ASEN 5114	Experiment 2
William Watkins	Spring 2022

1. Find the poles of the plant from the transfer function model in Experiment 1. Plot these in the complex plane.

From Experiment 1, the open loop transfer function model of the system is

$$\theta_L(s) = \frac{-1}{denominator}$$
$$\text{where}$$
$$denominator = s^3(\frac{J_{eq}L_M}{NK_\tau}) + s^2(\frac{J_{eq}R_M}{NK_\tau}) + s(NK_B) \tag{1}$$

The poles of the plant are found by setting the denominator equal to 0,

$$s^3(\frac{J_{eq}L_M}{NK_\tau}) + s^2(\frac{J_{eq}R_M}{NK_\tau}) + s(NK_B) = 0 \tag{2}$$

After factoring an s out, and then using the quadratic formula, the poles of the system are located at

$$\begin{aligned} s_1 &= 0 \\ s_2 &= -34.07 \\ s_3 &= -10,071.19 \end{aligned} \tag{3}$$

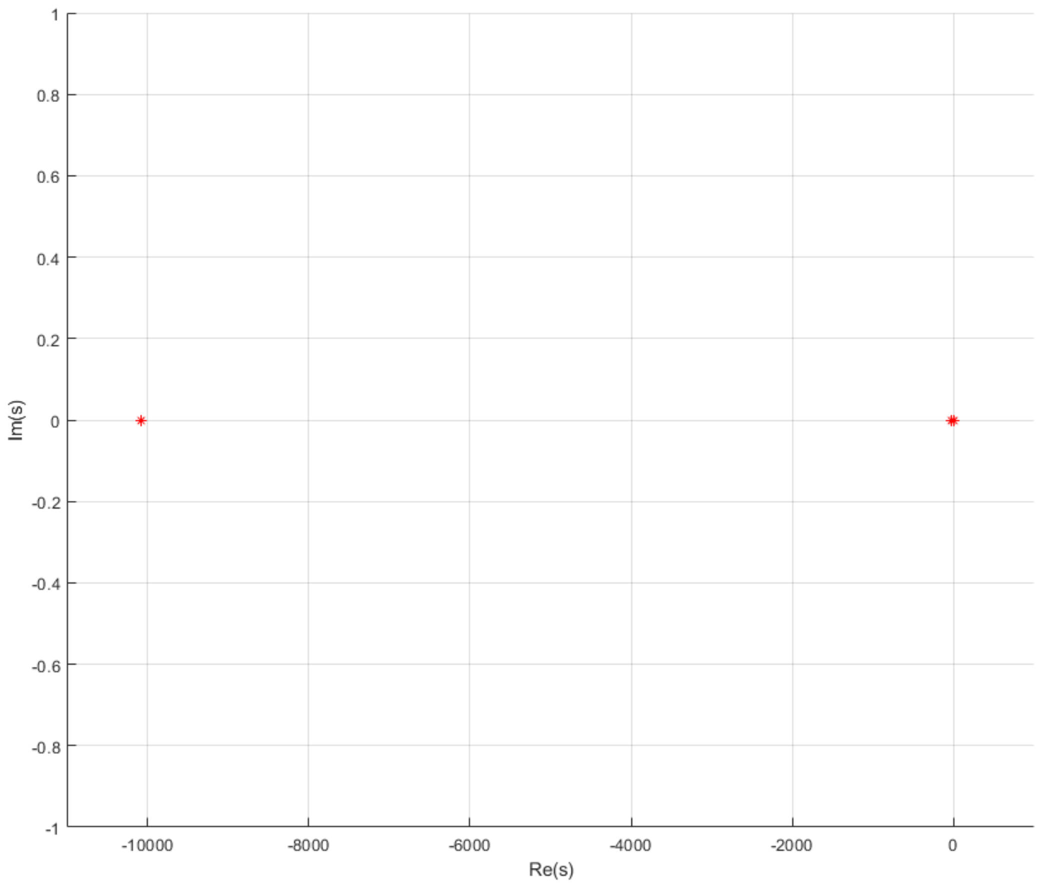

Figure 1: Open Loop Poles.

2. Find the poles of the closed loop system from part 5 of Experiment 1.

As in Part 1, finding the poles of the system is done by setting the denominator of the closed loop transfer function in Eqn. 4 equal to 0.

$$\begin{gathered}\theta_L(s) = -\frac{G_P + G_D s}{denominator}\theta_R(s) \\ \text{where} \\ denominator = s^3(\frac{J_{eq}L_M}{NK_\tau}) + s^2(\frac{J_{eq}R_M}{NK_\tau}) + s(NK_B - G_D) - G_P\end{gathered} \tag{4}$$

$$\begin{aligned} s_1 &= -10,068.54 \\ s_2 &= -26.72 \\ s_3 &= -10.00 \end{aligned} \tag{5}$$

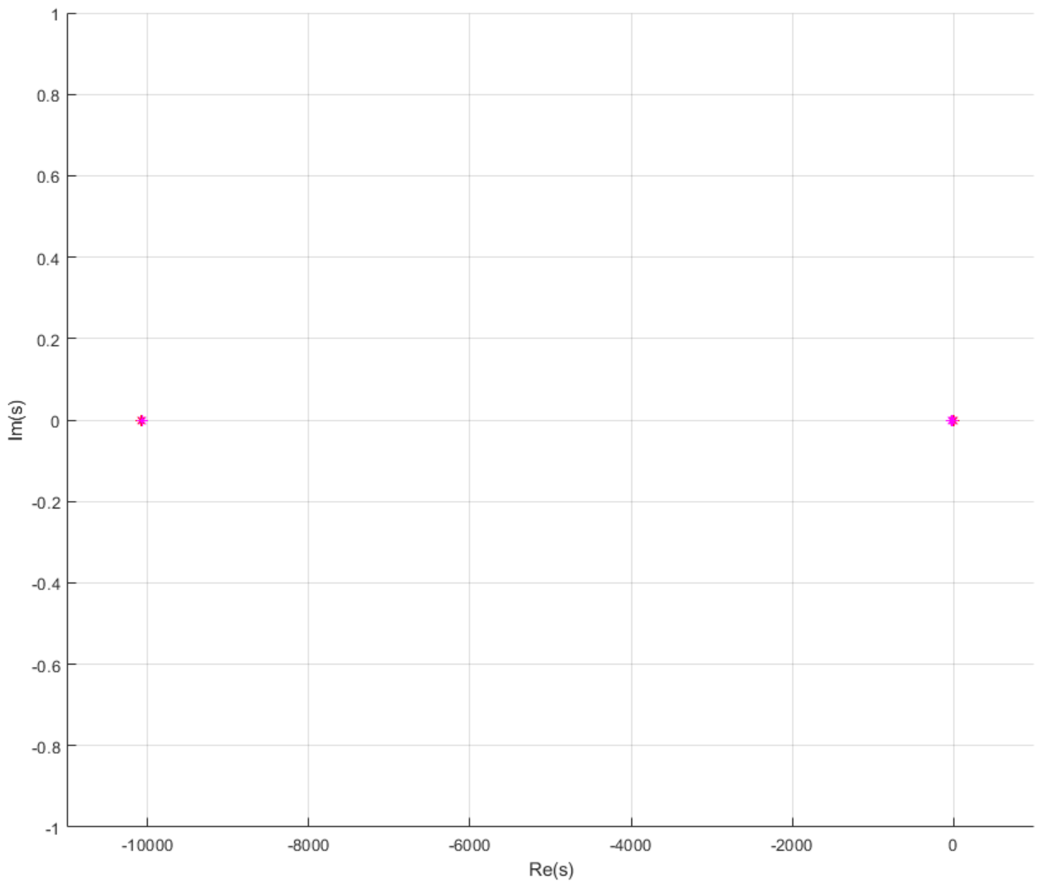

Figure 2: Closed Loop Poles.

The closed loop poles are very similar to the open loop poles. Both trios of poles are comprised of real parts only, and so the response in the time domain does not have an oscillation.

3. Repeat part 2, using a gain factor g that varies between 0 and 1. Plot this "root locus with respect to g" on the same plot. Describe the effect of this control system gain on the closed loop poles.

The poles of the system after applying the gain factor g to both G_D and G_P are shown in Fig. 3 in green.

