{1}{R}(\ln[\boldsymbol{\varepsilon} - iR] - \ln\boldsymbol{\varepsilon}) = \frac{1}{L}t$$

$$-\frac{1}{R}\left(\ln\left[\frac{\boldsymbol{\varepsilon} - iR}{\boldsymbol{\varepsilon}}\right]\right) = \frac{1}{L}t$$

$$\ln\left[\frac{\boldsymbol{\varepsilon} - iR}{\boldsymbol{\varepsilon}}\right] = -\frac{Rt}{L}$$

$$\frac{\boldsymbol{\varepsilon} - iR}{\boldsymbol{\varepsilon}} = \boldsymbol{e}^{-\frac{Rt}{L}}$$

$$\boldsymbol{\varepsilon} - iR = \boldsymbol{\varepsilon}\boldsymbol{e}^{-\frac{Rt}{L}}$$

$$iR = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}\boldsymbol{e}^{-\frac{Rt}{L}}$$

$$\boxed{i = \frac{\boldsymbol{\varepsilon}}{R}\left(\mathbf{1} - \boldsymbol{e}^{-\frac{Rt}{L}}\right)}$$

(current in an *R-L* circuit with emf)

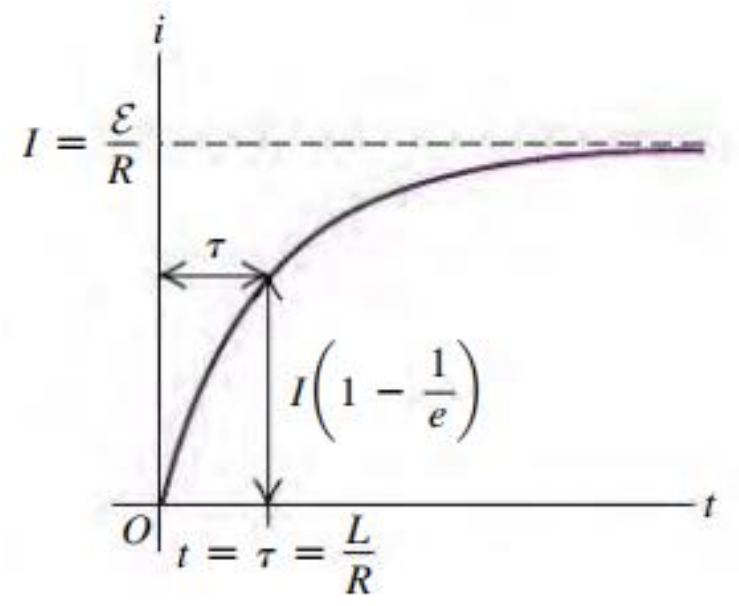


Figure 2 The behavior of the current as a function of time

Taking the derivative of Eq. (i), $i = \frac{\boldsymbol{\varepsilon}}{R}\left(\mathbf{1} - \boldsymbol{e}^{-\frac{Rt}{L}}\right)$ we find

$$\frac{di}{dt} = \frac{\boldsymbol{\varepsilon}}{L}\boldsymbol{e}^{-\frac{Rt}{L}}$$ ➡ Kecepatan arus

at time $t = 0$ ➡ $i = 0$

$$\frac{di}{dt} = \frac{\boldsymbol{\varepsilon}}{L}$$

at time $t = \infty$ ➡ $I = \frac{\boldsymbol{\varepsilon}}{R}$

$$\frac{di}{dt} = 0$$

at time $t = \frac{L}{R}$ ➡ $i = I\left(1 - \frac{1}{e}\right)$

⬇

$t = \tau$

this quantity is called the **time constant** for the circuit

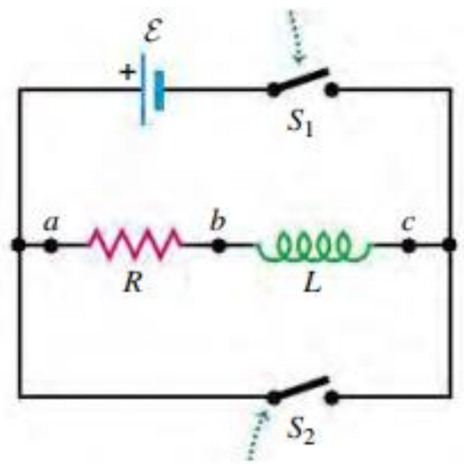
Example

A sensitive electronic device of resistance R = 175 Ω is to be connected to a source of emf (of negligible internal resistance) by a switch. The device is designed to operate with a 36-mA current, but to avoid damage to the device, the current can rise to no more than 4.9 mA in the first 58 μs after the switch is closed. An inductor is therefore connected in series with the device, as in Fig. 30.11; the switch in question is S_1.

(a) What is the required source emf ε?

(b) What is the required inductance L?

(c) What is the R-L time constant t?



Solution

a. The source emf ε

$$I = \frac{\varepsilon}{R} \Rightarrow \boldsymbol{\varepsilon = IR}$$

$$\boldsymbol{\varepsilon = 0,036} \times 175$$

$$\boldsymbol{\varepsilon = 6,3V}$$

a. The inductance L

$$i = \frac{\varepsilon}{R}\left(\mathbf{1} - e^{-\frac{Rt}{L}}\right)$$

$$i - \frac{\varepsilon}{R} = \mathbf{1} - e^{-\frac{Rt}{L}}$$

$$i - \frac{\varepsilon}{R} = -\frac{\varepsilon}{R} e^{-\frac{Rt}{L}}$$

$$\frac{\varepsilon}{R} - i = \frac{\varepsilon}{R} e^{-\frac{Rt}{L}}$$

$$1 - i\frac{R}{\varepsilon} = e^{-\frac{Rt}{L}}$$

$$\ln\left(1 - i\frac{R}{\varepsilon}\right) = \ln\left(\boldsymbol{e}^{-\frac{Rt}{L}}\right)$$

$$\ln\left(1 - i\frac{R}{\varepsilon}\right) = -\frac{Rt}{L}\ln \mathrm{e}$$

$$\ln\left(1 - i\frac{R}{\varepsilon}\right) = -\frac{Rt}{L}$$

$$\boldsymbol{L} = -\frac{Rt}{\ln\left(1 - i\frac{R}{\varepsilon}\right)}$$

$$\boldsymbol{L} = -\frac{175 \times 58 \times 10^{-6}}{\ln\left(1 - \frac{4{,}9 \times 10^{-3} \times 175}{6.3}\right)}$$

$\mathrm{L} = 69\mathrm{mH}$

c. the *R-L* time constant t

$$\tau = \frac{L}{R}$$

$$\tau = \frac{69 \times 10^{-3}}{175}$$

$$\tau = \frac{69 \times 10^{-3}}{175}$$

$\mathrm{t} = 3{,}9 \times 10^{-4} = 390\mu\mathrm{s}$

The current decay in an *R-L* circuit (Pelucutan Arus)

Now suppose switch $S1$ in the circuit of Fig. 1 has been closed for a while and the current has reached the value I_0. Resetting our stopwatch to redefine the initial time, we close switch S_2 at time $t = 0$, by passing the battery. (At the same time we should open S_1 to protect the battery.) The current through R and L does not instantaneously go to zero but decays smoothly, as shown in **Fig. 3**

