$$||\mathbf{v}|| = \sqrt{v_x^2 + v_y^2 + v_z^2 + ...}$$

$$||\mathbf{v}|| = \sqrt{\mathbf{v} \cdot \mathbf{v}}$$

$$||t\mathbf{v}|| = |t|||\mathbf{v}||$$

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_x b_z + ...$$

$$\mathbf{a} \cdot \mathbf{b} = ||\mathbf{a}||||\mathbf{b}||cos\theta$$

$$\mathbf{a} \cdot \mathbf{a} = a^2$$

$$\mathbf{a} \cdot \mathbf{b} = 0$$

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix}$$

$$\mathbf{a} \times \mathbf{b} = ||\mathbf{a}||||\mathbf{b}||cos\theta$$

$$A = \frac{1}{2}||(\mathbf{p}_1 - \mathbf{p}_0) \times (\mathbf{p}_2 - \mathbf{p}_0)||$$

$\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2$

$$||\mathbf{a} \times \mathbf{b}|| = ||\mathbf{a}||||\mathbf{b}||sin\theta$$

$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{||\mathbf{v}||}$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$$

$$[\mathbf{a}, \mathbf{b}, \mathbf{c}] = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = (\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} = (\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b}$$

$$[\mathbf{a}, \mathbf{b}, \mathbf{c}] = -[\mathbf{c}, \mathbf{b}, \mathbf{a}]$$

$$\mathbf{a}_{||\mathbf{b}} = \frac{\mathbf{a}\cdot\mathbf{b}}{b^2}\mathbf{b}$$

$$\mathbf{a}_{||\mathbf{b}} = \frac{1}{b^2} \begin{bmatrix} b_x^2 & b_x b_y & b_x b_z \\ b_x b_y & b_y^2 & b_y b_z \\ b_x b_z & b_y b_z & b_z^2 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$\mathbf{a}_{\perp \mathbf{b}} = \mathbf{a} - \mathbf{a}_{||\mathbf{b}}$$

$$(\mathbf{a}_{||\mathbf{b}})^2 + (\mathbf{a}_{\perp\mathbf{b}})^2 = a^2$$

$$\mathbf{u}_1 = \mathbf{v}_1, \mathbf{u}_2 = \mathbf{v}_2 - \left(\mathbf{v}_2\right)_{||\mathbf{u}_1}, \mathbf{u}_3 = \mathbf{v}_3 - \left(\mathbf{v}_3\right)_{||\mathbf{u}_1} - \left(\mathbf{v}_3\right)_{||\mathbf{u}_2}, ...$$

$$\mathbf{a} \otimes \mathbf{b} = \mathbf{a}\mathbf{b}^T = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \begin{bmatrix} b_x & b_y & b_z \end{bmatrix}$$

$$= \begin{bmatrix} a_x b_z & a_x b_y & a_x b_z \\ a_y b_x & a_y b_y & a_y b_z \\ a_z b_x & a_z b_y & a_z b_z \end{bmatrix}$$

$$\mathbf{v} = (v_x, v_y, v_z, 0)$$

$$\boldsymbol{p} = (v_x, v_y, v_z, 1)$$

$$\mathbf{A}_{n,m}\mathbf{B}_{m,p} = \mathbf{C}_{n,p}$$

$$\mathbf{M}_{i,j}^{T} = \mathbf{M}_{j,i}$$

$$\mathbf{M} = [\mathbf{abc}]$$

$$\mathbf{M} = \begin{bmatrix} X & a & b \\ a & X & c \\ b & c & X \end{bmatrix}$$

$$\mathbf{M}^T = \mathbf{M}$$

$$\mathbf{M} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$\mathbf{M}^T = -\mathbf{M}$$

$$\mathbf{M}\mathbf{v} = v_x\mathbf{a} + v_y\mathbf{b} + v_z\mathbf{c}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$det(\mathbf{M}) = \sum_{i=0}^{n-1} M_{ik}(-1)^{i+k}|\mathbf{M}_{\bar{i}\bar{k}}|$$