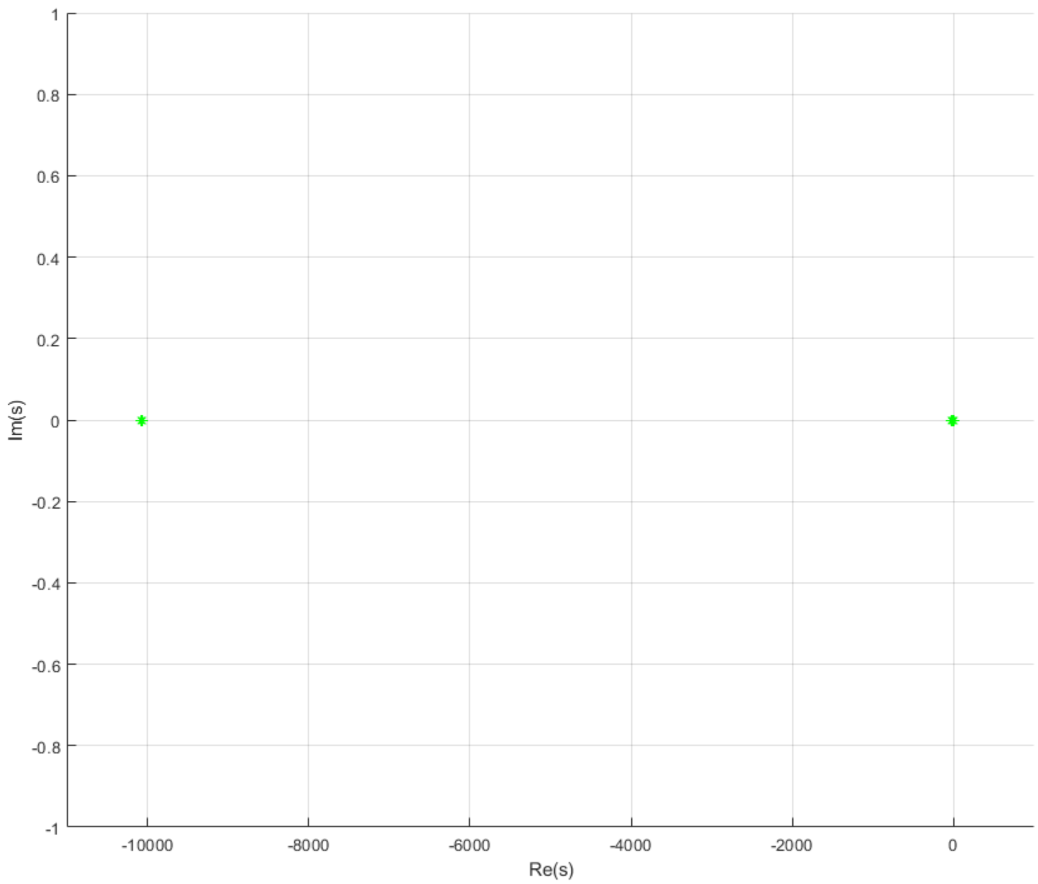

Figure 3: Closed Loop Poles with Gain Factor Applied ($0 \leq g \leq 1$).

Zooming into the right side of the graph in Fig. 4, we can see that the open loop pole s_1 is made more negative as the gain factor is increased from 0 to 1, whereas the opposite is true for the open loop pole s_2. On the left side of the plot, Fig. 5, we see that the pole s_3 is made more positive as the gain factor is increased.

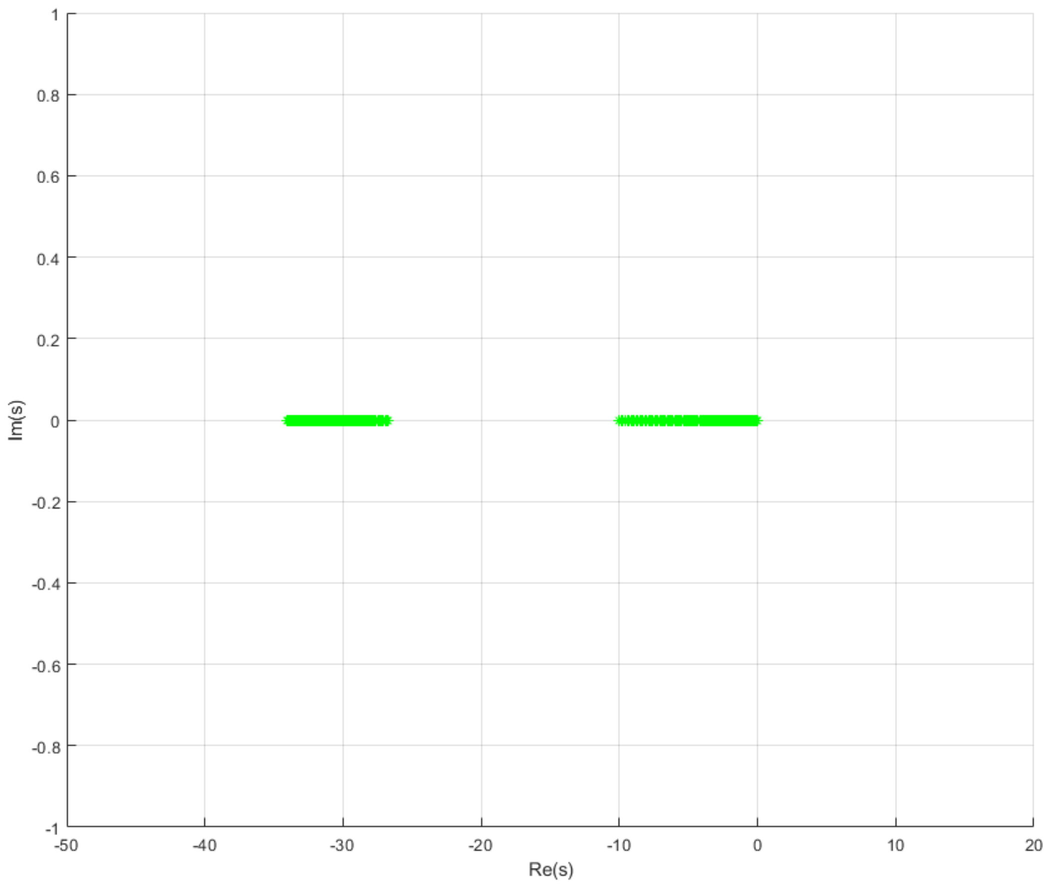

Figure 4: Closed Loop Poles with Gain Factor Applied ($0 \leq g \leq 1$).

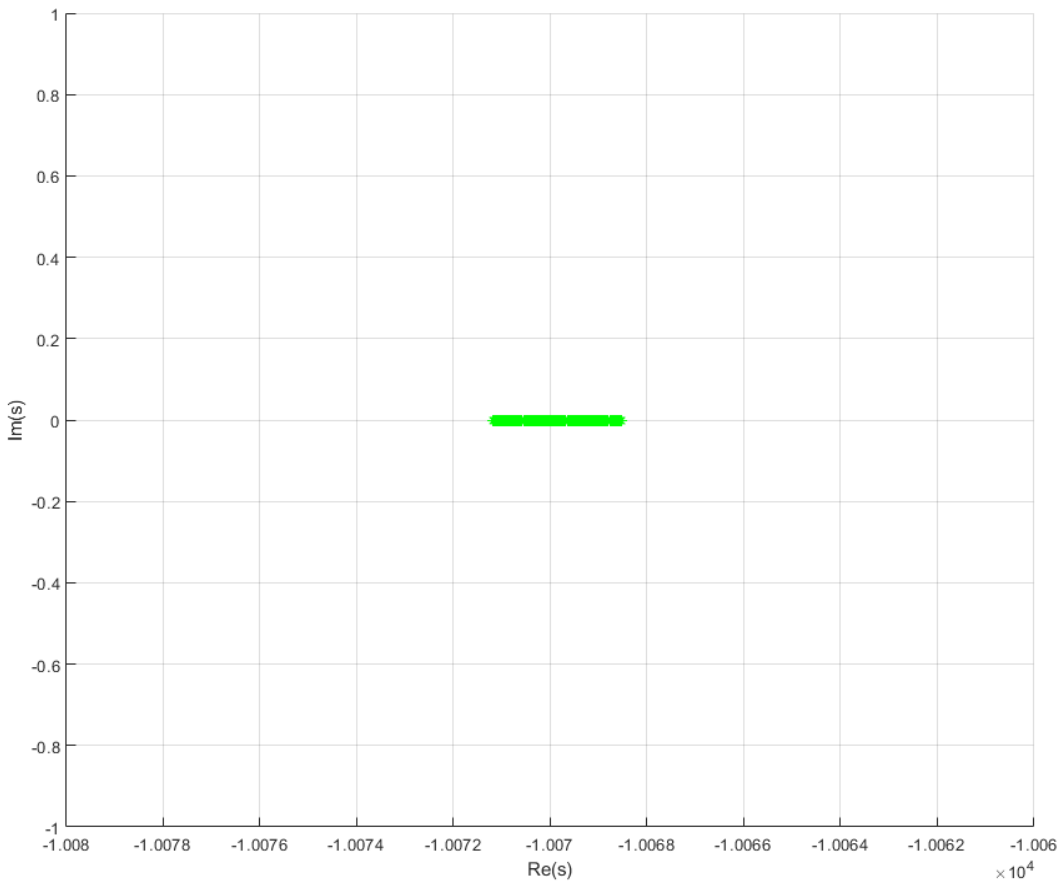

Figure 5: Closed Loop Poles with Gain Factor Applied ($0 \leq g \leq 1$).

