$$A_{n\times d}\ \nearrow$$
(matrix multiplications)

$$X \longrightarrow \begin{pmatrix} AB_{n\times n} & I_n \\ I_n & 0_n \end{pmatrix}_{2n\times 2n} := M = M_I \longrightarrow \boxed{T_M} \longrightarrow T_M(M_I)$$

$$B_{d\times n}\ \searrow$$

"$\longleftarrow$": gradients.

$$\odot$$

$$(T_M(M_I) \odot T_M(M_I)) = M_I\, C' := M_I'$$

$$C_{2n\times 2n} = C'_{2n\times 2n} \longrightarrow \boxed{T_M} \longrightarrow T_M(C')$$

↑   ↓ output AB

$$(M_I' \cdot C'^{-1})\big|_{n\times n}$$
restrict to n×n

$$\|$$

$$(T_M M_I \odot T_M C') \cdot C'^{-1}\big|_{n\times n}$$
$$\underset{AB}{\smile}$$

$$A_{n\times d}\ \nearrow$$

$$X \longrightarrow \begin{pmatrix} AB_{n\times n} & I_n \\ I_n & 0_n \end{pmatrix}_{2n\times 2n} := M_{2n\times 2n} \longrightarrow \begin{pmatrix} M_{2n\times 2n} & I_n \\ I_n & 0_n \end{pmatrix}_{4n\times 4n} := M_I \longrightarrow \boxed{T_M} \longrightarrow T_M(M_I)$$

$$B_{d\times n}\ \searrow$$

"$\longleftarrow$": gradients.

$$\odot$$

$$(T_M(M_I) \odot T_M(M_I)) = M_I\, C' := M_I'$$

$$C_{2n\times 2n} \longrightarrow \begin{pmatrix} C_{2n\times 2n} & I_{2n} \\ I_{2n} & 0_{2n} \end{pmatrix}_{4n\times 4n} := C' \longrightarrow \boxed{T_M} \longrightarrow T_M(C')$$

↑   ↓ output AB

$$(M_I' \cdot C'^{-1})\big|_{n\times n}$$
restrict to n×n

$$\|$$

$$(T_M M_I \odot T_M C') \cdot C'^{-1}\big|_{n\times n}$$
$$\underset{AB}{\smile}$$

"$\longleftarrow$": gradients.

$$A \longrightarrow \boxed{T_M} \longrightarrow T_M(A)$$

$$\odot$$

output AB

$$T_M(A) \odot T_M(B) = AB$$

$$B \longrightarrow \boxed{T_M} \longrightarrow T_M(B)$$

Case 2. Method 3.

$A_{n \times d}$

$B_{d \times n}$

$\xrightarrow{\times} AB := M_{n \times n} := M_{\mathcal{I}} \to (T_M) \to T_M(M_{\mathcal{I}})$

$C_{2n \times 2n} := C'_{2n \times 2n} \to (T_M) \to T_M(C')$

① $\to (T_M(M_{\mathcal{I}}) \oslash T_M(M_{\mathcal{I}})) = M_{\mathcal{I}} C' := M'_{\mathcal{I}}$

output AB

$(M'_{\mathcal{I}} \cdot C'^{-1})\big|_{n \times n}$ \quad restrict to $n \times n$

$\|$

$(T_M M_{\mathcal{I}} \oslash T_M C') \cdot C'^{-1}\big|_{n \times n}$

$\underset{AB}{\|}$