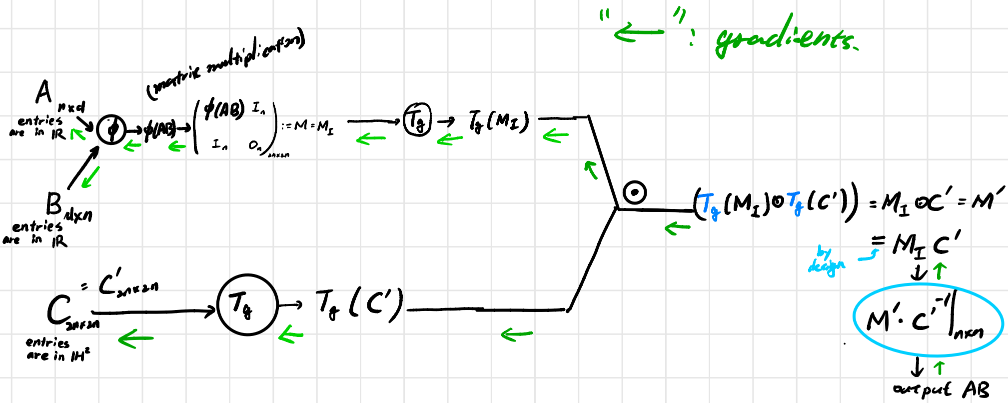
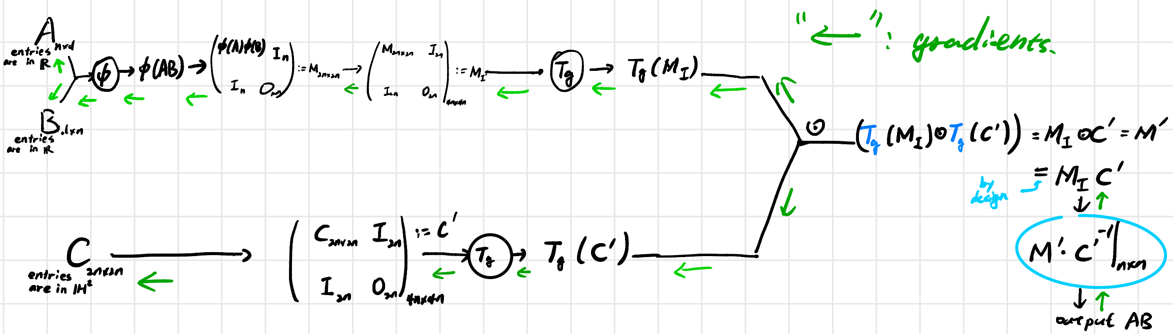
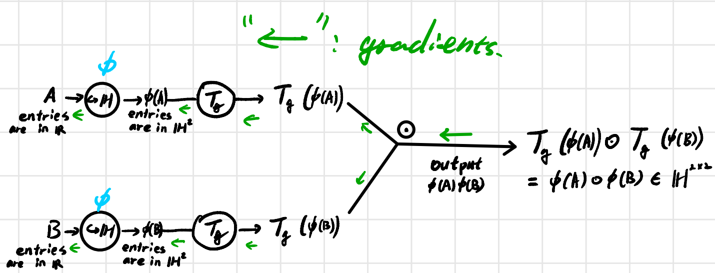
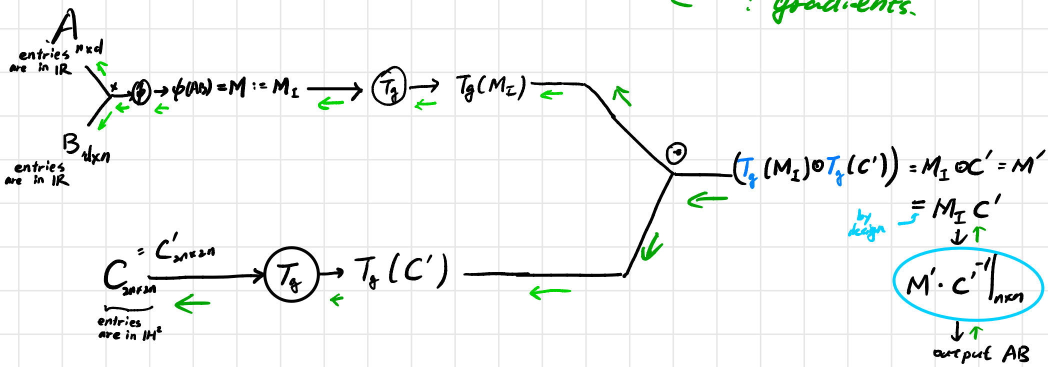


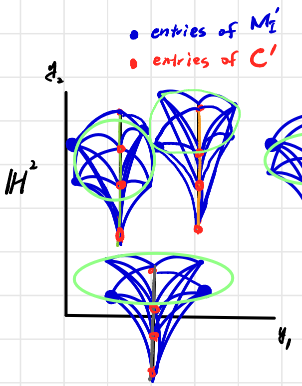


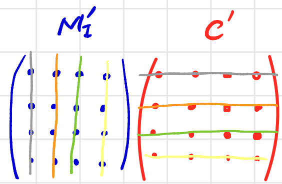


Case 2. Method 3.

"←": gradients.







$= M_I' C'$

$= M_I' \odot_{\mathbb{H}^2} C'$

by design

1. map M_I' entries to $\mathbb{H}^2$ randomly.
2. randomly generate M'' (invertible and real)
3. $C' := M_I'^{-1} \cdot M''$

$x \in T_m(M_I')$ $\quad x, y \in H^2$ and they are entries of $T_m(M_I')$

$y \in T_m(C')$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad T_m(C')$.

$$d_H(x,y) = d_H(\underbrace{S(x),\ S(y)}_{\text{on } y_2\text{-axis}}) = \ln\left(\frac{S(x)}{S(y)}\right)$$

$$\frac{\partial d_H}{\partial y} = \frac{\partial}{\partial y} \ln\left(\frac{Sx}{Sy}\right)$$

$$= \frac{Sy}{\cancel{Sx}} \frac{(-\cancel{Sx})\left(\frac{\partial S}{\partial y}\right)}{(Sy)^2}$$

$$= \frac{-1}{Sy} \frac{\partial S}{\partial y} = \frac{-1}{Sy} \cdot \frac{1}{(cy+d)^2} = \boxed{\frac{-1}{(ay+b)(cy+d)}}$$

$$\frac{\partial d_H}{\partial x} = \frac{Sy}{Sx} \frac{\frac{\partial S}{\partial x} - 0}{(Sy)^2}$$

$$= \frac{1}{Sx\,Sy} \cdot \frac{\partial S}{\partial x} = \frac{1}{SxSy} \frac{1}{(cx+d)^2} = \frac{1}{\left(\frac{ax+b}{cx+d}\right)\left(\frac{ay+b}{cy+d}\right)(cx+d)^2}$$

$$= \boxed{\frac{(cx+d)}{(ax+b)(ay+b)(cx+d)}}$$

$$Sx = \frac{ax+b}{cx+d} \qquad \frac{\partial S}{\partial x} = \frac{a(\cancel{cx}+d) - c(\cancel{ax}+b)}{(cx+d)^2}$$

$$Sy = \frac{ay+b}{cy+d} \qquad = \frac{ad-bc}{(cx+d)^2} = \frac{1}{(cx+d)^2}$$

$$= \frac{1}{(cy+d)^2}$$