While no oscillations are created in the time domain as a result of the poles moving because they do not have a complex part, the fact that s_1 and s_2 are moving towards each other implies that with enough gain factor, there will be a set of complex conjugate poles created. However, for gain factors between 0 and 1, the net result is that the time response now decays a bit slower than in the open loop response.

4. Repeat part 3 but use a gain factor g between 1 and 100. Describe the effect on the closed loop poles.

The closed loop poles for a gain factor between 1 and 100 can be seen in Fig. 6 in blue.

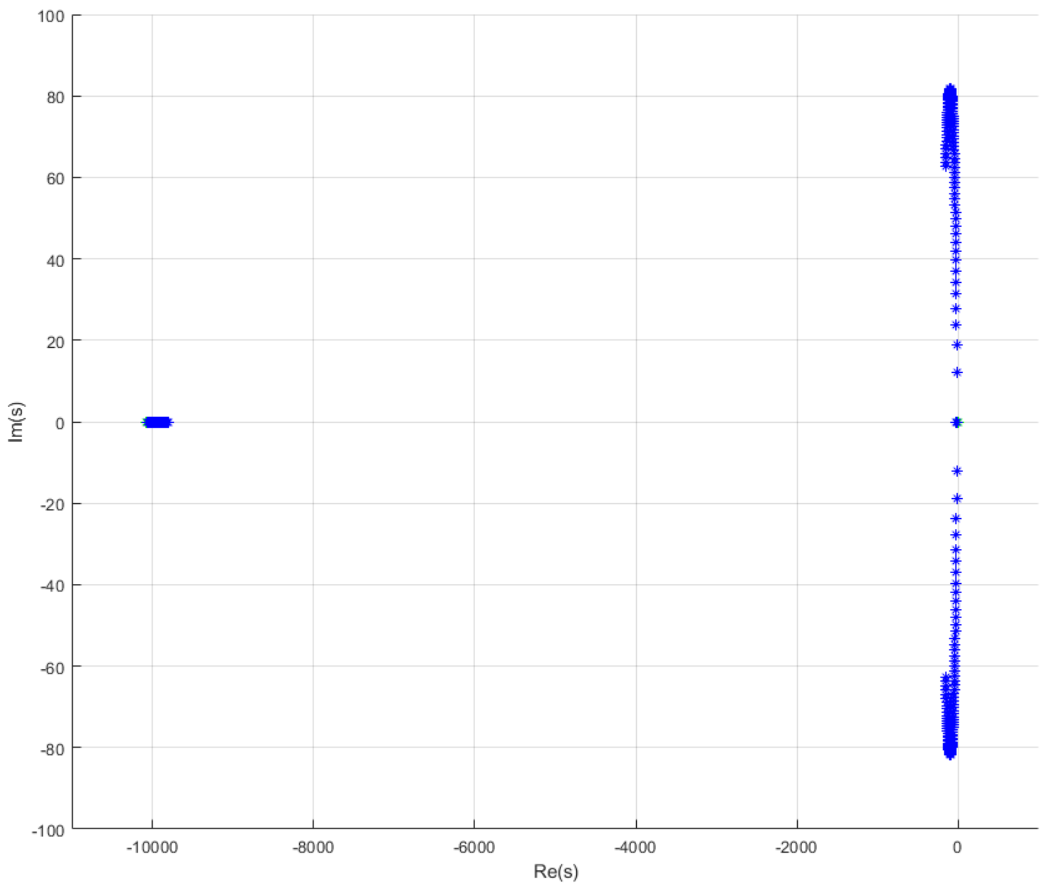

Figure 6: Closed Loop Poles with Gain Factor Applied ($1 \leq g \leq 100$).

Again zooming into the right side in Fig. 7, we see that the poles s_1 and s_2 of the system become complex conjugates once $g = 2$ is applied. In the time domain, this is represented by an oscillation that decays more rapidly as g is increased.

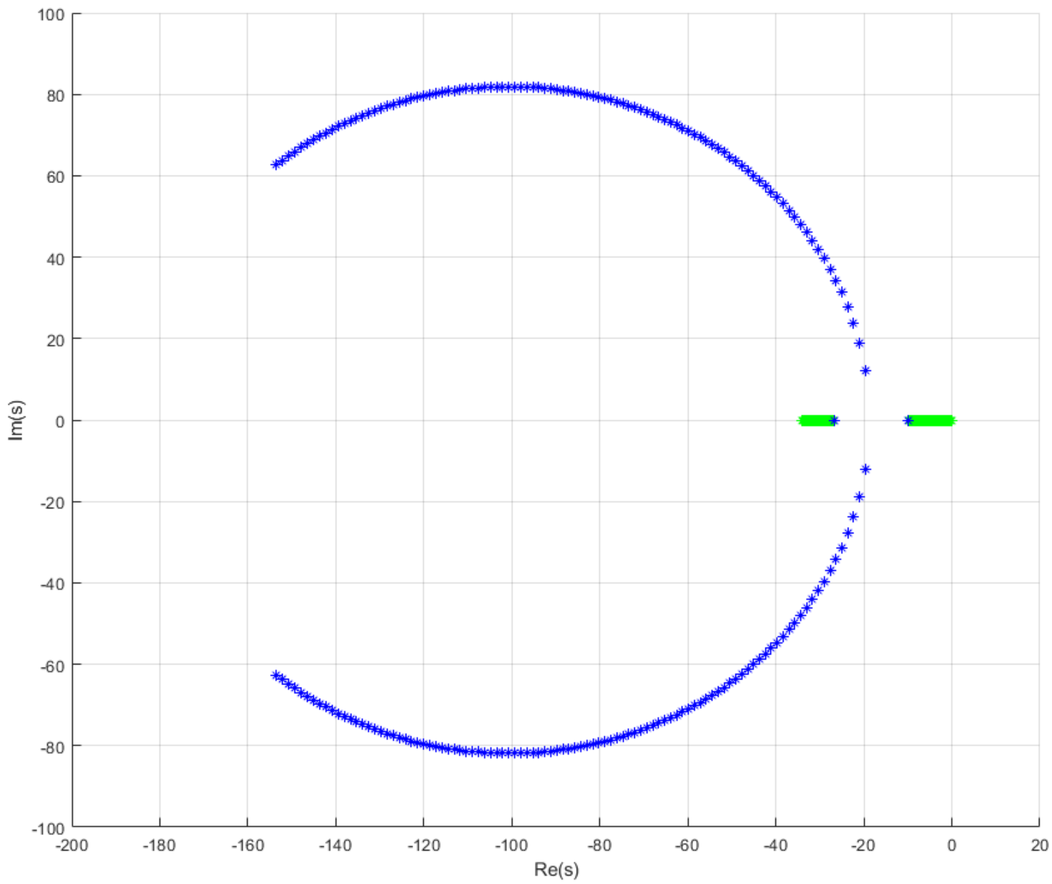

Figure 7: Closed Loop Poles with Gain Factor Applied ($1 \leq g \leq 100$).

5. Repeat part 4 but use negative g values between 0 and -1.

The closed loop poles for a gain factor between 0 and -1 can be seen in Fig. 8 in cyan.