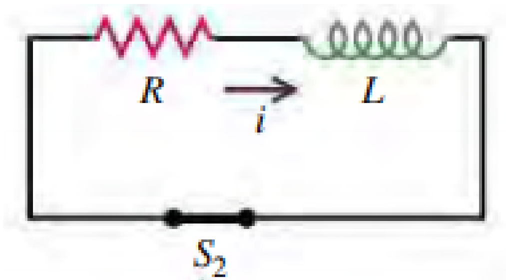


Fig. 3

We apply Kirchhoff's loop rule

$$-L\frac{di}{dt} - iR = 0$$

$$iR = -L\frac{di}{dt}$$

$$\frac{di}{i} = -\frac{R}{L}dt$$

$$\int \frac{di}{i} = -\int \frac{R}{L}dt$$

$$\ln i = \frac{R}{L}t + C$$

at $t = 0$ ⇒ $I_0 = \frac{\boldsymbol{\varepsilon}}{R}$

$$\ln I_0 = C$$

$$\ln\left(\frac{\boldsymbol{\varepsilon}}{\mathrm{R}}\right) = \mathrm{C}$$

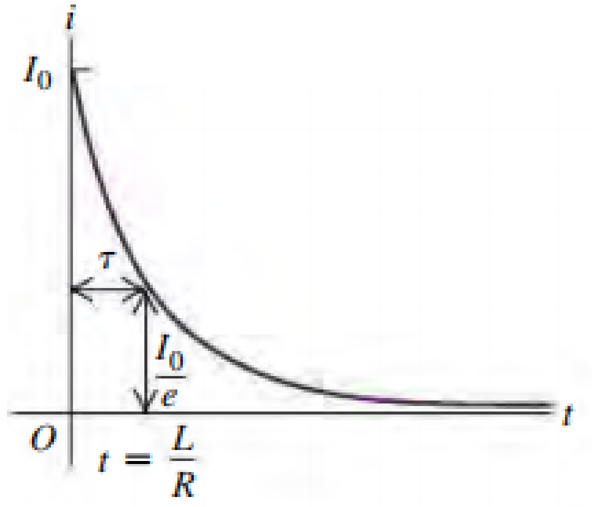
$$\ln i = \frac{R}{L}t + \ln\left(\frac{\boldsymbol{\varepsilon}}{\mathrm{R}}\right)$$

$$\ln i - \ln\left(\frac{\boldsymbol{\varepsilon}}{\mathrm{R}}\right) = \frac{R}{L}t$$

$$\ln\left(\frac{i\mathrm{R}}{\boldsymbol{\varepsilon}}\right) = \frac{R}{L}t$$

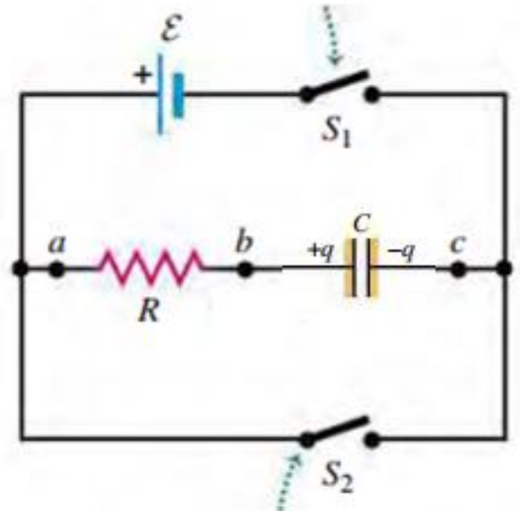
$$\frac{i\mathrm{R}}{\boldsymbol{\varepsilon}} = e^{-\frac{R}{L}t}$$

$$i = \frac{\boldsymbol{\varepsilon}}{R}e^{-\frac{R}{L}t}$$



The *R-C* circuit

Pengisian Muatan



Ketika saklar S_1 ditutup dan saklar S_2 dibuka maka:

We apply Kirchhoff's loop rule

$$\boldsymbol{\varepsilon} - \mathrm{iR} - \frac{q}{C} = 0$$

$$\boldsymbol{\varepsilon} = \frac{dq}{dt}\mathrm{R} + \frac{q}{C}$$

$$\mathbf{C\varepsilon} = \frac{\mathrm{dq}}{\mathrm{dt}}\mathrm{RC} + \mathrm{q}$$

$$\mathbf{C\varepsilon} - \mathbf{q} = \frac{\mathrm{dq}}{\mathrm{dt}}\mathrm{RC}$$

$$\frac{\mathrm{dq}}{\mathbf{C\varepsilon} - \mathbf{q}} = \frac{\mathrm{dt}}{\mathrm{RC}}$$

$$\int \frac{\mathrm{dq}}{\mathbf{C\varepsilon} - \mathbf{q}} = \int \frac{\mathrm{dt}}{\mathrm{RC}}$$

misal $\mathrm{u} = \mathbf{C\varepsilon} - \mathbf{q}$

$$\mathrm{du} = -dq$$

$$d\mathrm{q} = -\mathrm{du}$$

$$-\int \frac{\mathrm{du}}{\mathbf{u}} = \int \frac{\mathrm{dt}}{\mathrm{RC}}$$

$$\ln u = -\frac{\mathrm{t}}{\mathrm{RC}} + \mathrm{C}$$

$$\ln(\mathbf{C\varepsilon} - \mathbf{q}) = -\frac{\mathrm{t}}{\mathrm{RC}} + \mathrm{C}$$

at $t = 0$ $\quad$ $\mathrm{q} = 0$

$$\ln(\mathbf{C\varepsilon}) = \mathrm{C}$$

$$\ln(\mathbf{C\varepsilon} - \mathbf{q}) = -\frac{\mathrm{t}}{\mathrm{RC}} + \ln(\mathbf{C\varepsilon})$$

$$\ln(\mathbf{C\varepsilon} - \mathbf{q}) - \ln(\mathbf{C\varepsilon}) = -\frac{\mathrm{t}}{\mathrm{RC}}$$

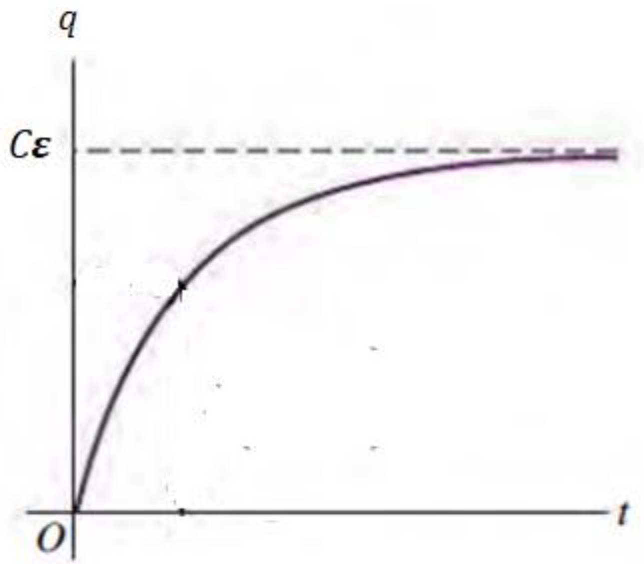
$$\ln\left(\frac{\mathbf{C\varepsilon} - \mathbf{q}}{\mathbf{C\varepsilon}}\right) = -\frac{\mathrm{t}}{\mathrm{RC}}$$

$$\frac{\mathrm{C\varepsilon} - \mathrm{q}}{\mathrm{C\varepsilon}} = e^{-\frac{t}{RC}}$$

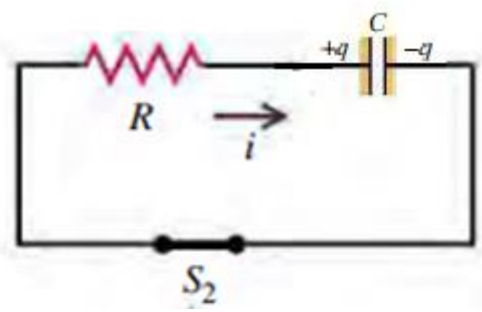
$$\mathrm{C\varepsilon} - \mathrm{q} = \mathrm{C\varepsilon} e^{-\frac{t}{RC}}$$

$$\mathrm{q} = \mathrm{C\varepsilon} - \mathrm{C\varepsilon} e^{-\frac{t}{RC}}$$

$$\boxed{\mathrm{q} = \mathrm{C\varepsilon}\left(1 - e^{-\frac{t}{RC}}\right)}$$



Pelucutan Muatan



Setelah beberapa saat saklar S_1 dibuka dan saklar S_2 ditutup maka:

Setelah beberapa saat saklar S_1 dibuka dan saklar S_2 ditutup maka:

at $t = 0$ ⇨ $q = C\varepsilon$

We apply Kirchhoff's loop rule

$$\boldsymbol{\varepsilon} - \mathrm{iR} - \frac{q}{C} = 0$$

$$\mathrm{iR} - \frac{q}{C} = 0$$

$$\frac{dq}{dt}\mathrm{R} + \frac{q}{C} = 0$$

$$\frac{dq}{dt}\mathrm{R} = -\frac{q}{C}$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

$$\int \frac{dq}{q} = -\int \frac{dt}{RC}$$

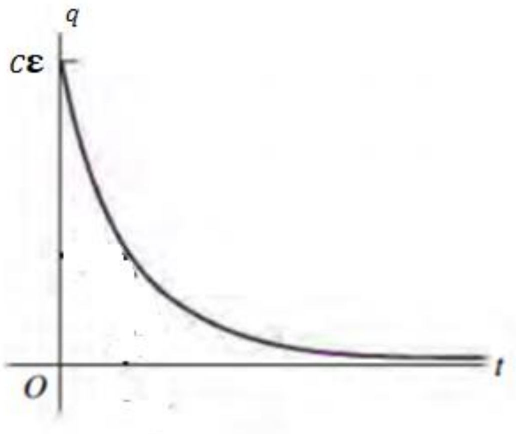
$$\ln q = -\frac{t}{RC} + \mathrm{C}$$

at $t = 0$ $\Rightarrow$ $\mathrm{q} = \mathrm{C}\varepsilon$

$$\ln \mathrm{C}\varepsilon = \mathrm{C}$$

$$\ln q = -\frac{t}{RC} + \ln \mathrm{C}\varepsilon$$

$$\ln \frac{q}{\mathrm{C}\varepsilon} = -\frac{t}{RC} \Rightarrow q = \mathrm{C}\varepsilon e^{-\frac{t}{RC}}$$



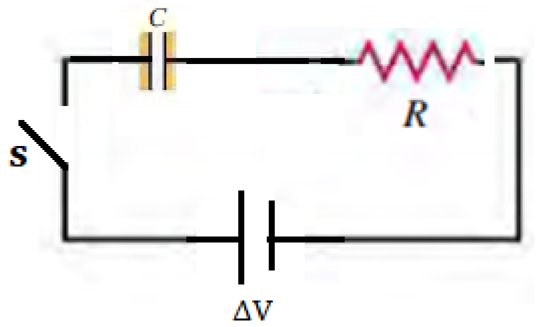
Contoh soal

Soal di Buku hal 156 no. 2

Suatu rangkaian RC-seri dihubungkan dengan batere $\varepsilon = 30$ volt, $\mathrm{R} = 15\ \mathrm{k}\Omega$ dan $\mathrm{C} = 6\mu\mathrm{F}$. Pada keadaan awal kapasitor tidak bermuatan.

a. Setelah berapa lama sejak saklar ditutup muatan dalam kapasitor sepertiga dari muatan maksimum

b. Tentukan besar muatan saat itu



Peneyelesaian:

a. $\mathrm{q} = \frac{1}{3}\mathrm{q}_{\mathrm{maks}}$

$$\mathrm{q} = \mathrm{C}\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\frac{1}{3}q_{\text{maks}} = \text{C}\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\frac{1}{3}\text{C}\varepsilon = \text{C}\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\frac{1}{3} = 1 - e^{-\frac{t}{RC}}$$

$$\frac{1}{3} - 1 = -e^{-\frac{t}{RC}}$$

$$\frac{2}{3} = e^{-\frac{t}{RC}}$$

$$\ln\left(\frac{2}{3}\right) = \ln e^{-\frac{t}{RC}}$$

$$\ln\left(\frac{2}{3}\right) = -\frac{t}{RC}\ln e$$

$$t = -RC\ \ln\left(\frac{2}{3}\right)$$

$$t = -1500 \times 6.10^{-6} \times (-0{,}405)$$

$$\boxed{t = 3{,}465\text{ ms}}$$

b. Besar muatan saat itu

$$\text{q} = \frac{1}{3}C\varepsilon$$

$$\text{q} = \frac{1}{3}6.10^{-6} \times 30$$

$$\text{q} = 60.10^{-6}\text{C}$$

$$\boxed{\text{q} = 60\ \mu\text{C}}$$

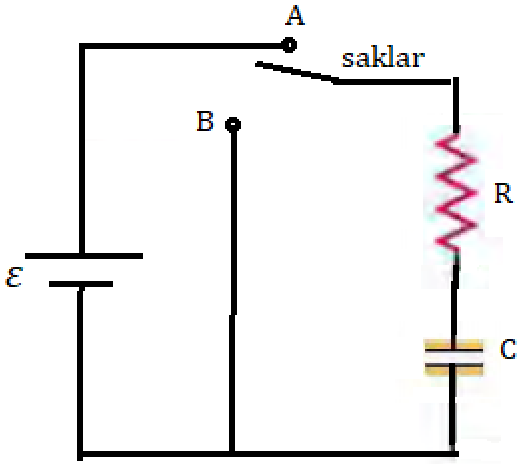
Soal di Buku hal 157 no. 8

Rangkaian RC-seri dengan $R = 1000\Omega, \mathrm{C} = 2\mu F$, dan dihubungkan batere 12 volt dengan menempakan saklar ke titik A. Tentukan:

a. Tegangan yang melewati resistor V_R pada saat $t = 1 \times 10^{-3} s$

b. Tegangan yang melewati kapasitor V_C pada saat $t = 1 \times 10^{-3} s$

c. Setelah kapasitor mencapakai muatan maksi um kemudian saklar didipindahkan ke titik B, sehingga RC-seri tidak terhubung batere, tentukan tegangan pada resistor V_R dan tegangan pada kapasitor V_C setelah 1 ms dari pencopotan batere tersebut.



Penyelesaian:

a. Tegangan yang melewati resistor V_R pada saat $t = 1 \times 10^{-3} s$

$$q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

$$V_R = iR = \varepsilon e^{-\frac{t}{RC}}$$

$$V_R = 12e^{-\frac{0,003}{1000\times2\times10^{-6}}}$$

$$V_R = 12e^{-\frac{0,003}{1000\times2\times10^{-6}}}$$

$$V_R = 2,68 \text{ volt}$$

b. Tegangan yang melewati kapasitor V_C pada saat $t = 1 \times 10^{-3} s$

$$q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

$$V_C = \frac{q}{C} = \varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

$$V_C = \frac{q}{C} = 12\left(1 - e^{-\frac{0,003}{1000\times2\times10^{-6}}}\right)$$

$$V_C = 9,32 \text{ volt}$$

c.Tegangan pada resistor V_R dan tegangan pada kapasitor V_C setelah 1 ms dari pencopotan batere

$$q = C\varepsilon e^{-\frac{t}{RC}}$$

$$i = \frac{dq}{dt} = -\frac{\varepsilon}{R}e^{-\frac{t}{RC}}$$

$$V_R = iR = -\varepsilon e^{-\frac{t}{RC}}$$

$$V_R = -12e^{-\frac{0,001}{1000\times2\times10^{-6}}}$$

$$V_R = -7,28 \text{ volt}$$

$$V_C = \frac{q}{C} = \varepsilon e^{-\frac{t}{RC}}$$

$$V_C = 12 \times e^{-\frac{0,001}{1000\times2\times10^{-6}}}$$

$$V_C = 12 \times e^{-\frac{0,001}{1000\times2\times10^{-6}}}$$

$$V_C = 7,28 \text{ volt}$$

Alternating Current

AC Sources

- Resistors in an AC Circuit
- Inductors in an AC Circuit
- Capacitors in an AC Circuit
- Power in an AC Circuit
- Resonance in a Series RLC Circuit
- The root-mean-square Current and Voltage

Part 2

Alternating Current (AC)

Alternating current is the current generated by an AC generator

The emf of generator:

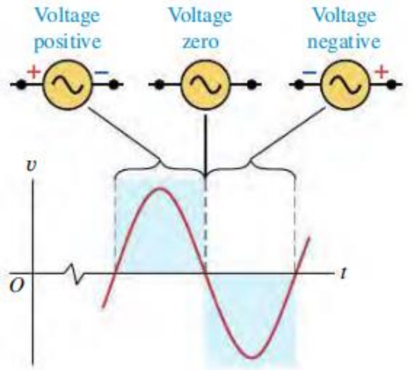
$$v = NBA\omega \sin \omega t$$

$$V_{max} = NBA\omega$$

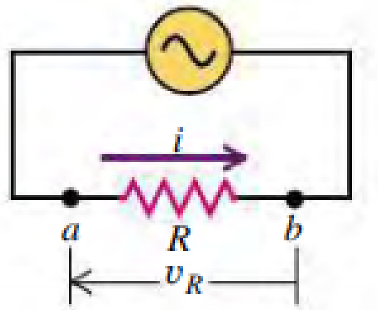
An AC circuit consists of circuit elements and a power source that provides an alternating voltage v. This time-varying voltage is described by:

$$v = V_{max} \sin \omega t$$

where *V*max is the maximum output voltage of the AC source, or the voltage amplitude.