$$|M_{ik}|$$

$$C_{ij}(\mathbf{M}) = (-1)^{i+j}|\mathbf{M}_{\bar{i}\bar{j}}|$$

$$\mathbf{M}^{-1} = \frac{1}{det(\mathbf{M})}\mathbf{C}^T(\mathbf{M})$$

$$C^T(M) =$$

$$\mathbf{A}^{-1} = \frac{1}{A_{00}A_{11} - A_{01}A_{10}} \begin{bmatrix} A_{11} & -A_{01} \\ -A_{10} & A_{00} \end{bmatrix}$$

$$\mathbf{B}^{-1} = \frac{1}{det(\mathbf{B})} \begin{bmatrix} B_{11}B_{22} - B_{12}B_{21} & B_{02}B_{21} - B_{01}B_{22} & B_{01}B_{12} - B_{02}B_{11} \\ B_{12}B_{20} - B_{10}B_{22} & B_{00}B_{22} - B_{02}B_{20} & B_{02}B_{10} - B_{00}B_{12} \\ B_{10}B_{21} - B_{11}B_{20} & B_{01}B_{20} - B_{00}B_{21} & B_{00}B_{11} - B_{01}B_{10} \end{bmatrix}$$

$$\mathbf{M}^{-1} = \frac{1}{[\mathbf{a},\mathbf{b},\mathbf{c}]} \begin{bmatrix} \mathbf{b} \times \mathbf{c} \\ \mathbf{c} \times \mathbf{a} \\ \mathbf{a} \times \mathbf{b} \end{bmatrix}$$

$$\mathbf{M}^{-1} = \frac{1}{\mathbf{s}\cdot\mathbf{v}+\mathbf{t}\cdot\mathbf{u}} \begin{bmatrix} \mathbf{b}\times\mathbf{v}+y\mathbf{t} | -\mathbf{b}\cdot\mathbf{t} \\ \mathbf{v}\times\mathbf{a}-x\mathbf{t} | \mathbf{a}\cdot\mathbf{t} \\ \mathbf{d}\times\mathbf{u}+w\mathbf{s} | -\mathbf{d}\cdot\mathbf{s} \\ \mathbf{u}\times\mathbf{c}-z\mathbf{s} | \mathbf{c}\cdot\mathbf{s} \end{bmatrix}$$

$$a \times b$$

$$c \times d$$

$$ya - xb$$

$$wc - zd$$

$$\mathbf{M}^T\mathbf{M} = \begin{bmatrix} a^2 & \mathbf{a}\cdot\mathbf{b} & \mathbf{a}\cdot\mathbf{c} \\ \mathbf{b}\cdot\mathbf{a} & b^2 & \mathbf{b}\cdot\mathbf{c} \\ \mathbf{c}\cdot\mathbf{a} & \mathbf{c}\cdot\mathbf{b} & c^2 \end{bmatrix}$$

$$M^T = M^{-1}$$

$$\mathbf{B} = \mathbf{M}\mathbf{A}\mathbf{M}^{-1}$$

$$M_{rot_x}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}$$

$$M_{rot_y}(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix}$$

$$M_{rot_z}(\theta) = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{rot}(\theta, \mathbf{a}) = \begin{bmatrix} c+(1-c)a_x^2 & (1-c)a_xa_y - sa_z & (1-c)a_xa_z + sa_y \\ (1-c)a_xa_y + sa_z & c+(1-c)a_y^2 & (1-c)a_ya_z - sa_x \\ (1-c)a_xa_z - sa_y & (1-c)a_ya_z + sa_x & c+(1-c)a_z^2 \end{bmatrix}$$

$cos\theta$

$sin\theta$

$$M_{reflect}(\mathbf{a}) = \begin{bmatrix} 1-2a_x^2 & -2a_xa_y & -2a_xa_z \\ -2a_xa_y & 1-2a_y^2 & -2a_ya_z \\ -2a_xa_z & -2a_ya_z & 1-2a_z^2 \end{bmatrix}$$

$$M_{invol}(\mathbf{a}) = \begin{bmatrix} 2a_x^2 - 1 & 2a_x a_y & 2a_x a_z \\ 2a_x a_y & 2a_y^2 - 1 & 2a_y a_z \\ 2a_x a_z & 2a_y a_z & 2a_z^2 - 1 \end{bmatrix}$$

