ASEN 5114: Experiment 4

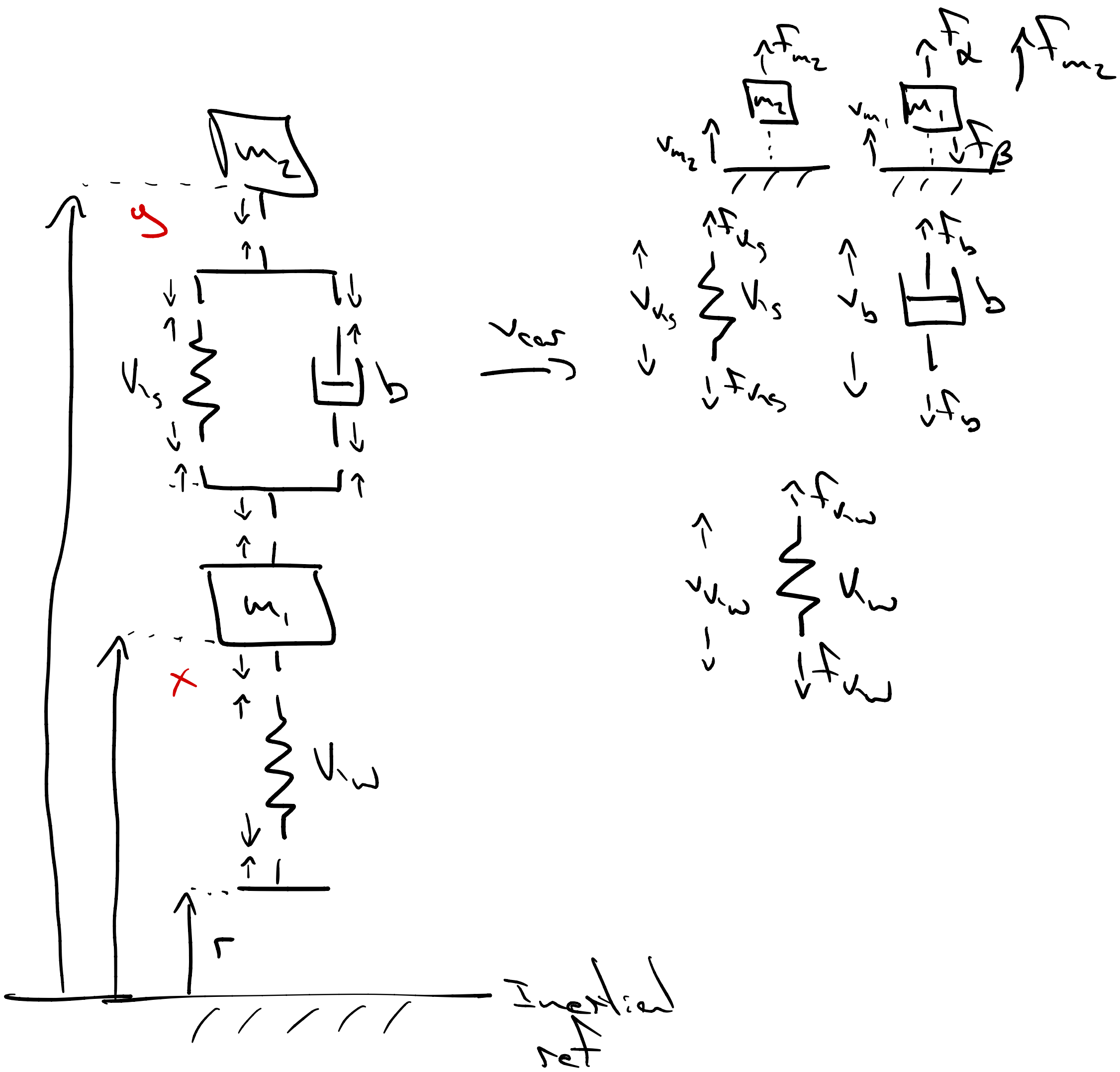
William
Watkins

2.5

For suspension in Ex 2.2, plot position of car & wheel after a "unit bump" (ie r is a unit step).