Resistors in an AC Circuit



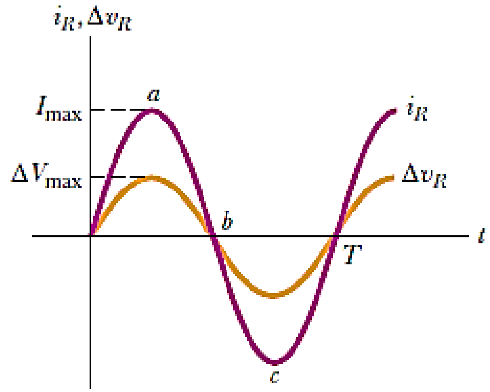
$$v = V_{max} \sin \omega t$$

$$i = \frac{V}{R} = \frac{V_{max} \sin \omega t}{R}$$

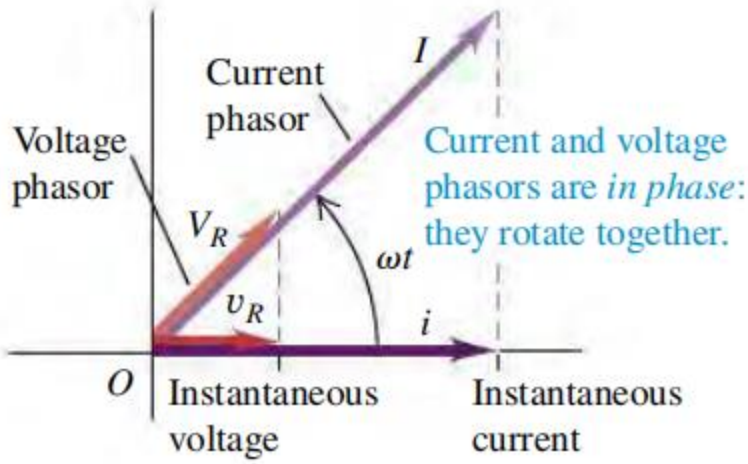
$$i_{max} = \frac{V_{max}}{R}$$

$$i = i_{max} \sin \omega t$$

instantaneous current and voltage in phase

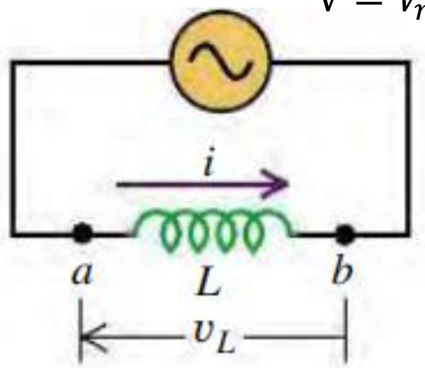


Phasor diagram



Inductors in an AC Circuit

$$V = V_{max} \sin \omega t$$



$$\varepsilon_L = v_L = L\,\frac{di}{dt}$$

$$V_{max} \sin \omega t = L\,\frac{di}{dt}$$

$$\int di = \int \frac{V_{max}}{L} \sin \omega t \;\, dt$$

$$i = \frac{-V_{max}}{\omega L} \cos \omega t$$

where,

$$\cos\omega t = -\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = \frac{V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = \; i_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\boxed{i_{max} = \frac{V_{max}}{\omega L}}$$

inductive reactance, X_L :

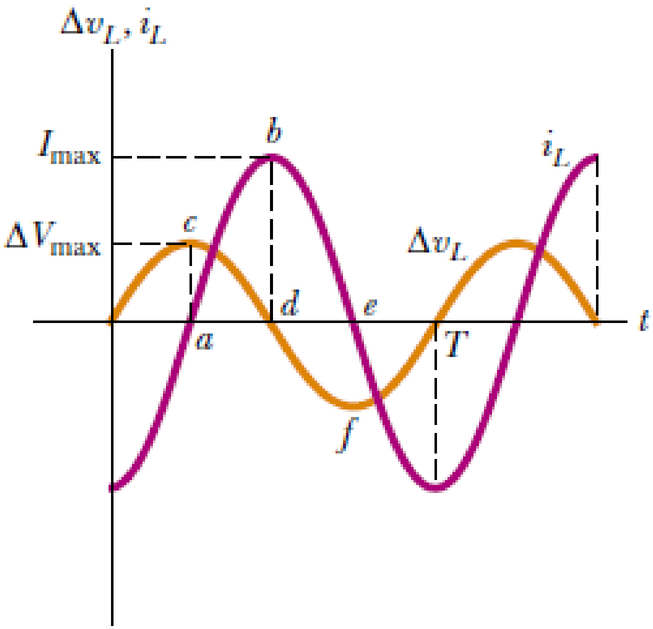
$$\boxed{X_L = \;\omega L}$$

$v = V_{max} \sin \omega t$

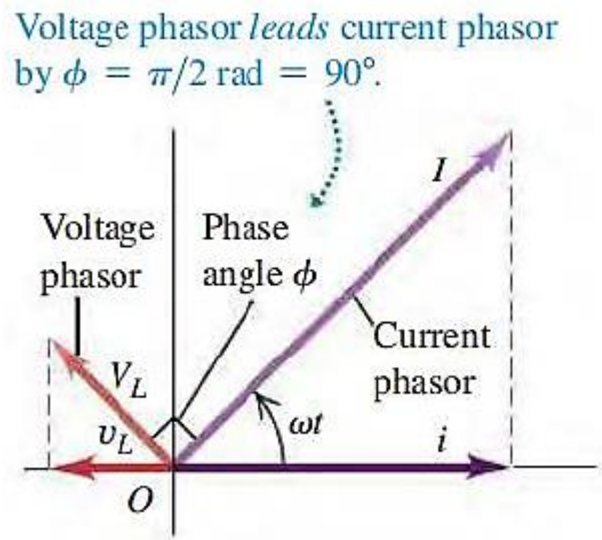
$$i = i_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$$

The current lags behind the voltage by $\frac{\pi}{2}$ or 90°

Graphs of current and voltage versus time



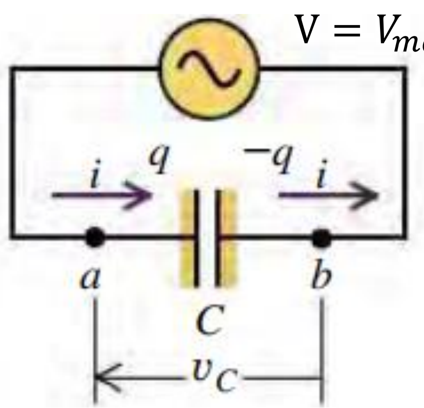
Phasor diagram



Phasor diagram for the inductive circuit, showing that the current lags behind the voltage by 90°.