$$M_{scale}(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$$

$$M_{scale}(s, \mathbf{a}) = \begin{bmatrix} (s-1)a_x^2 + 1 & (s-1)a_x a_y & (s-1)a_x a_z \\ (s-1)a_x a_y & (s-1)a_y^2 + 1 & (s-1)a_y a_z \\ (s-1)a_x a_z & (s-1)a_y a_z & (s-1)a_z^2 + 1 \end{bmatrix}$$

$$M_{skew}(\theta, \mathbf{a}, \mathbf{b}) = \begin{bmatrix} a_x b_x tan\theta + 1 & a_x b_y tan\theta & a_x b_z tan\theta \\ a_y b_x tan\theta & a_y b_y tan\theta + 1 & a_y b_z tan\theta \\ a_z b_x tan\theta & a_z b_y tan\theta & a_z b_z tan\theta + 1 \end{bmatrix}$$

$$M_{skew}(\theta, \mathbf{a}, \mathbf{b}) = \mathbf{I} + tan\theta(\mathbf{a} \otimes \mathbf{b})$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{M} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\mathbf{H}^{-1} = \begin{bmatrix} \mathbf{M}^{-1} & -\mathbf{M}^{-1}\mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\mathbf{q} = xi + yj + zk + w$$

$$\mathbf{q} = \mathbf{v} + s$$

$$\mathbf{q} = (sin\frac{\theta}{2})\mathbf{a} + cos\frac{\theta}{2}$$

θ

$$\mathbf{q}_1\mathbf{q}_2 = \mathbf{v}_1 \times \mathbf{v}_2 + s_1\mathbf{v}_2 + s_2\mathbf{v}_1 + s_1 s_2 - \mathbf{v}_1 \cdot \mathbf{v}_2$$

$$\mathbf{q}_2\mathbf{q}_1 = \mathbf{q}_1\mathbf{q}_2 - 2(\mathbf{v}_1 \times \mathbf{v}_2)$$

$$\mathbf{q}^* = -\mathbf{v} + s$$

$$\mathbf{q}\mathbf{q}^* = \mathbf{q}^*\mathbf{q} = v^2 + s^2$$

$$||\mathbf{q}|| = \sqrt{\mathbf{q}\mathbf{q}^*} = \sqrt{v^2 + s^2}$$

$$\mathbf{q}^{-1} = \frac{\mathbf{q}^*}{\mathbf{q}\mathbf{q}^*} = \frac{-\mathbf{v}+s}{v^2+s^2}$$

$$\mathbf{v}^{'} = \mathbf{q}\mathbf{v}\mathbf{q}^{-1}$$

$$v_x i + v_y j + v_z k + 0$$

$$\mathbf{v}' = \mathbf{q}\mathbf{v}\mathbf{q}^*$$

$$\mathbf{v}^{`} = (\mathbf{q}_2\mathbf{q}_1)\mathbf{v}(\mathbf{q}_2\mathbf{q}_1)^*$$

$$\mathbf{M}_{rot}(\mathbf{q}) = \begin{bmatrix} 1-2y^2-2z^2 & 2(xy-wz) & 2(xz+wy) \\ 2(xy+wz) & 1-2x^2-2z^2 & 2(yz-wx) \\ 2(xz-wy) & 2(yz+wx) & 1-2x^2-2y^2 \end{bmatrix}$$

$$\mathbf{n} = (\mathbf{p}_1 - \mathbf{p}_0) \times (\mathbf{p}_2 - \mathbf{p}_0)$$

$$\mathbf{n}^B = \mathbf{n}^A \mathbf{M}^{-1}$$

$$\mathbf{n}^B = \mathbf{n}^A adj(\mathbf{M}) = \mathbf{n}^A det(\mathbf{M})\mathbf{M}^{-1}$$

$$d = \sqrt{u^2 - \frac{(\mathbf{u}\cdot\mathbf{v})^2}{v^2}}$$

$$d = \sqrt{\frac{(\mathbf{u} \times \mathbf{v})^2}{v^2}}$$

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \frac{1}{(\mathbf{v}_1 \cdot \mathbf{v}_2)^2 - v_1^2 v_2^2} \begin{bmatrix} -v_2^2 & \mathbf{v}_1 \cdot \mathbf{v}_2 \\ -\mathbf{v}_1 \cdot \mathbf{v}_2 & v_1^2 \end{bmatrix} \begin{bmatrix} (\mathbf{p}_2 - \mathbf{p}_1) \cdot \mathbf{v}_1 \\ (\mathbf{p}_2 - \mathbf{p}_1) \cdot \mathbf{v}_2 \end{bmatrix}$$

