# Outline

- Examples of Applications of Sparsity
- $l_{2,}$ $l_{0,}$ and $l_1$ Norms
- Solution Approaches
  - Matching Pursuit
  - Smooth Reformulations
- Sparse Solutions to Some Applications

# What is Sparsity?

A vector is said to be sparse if it only has *"a few"* non-zero components

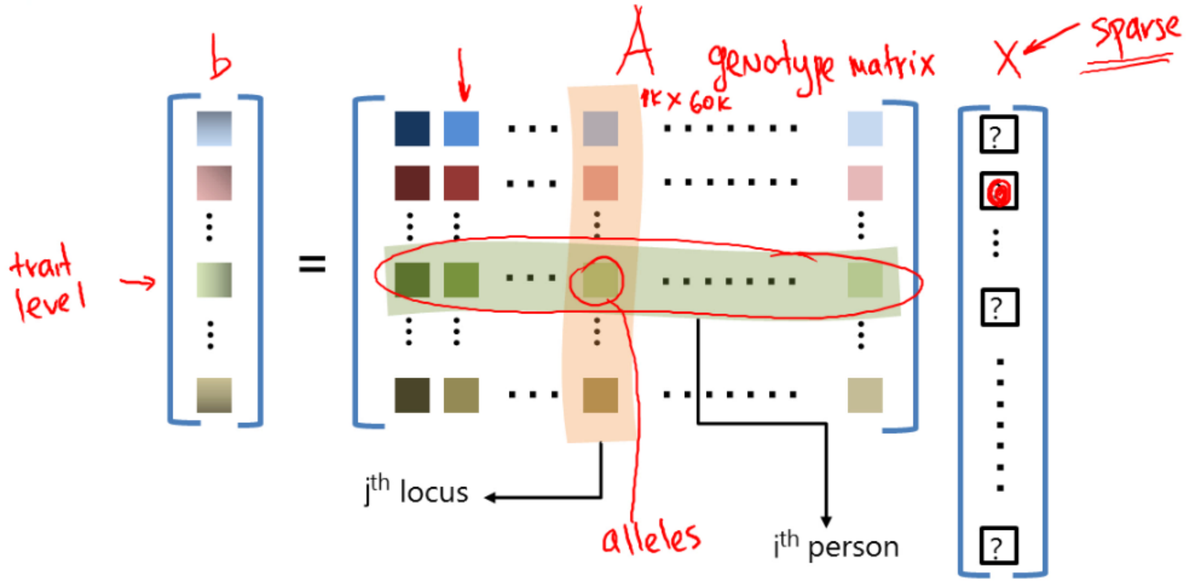
The vector can represent a signal (image), which may be sparse in its native domain (e.g., image of sky at night) or can be made sparse in another domain (e.g., natural images in the DFT domain)

A sparse vector may originate in numerous applications

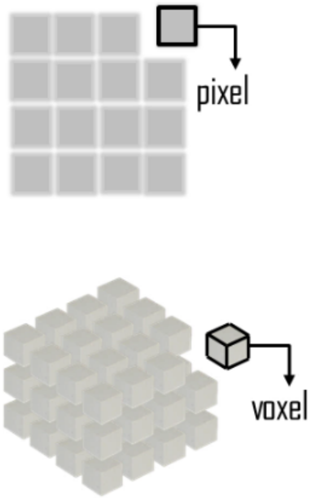
# Applications

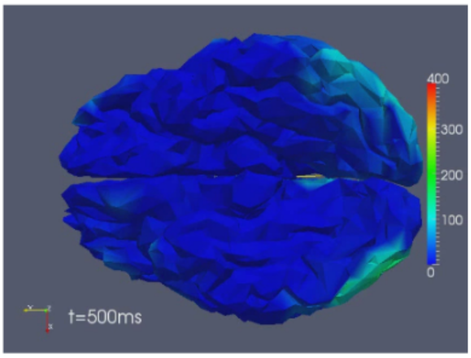
- Image and Video Processing
- Machine Learning
- Statistics
- Genetics
- Econometrics
- Neuroscience
- ...

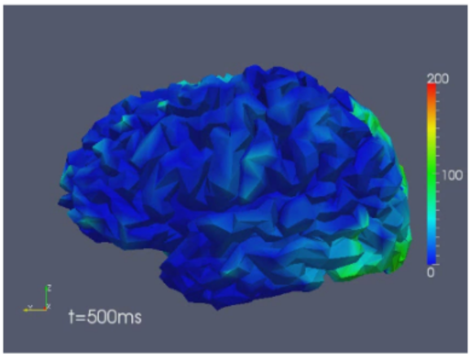
# Genetics


b
A
genotype matrix
1K x 60K
X
sparse
trait level
=
?
?
?
jth locus
alleles
ith person

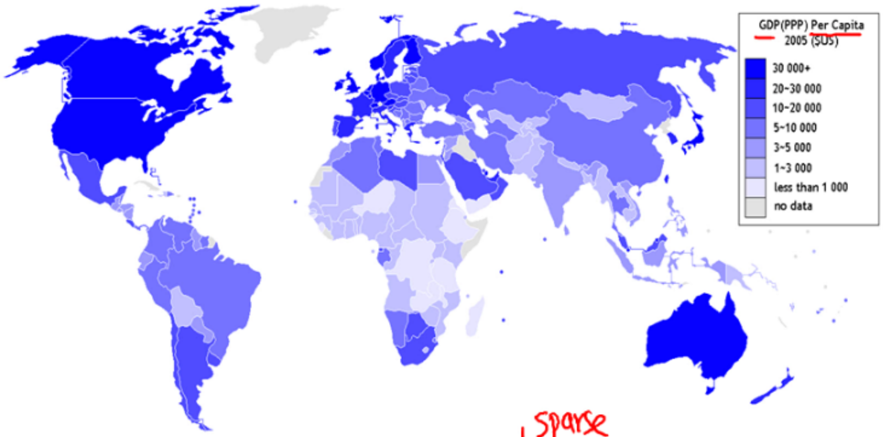
# Neuroscience



# Econometrics



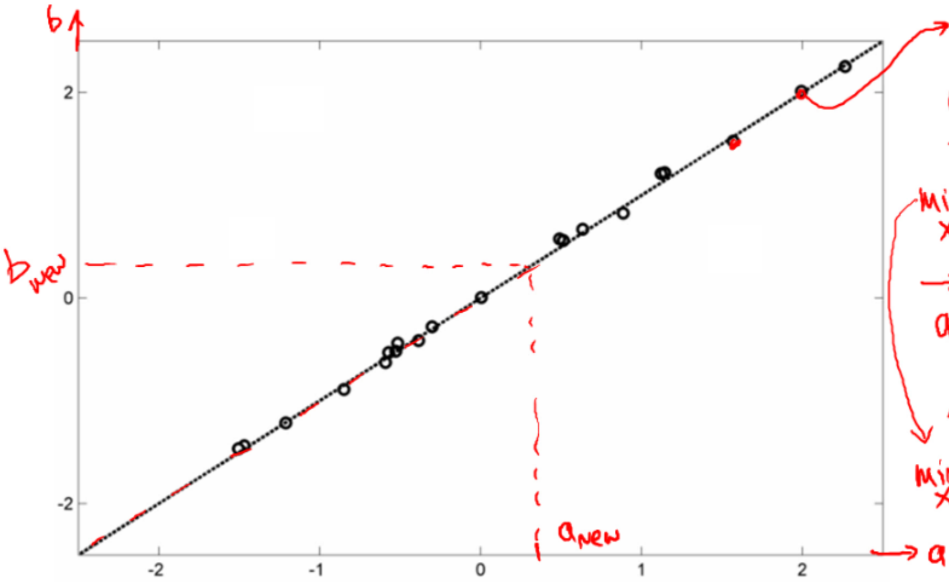

Population density
Fraction of tropical area
Size of economy
Defense spending
Life expectancy
Public investment
⋮
Land area