Assume $m_1 = 10\,kg$, $m_2 = 350\,kg$, $k_w = 500{,}000\,N/m$, $k_s = 10{,}000\,N/m$

Find value of b you would prefer if you were a passenger



Topology

$f_\beta = f_{k_w}$

$f_\alpha = f_{k_s} + f_b$

$f_{m_2} = -f_{k_s} - f_b$

$f_{m_1} = f_\alpha - f_\beta$

$v_{k_w} = \dot{x} - \dot{r}$

$v_{m_1} = \dot{x}$

$v_{k_s} = v_B = \dot{y} - \dot{x}$

$v_{m_2} = \dot{y}$

Element

$\dot{v}_{m_2} = \frac{1}{m_2} f_{m_2}$

$\dot{v}_{m_1} = \frac{1}{m_1} F_{m_1}$

$\dot{f}_{k_w} = K_w \, v_{k_w}$

$\dot{f}_{k_s} = K_s \, v_{k_s}$

$f_b = B \, v_b$

Inputs

r

Outputs

$\dot{y}, \dot{x}$ OR $\ddot{y}, \ddot{x}$

$\cancel{y_B}\, y_2 = \dot{y} = v_{m_2} = x_2$

$y_1 = \dot{x} = v_{m_1} = x_1$

SSM Method

1)
- Inputs - r
- Outputs - y, x
- States - 4 energy storage elements, so 4 states
  - mass m_1 - $v_{m_1} = x_1$
  - mass m_2 - $v_{m_2} = x_2$

  - spring K_w - $f_{k_w} = x_3$
  - spring K_s - $f_{k_s} = x_4$

2)