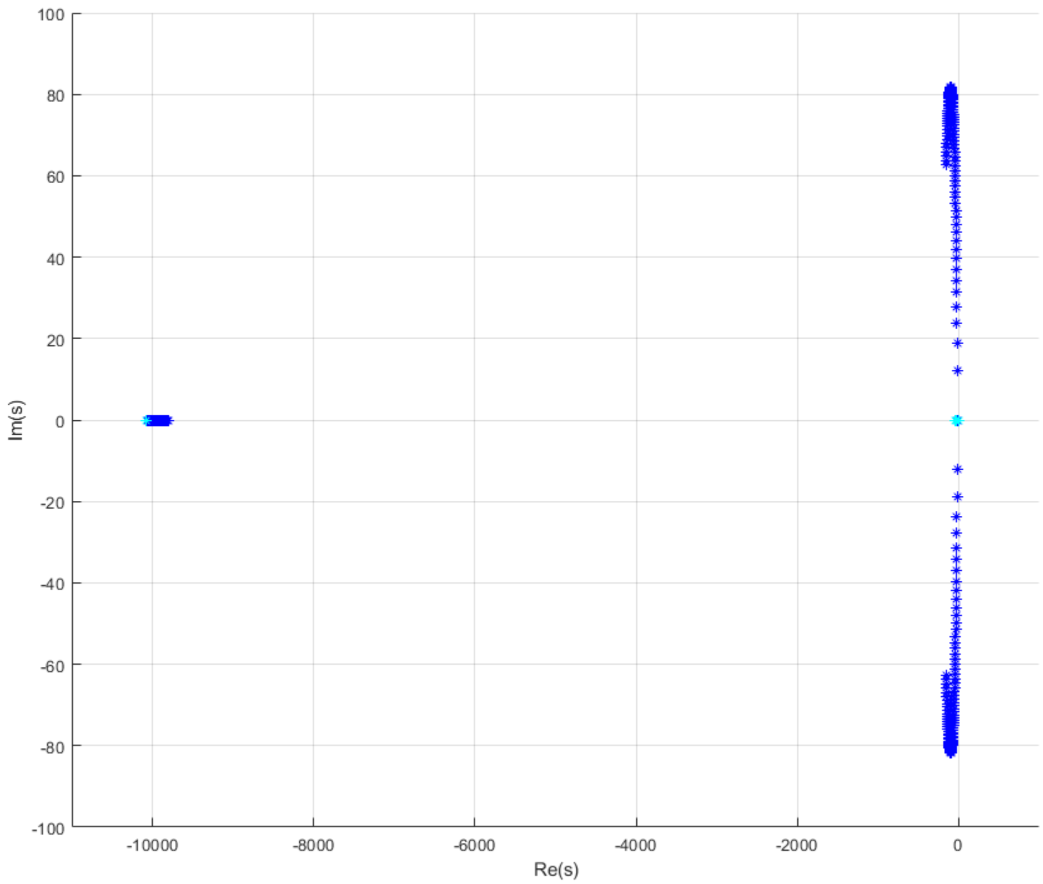

Figure 8: Closed Loop Poles with Gain Factor Applied (-1 $\leq g \leq 0$).

Zooming into the right side first in Fig. 9, we can see that the use of a negative gain factor makes the system unstable by increasing s_1 above 0, which results in a time domain response that increases without bound. However, on both s_2 and s_3 (seen in Fig. 10), the negative gain factor actually decreases the settling time for their respective time domain responses.

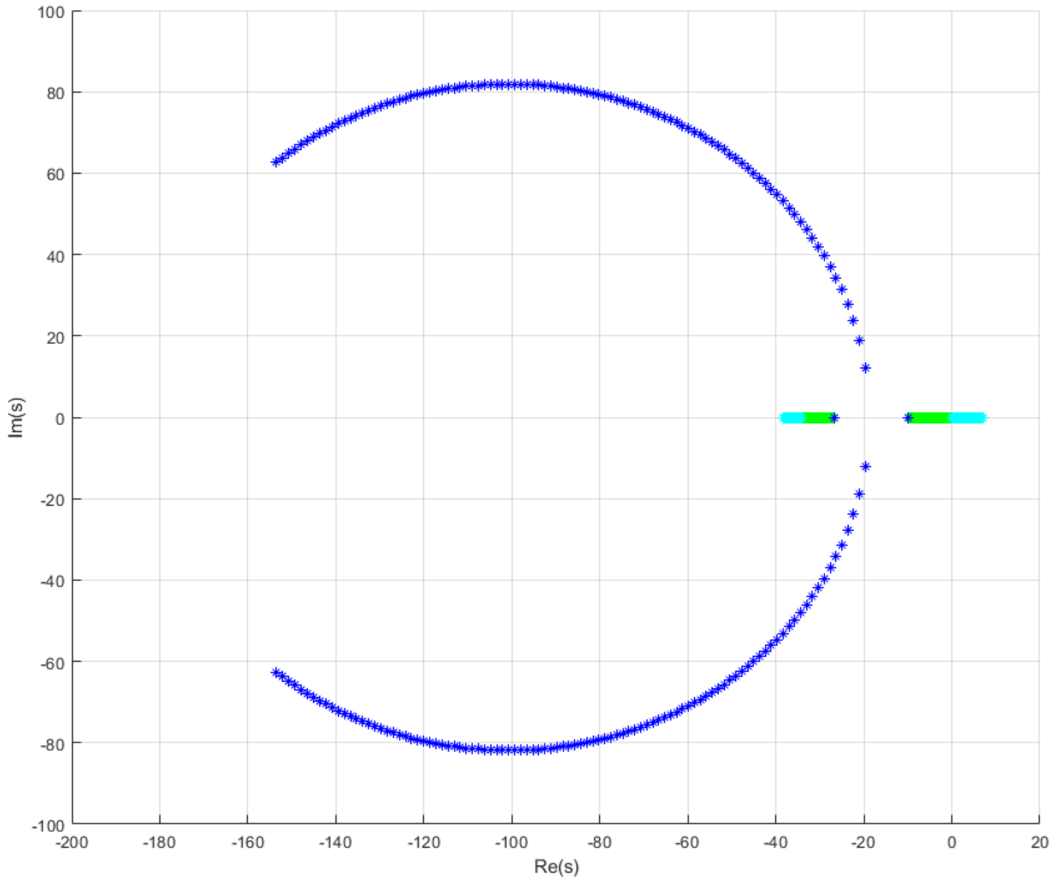

Figure 9: Closed Loop Poles with Gain Factor Applied (-1 $\leq g \leq 0$).

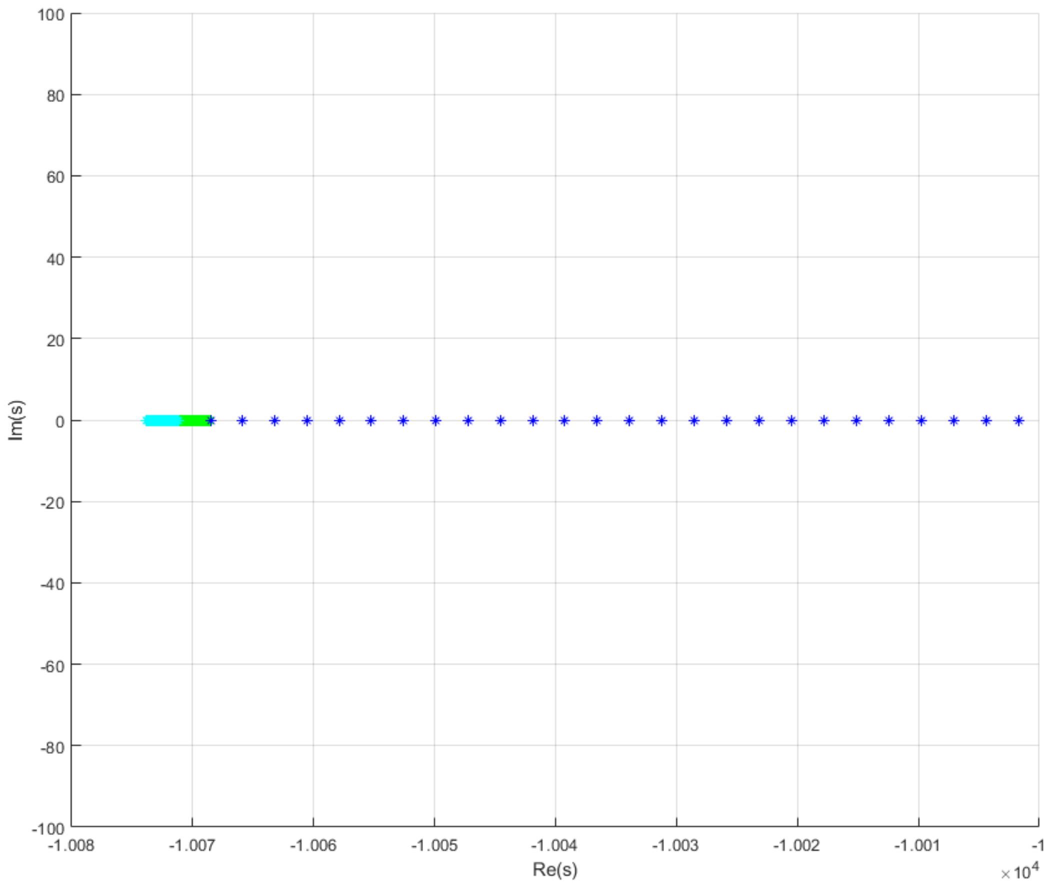

Figure 10: Closed Loop Poles with Gain Factor Applied (-1 $\leq g \leq$ 0).