sparse

$$Ax = b$$

SOURCE: mapsof.net

# Robust Regression




$(a_i, b_i)$

$a_i x \approx b_i$

$\min_x \sum_i (a_i x - b_i)^2$

$\to x^*$

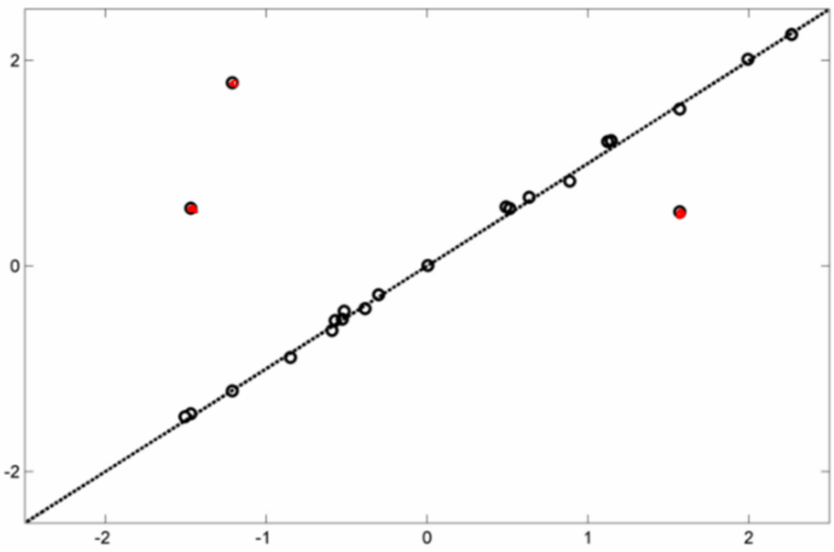
$a_{new} x^* = b_{new}$

$Ax = b$

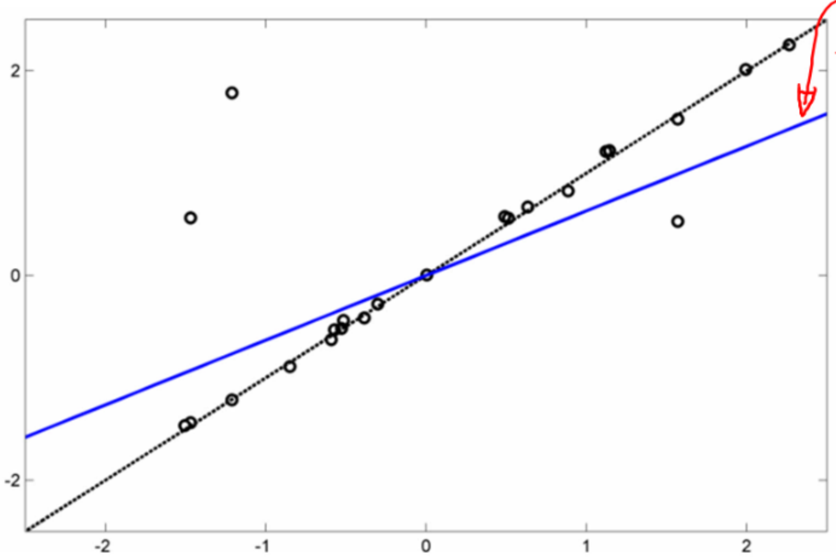
$\min_x \|Ax - b\|_2^2$

$\ell_2$-norm

# Robust Regression



# Robust Regression



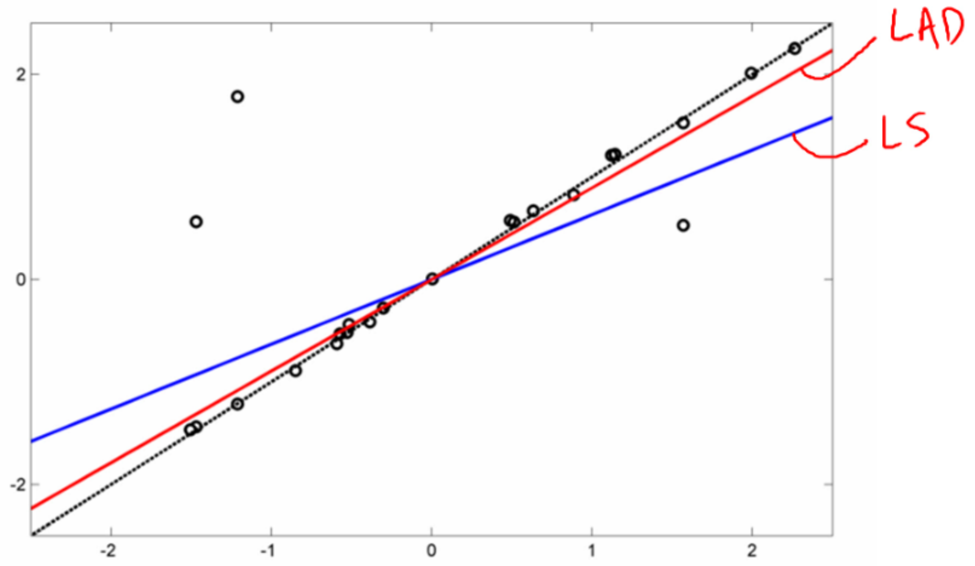

$$Ax + e \simeq b$$

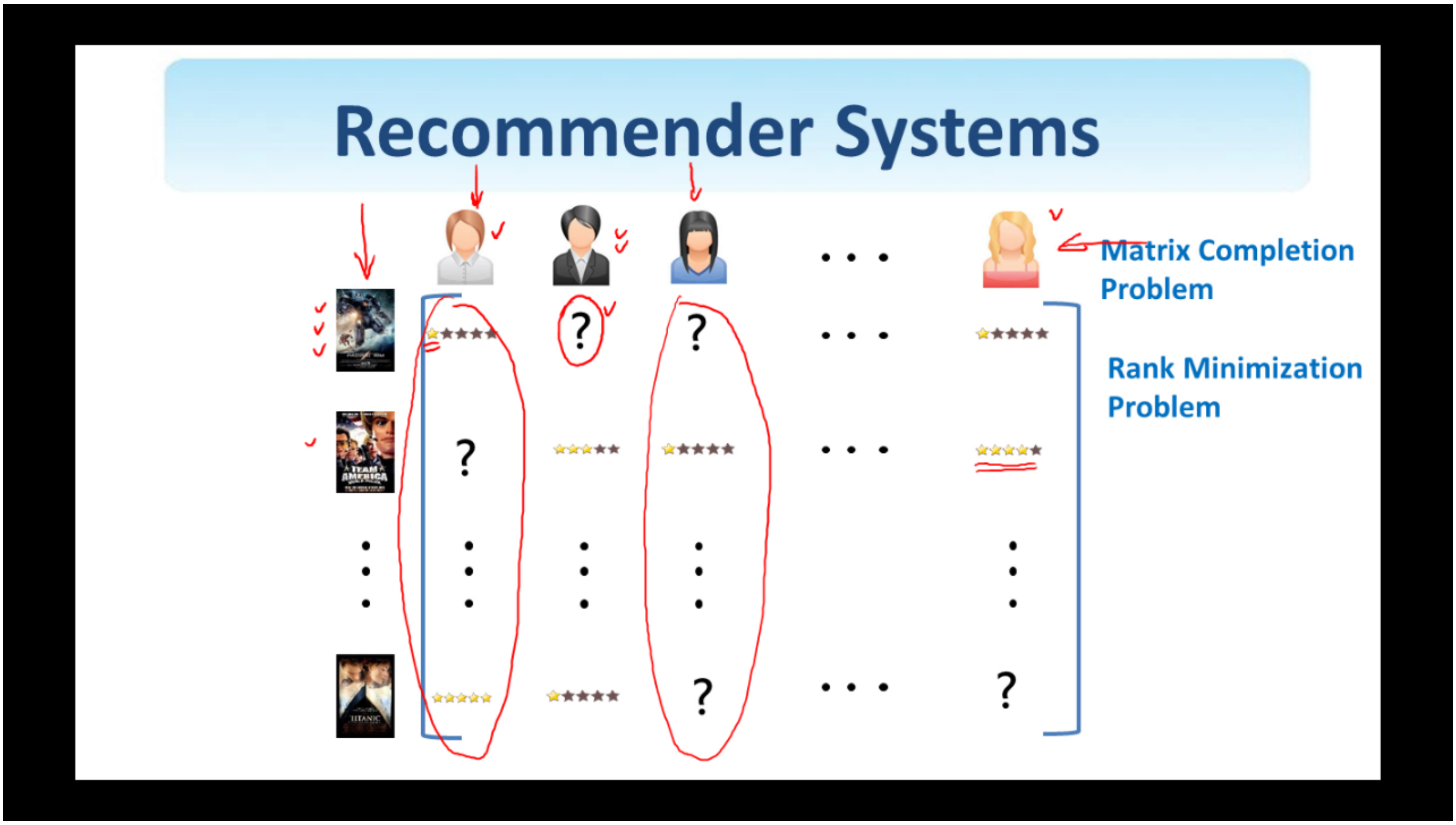
$x^*$

sparse

Least Absolute Deviations

# Robust Regression




Recommender Systems
Matrix Completion Problem
Rank Minimization Problem

# Image Denoising

$y_i$

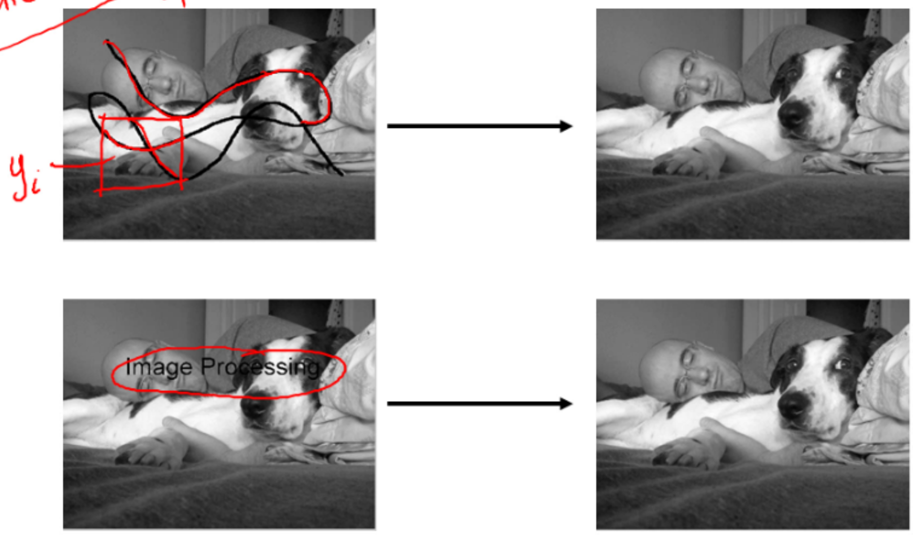
$y_i \cong A\, x_i$ ← sparse

$\min_{x_i} \|y_i - A x_i\|_2^2 + \lambda \|x_i\|_1$ ← $\ell_1$-Norm

# Image Inpainting

mask, dictionary, sparse

$$y_i \approx R A x_i$$

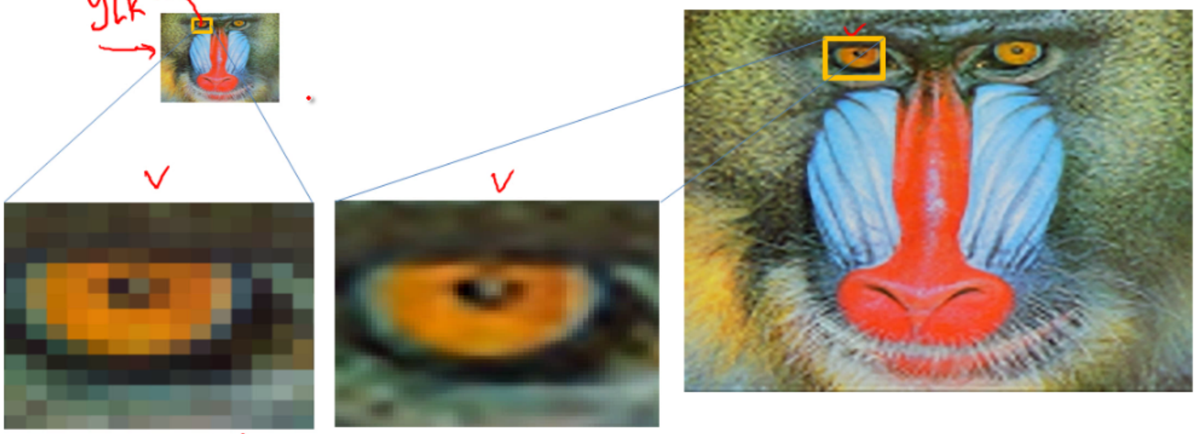



$$x_i^* = \arg\min_{x_i} \|y_i - R A x_i\|_2^2 + \lambda \|x_i\|_1$$

$$A x_i^*$$

# Image Super-Resolution

$y_{LR}$



$X^*_{SPARSE}$

$y_{LR} = A_{LR} X^*_{SPARSE}$

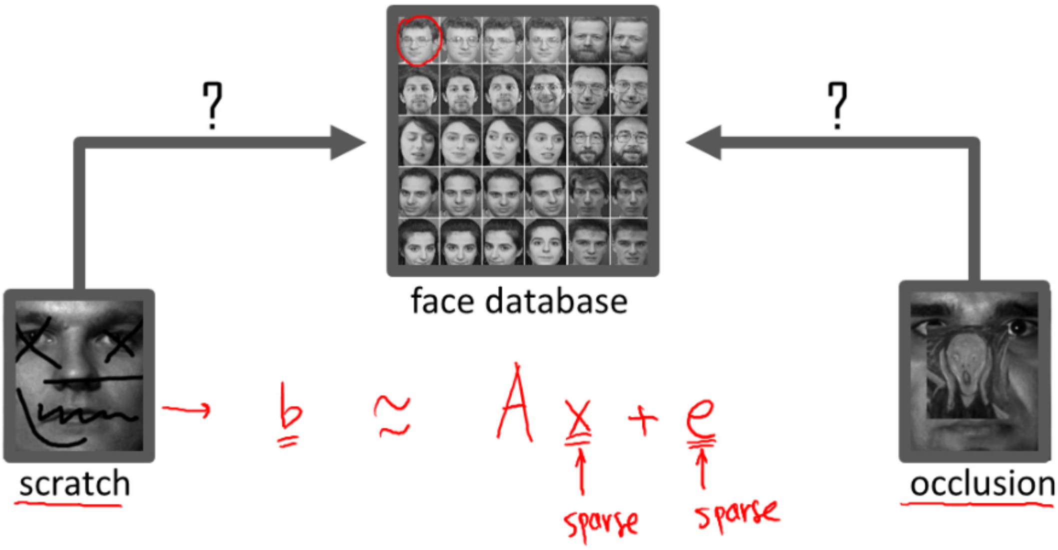
$y_{HR} = A_{HR} X^*_{SPARSE}$

# Video Surveillance

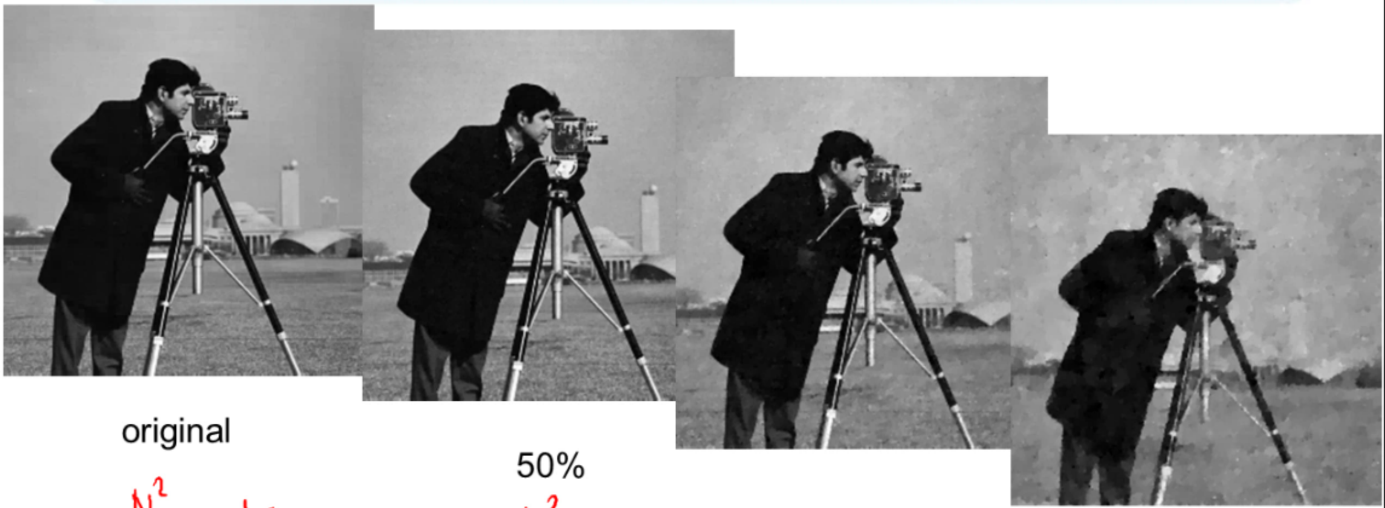
Background
Low-rank matrix

Foreground
Sparse matrix

# Robust Face Recognition



# Compressive Sensing



original

$N^2$ samples

50%

$N^2/2$

25%

$N^2/4$

$N^2/10$

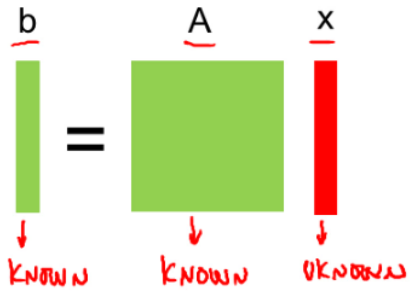
10%

# Linear Inverse Problems

- Fully-determined system of equations
  \# equations = # unknowns

- Unique solution (if A is full rank):

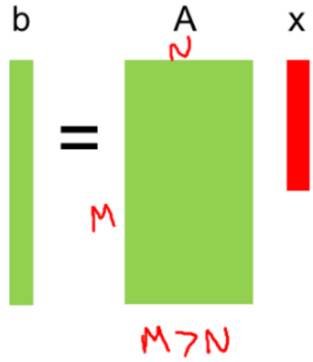
$$\mathbf{x}^* = A^{-1}\mathbf{b}$$



# Linear Inverse Problems

- Over-determined system of equations
  
  # equations > # unknowns
- The Least Squares solution is given by

$$\mathbf{x}^* = \underbrace{\left(A^T A\right)}_{N \times N}^{-1} A^T \mathbf{b}$$




$$\min_x \|Ax-b\|_2^2 \; ; \; \nabla_x \|Ax-b\|_2^2 = 0 \rightarrow \nabla_x \left\{ (Ax-b)^T (Ax-b) \right\} = 0$$

$$\rightarrow \nabla_x \left\{ x^T A^T A x - 2 x^T A^T b + b^T b \right\} = 0 \rightarrow 2A^T A x - 2A^T b = 0$$

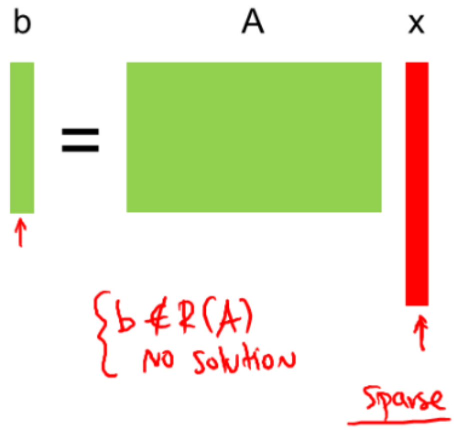
# Linear Inverse Problems

- under-determined system of equations

  # equations < # unknowns

- Infinitely many solutions (usually!)
- How to pick x? it depends on the application.
- Regularization

$$\min_{\mathbf{x}} J(\mathbf{x}) \ \text{ subject to } \ \mathbf{b} = \mathbf{A}\mathbf{x}$$



# Minimum $l_2$ Norm Solution

- We want x to be 'small' (in the $l_2$ sense)

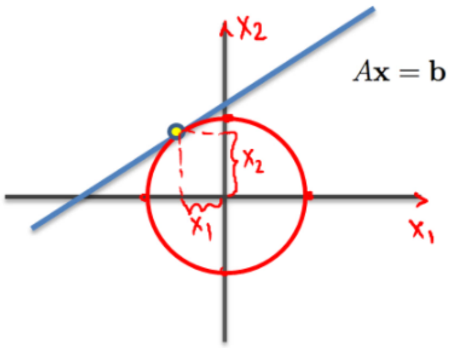
$$\|\mathbf{x}\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}$$