$x_1^* = f_{m_1}$

$x_2^* = f_{m_2}$

$x_3^* = v_{k_w}$

$x_4^* = v_{k_s}$

3) $x_1^* = g(x_1, x_2, x_3, x_4) = F_{m_1}$

$f_{m_1} = F_\alpha - f_\beta$

$= (f_{k_s} + f_b) - (f_{k_w})$

$= -f_{k_w} + (f_{k_s} + f_b)$

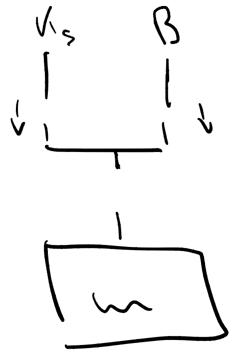
$= f_{k_s} + f_b - f_{k_w}$

$= f_{k_s} + B v_b - f_{k_w}$

$= f_{k_s} + B[\dot{y} - \dot{x}] - f_{k_w}$

$= f_{k_s} + B[v_{m_2} - v_{m_1}] - f_{k_w}$

$x_1^* = f_{m_1} = x_4 + B[x_2 - x_1] - x_3$



$x_2^* = F_{m_2} = (-f_{k_s} - f_b)$

$= (-f_{k_s} - B v_b)$

$= (-f_{k_s} - B[\dot{y} - \dot{x}])$

$= (-f_{k_s} - B[v_{m_2} - v_{m_1}])$

$x_2^* = f_{m_2} = (-x_4 - B[x_2 - x_1])$

$x_3^* = v_{k_w} = \dot{x} - \dot{r}$

$= v_{m_1} - \dot{r}$

$x_3^* = v_{k_w} = x_1 - \dot{r}$

$x_4^* = v_{k_s} = \dot{y} - \dot{x}$

$= v_{m_2} - v_{m_1}$

$x_4^* = v_{k_s} = x_2 - x_1$

4) $\dot{x}_1 = \frac{1}{m_1} x_1^*$

$$\dot{x}_1 = \frac{1}{m_1}[x_3 - x_4 - \overbrace{B[x_2 - x_1])}^{pos}]$$

$$\dot{x}_2 = \frac{1}{m_2} x_2^*$$

$$\dot{x}_2 = \frac{1}{m_2}[(-x_4 - B[x_2 - x_1])]$$

$$\dot{x}_3 = K_w x_3^*$$

$$\dot{x}_3 = K_w[x_1 - \dot{r}]$$

$$\dot{x}_4 = K_s[x_2 - x_1]$$

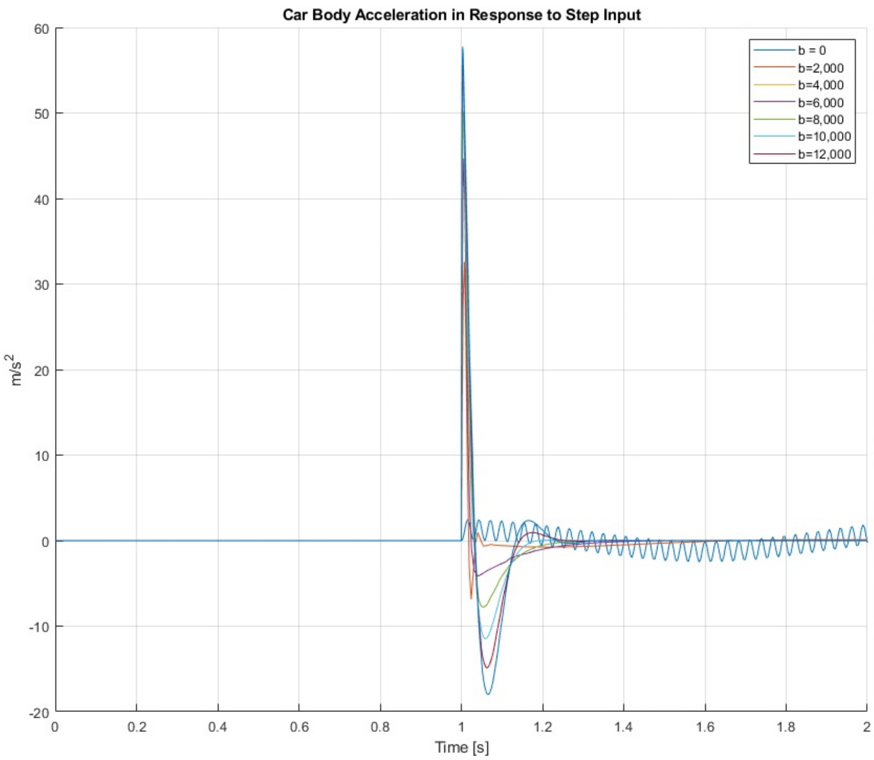
$y_1 = \dot{x} = v_{m_1} = x_1$
$y_2 = \dot{y} = v_{m_2} = x_2$

5) $\dot{x} = Ax + Bu$
$y = Cx + Du$

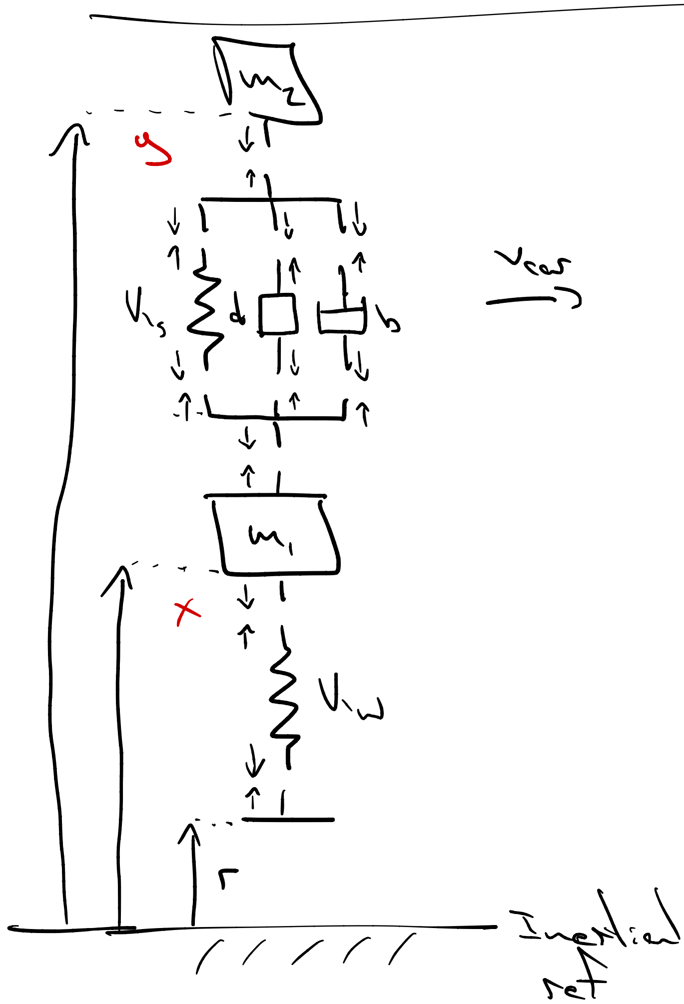
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -B/m_1 & B/m_1 & -1/m_1 & 1/m_1 \\ B/m_2 & -B/m_2 & 0 & -1/m_2 \\ K_w & 0 & 0 & 0 \\ -K_s & K_s & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -K_w \\ 0 \end{bmatrix} [\dot{r}]$$