$p_1, p_2(L1(t) = p_1 + t_1 v_1)$

$$d = ||L_2(t_2) - L_1(t_1)||$$

$$d = \sqrt{\frac{[(\mathbf{p}_2 - \mathbf{p}_1) \times \mathbf{v}_1]^2}{v_1^2}}$$

$$\mathbf{f} \cdot \mathbf{p} = 0, \mathbf{f} = [n_x n_y n_z d] = [\mathbf{n}|d]$$

$$d = \mathbf{f} \cdot \mathbf{p}$$

$$\mathbf{p}' = \mathbf{p} - 2(\mathbf{f} \cdot \mathbf{p})\mathbf{n}$$

$$\mathbf{H}_{reflect}(\mathbf{f}) = \begin{bmatrix} 1-2n_x^2 & -2n_xn_y & -2n_xn_z & -2n_xd \\ -2n_xn_y & 1-2n_y^2 & -2n_yn_z & -2n_yd \\ -2n_xn_z & -2n_yn_z & 1-2n_z^2 & -2n_zd \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{q} = \mathbf{p} - \frac{\mathbf{f} \cdot \mathbf{p}}{\mathbf{f} \cdot \mathbf{v}} \mathbf{v}$$

$$\mathbf{p} = \frac{d_1(\mathbf{n}_3 \times \mathbf{n}_2) + d_2(\mathbf{n}_1 \times \mathbf{n}_3) + d_3(\mathbf{n}_2 \times \mathbf{n}_1)}{[\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3]}$$

$$\mathbf{p} = \frac{d_1(\mathbf{v}\times\mathbf{n}_2)+d_2(\mathbf{n}_1\times\mathbf{v})}{v^2}$$

$$\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2$$

$$\mathbf{f}^B = \mathbf{f}^A det(\mathbf{H})\mathbf{H}^{-1} = \mathbf{f}^A adj(\mathbf{H})$$

$$\{\mathbf{v}|\mathbf{m}\}, \mathbf{m} = \mathbf{p}_1 \times \mathbf{p}_2$$

$$(\mathbf{p}|w)$$

$$d = \frac{|\mathbf{v}_1 \cdot \mathbf{m}_2 + \mathbf{v}_2 \cdot \mathbf{m}_1|}{||\mathbf{v}_1 \times \mathbf{v}_2||}$$

$$\{\mathbf{v}^B|\mathbf{m}^B\} = \{\mathbf{M}\mathbf{v}^A|\mathbf{m}^A adj(\mathbf{M}) + \mathbf{t} \times (\mathbf{M}\mathbf{v}^A)\}$$

$$\mathbf{a} \wedge \mathbf{b} = (a_y b_z - a_z b_y)\mathbf{e}_{23} + (a_z b_x - a_x b_z)\mathbf{e}_{31} + (a_x b_y - a_y b_x)\mathbf{e}_{12}$$

$$\mathbf{e}_i \wedge \mathbf{e}_j = \mathbf{e}_{ij}$$

$$\mathbf{a} \vee \mathbf{b} = (a_y b_z - a_z b_y)\mathbf{e}_1 + (a_z b_x - a_x b_z)\mathbf{e}_2 + (a_x b_y + a_y b_x)\mathbf{e}_3$$

$$\mathbf{a} \wedge \mathbf{b} = -\mathbf{b} \wedge \mathbf{a}$$

$$\mathbf{a} \wedge \mathbf{a} = 0$$

$$\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c} = (a_x b_y c_z + a_y b_z c_x + a_z b_x c_y - a_x b_z c_y - a_y b_x c_z - a_z b_y c_x)(\mathbf{e}_1 \wedge \mathbf{e}_2 \wedge \mathbf{e}_3)$$