Capacitors in an AC Circuit

$V = V_{max} \sin \omega t$



We know from the definition of capacitance that $C = q / v$

$$q = Cv$$

$$q = CV_{max} \sin \omega t$$

Because $i = dq / dt$

$$i = \frac{dq}{dt} = \omega\, C\, V_{max} \cos \omega t$$

$$i = \omega\, C\, V_{max}\, \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = \frac{V_{max}}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $i_{max} = \frac{V_{max}}{1/\omega C}$

$$i = i_{max}\, \sin\left(\omega t + \frac{\pi}{2}\right)$$

capacitive reactance, X_C :

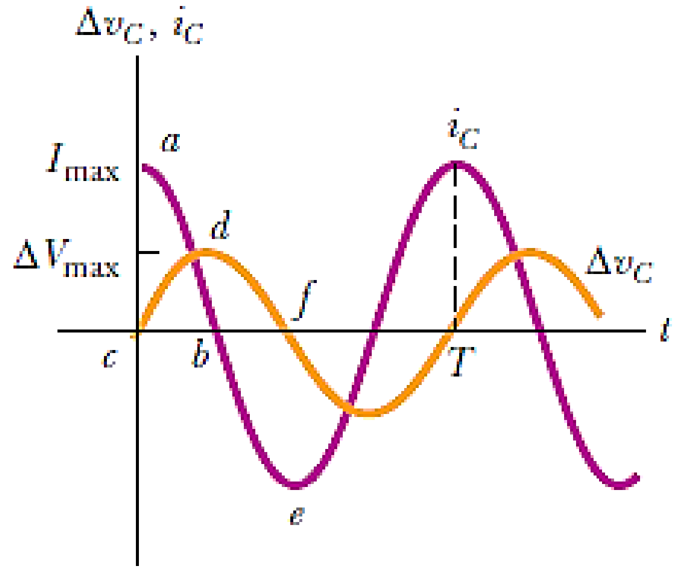
$$\frac{1}{\omega C} = X_C$$

$$v = V_{max} \sin \omega t$$

$$i = i_{max}\, \sin\left(\omega t + \frac{\pi}{2}\right)$$

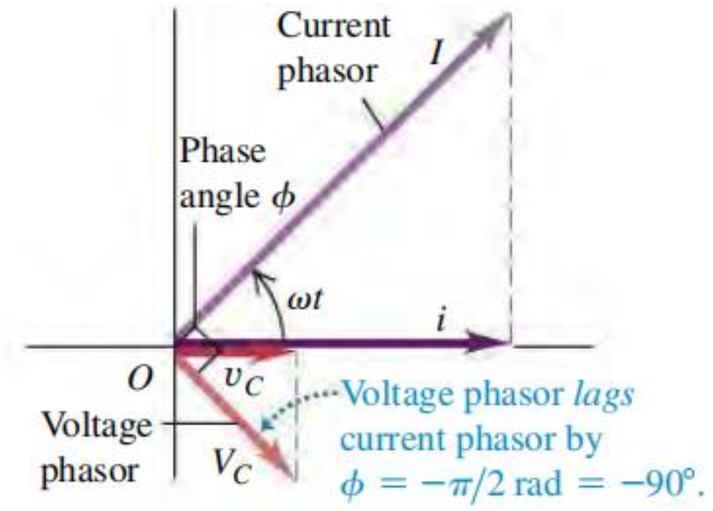
The voltage lags behind the current by 90° $or\ \frac{\pi}{2}$

Graphs of current and voltage versus time



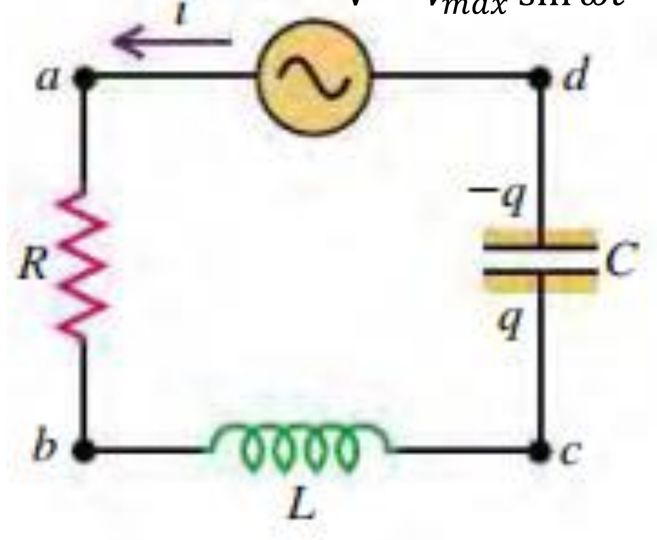
Phasor diagram

Phasor diagram for the capacitive circuit, showing that the current leads the voltage by 90°.



The RLC Series Circuit

$$V = V_{max} \sin \omega t$$



The current at all points in a series AC circuit has the same amplitude and phase.

Based on the preceding sections, we know that the voltage across each element has a different amplitude and phase.

Resistors

$$v_R = V_{max} \sin \omega t$$

$$i_R = i_{max} \sin \omega t$$

Inductors

$$v_L = V_{max} \sin \omega t$$

$$i_L = i_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Capacitors

$$v_C = V_{max} \sin \omega t$$

$$i_C = i_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

the maximum voltage values across the elements

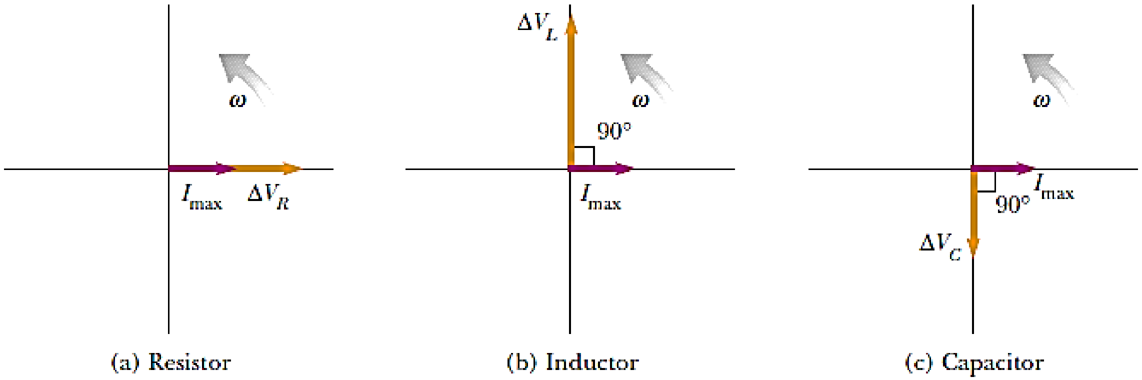
$$V_{maxR} = i_{max}\, \mathrm{R}$$

$$V_{maxL} = i_{max}\, X_L$$

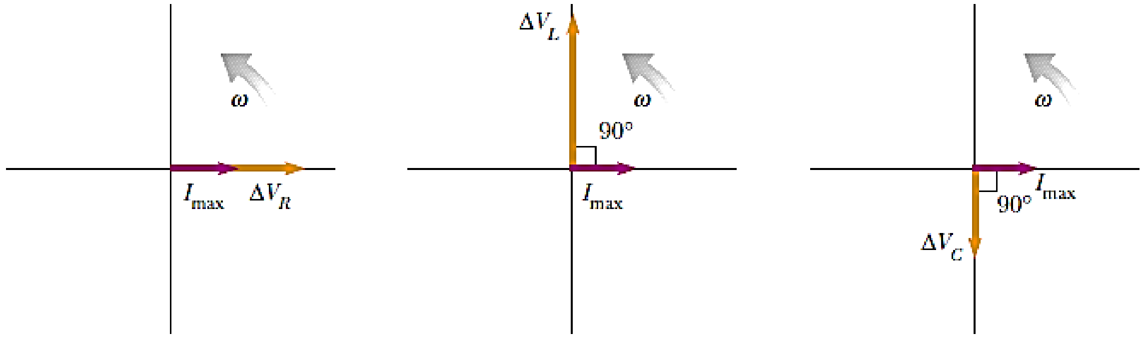
$$V_{maxC} = i_{max}\, X_C$$

we can express the instantaneous voltages across the three circuit elements as

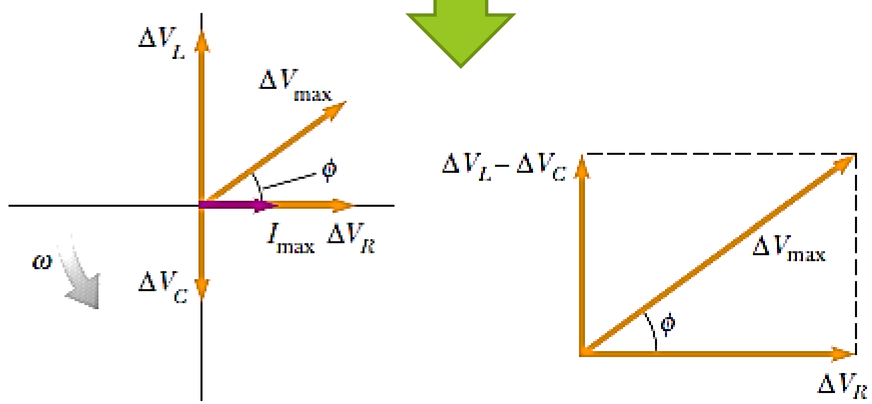
$$v = v_R + v_L + v_C$$



Phasor diagram for the series *RLC* circuit



(a) Resistor (b) Inductor (c) Capacitor



(a) (b)