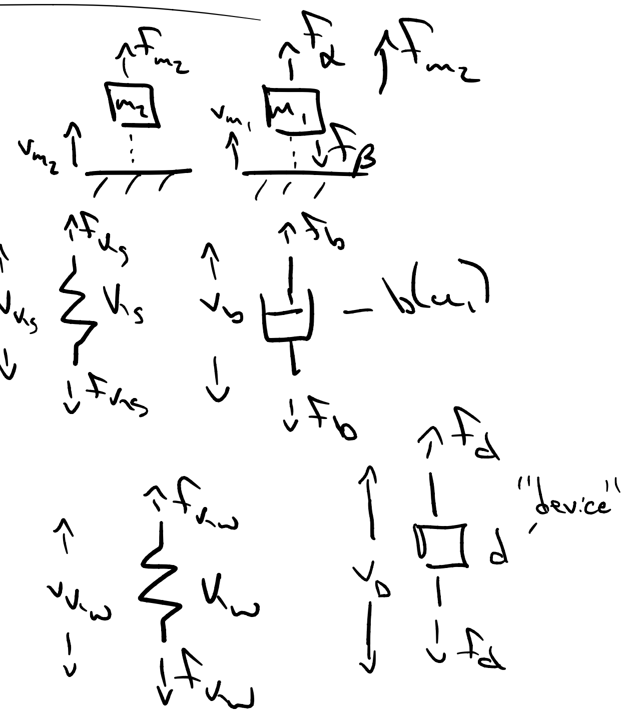
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [\dot{r}]$$

The response to a unit step input can be seen below. The acceleration of y is plotted, as that is what humans define "comfort" as relating to. A comfortable damping coefficient is one that minimizes the acceleration & doesn't rapidly oscillate. So, a damping coefficient b between 6,000 & 8,000 would be most desireable.


Car Body Acceleration in Response to Step Input
b = 0
b=2,000
b=4,000
b=6,000
b=8,000
b=10,000
b=12,000
m/s^2
Time [s]
60
50
40
30
20
10
0
-10
-20
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

2)





Topology

$f_\beta = f_{k_w}$

$f_\alpha = f_{k_s} + f_d + f_b$

$f_{m_2} = -f_{k_s} - f_b - f_d$

$f_{m_1} = f_\alpha - f_\beta$

$v_{k_w} = \dot{x} - \dot{r}$

$v_{m_1} = \dot{x}$

$v_{k_s} = v_D = \dot{y} - \dot{x} = v_b$

$v_{m_2} = \dot{y}$

Element

$\dot{v}_{m_2} = \frac{1}{m_2} f_{m_2}$

$\dot{v}_{m_1} = \frac{1}{m_1} f_{m_1}$

$\dot{f}_{k_w} = k_w v_{k_w}$

$\dot{f}_{k_s} = k_s v_{k_s}$

$f_b = b(u_1) \cdot v_b$

$f_d = u_2$

Inputs

r

Outputs

y, x

$y_2 = \dot{y} = v_{m_2} = x_2$
$y_1 = \dot{x} = v_{m_1} = x_1$

SSM Method

1)
- Inputs – r
- Outputs – y, x
- States – 4 energy storage elements, so 4 states
  - mass m_1 – $v_{m_1} = x_1$
  - mass m_2 – $v_{m_2} = x_2$

