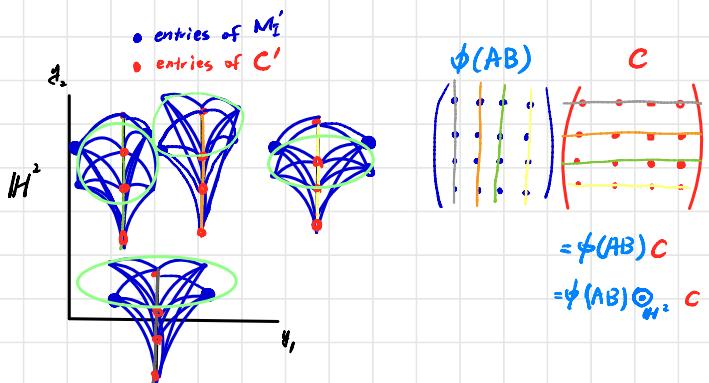
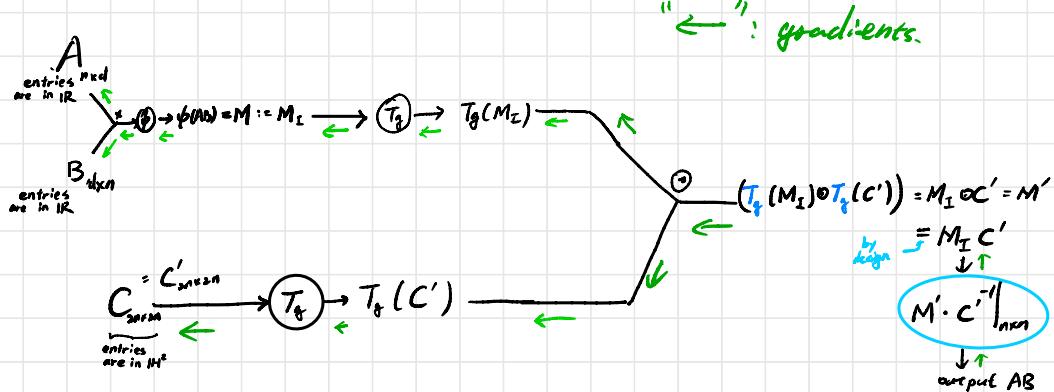


Case 2. Method 3.



1. map M_I' entries to H^2 randomly.
2. randomly generate M'' (invertible and real)
3. $C' := M_I'^{-1} \cdot M''$

$x \in T_m(M_2')$ $x, y \in \mathbb{H}^2$ and they are entries of $T_m(M_2')$
 $y \in T_m(C')$ $T_m(C')$.

$$d_{\mathbb{H}}(x, y) = \underbrace{d_{\mathbb{H}}(S(x), S(y))}_{\text{on } \mathbb{H}_2\text{-axis}} = \ln \left(\frac{s(x)}{s(y)} \right)$$

$$\begin{aligned} \frac{\partial d_{\mathbb{H}}}{\partial y} &= \frac{\partial}{\partial y} \ln \left(\frac{s(x)}{s(y)} \right) \\ &= \frac{s(y)(-s'(y))(\frac{\partial s}{\partial y})}{s(x)(s(y))^2} \\ &= \frac{-1}{s(y)} \frac{\frac{\partial s}{\partial y}}{s(y)} = \frac{-1}{s(y)} \cdot \frac{1}{(cx+d)^2} = \boxed{\frac{-1}{(ax+b)(cy+d)}} \\ \frac{\partial d_{\mathbb{H}}}{\partial x} &= \frac{s(y)}{s(x)} \frac{\frac{\partial s}{\partial x} - 0}{(s(y))^2} \\ &= \frac{1}{s(x)s(y)} \cdot \frac{\frac{\partial s}{\partial x}}{(cx+d)^2} = \frac{1}{s(x)s(y)} \frac{1}{(cx+d)^2} = \boxed{\frac{1}{(\frac{ax+b}{cx+d})(\frac{ay+b}{cy+d})(cx+d)}} \\ s(x) &= \frac{ax+b}{cx+d} \quad \frac{\partial s}{\partial x} = \frac{a(cx+d) - c(ax+b)}{(cx+d)^2} \\ s(y) &= \frac{ay+b}{cy+d} \quad = \frac{ad - bc}{(cx+d)^2} = \frac{1}{(cx+d)^2} \\ &= \boxed{\frac{(cx+d)}{(ax+b)(ay+d)(cx+d)}} \end{aligned}$$

$$= \frac{1}{(cy+d)^2}$$