The phasor V_R is in phase with the current phasor i_{max}, the phasor V_L leads i_{max} by 90°, and the phasor V_c lags i_{max} by 90°. The total voltage V_{max} makes an angle - with i_{max}. (b) Simplified version of the phasor diagram shown in part (a).

in Figure b, we see that

$$V_{max} = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(I_{max} R)^2 + (I_{max} X_L - I_{max} X_C)^2}$$

$$V_{max} = I_{max} \sqrt{R^2 + (X_L - X_C)^2}$$

Therefore, we can express the maximum current as

$$I_{max} = \frac{V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

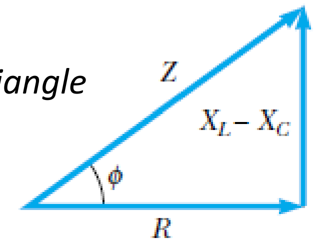
Once again, this has the same mathematical form as Equation 27.8. The denominator of the fraction plays the role of resistance and is called the impedance *Z* of the circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where impedance also has units of ohms. Therefore, we can write:

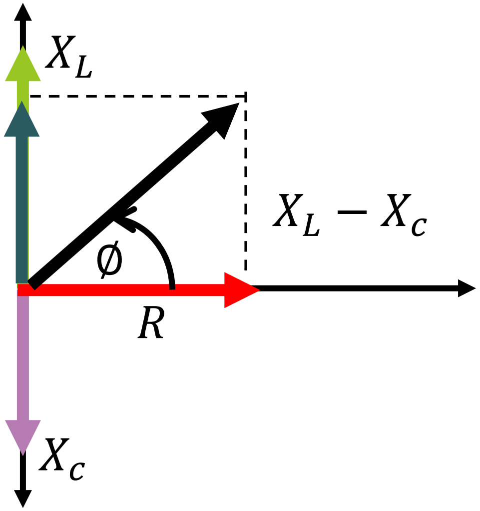
$$V_{max} = i_{max}\,\mathrm{Z}$$

impedance triangle



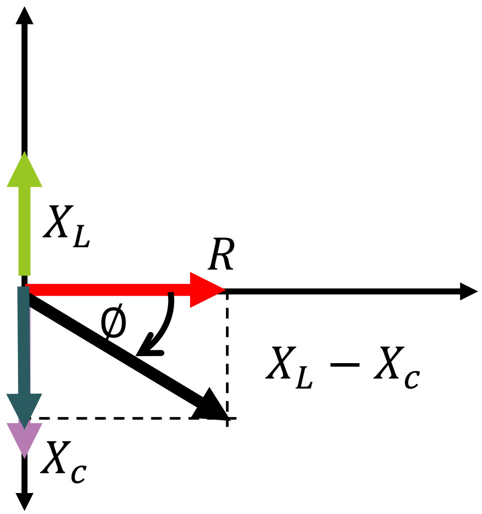
Phasor diagram

The curcuit more inductive than capacitive **(X_L > X_C)**



$$\tan \emptyset = \frac{X_L - X_C}{R}$$

***The curcuit more capacitive than inductive* ($X_C > X_L$)**



$$\tan \emptyset = \frac{X_L - X_C}{R}$$

the phase angle is negative

***The circuit purely resistive* ($X_C = X_L$)**

Resonance

$$X_L = X_C$$

$$Z = R$$

Minimum

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \sqrt{\frac{1}{LC}}$$

$$2\pi f = \sqrt{\frac{1}{LC}}$$

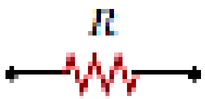
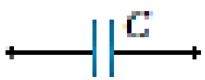
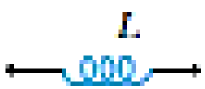
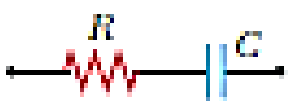
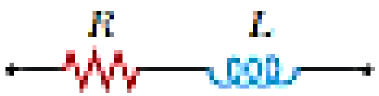
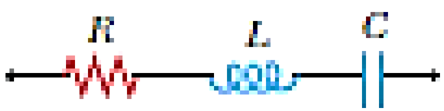
$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

resonance frequency

Maximum current

Table 33.1

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C{}^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L{}^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

[a] In each case, an AC voltage (not shown) is applied across the elements.

$$V = i\,Z$$

$$V = i\sqrt{R^2 + (X_L - X_C)^2}$$

$$V = \sqrt{i^2\,R^2 + i^2(X_L - X_C)^2}$$

$$\text{V} = \sqrt{{V_R}^2 + (V_L^2 - V_C^2)}$$

Power in an AC Circuit

<u>Root-mean-square of Current and Voltage</u>

The ammeters and voltmeters are desgined to read rms values

Direct Current

$$P = i_{ef}^2 R$$

$$\frac{\text{W}}{\text{t}} = i_{ef}^2 R \qquad \text{t} = \text{T}$$

$$W = i_{ef}^2 RT$$

Alternating Current

$$i = i_{max}\sin\omega t$$

$$P = i^2 R$$

$$P = (i_{max}\sin\omega t)^2 R$$

$$\frac{dW}{dt} = (i_{max}\sin\omega t)^2 R$$

$$\frac{dW}{dt} = i_{max}^2\sin^2(\omega t)\,R$$

$$W = \int_0^T i_{max}^2 \sin^2(\omega t)\, R\, dt$$

$$W = i_{max}^2 R \int_0^T \sin^2(\omega t)\, dt$$

$$W = i_{max}^2 R \int_0^T \left[\frac{1}{2} - \frac{1}{2}\cos(2\omega t)\right] dt$$

$$W = i_{max}^2 R \left[\frac{1}{2} t - \frac{1}{4\omega}\sin(2\,\omega t)\right]_0^T$$

$$W = i_{max}^2 R \left[\frac{1}{2} T - \frac{1}{4\omega}\sin(2\,\omega T) - 0\right]$$

$$W = i_{max}^2 R \left[\frac{1}{2} T - \frac{1}{4\omega}\sin(2\frac{2\pi}{T}T)\right]$$

$$W = i_{max}^2 R \left[\frac{1}{2} T - \frac{1}{4\omega}\sin(4\pi)\right]$$

$$W = \frac{1}{2} i_{max}^2 R T$$

$$i_{ef}^2 R T = \frac{1}{2} i_{max}^2 R T$$

$$i_{ef}^2 = \frac{1}{2} i_{max}^2$$

$$i_{eff} = \sqrt{\frac{1}{2} i_{max}^2}$$

$$i_{eff} = i_{max}\sqrt{\frac{1}{2}}$$

$$\boxed{i_{eff} = \frac{i_{max}}{\sqrt{2}}} \Rightarrow \boxed{V_{eff} = \frac{V_{max}}{\sqrt{2}}}$$

Power in an AC Circuit

$$P = i_{ef}^{2} R$$

$$P = i_{eff} i_{eff} R$$

$$P = i_{eff} \frac{V_{eff}}{Z} R$$

$$P = V_{eff} i_{eff} \cos \emptyset$$

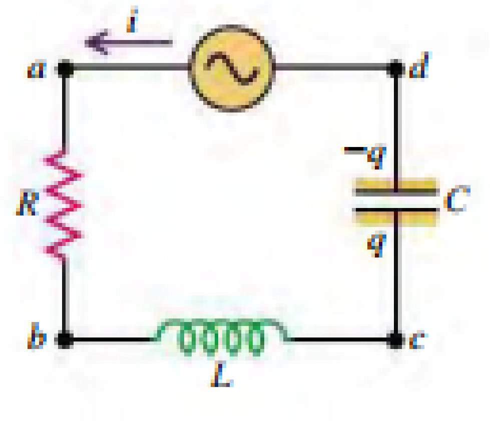
quantity $\cos \emptyset =$ the power factor

$$P = \frac{V_{max}}{\sqrt{2}} \frac{i_{max}}{\sqrt{2}} \cos \emptyset$$

$$P = \frac{1}{2} V_{max} i_{max} \cos \emptyset$$

Example

In the series circuit of Fig, suppose R = 300 Ω, L = 60 mH, C = 0.50 μF, V = 50 V, and **ω** = 10000 rad/s. Find the reactances X_L and X_C, the impedance Z, the current amplitude I, the phase angle Φ, and the voltage amplitude across each circuit element.