  - spring V_{1w} – $F_{V_{1w}} = x_3$
  - spring V_s – $F_{V_s} = x_4$

2)
$$\dot{x}_1 = F_{m_1}$$
$$\dot{x}_2 = F_{m_2}$$

$$\dot{x}_3 = v_{V_{1w}}$$
$$\dot{x}_4 = v_{V_s}$$

3)
$$\dot{x}_1 = g(x_1, x_2, x_3, x_4) = F_{m_1}$$

$$F_{m_1} = F_\alpha - F_\beta$$
$$= (F_{V_s} + F_b + F_d) - F_{V_{1w}}$$
$$= F_{V_s} + b(u_1)v_b + F_d - F_{V_{1w}}$$
$$= F_{V_s} + b(u_1)[\dot{y} - \dot{x}] + F_d - F_{V_{1w}}$$
$$= F_{V_s} + b(u_1)[v_{m_2} - v_{m_1}] + F_d - F_{V_{1w}}$$

$$\dot{x}_1 = F_{m_1} = x_4 + b(u_1)[x_2 - x_1] + u_2 - x_3$$

$$x_2^* = F_{m_2} = (-F_{k_s} - F_b - F_d)$$

$$= -F_{k_s} - b(u_1)\, v_b - F_d$$

$$= -F_{k_s} - b(u_1)[\dot{y} - \dot{x}] - F_d$$

$$= -F_{k_s} - b(u_1)[v_{m_2} - v_{m_1}] - F_d$$

$$x_2^* = F_{m_2} = -x_4 - b(u_1)[x_2 - x_1] - u_2$$

$$x_3^* = v_{k_w} = \dot{x} - \dot{r}$$

$$= v_{m_1} - \dot{r}$$

$$x_3^* = v_{k_w} = x_1 - \dot{r}$$

$$x_4^* = v_{k_s} = \dot{y} - \dot{x}$$

$$= v_{m_2} - v_{m_1}$$

$$x_4^* = v_{k_s} = x_2 - x_1$$

4) $\dot{x}_1 = \frac{1}{m_1} x_1^*$

$$\dot{x}_1 = \frac{1}{m_1}\left[x_4 + b(u_1)[x_2 - x_1] + u_2 - x_3\right]$$

$$\dot{x}_2 = \frac{1}{m_2} x_2^*$$

$$\dot{x}_2 = \frac{1}{m_2}\left[-x_4 - b(u_1)[x_2 - x_1] - u_2\right]$$

$$\dot{x}_3 = k_w\, x_3^*$$

$$\dot{x}_3 = k_w[x_1 - \dot{r}]$$

$$\dot{x}_4 = k_s[x_2 - x_1]$$

$$y_1 = \dot{x} = v_{m_1} = x_1$$

$$y_2 = \dot{y} = v_{m_2} = x_2$$

5)