6. What does the above root locus analysis tell you about the effect of the gain g on the closed loop system's natural response? Verify your conclusions by simulating the closed loop step response for several values of g.

The root locus analysis gives several insights on the behavior of the system. First, for a gain factor between 0 and 1, the system does not oscillate in response to an input. However, for a gain factor of 2 or above (up to at least 100), oscillations are present with an increasing decay rate as g is increased. Finally, any negative gain factor will make the system unstable.

The system was simulated seven times, with gain factors of -1, -0.5, 0.5, 1, 10, 50, and 100. The results of these simulations are below, and they are consistent with the predictions drawn from the root locus analysis.

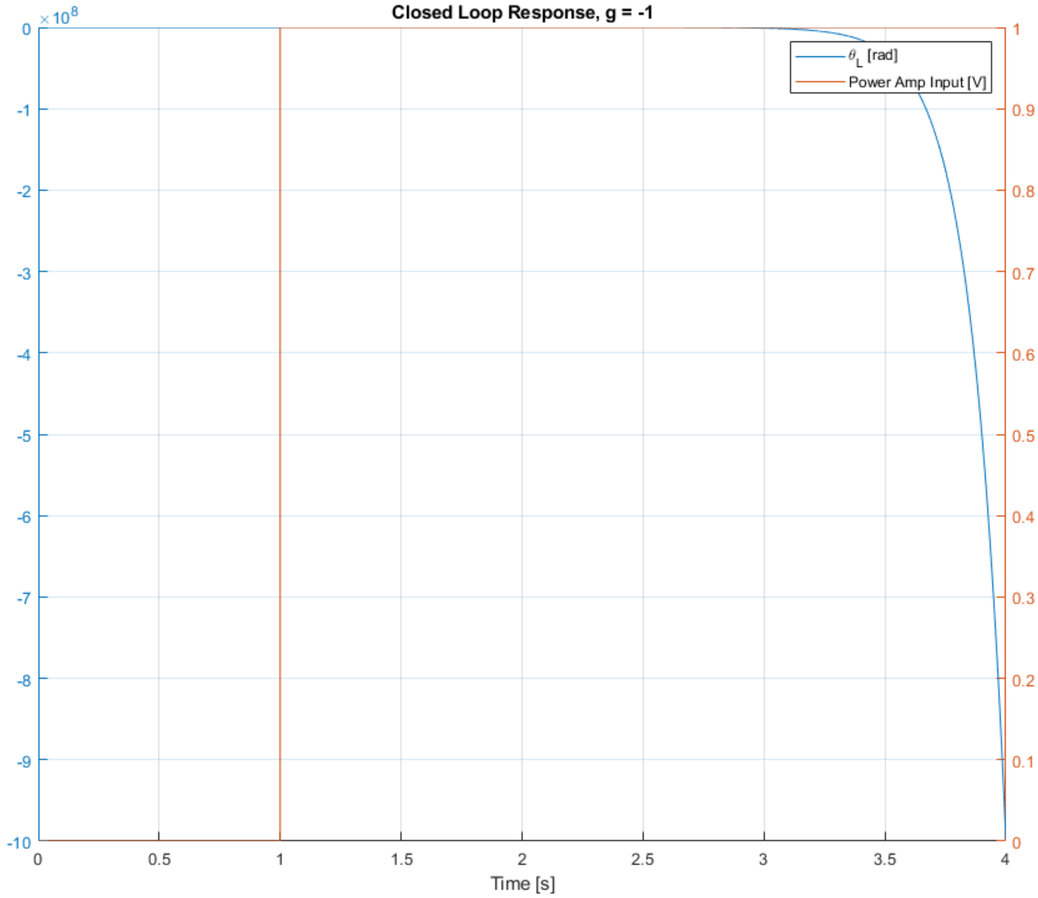

Figure 11: Closed Loop Response, g = -1.

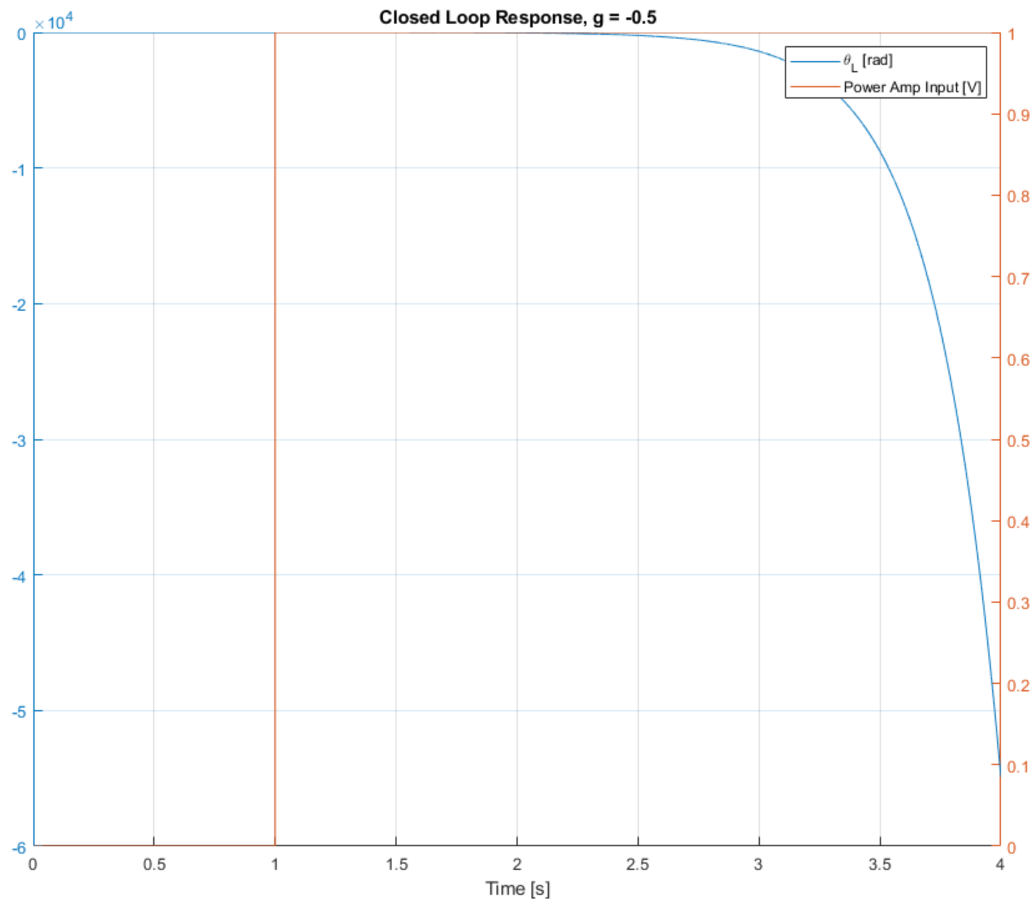

Figure 12: Closed Loop Response, g = -0.5.