Solution

$$X_L = \omega L$$

$$X_L = 10000 \times 60 \times 10^{-3}$$

$$\boldsymbol{X_L = 600\Omega}$$

$$X_C = \frac{1}{\omega C}$$

$$X_C = \frac{1}{10000 \times 5 \times 10^{-7}}$$

$$\boldsymbol{X_C = 200\Omega}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{300^2 + (600 - 200)^2}$$

$$Z = \sqrt{300^2 + (400)^2}$$

$$\boldsymbol{Z = 500\Omega}$$

$$i_{max} = \frac{50}{500} = 0{,}1\ A$$

$$\tan \emptyset = \frac{X_L - X_C}{R}$$

$$\tan \emptyset = \frac{600 - 200}{300}$$

$$\tan \emptyset = \frac{400}{300}$$

$$\tan \emptyset = \frac{4}{3}$$

$$V_L = i_{max} X_L$$

$$V_L = 0{,}1 \times 600 = 60 \text{ volt}$$

$$V_C = 0{,}1 \times 200 = 20 \text{ volt}$$

$$V_R = 0{,}1 \times 300 = 30 \text{ volt}$$

Soal buku hal 156 no. 3

Suatu rangkaian RLC-seri dengan R = 400Ω, L = 0,9 H, dan C = 2 μF. Dihubungkan dengan sumber tegangan bolak-balik yang beroperasi pada tegangan maksimum $\varepsilon_{max} = 110\,V$. Hitunglah:

a. Tegangan efektif pada resistor, inductor, dan kapasitor pada kondisi resonansi
b. Tegangan efektif sumber tegangan
c. Tegangan efektif pada gabungan LC-seri

Peneyelesaian:

a. Tegangan efektif pada resistor, inductor, dan kapasitor pada kondisi resonansi

Kondisi resonansi

$$\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{0{,}9 \times 2 \times 10^{-6}}} = 745{,}35\text{rad/s}$$

$$X_L = \omega L = 745{,}35 \times 0{,}9 = 670{,}82\Omega$$

$$X_C = \frac{1}{745{,}35 \times 2 \times 10^{-6}} = 670{,}82\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{400^2 + (670{,}82 - 670{,}82)^2}$$

Z= 400Ω

$$i_{max} = \frac{\varepsilon_{max}}{Z} = \frac{110}{400} = 0{,}275\,A$$

$$i_{eff} = \frac{0{,}275}{\sqrt{2}} = 0{,}196A$$

$$V_{efR} = i_{eff}R = 0{,}196 \times 400 = 78{,}4 \text{ volt}$$

$$V_{efL} = i_{eff}X_L$$

$$V_{efL} = 0{,}196 \times 670{,}82 = 131{,}48\text{volt}$$

$$V_{efC} = i_{eff}X_C$$

$$V_{efC} = 0{,}196 \times 670{,}82 = 131{,}48\text{volt}$$

b. Tegangan efektif sumber tegangan

$$V_{eff} = \frac{V_{max}}{\sqrt{2}}$$

$$V_{eff} = \frac{110}{\sqrt{2}} = 55\sqrt{2}\text{volt}$$

c. Tegangan efektif pada gabungan LC-seri

$$V_{efLC} = i_{eff}(X_L - X_C)$$

$$V_{efLC} = 0{,}196 \times (670{,}82 - 670{,}82)$$

$$V_{efLC} = 0$$

Soal buku hal 157 no. 4

Pada rangkaian RLC-seri dikenai sumber tegangan dengan tegangan efektif 240 volt danfrekuensi 50Hz. R = 80 Ω, L = 60 mH, dan C = 20μF. Ditanyakan:

a. Reaktansi induktif, kapasitif
b. Impedansi rangkaian
c. Nilai efektif arus dalam rangkaian
d. Daya yang hilang dalam resistor
e. Tegangan efektif pada inductor
f. Tegangan maksimum pada inductor
g. Frekuensi resonansi rangkain

Penyelesaian:

a. Reaktansi induktif, kapasitif

$$X_L = \omega L = 2\pi f L$$

$$X_L = 2 \times 3{,}14 \times 50 \times 60 \times 10^{-3}$$

$$\boldsymbol{X_L = 18{,}84\Omega}$$

$$X_C = \frac{1}{\omega C}$$

$$X_C = \frac{1}{2 \times 3{,}14 \times 50 \times 20 \times 10^{-6}}$$

$$\boldsymbol{X_C = 159{,}24\ \Omega}$$

b. Impedansi rangkaian

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{80^2 + (18{,}84 - 159{,}24)^2}$$

$$Z = \sqrt{6400 + 19{,}712.16}$$

$$\boldsymbol{Z = 161,59\Omega}$$

c. Nilai efektif arus dalam rangkaian

$$i_{eff} = \frac{\varepsilon_{eff}}{Z} = \frac{240}{161{,}59} = 1{,}49\ A$$

d. Daya yang hilang dalam resistor

$$P = i_{ef}^2 R = 1{,}49^2 \times 80$$

$$\boldsymbol{P = 1,49^2 \times 80 = 177,61\ \text{watt}}$$

e. Tegangan efektif pada inductor

$$V_{efL} = i_{eff} X_L$$

$$\boldsymbol{V_{efL} = 1,49 \times 18,84 = 28,07\ \text{volt}}$$

f. Tegangan maksimum pada inductor

$$V_{maxL} = V_{effL}\sqrt{2}$$

$$\boldsymbol{V_{max} = 28,07\sqrt{2}}$$

g. Frekuensi resonansi rangkain

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$f = \frac{1}{2 \times 3{,}14}\sqrt{\frac{1}{60 \times 10^{-3} \times 20 \times 10^{-6}}}$$

$$\mathbf{f = 145,36\ Hz}$$

Example

A series RLC AC circuit has $R = 425\ \Omega$, $L = 1.25$ H, $C = 3.50\ \mu$F, $\omega = 377\ \text{s}^{-1}$, and $\Delta V_{max} = 150$ V.

(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

Solution The reactances are $X_L = \omega L = \boxed{471\ \Omega}$ and $X_C = 1/\omega C = \boxed{758\ \Omega.}$

The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(425\ \Omega)^2 + (471\ \Omega - 758\ \Omega)^2} = \boxed{513\ \Omega}$$

(B) Find the maximum current in the circuit.

Solution

$$I_{max} = \frac{\Delta V_{max}}{Z} = \frac{150\ \text{V}}{513\ \Omega} = \boxed{0.292\ \text{A}}$$

(C) Find the phase angle between the current and voltage.

Solution

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{471\ \Omega - 758\ \Omega}{425\ \Omega}\right)$$

$$= \boxed{-34.0^\circ}$$

Because the capacitive reactance is larger than the inductive reactance, the circuit is more capacitive than inductive. In this case, the phase angle ϕ is negative and the current leads the applied voltage.

(D) Find both the maximum voltage and the instantaneous voltage across each element.

Solution The maximum voltages are

$$\Delta V_R = I_{max}R = (0.292\ \text{A})(425\ \Omega) = \boxed{124\ \text{V}}$$

$$\Delta V_L = I_{max}X_L = (0.292\ \text{A})(471\ \Omega) = \boxed{138\ \text{V}}$$

$$\Delta V_C = I_{max}X_C = (0.292\ \text{A})(758\ \Omega) = \boxed{221\ \text{V}}$$

Using Equations 33.21, 33.22, and 33.23, we find that we can write the instantaneous voltages across the three elements as

$$\Delta v_R = \boxed{(124\ \text{V})\sin 377t}$$

$$\Delta v_L = \boxed{(138\ \text{V})\cos 377t}$$

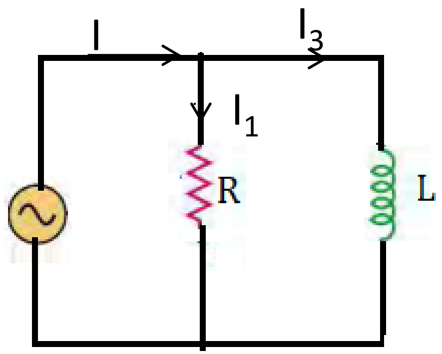
$$\Delta v_C = \boxed{(-221\ \text{V})\cos 377t}$$

What If? **What if you added up the maximum voltages across the three circuit elements? Is this a physically meaningful quantity?**

Answer The sum of the maximum voltages across the elements is $\Delta V_R + \Delta V_L + \Delta V_C = 484$ V. Note that this sum is much greater than the maximum voltage of the source, 150 V. The sum of the maximum voltages is a meaningless quantity because when sinusoidally varying quantities are added, *both their amplitudes and their phases* must be taken into account. We know that the maximum voltages across the various elements occur at different times. That is, the voltages must be added in a way that takes account of the different phases.