- The problem to solve is

$$\begin{cases} \min_{\mathbf{x}} \|\mathbf{x}\|_2 \\ subject\,to\; A\mathbf{x} = \mathbf{b} \end{cases}$$

- The closed form solution is given by

$$\underline{\mathbf{x}^* = A^T \left(AA^T\right)^{-1} \mathbf{b}}$$




$$x = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} \quad \|x\|_2 = \sqrt{9+16} = \sqrt{25} = 5$$

$$\|x\|_2^2 = 1 \Rightarrow x_1^2 + x_2^2 = 1$$

# Minimum $l_2$ Norm Solution

Derivation of closed form solution

$$\boxed{\begin{aligned} &\min_x \|x\|_2 \\ &\text{s.t. } Ax-b \end{aligned}} \Bigg\} \quad \min_x \Big( \underbrace{\|x\|_2 + \lambda^T (Ax-b)}_{L(x)} \Big) = \min_x L(x)$$

$$\lambda^T A x = \underline{x^T A^T \lambda}$$

KKT

$$\nabla_x L(x) = 0 \rightarrow x + A^T\lambda = 0 \Rightarrow x = -A^T\lambda \Rightarrow \boxed{x^* = A^T(AA^T)^{-1} b}$$

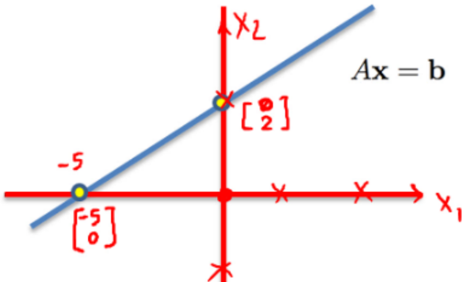
$$\nabla_\lambda L(x) = 0 \rightarrow Ax - b = 0 \rightarrow -AA^T\lambda = b \Rightarrow \lambda = -(AA^T)^{-1} b$$

# Minimum $l_0$ Norm Solution

- We want x to be 'sparse' $\longrightarrow$ it should have few non-zero entries.
- Sparsity can be modeled via the $l_0$ norm*

$\|\alpha x\|_0 \neq \alpha \|x\|_0$

$$\|\mathbf{x}\|_0 = \#\, non-zero\ entries\, in\, \mathbf{x}$$




- The problem to solve is now

$$\boxed{\begin{aligned} &\min_{\mathbf{x}} \underline{\|\mathbf{x}\|_0} \\ &subject\, to\, A\mathbf{x} = \mathbf{b} \end{aligned}}$$

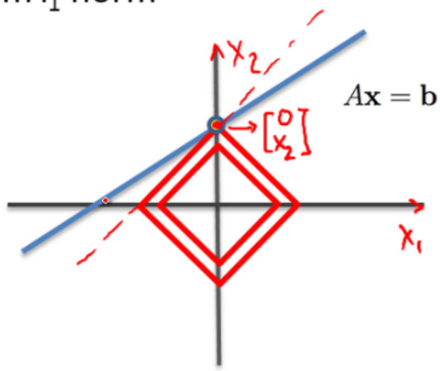
$x = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}, \quad \|x\|_0 = 2$

# Minimum $l_1$ Norm Solution

- Another special solution is the one with minimum $l_1$ norm

$$\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i|$$

- The problem to solve is now




$x = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$ $\|x\|_1 = 3+4=7$

$\|x\|_2 = 5$

$\|x\|_0 = 2$

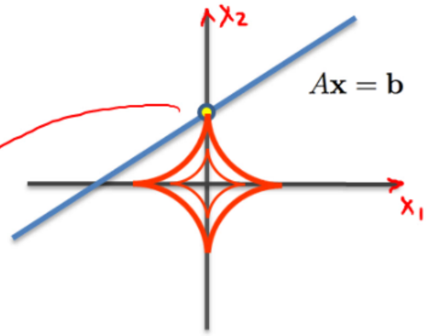
$$\boxed{\begin{aligned} &\min_{\mathbf{x}} \|\mathbf{x}\|_1 \\ &subject\,to\; \underline{A\mathbf{x} = \mathbf{b}} \end{aligned}}$$

Basis pursuit

# Minimum $l_p$ Norm Solution

- Another class of special solutions minimizes the $l_p$ norm (0<p<1)

$$\longrightarrow \|\mathbf{x}\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p}$$

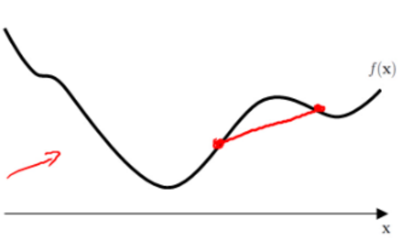


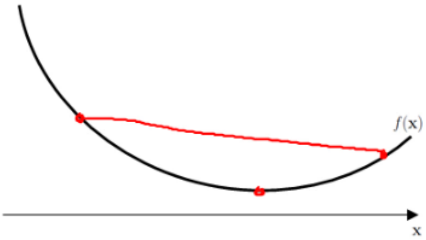

- The problem to solve becomes

$$\boxed{\begin{array}{c} \underset{\mathbf{x}}{min}\, \|\mathbf{x}\|_p \\ subject\,to\; A\mathbf{x} = \mathbf{b} \end{array}}$$

$X = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$ $\|X\|_{0.5} = (\sqrt{3} + \sqrt{4})^2$