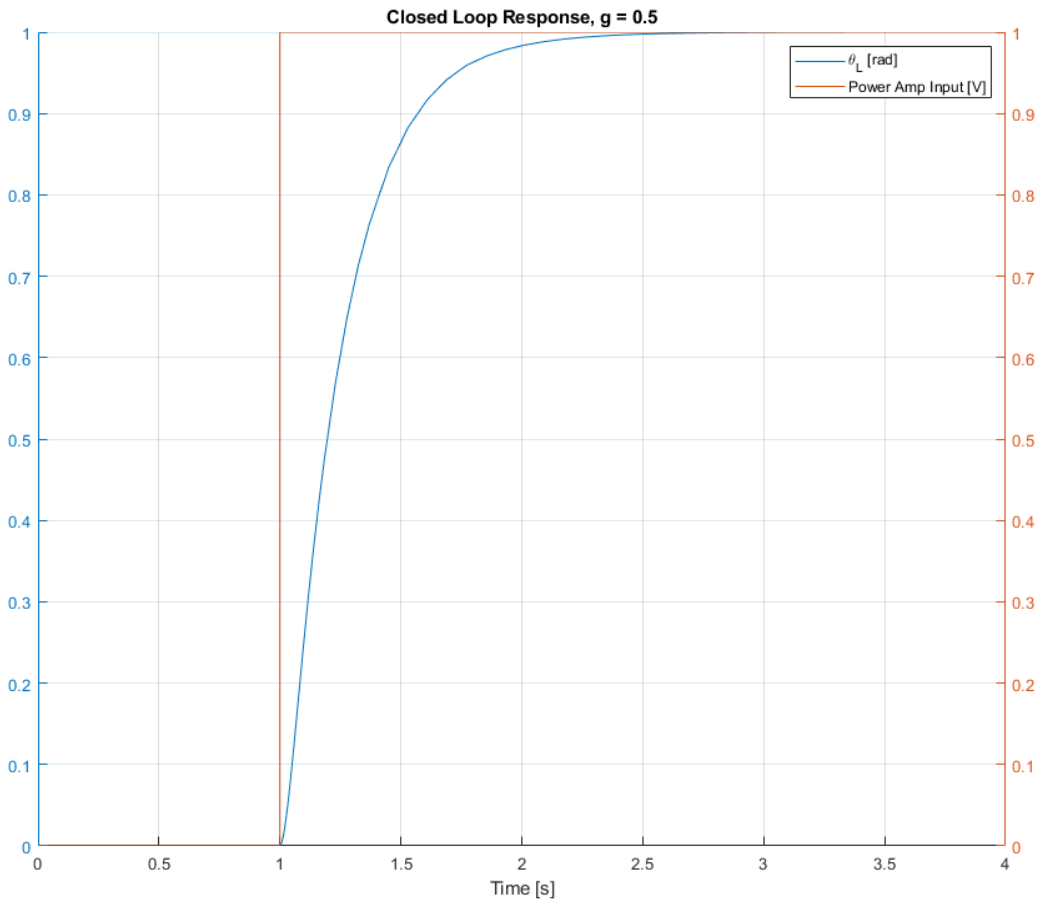

Figure 13: Closed Loop Response, g = 0.5.

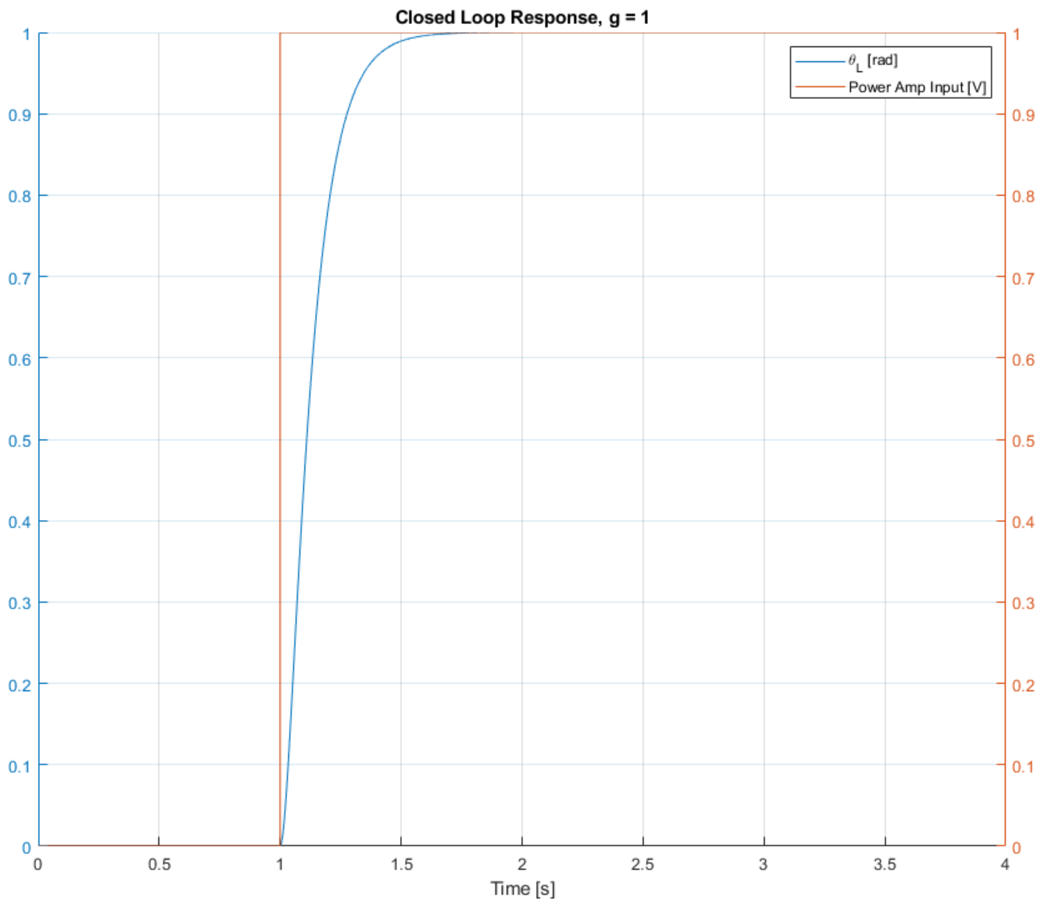

Figure 14: Closed Loop Response, g = 1.

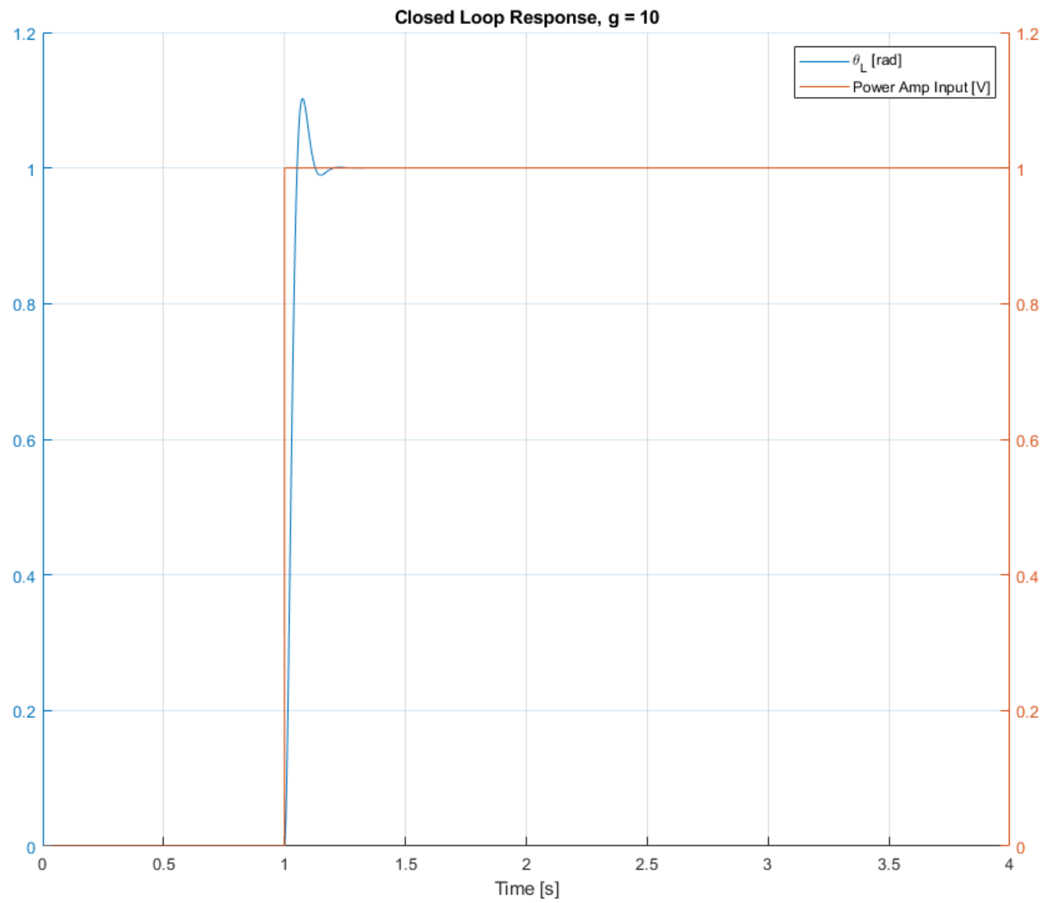

Figure 15: Closed Loop Response, g = 10.