$$gr(\mathbf{A} \wedge \mathbf{B}) = gr(\mathbf{A}) + gr(\mathbf{B})$$

$$gr(\mathbf{A} \wedge \mathbf{B}) = -1^{gr(\mathbf{A})gr(\mathbf{B})}(\mathbf{B} \wedge \mathbf{A})$$

$$\mathbf{E}_n = \mathbf{e}_{12...n} = \mathbf{e}_1 \wedge \mathbf{e}_2 \wedge ...\mathbf{e}_n$$

$$\mathbf{A} \wedge \overline{\mathbf{A}} = \mathbf{E}_n$$

$$\mathbf{a} \times \mathbf{b} = \overline{\mathbf{a} \wedge \mathbf{b}}$$

$$\underline{\mathbf{A}} = (-1)^{k(n-k)}\overline{\mathbf{A}}$$

$$\overline{\mathbf{A} \wedge \mathbf{B}} = \overline{\mathbf{A}} \vee \overline{\mathbf{B}}$$

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \vee \overline{\mathbf{b}}$$

$$\mathbf{p} \wedge \mathbf{q} = (q_x - p_x)\mathbf{e}_{41} + (q_y - p_y)\mathbf{e}_{42} + (q_z - p_z)\mathbf{e}_{43} + (p_y q_z - p_z q_y)\mathbf{e}_{23} + (p_z q_x - p_x q_z)\mathbf{e}_{31} + (p_x q_y - p_y q_x)\mathbf{e}_{12}$$

$$\mathbf{p} \wedge \mathbf{L} = (L_{vy}p_z - L_{vz}p_y + L_{mx})\overline{\mathbf{e_1}} + (L_{vz}p_x - L_{vx}p_z + L_{my})\overline{\mathbf{e_2}} + (L_{vx}p_y - L_{vy}p_x + L_{mz})\overline{\mathbf{e_3}} + (-L_{mx}p_x - L_{my}p_y - L_{mz}p_z)\overline{\mathbf{e_4}}$$

$q \wedge r$

$$\{v|m\}$$

$$\mathbf{f} \vee \mathbf{g} = (f_z g_y - f_y g_z)\mathbf{e}_{41} + (f_x g_z - f_z g_x)\mathbf{e}_{42} + (f_y g_x - f_x g_y)\mathbf{e}_{43} + (f_x g_w - f_w g_x)\mathbf{e}_{23} + (f_y g_w - f_w g_y)\mathbf{e}_{31} + (f_z g_w - f_w g_z)\mathbf{e}_{12}$$

$$\mathbf{f} \vee \mathbf{L} = (L_{my}f_z - L_{mz}f_y + L_{vx}f_w)\mathbf{e}_1 + (L_{mz}f_x - L_{mx}f_z + L_{vy}f_w)\mathbf{e}_2 + (L_{mx}f_y - L_{my}f_x + L_{vz}f_w)\mathbf{e}_3 + (-L_{vx}f_x - L_{vy}f_y - L_{vz}f_z)\mathbf{e}_4$$

$$d = \frac{\mathbf{L}_1 \vee \mathbf{L}_2}{||\mathbf{v}_1 \wedge \mathbf{v}_2||}$$

$$L = \{v|m\}$$

$$d = \frac{\mathbf{p} \vee \mathbf{f}}{||\mathbf{n}||}$$

$$f = \{\mathbf{n}|d\}$$

$$[\mathbf{abc}]^{-1} = \frac{1}{\underline{\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c}}} \begin{bmatrix} \underline{\mathbf{b} \wedge \mathbf{c}} \\ -\underline{\mathbf{a} \wedge \mathbf{c}} \\ \underline{\mathbf{a} \wedge \mathbf{b}} \end{bmatrix}$$

$$[\mathbf{abcd}]^{-1} = \frac{1}{\underline{\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c} \wedge \mathbf{d}}} \begin{bmatrix} \underline{\mathbf{b} \wedge \mathbf{c} \wedge \mathbf{d}} \\ -\underline{\mathbf{a} \wedge \mathbf{c} \wedge \mathbf{d}} \\ \underline{\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{d}} \\ -\underline{\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c}} \end{bmatrix}$$