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

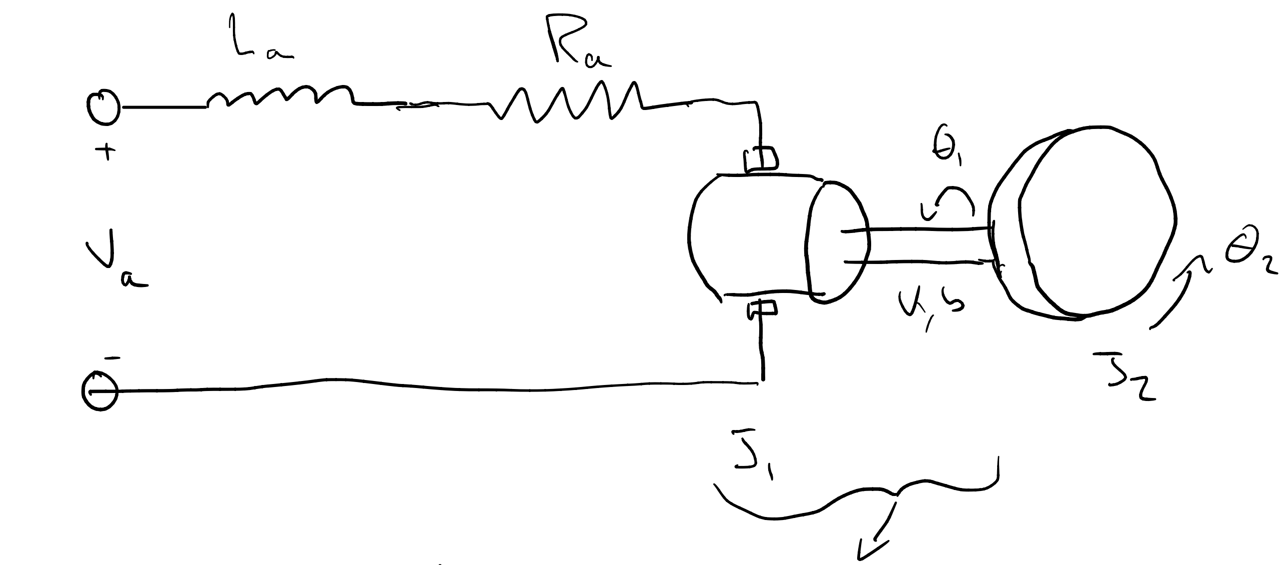
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -b(u_1)/m_1 & b(u_1)/m_1 & -1/m_1 & 1/m_1 \\ b(u_1)/m_2 & -b(u_1)/m_2 & \emptyset & -1/m_2 \\ V_w & \emptyset & \emptyset & \emptyset \\ -V_s & V_s & \emptyset & \emptyset \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \emptyset & 1/m_1 & \emptyset \\ \emptyset & \emptyset & \emptyset \\ -V_w & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \end{bmatrix} \begin{bmatrix} \dot{r} \\ u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & \emptyset & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset & \emptyset \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix} [\dot{r}]$$

b) The resulting system is <u>not</u> linear. While the input u_2 does not make the system nonlinear, using a changable damping coefficient that is tied to the inputs of the system does make it non-linear.

c) Yes, replacing the springs and shock absorber with u_2 is possible. However, it would require very large forces to do so, and even with hardware available today that would be difficult to implement without some sort of delay between the input & the output.

3)



$$\tau_m = J_1\ddot{\theta}_1 + B\dot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + K_1(\theta_1 - \theta_2)$$

$$\tau_m = K_t I_m = J_1\ddot{\theta}_1 + B\dot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + K_1(\theta_1 - \theta_2)$$

SSM

Inputs

V_a

Output

θ_1

States:

$x_1 = \theta_1$

$x_2 = \theta_2$

$x_3 = I_L$

Really not sure how to do this. I know you can rewrite inductor as a spring & resistor as a damper, but how can you connect those to the motor?

Experiment 4

Table of Contents

Clean up 1
Constants 1
Try in LTI SS model 1
Simulating 1

William Watkins 5 February 2022

Clean up

```
clear all
close all
clc
```

Constants

```
m1 = 10;
m2 = 350;
Kw = 500000;
Ks = 10000;
b = 0;
Input = 1;
tspan = 0:0.001:5;
Initial = [0; 0; 0; 0];
```

Try in LTI SS model

```
A = [-(b/m1) (b/m1) -(1/m1) (1/m1);
    (b/m2) -(b/m2) 0 -(1/m2);
    Kw 0 0 0;
    -Ks Ks 0 0];

B = [0;0;-Kw;0];
C = [1 0 0 0;
     0 1 0 0];
D = [0;0];
```