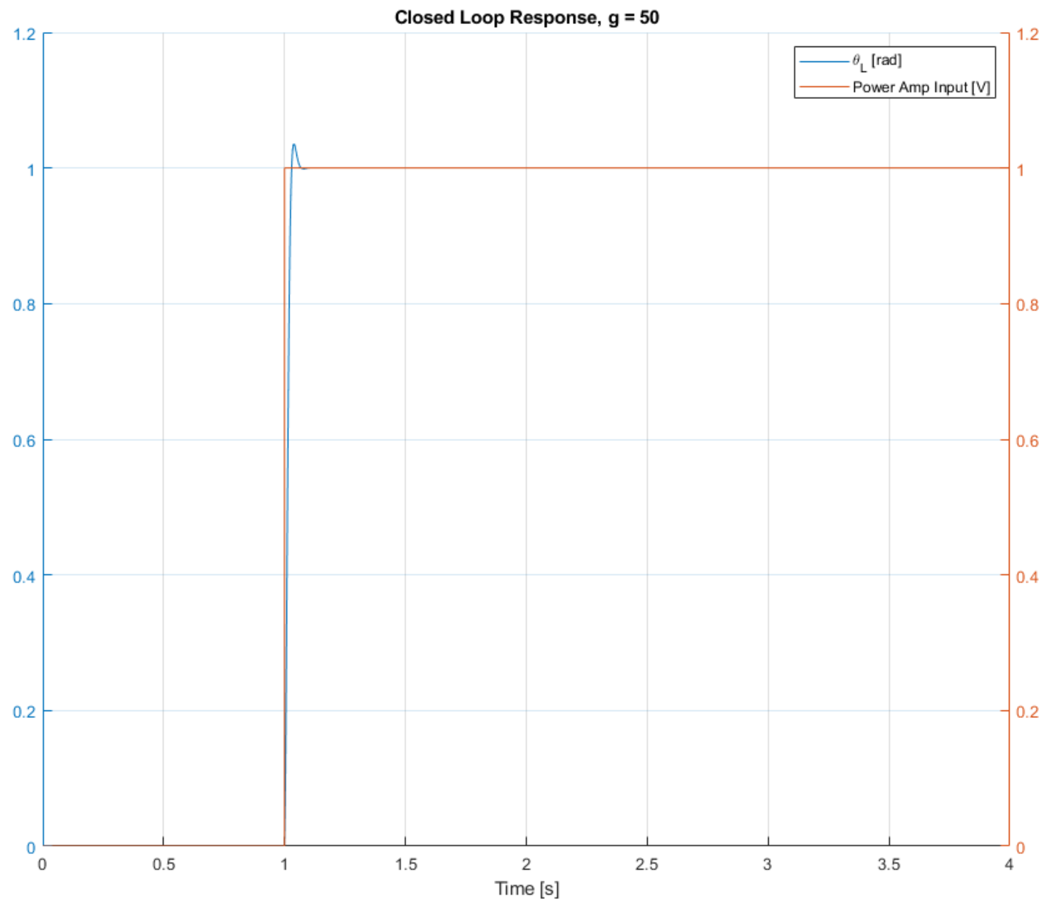

Figure 16: Closed Loop Response, g = 50.

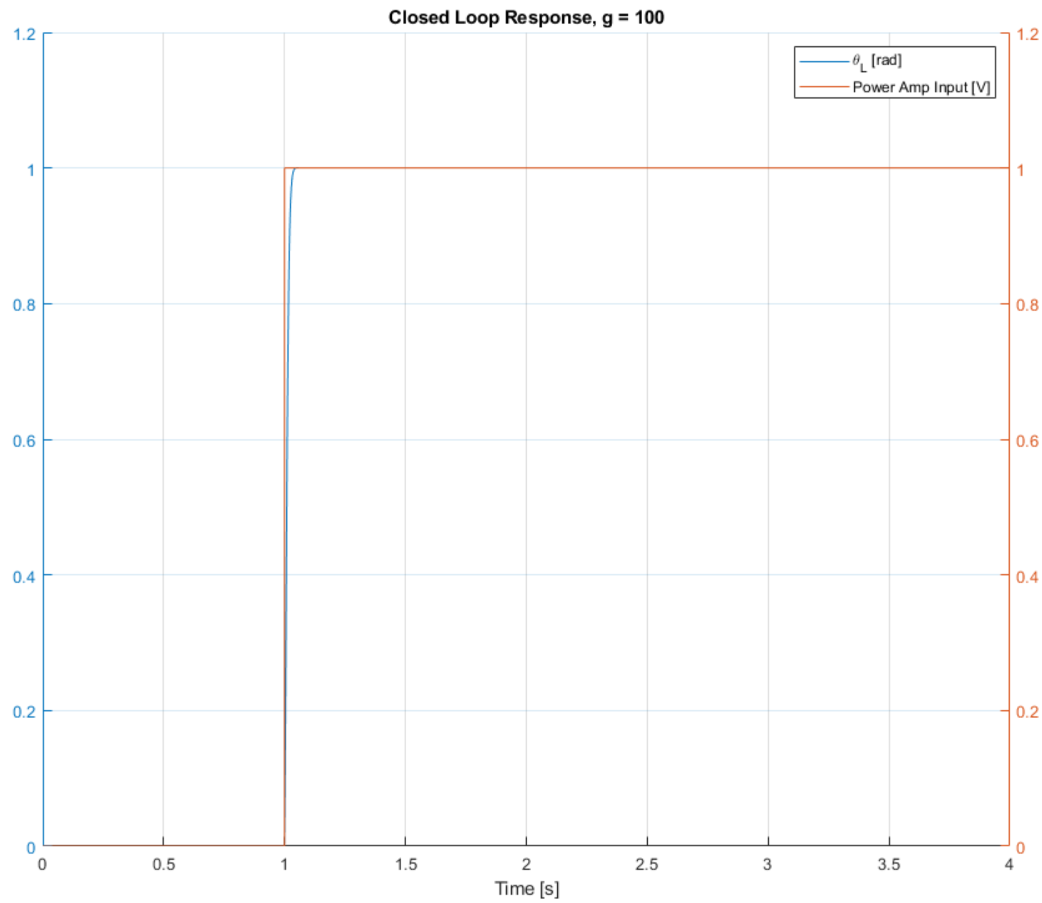

Figure 17: Closed Loop Response, g = 100.