Impedansi (Z) Parallel

RL parallel in an AC circuit



$$V = V_{max} \sin \omega t$$

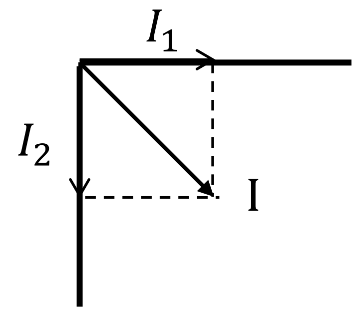
$$I = \frac{V_{max}}{Z}$$

$$I_1 = \frac{V_{max}}{R}$$

$$I_3 = \frac{V_{max}}{X_L}$$

$$i_1 = I_1 \sin \omega t$$

$$i_2 = I_2 \sin\left(\omega t - \frac{\pi}{2}\right)$$



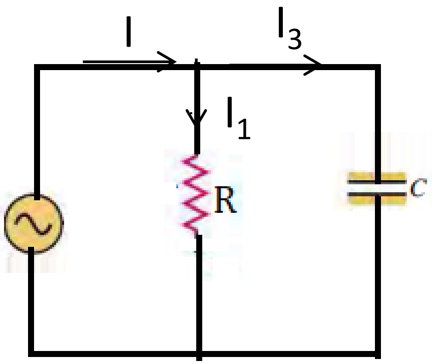
$$I = \sqrt{I_1^2 + I_2^2}$$

$$\frac{V_{max}}{Z} = \sqrt{\left(\frac{V_{max}}{R}\right)^2 + \left(\frac{V_{max}}{X_L}\right)^2}$$

$$\left(\frac{V_{max}}{Z}\right)^2 = \left(\frac{V_{max}}{R}\right)^2 + \left(\frac{V_{max}}{X_L}\right)^2$$

$$\frac{1}{Z^2} = \frac{1}{R^2} + \frac{1}{{X_L}^2}$$

RC Parallel in an AC Circuit



$$V = V_{max} \sin \omega t$$

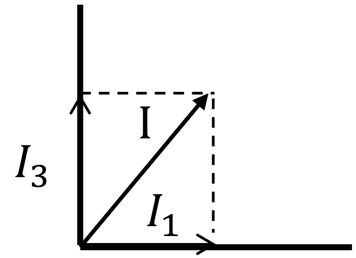
$$I = \frac{V_{max}}{Z}$$

$$I_1 = \frac{V_{max}}{R}$$

$$I_3 = \frac{V_{max}}{X_C}$$

$$i_1 = I_1 \sin \omega t$$

$$i_3 = I_3 \sin\left(\omega t + \frac{\pi}{2}\right)$$



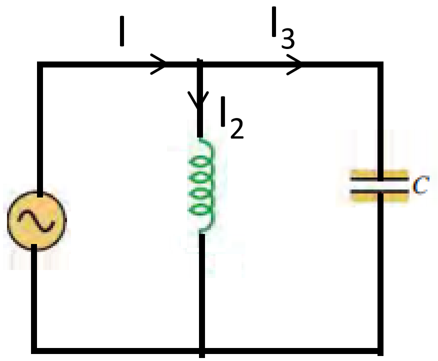
$$I = \sqrt{I_1^2 + I_3^2}$$

$$\frac{V_{max}}{Z} = \sqrt{\left(\frac{V_{max}}{R}\right)^2 + \left(\frac{V_{max}}{X_C}\right)^2}$$

$$\left(\frac{V_{max}}{Z}\right)^2 = \left(\frac{V_{max}}{R}\right)^2 + \left(\frac{V_{max}}{X_C}\right)^2$$

$$\frac{1}{Z^2} = \frac{1}{R^2} + \frac{1}{{X_C}^2}$$

LC Parallel in an AC Circuit



$$V = V_{max} \sin \omega t$$

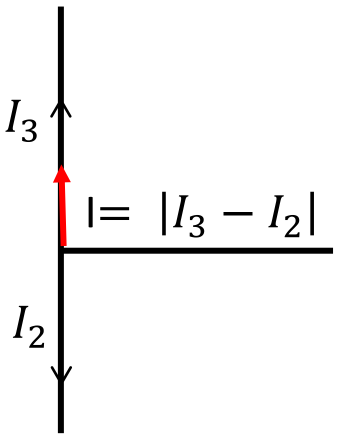
$$I = \frac{V_{max}}{Z}$$

$$I_2 = \frac{V_{max}}{X_L}$$

$$I_3 = \frac{V_{max}}{X_C}$$

$$i_2 = I_2 \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i_3 = I_3 \sin\left(\omega t + \frac{\pi}{2}\right)$$

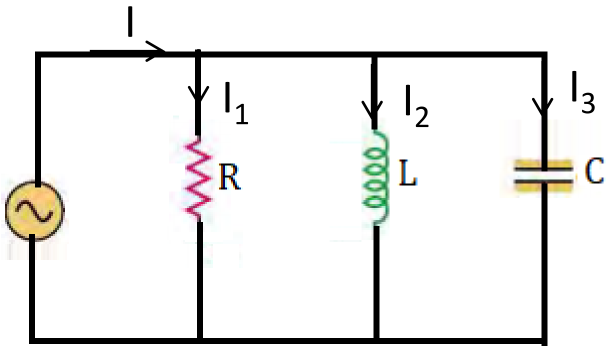


$$I = |I_3 - I_2|$$

$$\frac{V_{max}}{Z} = \left|\frac{V_{max}}{X_C} - \frac{V_{max}}{X_L}\right|$$

$$\frac{1}{Z} = \left|\frac{1}{X_C} - \frac{1}{X_L}\right|$$

RLC Parallel in an AC Circuit



$$I = \frac{V_{max}}{Z}$$

$$I_1 = \frac{V_{max}}{R}$$

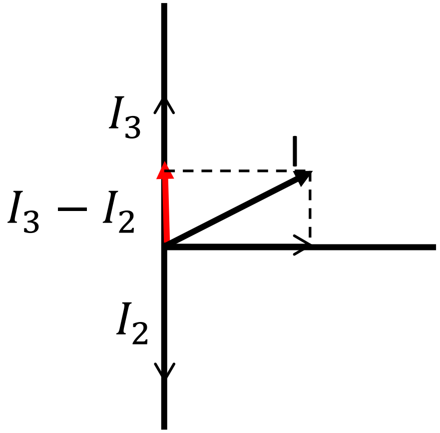
$$I_2 = \frac{V_{max}}{X_L}$$

$$I_3 = \frac{V_{max}}{X_C}$$

$$i_1 = I_1 \sin \omega t$$

$$i_2 = I_2 \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i_3 = I_3 \sin\left(\omega t + \frac{\pi}{2}\right)$$



$$I = \sqrt{I_1^2 + (I_3 - I_2)^2}$$

$$\frac{V_{max}}{Z} = \sqrt{\left(\frac{V_{max}}{R}\right)^2 + \left(\frac{V_{max}}{X_C} - \frac{V_{max}}{X_L}\right)^2}$$

$$\left(\frac{V_{max}}{Z}\right)^2 = \left(\frac{V_{max}}{R}\right)^2 + \left(\frac{V_{max}}{X_C} - \frac{V_{max}}{X_L}\right)^2$$

$$\frac{1}{Z^2} = \frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2$$