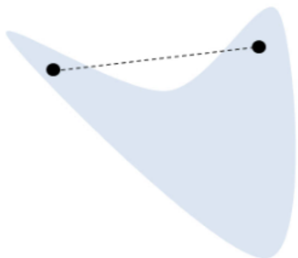
# On Convexity

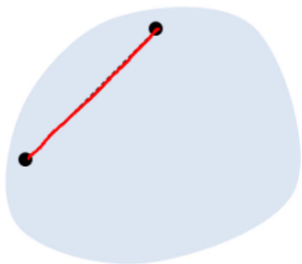


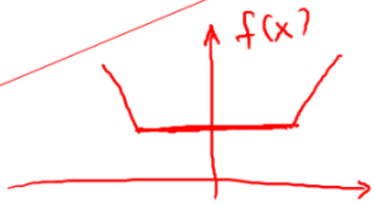



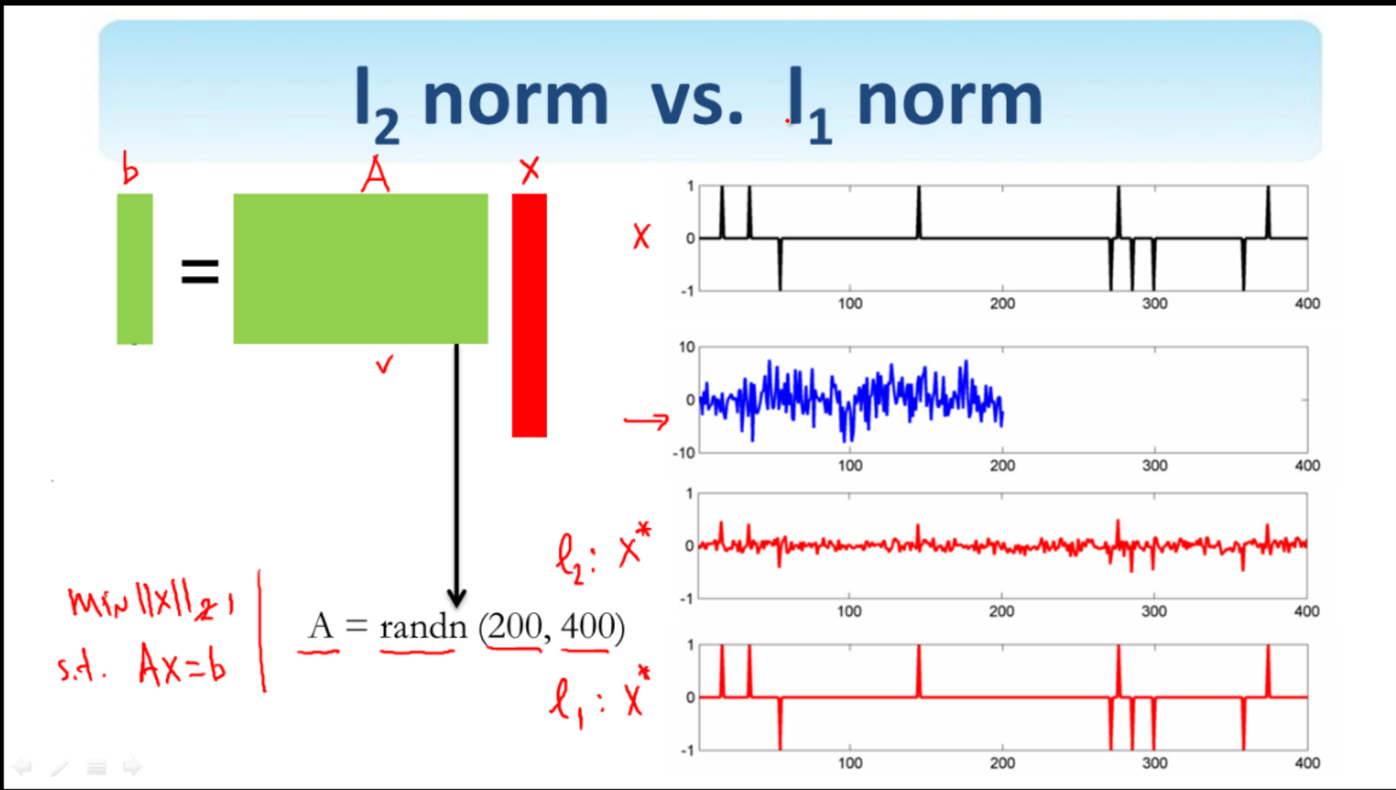

$$\min_{\mathbf{x}} f(\mathbf{x})$$

$$\text{subject to } \mathbf{x} \in \mathcal{S}$$







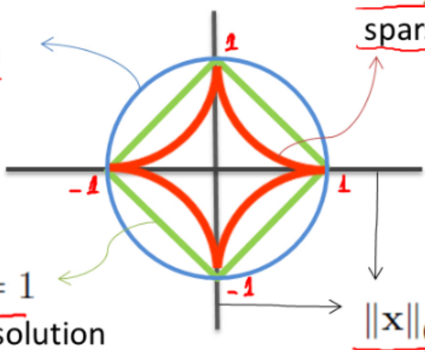

l2 norm vs. l1 norm
b
A
x
=
A = randn (200, 400)
min ||x||2,
s.t. Ax=b
x
l2: x*
l1: x*
100
200
300
400
1
0
-1
10
-10

# All Norm Balls in One Picture

convex
closed form
solution

$\|\mathbf{x}\|_2 = 1$
small solution

$\|\mathbf{x}\|_p = 1$ $0<p<1$
sparse solution
NON-convex




convex

$\|\mathbf{x}\|_1 = 1$
sparse solution

$\|\mathbf{x}\|_0 = 1$
sparse solution
NP-hard

# $l_0$ norm vs. $l_1$ norm

$$\min_{\mathbf{x}} ||\mathbf{x}||_0 \quad subject\,to\; A\mathbf{x} = \mathbf{b}$$

- Models sparsity directly
- Non-convex
- NP-hard
- Greedy approaches (Matching Pursuit) approximate the solution

$$\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad subject\,to\; A\mathbf{x} = \mathbf{b}$$

- Models sparsity indirectly
- Convex
- Non-smooth
- Can be solved via convex optimization algorithms

# Reformulation

$$\min_{\mathbf{x}} ||\mathbf{x}||_0$$
$$subject\,to\; A\mathbf{x} = \mathbf{b}$$

Noise in observation

$$\min_{\mathbf{x}} \|\mathbf{x}\|_0 \qquad \|C\mathbf{x}\|_2$$
$$subject\,to\; \|A\mathbf{x}-\mathbf{b}\|_2 \le \epsilon$$

$x(\epsilon)$

Swapping constraint & objective

$$\min_{\mathbf{x}} \|A\mathbf{x}-\mathbf{b}\|_2$$
$$subject\,to\; \|\mathbf{x}\|_0 \le S$$

$x(S)$

# Reformulation

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad subject\,to\; A\mathbf{x} = \mathbf{b}$$

Noise in observation

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad subject\,to\; \|A\mathbf{x} - \mathbf{b}\|_2 \le \epsilon$$

$$\min_{x} \{\|Ax-b\|_2^2 + \lambda\|Cx\|_2^2\}$$

$$x^* = (A^TA + \lambda C^TC)^{-1}A^Tb$$

Swapping constraint & objective

$$\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|_2 \quad subject\,to\; \|\mathbf{x}\|_1 \le S$$

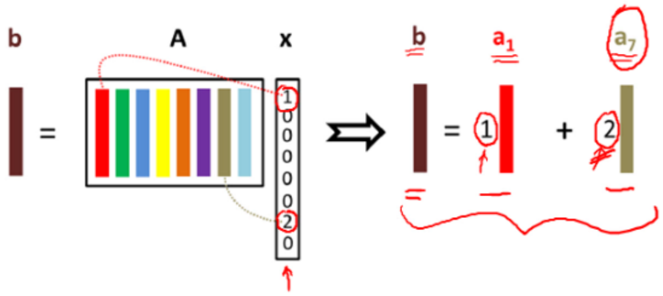
Bringing constraint to the objective

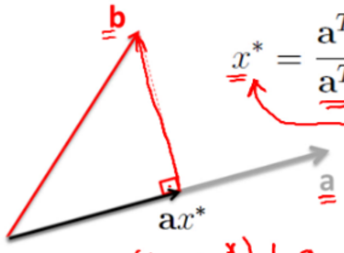
$$\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|_2 + \lambda\|\mathbf{x}\|_1$$

$$\underbrace{\|x(\lambda)\|_1 \le S}_{\text{LASSO}}$$

# Matching Pursuit

$$\min_{\mathbf{x}} \|A\mathbf{x}-\mathbf{b}\|_2$$
$$subject\,to\ \|\mathbf{x}\|_0 \le S$$




$$x^* = \frac{\mathbf{a}^T\mathbf{b}}{\mathbf{a}^T\mathbf{a}} = \mathbf{a}^T\mathbf{b}$$

$\|a\| = 1$

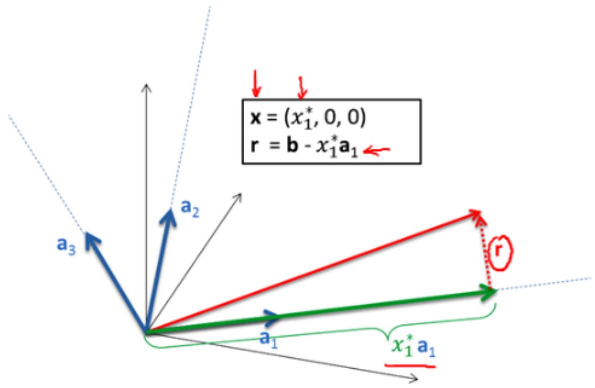
$$\|\mathbf{a}x^*\|_2 = |x^*|\underbrace{\|\mathbf{a}\|_2}_{=1} = |x^*|$$

$$i = \underset{k}{argmax}\ |x_k^*|$$

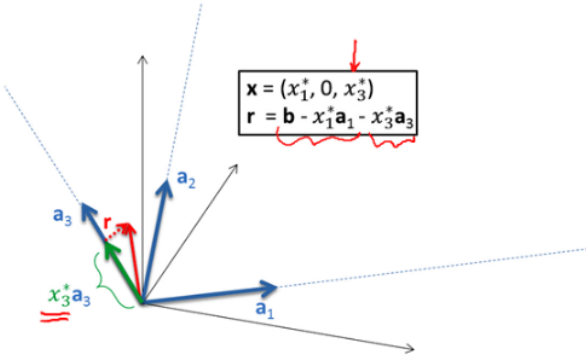
best column

$(b - ax^*) \perp a \Rightarrow (b-ax^*)^T a = 0$

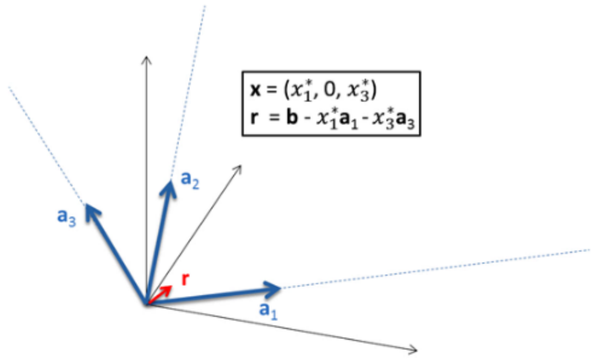
# Matching Pursuit



# Matching Pursuit



# Matching Pursuit



# Orthogonal Matching Pursuit

$$\boxed{\begin{array}{c}\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|_2 \\ subject\ to\ \|\mathbf{x}\|_0 \leq S\end{array}}$$

**Input:** $A$ (with unit norm columns), $\mathbf{b}$, and $S$.
Initialize $\mathbf{r} = \mathbf{b}$ and $\Omega = \emptyset$.
**While** $\|\mathbf{x}\|_0 < S$
compute $x_j = \mathbf{a}_j^T \mathbf{r}$ for all $j \notin \Omega$
$i = \underset{j \notin \Omega}{argmax} |x_j|$
$\Omega \longleftarrow \Omega \cup \{i\}$
$\boxed{\mathbf{x}_\Omega^* = \underset{\mathbf{x}}{argmin} \|A_\Omega \mathbf{x} - \mathbf{b}\|_2^2}$
$\mathbf{r} \longleftarrow \mathbf{b} - A_\Omega \mathbf{x}_\Omega^*$

# Orthogonal Matching Pursuit

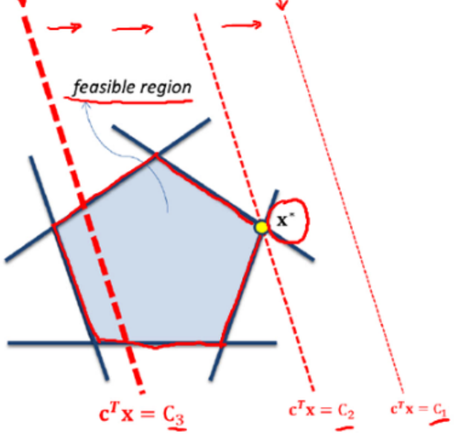
$$\min_{\mathbf{x}} \|\mathbf{x}\|_0$$
$$subject\, to \; \|A\mathbf{x} - \mathbf{b}\|_2 \leq \epsilon$$