Simulating

```
simOut = sim('Car_Suspension_Model.slx');
output(1,:) = simOut.yout{1}.Values.Data(:,2)';
tSpan(1,:) = simOut.yout{1}.Values.Time';
output(2,:) = simOut.yout{2}.Values.Data(:,2)';
tSpan(2,:) = simOut.yout{2}.Values.Time';
Final = simOut.yout{3}.Values.Data(:,2)';
tfinal = simOut.yout{3}.Values.Time';
```

```
output(4,:) = simOut.yout{4}.Values.Data';
tSpan(4,:) = simOut.yout{4}.Values.Time';

figure('Position', [200 200 1000 800]);
hold on;
grid on;
plot(tfinal,Final)

b = 2000;
A = [-(b/m1) (b/m1) -(1/m1) (1/m1);
    (b/m2) -(b/m2) 0 -(1/m2);
    Kw 0 0 0;
    -Ks Ks 0 0];
simOut = sim('Car_Suspension_Model.slx');
Final = simOut.yout{3}.Values.Data(:,2)';
tfinal = simOut.yout{3}.Values.Time';
plot(tfinal,Final)

b = 4000;
A = [-(b/m1) (b/m1) -(1/m1) (1/m1);
    (b/m2) -(b/m2) 0 -(1/m2);
    Kw 0 0 0;
    -Ks Ks 0 0];
simOut = sim('Car_Suspension_Model.slx');
Final = simOut.yout{3}.Values.Data(:,2)';
tfinal = simOut.yout{3}.Values.Time';
plot(tfinal,Final)

b = 4000;
A = [-(b/m1) (b/m1) -(1/m1) (1/m1);
    (b/m2) -(b/m2) 0 -(1/m2);
    Kw 0 0 0;
    -Ks Ks 0 0];
simOut = sim('Car_Suspension_Model.slx');
Final = simOut.yout{3}.Values.Data(:,2)';
tfinal = simOut.yout{3}.Values.Time';
plot(tfinal,Final)

b = 6000;
A = [-(b/m1) (b/m1) -(1/m1) (1/m1);
    (b/m2) -(b/m2) 0 -(1/m2);
    Kw 0 0 0;
    -Ks Ks 0 0];
simOut = sim('Car_Suspension_Model.slx');
Final = simOut.yout{3}.Values.Data(:,2)';
tfinal = simOut.yout{3}.Values.Time';
plot(tfinal,Final)

b = 8000;
A = [-(b/m1) (b/m1) -(1/m1) (1/m1);
    (b/m2) -(b/m2) 0 -(1/m2);
    Kw 0 0 0;
    -Ks Ks 0 0];
simOut = sim('Car_Suspension_Model.slx');
```

```
Final = simOut.yout{3}.Values.Data(:,2)';
tfinal = simOut.yout{3}.Values.Time';
plot(tfinal,Final)

b = 10000;
A = [-(b/m1) (b/m1) -(1/m1) (1/m1);