MATLAB Code

```
%% Experiment 2
% William Watkins
% 16 January 2022

%% Clean up
clear all
close all
clc

%% Constants/Properties
Jconv = 1/0.00034171718982094; %[g*cm^2 / lb*in^2]
Jc = 1*10^-7; % [kg*m^2 / g*cm^2]
Rm = 19.2;      % [Ohms]
Lm = 0.0019;    % [Henrys]
Ktau = 40.1*10^-3;  % [Nm/A]
Kb = 1/238 / (2*pi/60); % [V/(rad/s)]
rBig = 2.51; % [in]
rSmall = 0.79; % [in]
Ngearhead = 10;
Ngears = rBig/rSmall;
Jm = 12.5 * Jc; % [kg*m^2]
Jgearhead = 0.6 * Jc; % [kg*m^2]
N = Ngearhead * Ngears;

%% Computing Moments
Jtm = Jm; % [kg*m^2]
rhoAl = 0.097; % [lb/in^2]
h = 0.199; % [in]
wBeam = 0.504; % [in]
dBeam = 10; % [in]
sSquare = 1.75; % [in]
Vsq = sSquare^2 * h; % [in^3]
Vbeam = h * wBeam * dBeam; % [in^3]
mSquare = Vsq * rhoAl; % [lb]
mBeam = Vbeam * rhoAl; % [lb]
Jbeam = (1/12) * mBeam * (dBeam^2 + wBeam^2); % [lb*in^2]
Jsquare = (1/12) * mSquare * 2 * sSquare^2; % [lb*in^2]
z = dBeam/2 + sSquare/2; % [in]
JPA = Jbeam + mBeam*z^2; % [lb*in^2]
Jl = JPA + Jsquare; % [lb*in^2]
Jl = Jl * Jconv * Jc; % [kg*m^2]
Jeq = Jl + N^2 * Jtm; % [kg*m^2]

%% System Coefficients
s3 = (Jeq/(N*Ktau))*Lm;
s2 = (Jeq/(N*Ktau))*Rm;
s1 = N*Kb;
Gd_o = -0.1;
Gp_o = -10;
Gd = Gd_o;
```

```
Gp = Gp_o;

%% Problem 1
pole1 = 0;
pole2 = (-(Jeq*Rm/(N*Ktau)) + sqrt((Jeq*Rm/(N*Ktau))^2 - 4*(Jeq*Lm/(N*Ktau))*(N*Kb)))/(2*Je
pole3 = (-(Jeq*Rm/(N*Ktau)) - sqrt((Jeq*Rm/(N*Ktau))^2 - 4*(Jeq*Lm/(N*Ktau))*(N*Kb)))/(2*Je

figure('Position', [200 200 1000 800]);
hold on;
grid on;
plot([pole1 pole2 pole3], [0 0 0], '*','color','red');
xlabel('Re(s)');
ylabel('Im(s)');
xlim([-11000 1000])

%% Problem 2
p = [Jeq*Lm/(N*Ktau) (Jeq*Rm/(N*Ktau)) N*Kb-Gd -Gp];
r = roots(p);
plot(r, [0 0 0], '*','color','magenta');

%% Problem 3
g = 0:0.01:1;
pg = [Jeq*Lm/(N*Ktau) (Jeq*Rm/(N*Ktau)) N*Kb-g(1)*Gd -g(1)*Gp];
rg(:,1) = roots(pg(1,:));
for i = 2:length(g)
    pg(i,:) = [Jeq*Lm/(N*Ktau) (Jeq*Rm/(N*Ktau)) N*Kb-g(i)*Gd -g(i)*Gp];
    rg(:,i) = roots(pg(i,:));
end
reax = zeros(3,101);
plot(rg,reax,'*','color','green')

%% Problem 4
g4 = 1:1:100;
pg4 = [Jeq*Lm/(N*Ktau) (Jeq*Rm/(N*Ktau)) N*Kb-g4(1)*Gd -g4(1)*Gp];
rg4(:,1) = roots(pg4(1,:));
for i = 2:length(g4)
    pg4(i,:) = [Jeq*Lm/(N*Ktau) (Jeq*Rm/(N*Ktau)) N*Kb-g4(i)*Gd -g4(i)*Gp];
    rg4(:,i) = roots(pg4(i,:));
end

plot(rg4,'*','color','blue')

%% Problem 5
g5 = 0:-0.01:-1;
pg5 = [Jeq*Lm/(N*Ktau) (Jeq*Rm/(N*Ktau)) N*Kb-g5(1)*Gd -g5(1)*Gp];
rg5(:,1) = roots(pg5(1,:));
for i = 2:length(g5)
    pg5(i,:) = [Jeq*Lm/(N*Ktau) (Jeq*Rm/(N*Ktau)) N*Kb-g5(i)*Gd -g5(i)*Gp];
    rg5(:,i) = roots(pg5(i,:));
end

plot(rg5,reax,'*','color','cyan')

%% Problem 6
```

```
g = [-1 -0.5 0 0.5 1 10 50 100];
Gd = g(1)*Gd_o;
Gp = g(1)*Gp_o;
simOut6 = sim('Models/PD_Control_Step.slx');
simTime1(1,:) = simOut6.yout{1}.Values.Time';
simVolt1(1,:) = simOut6.yout{1}.Values.Data';
simTime1(2,:) = simOut6.yout{2}.Values.Time';
simVolt1(2,:) = simOut6.yout{2}.Values.Data';

Gd = g(2)*Gd_o;
Gp = g(2)*Gp_o;
simOut6 = sim('Models/PD_Control_Step.slx');
simTime2(1,:) = simOut6.yout{1}.Values.Time';
simVolt2(1,:) = simOut6.yout{1}.Values.Data';
simTime2(2,:) = simOut6.yout{2}.Values.Time';
simVolt2(2,:) = simOut6.yout{2}.Values.Data';

Gd = g(3)*Gd_o;
Gp = g(3)*Gp_o;
simOut6 = sim('Models/PD_Control_Step.slx');
simTime3(1,:) = simOut6.yout{1}.Values.Time';
simVolt3(1,:) = simOut6.yout{1}.Values.Data';
simTime3(2,:) = simOut6.yout{2}.Values.Time';
simVolt3(2,:) = simOut6.yout{2}.Values.Data';

Gd = g(4)*Gd_o;
Gp = g(4)*Gp_o;
simOut6 = sim('Models/PD_Control_Step.slx');
simTime4(1,:) = simOut6.yout{1}.Values.Time';
simVolt4(1,:) = simOut6.yout{1}.Values.Data';
simTime4(2,:) = simOut6.yout{2}.Values.Time';
simVolt4(2,:) = simOut6.yout{2}.Values.Data';

Gd = g(5)*Gd_o;
Gp = g(5)*Gp_o;
simOut6 = sim('Models/PD_Control_Step.slx');
simTime5(1,:) = simOut6.yout{1}.Values.Time';
simVolt5(1,:) = simOut6.yout{1}.Values.Data';
simTime5(2,:) = simOut6.yout{2}.Values.Time';
simVolt5(2,:) = simOut6.yout{2}.Values.Data';

Gd = g(6)*Gd_o;
Gp = g(6)*Gp_o;
simOut6 = sim('Models/PD_Control_Step.slx');
simTime6(1,:) = simOut6.yout{1}.Values.Time';
simVolt6(1,:) = simOut6.yout{1}.Values.Data';
simTime6(2,:) = simOut6.yout{2}.Values.Time';
simVolt6(2,:) = simOut6.yout{2}.Values.Data';

Gd = g(7)*Gd_o;
Gp = g(7)*Gp_o;
simOut6 = sim('Models/PD_Control_Step.slx');
simTime7(1,:) = simOut6.yout{1}.Values.Time';
simVolt7(1,:) = simOut6.yout{1}.Values.Data';
```

```
simTime7(2,:) = simOut6.yout{2}.Values.Time';
simVolt7(2,:) = simOut6.yout{2}.Values.Data';

Gd = g(8)*Gd_o;
Gp = g(8)*Gp_o;
simOut6 = sim('Models/PD_Control_Step.slx');
simTime8(1,:) = simOut6.yout{1}.Values.Time';
simVolt8(1,:) = simOut6.yout{1}.Values.Data';
simTime8(2,:) = simOut6.yout{2}.Values.Time';
simVolt8(2,:) = simOut6.yout{2}.Values.Data';

%% PLots for 6
figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
plot(simTime1(1,:),simVolt1(1,:))
yyaxis right
plot(simTime1(2,:),simVolt1(2,:))
legend({'\theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Closed Loop Response, g = -1')
xlabel('Time [s]')

figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
plot(simTime2(1,:),simVolt2(1,:))
yyaxis right
plot(simTime2(2,:),simVolt2(2,:))
legend({'\theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Closed Loop Response, g = -0.5')
xlabel('Time [s]')

figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
plot(simTime3(1,:),simVolt3(1,:))
yyaxis right
plot(simTime3(2,:),simVolt3(2,:))
legend({'\theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Closed Loop Response, g = 0')
xlabel('Time [s]')

figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
plot(simTime4(1,:),simVolt4(1,:))
yyaxis right
plot(simTime4(2,:),simVolt4(2,:))
legend({'\theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Closed Loop Response, g = 0.5')
```

```
xlabel('Time [s]')

figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
plot(simTime5(1,:),simVolt5(1,:))
yyaxis right
plot(simTime5(2,:),simVolt5(2,:))
legend({'\theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Closed Loop Response, g = 1')
xlabel('Time [s]')

figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
plot(simTime6(1,:),simVolt6(1,:))
yyaxis right
plot(simTime6(2,:),simVolt6(2,:))
ylim([0 1.2])
legend({'\theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Closed Loop Response, g = 10')
xlabel('Time [s]')

figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
plot(simTime7(1,:),simVolt7(1,:))
yyaxis right
plot(simTime7(2,:),simVolt7(2,:))
ylim([0 1.2])
legend({'\theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Closed Loop Response, g = 50')
xlabel('Time [s]')

figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
plot(simTime8(1,:),simVolt8(1,:))
yyaxis right
plot(simTime8(2,:),simVolt8(2,:))
ylim([0 1.2])
legend({'\theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Closed Loop Response, g = 100')
xlabel('Time [s]')
```