**Input:** $A$ (with unit norm columns), $\mathbf{b}$, and $\epsilon$.
Initialize $\mathbf{r} = \mathbf{b}$ and $\Omega = \emptyset$.
**While** $\|\mathbf{r}\|_2^2 > \epsilon$
compute $x_j = \mathbf{a}_j^T \mathbf{r}$ for all $j \notin \Omega$
$i = \underset{j \notin \Omega}{argmax} |x_j|$
$\Omega \longleftarrow \Omega \cup \{i\}$
$\boxed{\mathbf{x}_\Omega^* = \underset{\mathbf{x}}{argmin} \|A_\Omega \mathbf{x} - \mathbf{b}\|_2^2}$
$\mathbf{r} \longleftarrow \mathbf{b} - A_\Omega \mathbf{x}_\Omega^*$

# Linear Programs

$$\min_{\mathbf{x}} \ \mathbf{c}^T\mathbf{x}$$
$$subject\ to\ F_i\mathbf{x} + \mathbf{g}_i \leq \mathbf{0}\ \forall i$$

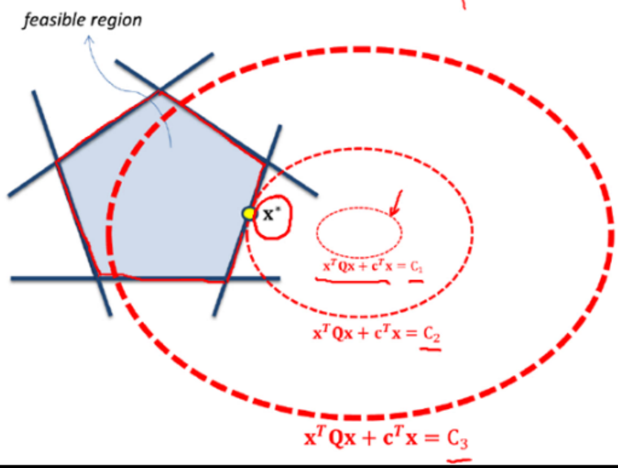
Linprog



# Quadratic Programs

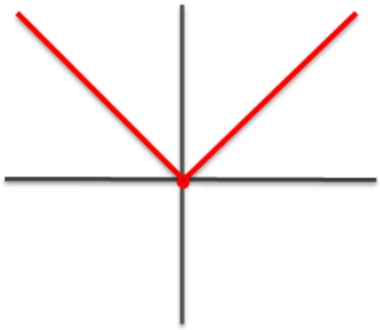
$$\min_{\mathbf{x}} \mathbf{x}^T Q\mathbf{x} + \mathbf{c}^T\mathbf{x}$$
$$subject\ to\ F_i\mathbf{x} + \mathbf{g}_i \leq \mathbf{0}\ \forall i$$

quadprog



# Smooth Reformulation Tricks

The $L_1$ norm is non-differentiable at the origin



We introduce two reformulation tricks that transform sparse optimization problems into well-studied Linear and Quadratic programs

# Positive-Negative Split Trick

$$p_i = \begin{cases} x_i & if\ x_i > 0 \\ 0 & else \end{cases} \qquad n_i = \begin{cases} -x_i & if\ x_i < 0 \\ 0 & else \end{cases}$$

$$x_i = p_i - n_i$$

$$x = \begin{bmatrix} 2 \\ -3 \\ 0 \\ 5 \end{bmatrix} \quad p_1 = 2,\ n_2 = 3,\ p_4 = 5$$

$$x = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

$$x = p - n$$

$$\|\mathbf{x}\|_1 = \mathbf{1}^T(\mathbf{p} + \mathbf{n}) = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \left( \begin{bmatrix} 2 \\ 0 \\ 0 \\ 5 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$p^T n = 0$$

# Positive-Negative Split Trick

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1$$
$$subject\ to\ A\mathbf{x} = \mathbf{b}$$

$\rightarrow LP$

$x = p - n$

$$\min_{p,n}\ 1^T(p+n)$$
$$\text{s.t.}\ A(p-n) = b$$
$$p, n \geq 0$$
$$\boxed{\cancel{p^T n = 0}}$$

$$z = \begin{bmatrix} p \\ n \end{bmatrix}$$

$$\boxed{\begin{aligned} &\min_{z}\ 1^T z \leftarrow \\ &\text{s.t.}\ Cz = b \\ &\quad z \geq 0 \end{aligned}}$$

$C = AF$

$F = [I_{N\times N} \mid -I_{NN}]$

$N \times 2N$

# Positive-Negative Split Trick

$\underline{x} = p - n$

$$\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|_2^2 + \lambda \|\mathbf{x}\|_1$$

LASSO → QP

$$\min_{p,n} \|A(p-n) - b\|_2^2 + \lambda 1^T p + \lambda 1^T n$$

$$p, n \geq 0$$

$$\boxed{\cancel{p^T n = 0}}$$

$$z = \begin{bmatrix} p \\ n \end{bmatrix}$$

$$\Rightarrow \boxed{\begin{array}{c} \min_{z} z^T B z + c^T z \\ z \geq 0 \end{array}}$$

$$\leftarrow B = \begin{bmatrix} A^T A & -A^T A \\ -A^T A & A^T A \end{bmatrix}$$

$$\leftarrow c = \lambda 1 + 2 \begin{bmatrix} -A^T b \\ A^T b \end{bmatrix}$$

# Suppression Trick

$s, \; s_k$

$|x_k| \le s_k$

$$\begin{cases} \min_{\mathbf{x}} \|\mathbf{x}\|_1 \\ \text{subject to } A\mathbf{x} = \mathbf{b} \end{cases}$$

$$\left.\begin{aligned} &\min_{x,s} \; 1^T s \\ &\text{s.t. } Ax = b \\ &\quad |x_k| \le s_k, \; \forall k \\ &\quad s \ge 0 \end{aligned}\right\} \rightarrow \left\{\begin{aligned} &\min_{x,s} \; \underline{1^T s} \\ &\text{s.t. } Ax = b \\ &\quad x_k \le s_k, \; \forall k \\ &\quad x_k \ge -s_k, \; \forall k \\ &\quad s \ge 0 \end{aligned}\right\} \; \underline{\underline{LP}}$$

# Advanced Methods

- Stagewise OMP (StOMP), compressive sampling matching pursuit (CoSaMP).
- FISTA (Fast Iterative Shrinkage Algorithm)
- ADMM (Alternating Direction Method of Multipliers)

$$\min_{x,y} f(x) + g(y)$$
$$\text{subject to } Ax + By = c$$

# Dictionary Learning

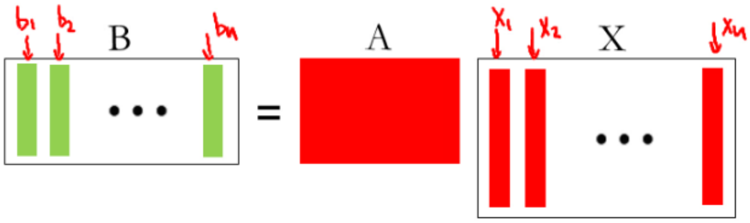
$$\begin{cases} \min_{\mathbf{x}} \|A\mathbf{x}_1 - \mathbf{b}_1\|_2^2 \\ \text{subject to } \|\mathbf{x}_1\|_0 \le s \end{cases}$$

$$\vdots$$

$$\begin{cases} \min_{\mathbf{x}} \|A\mathbf{x}_n - \mathbf{b}_n\|_2^2 \\ \text{subject to } \|\mathbf{x}_n\|_0 \le s \end{cases}$$

$$\min_{X} \|AX - B\|_F^2$$
$$\text{subject to } \|\mathbf{x}_i\|_0 \le s \quad 1 \le i \le n$$

$$\|A\|_F = \sqrt{\sum_i \sum_j |a_{ij}|^2}$$




What if we can choose A too?

$$\min_{A,X} \|AX - B\|_F^2$$
$$\text{subject to } \|\mathbf{x}_i\|_0 \le s \quad 1 \le i \le n$$

# Method of Optimal Directions

(MOD)

$$\min_{A,X} \|AX - B\|_F^2$$
$$subject\, to\ \|\mathbf{x}_i\|_0 \le S\ \forall i$$

Alternating minimization

Keep A fixed; solve for X

$$\min_{\mathbf{x}_i} \|A\mathbf{x}_i - \mathbf{b}_i\|_2^2$$
$$subject\, to\ \|\mathbf{x}_i\|_0 \le S$$