    (b/m2) -(b/m2) 0 -(1/m2);
    Kw 0 0 0;
    -Ks Ks 0 0];
simOut = sim('Car_Suspension_Model.slx');
Final = simOut.yout{3}.Values.Data(:,2)';
tfinal = simOut.yout{3}.Values.Time';
plot(tfinal,Final)

b = 12000;
A = [-(b/m1) (b/m1) -(1/m1) (1/m1);
    (b/m2) -(b/m2) 0 -(1/m2);
    Kw 0 0 0;
    -Ks Ks 0 0];
simOut = sim('Car_Suspension_Model.slx');
Final = simOut.yout{3}.Values.Data(:,2)';
tfinal = simOut.yout{3}.Values.Time';
plot(tfinal,Final)

legend({'b = 0','b=2,000','b=4,000','b=6,000','b=8,000','b=10,000'...
    ,'b=12,000'},'Location','northeast')
title('Car Body Acceleration in Response to Step Input')
xlabel('Time [s]')
ylabel('m/s^2')
```

```
Warning: Solver is encountering difficulty in simulating model '<a
href="matlab:open_system ('Car_Suspension_Model')">Car_Suspension_Model</a>'
 at
time 1.0000000000000038. Simulink will continue to simulate with warnings.
Please check the model for errors.
Warning: Solver was unable to reduce the step size without violating minimum
step size of 3.55271E-15 for 1 consecutive times at time 1.  Solver will
continue simulation with the step size restricted to 3.55271E-15 and using an
effective relative error tolerance of 0.0135238, which is greater than the
specified relative error tolerance of 0.001. This usually may be caused by the
high stiffness of the system. Please check the system or increase the solver
 <a
href="matlab:configset.internal.open('Car_Suspension_Model','MaxConsecutiveMinStep');">Num
of consecutive min steps</a> violation parameter.
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 at
time 1.0000000000000038. Simulink will continue to simulate with warnings.
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```

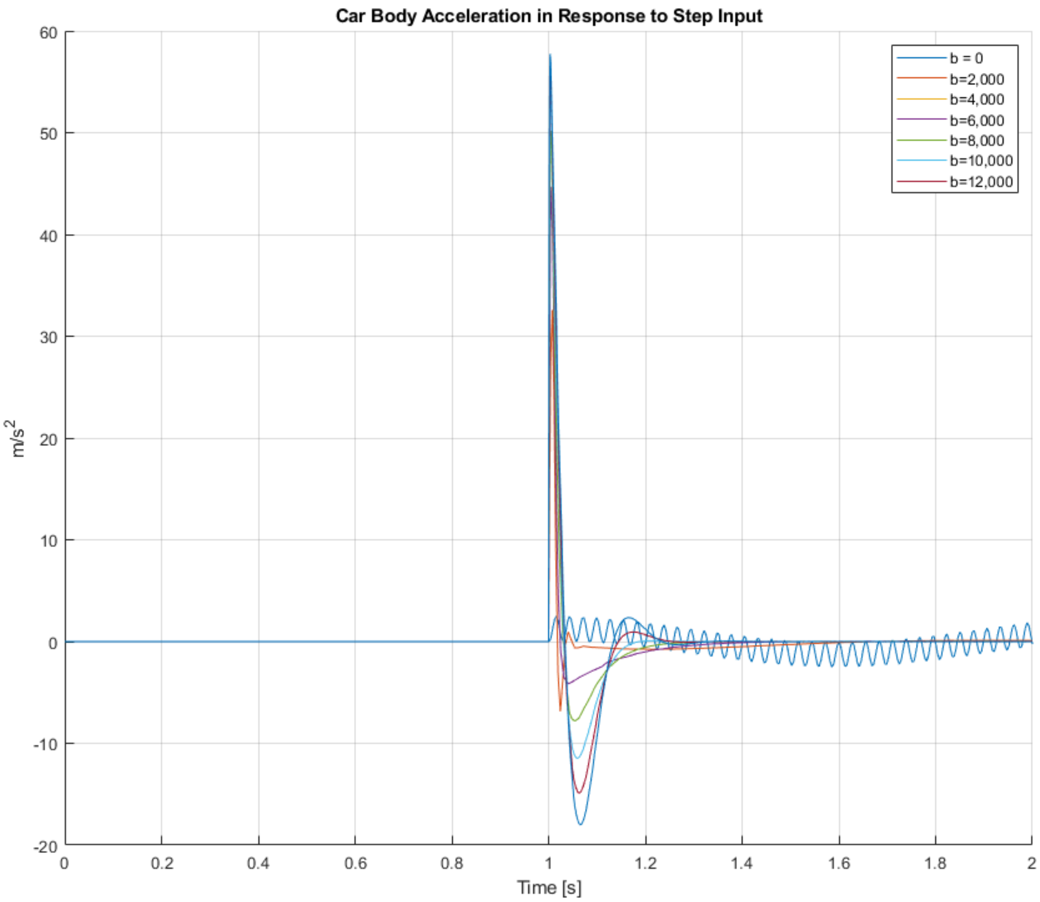

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