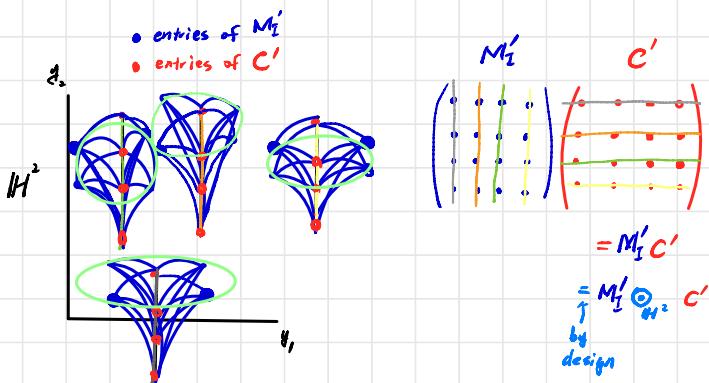
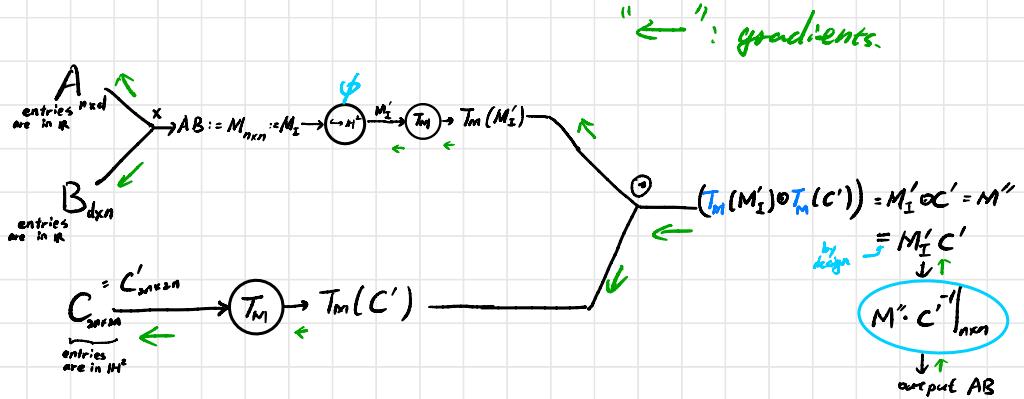


Case 2. Method 3.



1. map M'_I entries to H^2 randomly.
2. randomly generate M'' (invertible and real)
3. $C' := M'^{-1}_I \cdot M''$

$$d_H(x, y) = \underbrace{d_H(S(x), S(y))}_{\text{on } \mathbb{A}_3} = \ln \left(\frac{S(x)}{S(y)} \right)$$

$$\begin{aligned}\frac{\partial d_1}{\partial y} &= \frac{\partial}{\partial y} \ln \left(\frac{sy}{sx} \right) \\&= \frac{sy}{sx} \frac{(-sx)(\frac{\partial s}{\partial y})}{(sy)^2} \\&= \frac{-1}{sy} \frac{\partial s}{\partial y} = \frac{-1}{sy} \cdot \frac{1}{(cy+d)^2} = \boxed{\frac{-1}{(ay+b)(cy+d)}} \\[10pt]\frac{\partial d_1}{\partial x} &= \frac{sy}{sx} \frac{\frac{\partial s}{\partial x} - 0}{(sy)^2} \\&= \frac{1}{sy} \frac{1}{\frac{\partial s}{\partial x}} = \frac{1}{sy} \frac{1}{(cx+d)^2} = \frac{1}{\frac{(ax+b)}{(cx+d)} \left(\frac{ay+b}{cx+d} \right) (cx+d)} \\[10pt]\frac{1}{x+d} &\quad \frac{\partial s}{\partial x} = \frac{a(cx+d) - c(ax+b)}{(cx+d)^2} \\&= \boxed{\frac{(cx+d)}{(ax+b)(ay+b)(cx+d)}}\end{aligned}$$

$$Sx = \frac{ax+b}{cx+d} \quad \frac{\partial S}{\partial x} = \frac{a(cx+d) - c(ax+b)}{(cx+d)^2}$$

$$sy = \frac{ay+bd}{cy+cd} = \frac{ad-bc}{(cx+d)^2} = \frac{1}{(cx+d)^2}$$

$$= \frac{1}{(cg+d)^2}$$