Keep X fixed; solve for A

$$\min_{A} \|AX - B\|_F^2$$

$$\boxed{A = BX^T(XX^T)^{-1}}$$

# Bi-convex Dictionary Learning

$$\min_{A,X} \|AX - B\|_F^2 + \lambda \|X\|_1$$

Alternating minimization

Keep A fixed; solve for X

$$\min_{X} \|AX - B\|_F^2 + \lambda \|X\|_1$$

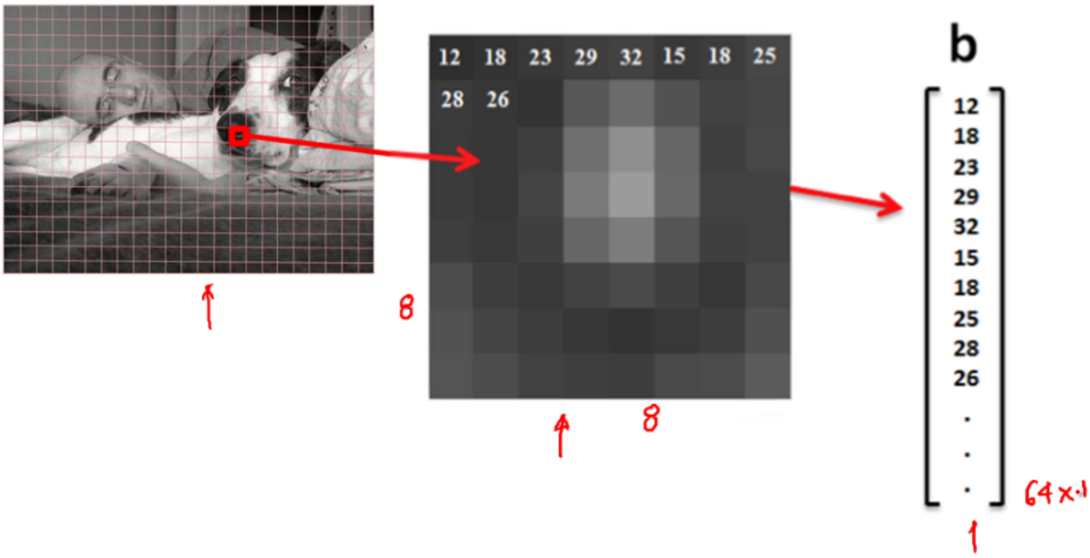
A series of LASSO problems

Keep A fixed; solve for X

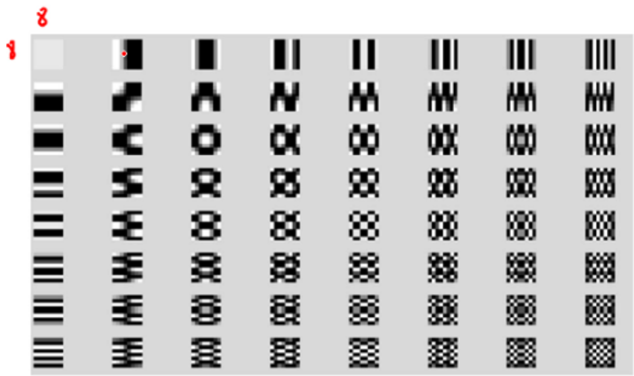
$$\min_{A} \|AX - B\|_F^2$$

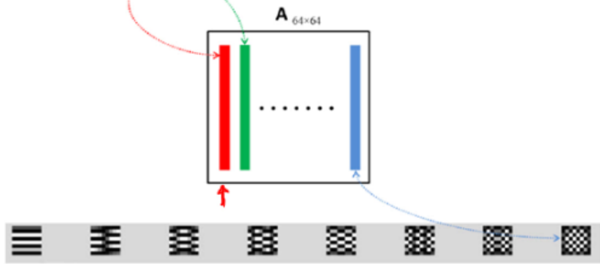
Least Squares: $A = BX^T(XX^T)^{-1}$

# Forming b

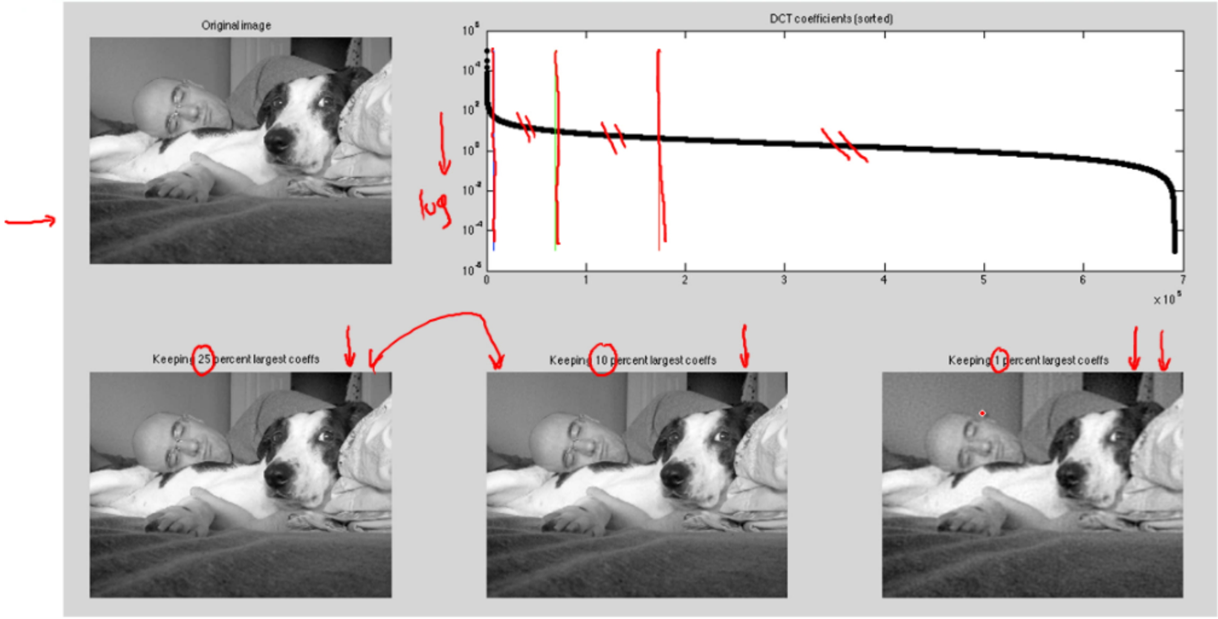


# Forming A: DCT Dictionary

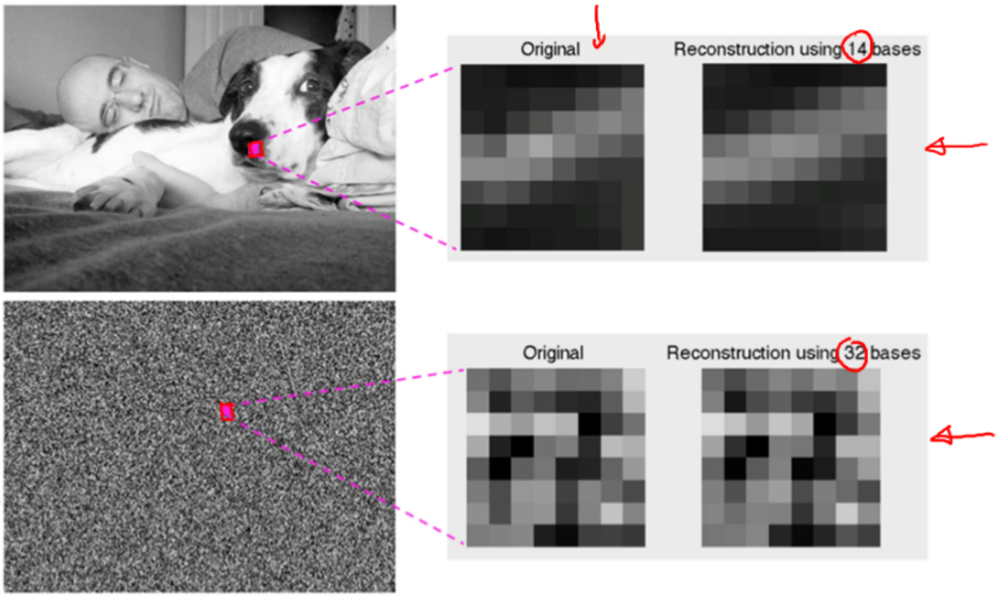


# Experiment


Original image
DCT coefficients (sorted)
log
Keeping 25 percent largest coeffs
Keeping 10 percent largest coeffs
Keeping 1 percent largest coeffs

# Image Denoising



# Image Denoising

$$\min_{A,X} \|AX - B\|_F^2 + \lambda \|X\|_1$$

Dictionary

Input noisy image

$AX^*$ → Recovered image

# Image Denoising

PSNR = 22.1 dB

PSNR = 33.4 dB

# Image Inpainting

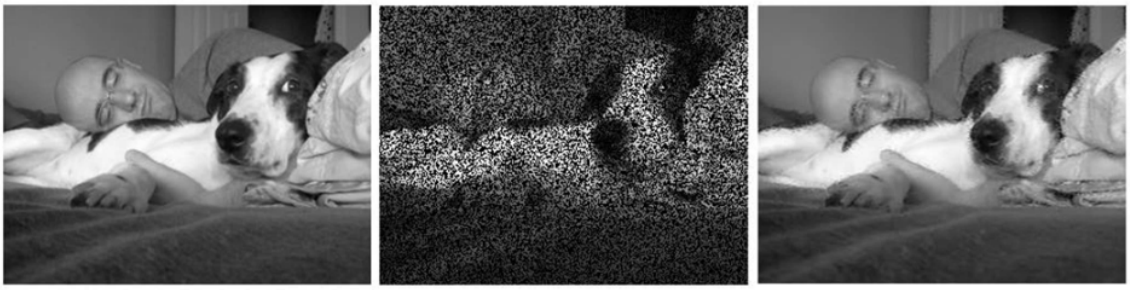
$$\underset{X}{min} \; \|RAX - B\|_F^2 + \lambda \|X\|_1$$

Degradation matrix

Input image w\ missing pixels

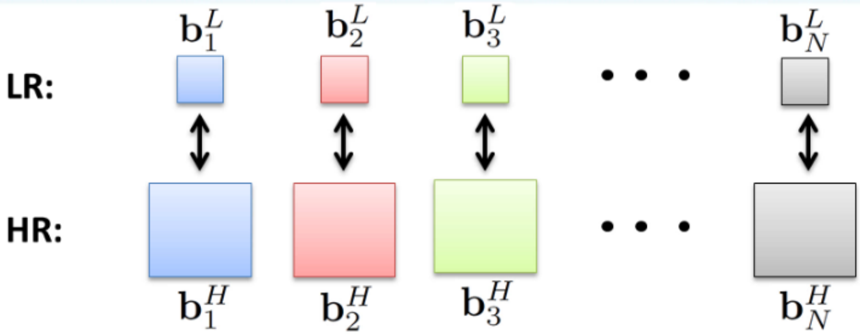
$AX^*$ → Recovered image

# Image Inpainting



50%

# Image Super-Resolution

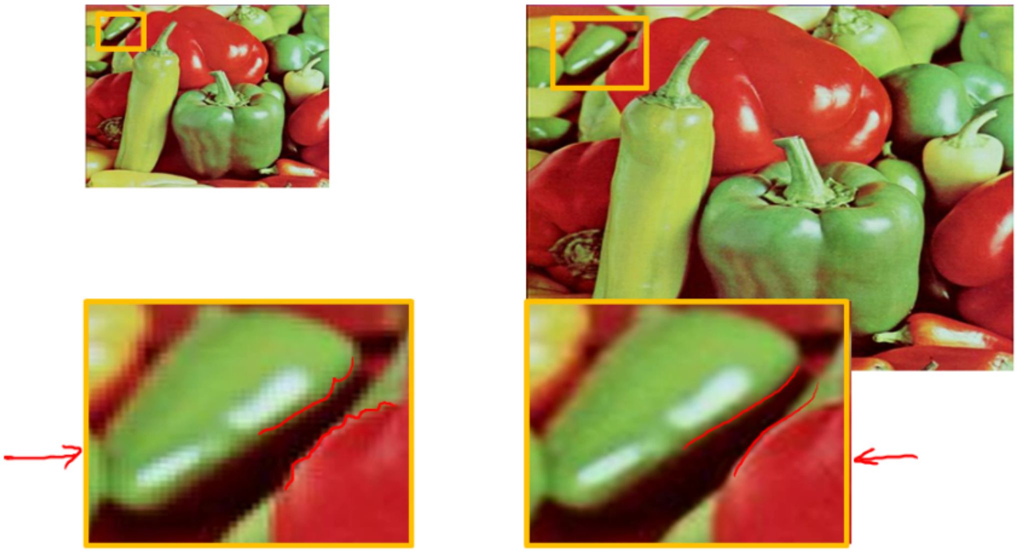



**Training phase:** $$\min_{A^L, A^H, X} \underbrace{\|A^L X - B^L\|_F^2} + \mu \underbrace{\|A^H X - B^H\|_F^2} + \lambda \|X\|_1$$

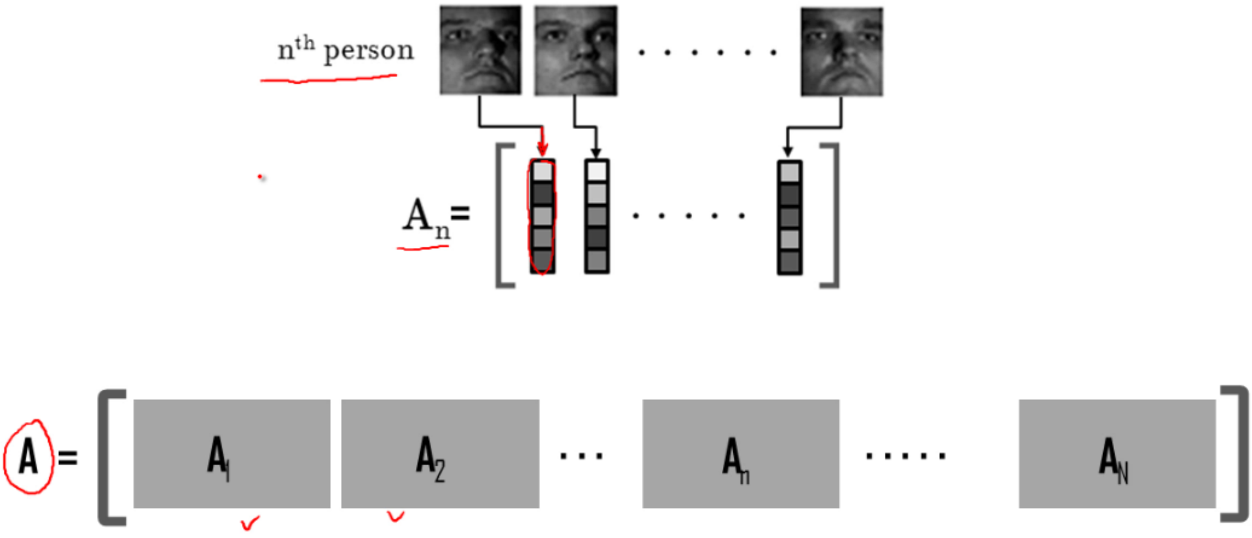
**Reconstruction phase:** $$X^* = \underset{X}{argmin} \|A^L X - B^{new}\|_F^2 + \lambda \|X\|_1 \longrightarrow A^H X^*$$

Super-resolved image

# Image Super-Resolution



# Robust Face Recognition



# Robust Face Recognition

$\mathbf{b} = A\mathbf{x} + \mathbf{e}$

sparse sparse

$$\min_{\mathbf{x},\mathbf{e}} \ \|\mathbf{x}\|_1 + \lambda\|\mathbf{e}\|_1 \quad \text{subject to } A\mathbf{x} + \mathbf{e} = \mathbf{b}$$

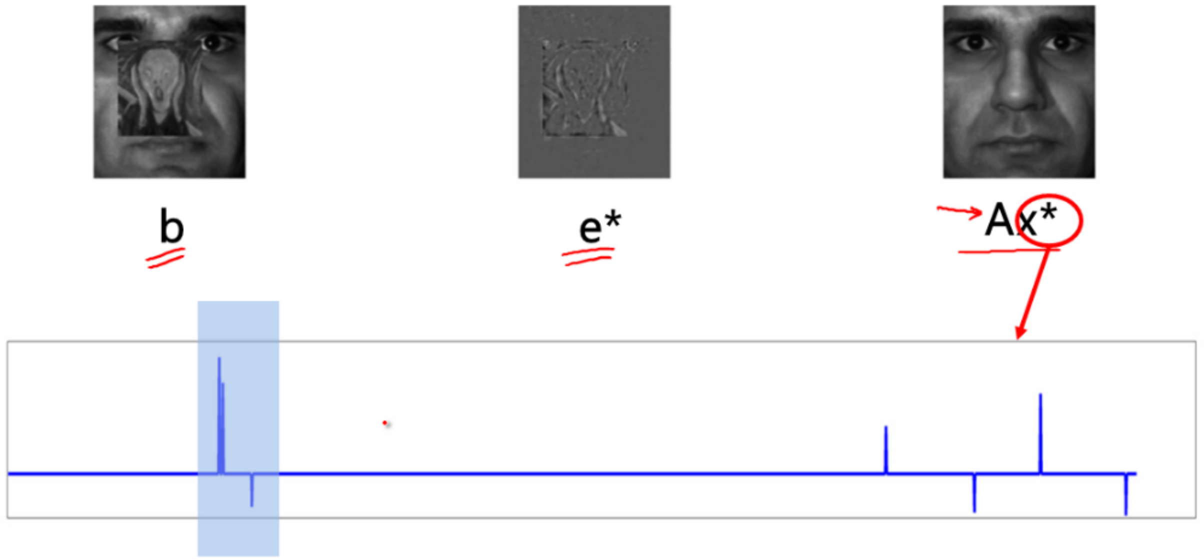
$$\mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \lambda\mathbf{e} \end{bmatrix}$$

$$F = [A \ \tfrac{1}{\lambda}I]$$

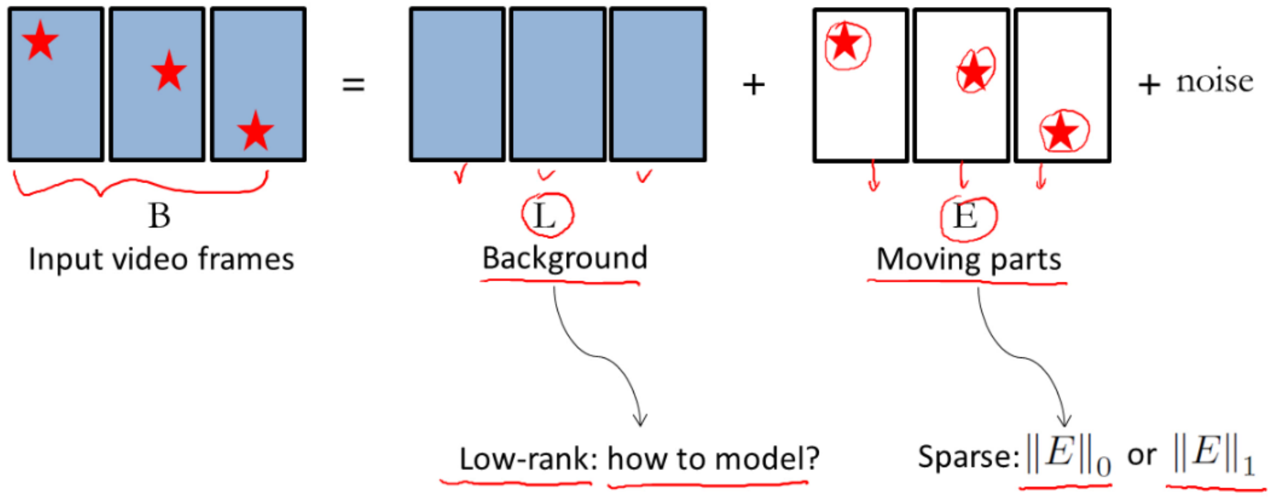
$$\min_{\mathbf{z}} \ \|\mathbf{z}\|_1 \quad \text{subject to } F\mathbf{z} = \mathbf{b}$$

Basis pursuit

# Robust Face Recognition



# Video Surveillance

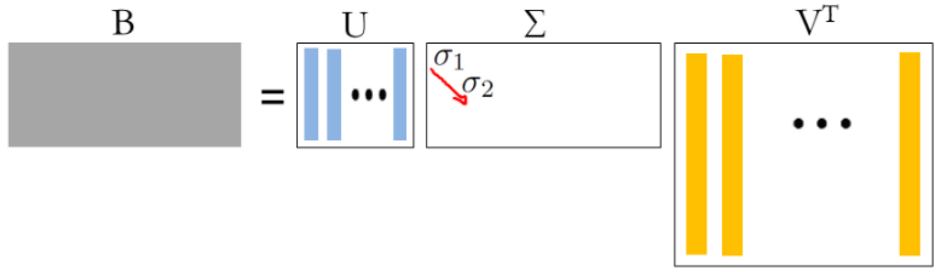


# Singular Value Decomposition

$$\min_{L} \|B - L\|_F$$

$$\text{subject to } rank\,(L) \leq k$$

$$B = \sum_{i=1}^{r} \mathbf{u}_i \sigma_i \mathbf{v}_i^T \qquad\qquad L = \sum_{i=1}^{k} \mathbf{u}_i \sigma_i \mathbf{v}_i^T$$



# Video Surveillance

$$\min_{L,E} \|B - L - E\|_F^2 + \lambda \|E\|_1$$
$$subject\,to\, rank(L) \leq k$$

NxM

Nxk

kxM

$$L = U\Sigma V^T$$

$$L = AX$$

$$\min_{L,E} \|B - L - E\|_F^2 + \lambda \|E\|_1$$
$$subject\,to\, \|\Sigma\|_0 \leq k$$

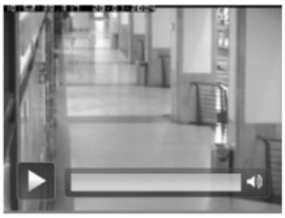
$$\min_{A,X,E} \|B - AX - E\|_F^2 + \lambda \|E\|_1$$

$$\min_{L,E} \|B - L - E\|_F^2 + \lambda \|E\|_1 + \mu \|L\|_*$$

Nuclear Norm

# Video Surveillance

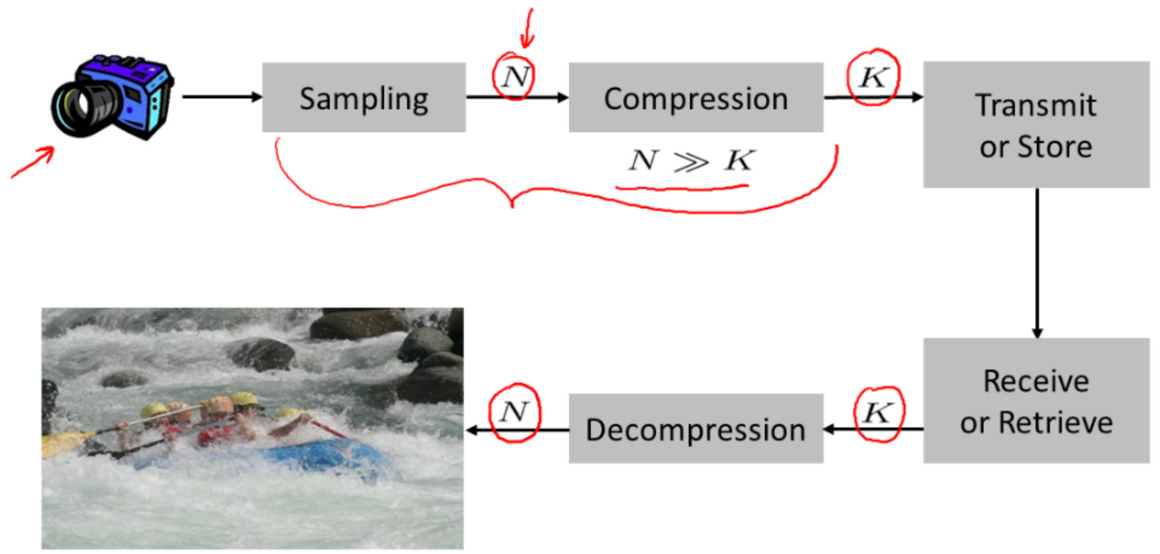
B



L

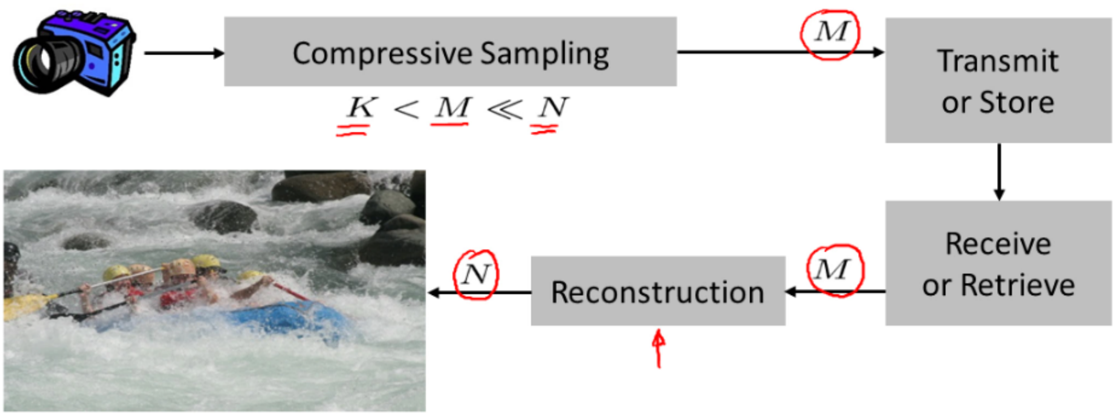
E

# Sensing by Sampling



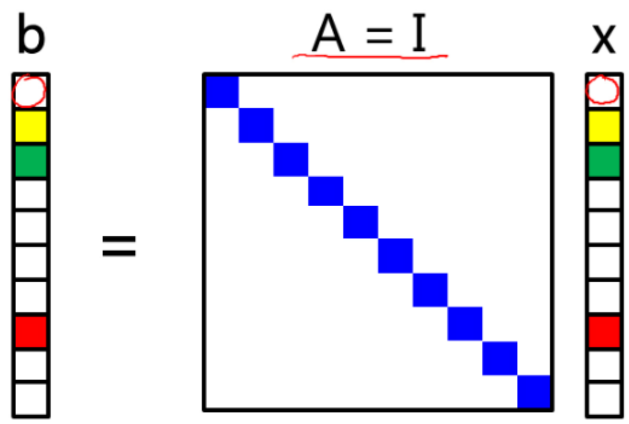
# Compressive Sensing

- Directly acquire *"compressed" data*
- Replace samples by more general "measurements"



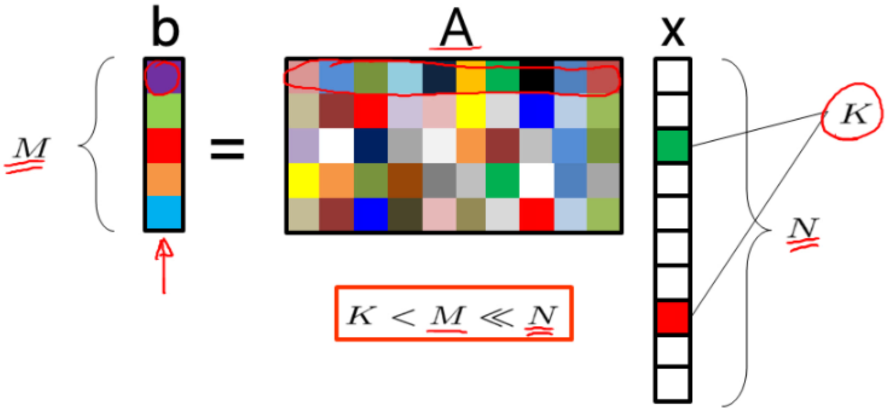
# Sampling

- Signal $x$ is K-*sparse* in basis/dictionary $A$
  - WLOG assume sparse in space domain



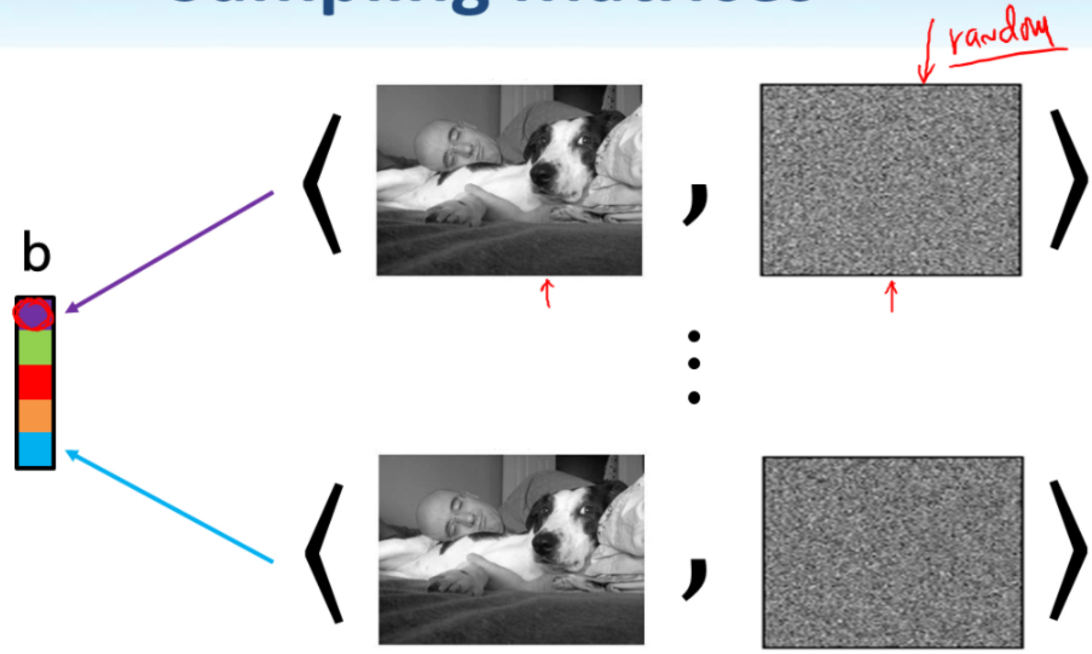
# Compressive Data Acquisition

- When data is sparse/compressible, can directly acquire a condensed representation with no/little information loss through dimensionality reduction

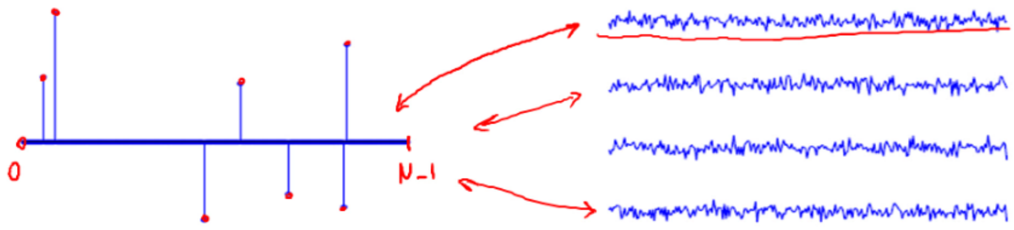



$$\boxed{K < M \ll N}$$

# Sampling Matrices



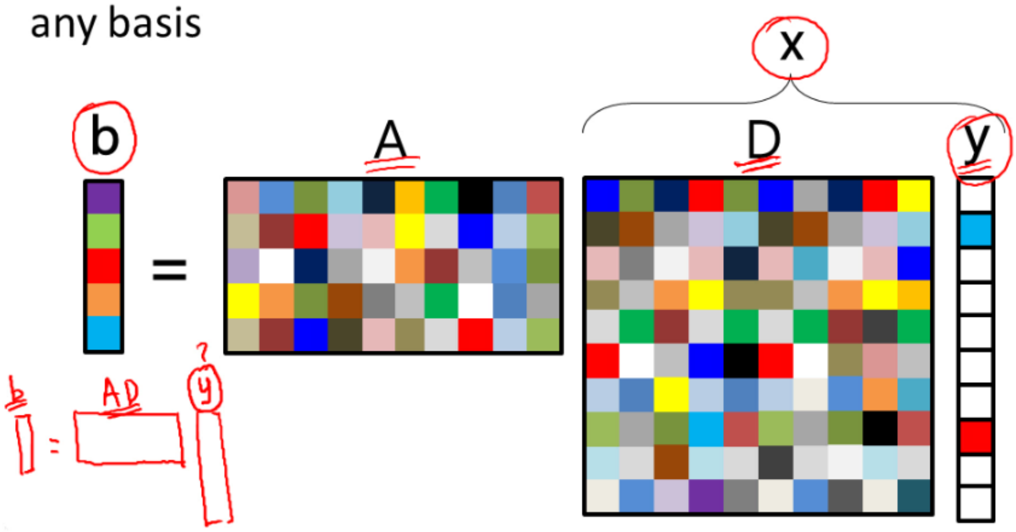
# Intuition



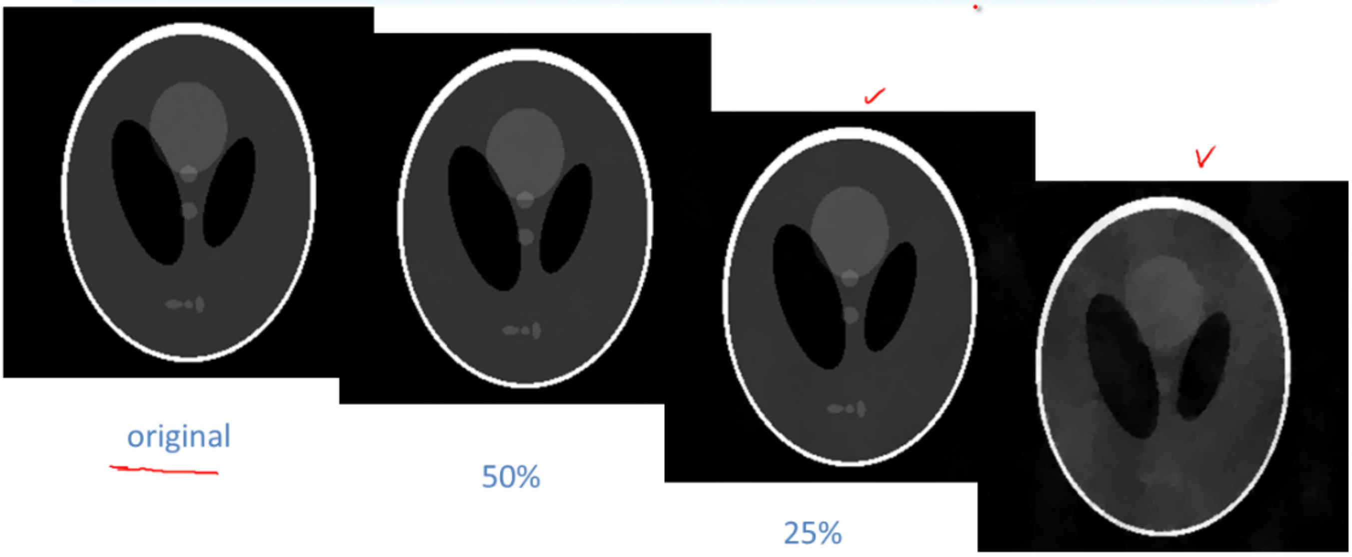

- Signal is local, measurements are global
- Each measurement picks up a little information about each component

# Universality

- Random measurements can be used for signals sparse in any basis



# Results Compressive Sensing



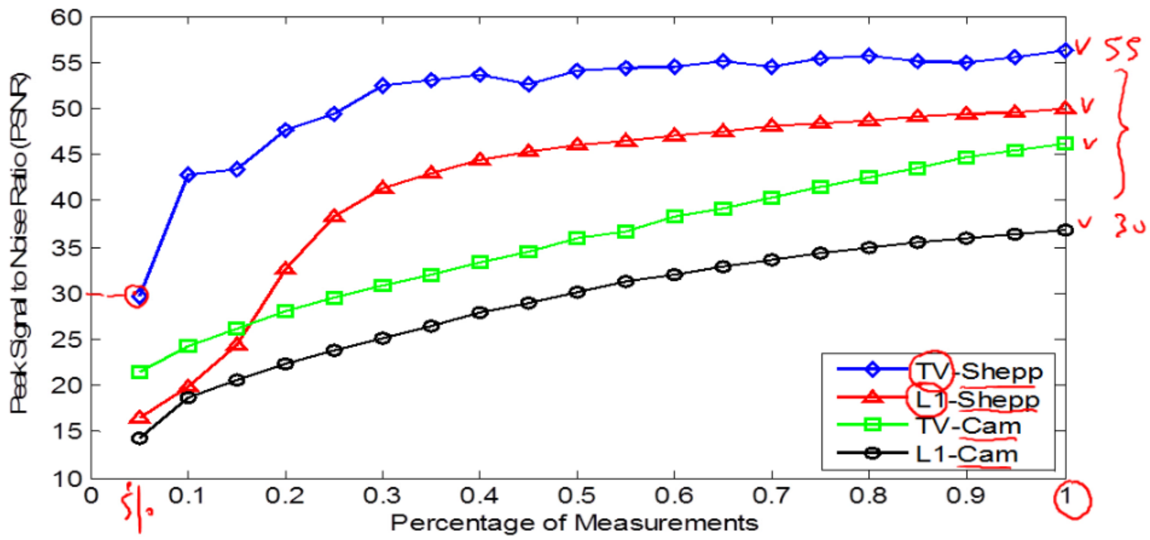
original

50%

25%

5%

# Results


Peak Signal to Noise Ratio (PSNR)
Percentage of Measurements
TV-Shepp
L1-Shepp
TV-Cam
L1-Cam
55
30
5%