ASEN 5114: Experiment 4

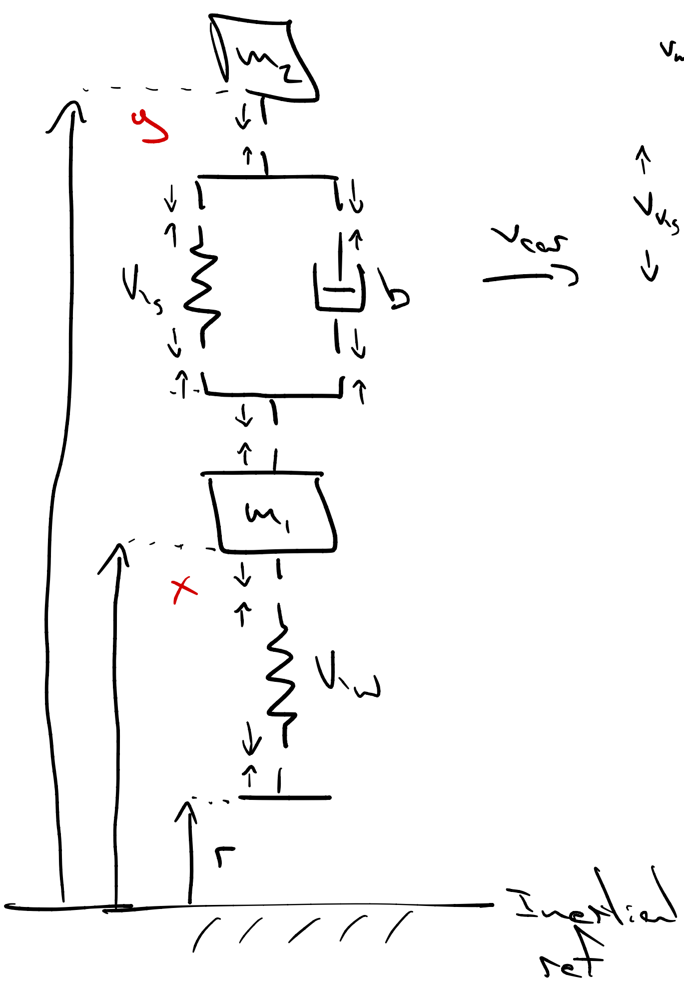
William
Watkins

2.5

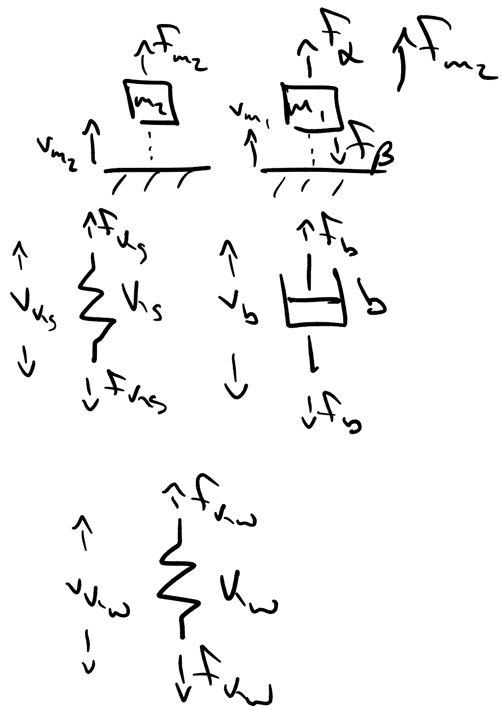
For suspension in Ex 2.2, plot position of car & wheel after a "unit bump" (i.e. r is a unit step).

Assume $m_1 = 10\,kg$, $m_2 = 350\,kg$, $k_w = 500{,}000\,N/m$, $k_s = 10{,}000\,N/m$

Find value of b you would prefer if you were a passenger



$\xrightarrow{\text{yields}}$



Topology

$f_\beta = f_{k_w}$

$f_\alpha = f_{k_s} + f_b$

$f_{m_2} = -f_{k_s} - f_b$

$f_{m_1} = f_\alpha - f_\beta$

$v_{k_w} = \dot{x} - \dot{r}$

$v_{m_1} = \dot{x}$

$v_{k_s} = v_B = \dot{y} - \dot{x}$

$v_{m_2} = \dot{y}$

Element

$\dot{v}_{m_2} = \frac{1}{m_2} f_{m_2}$

$\dot{v}_{m_1} = \frac{1}{m_1} f_{m_1}$

$\dot{f}_{k_w} = k_w v_{k_w}$

$\dot{f}_{k_s} = k_s v_{k_s}$

$f_b = B v_b$

Inputs

r

Outputs

$\dot{y}, \dot{x}$ OR $\ddot{y}, \ddot{x}$

$y_2 = \dot{y} = v_{m_2} = x_2$
$y_1 = \dot{x} = v_{m_1} = x_1$

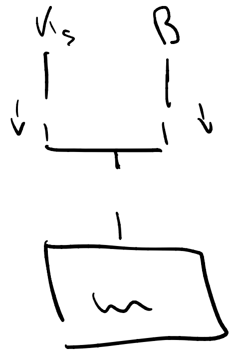
SSM Method

1)
- Inputs – r
- Outputs – y, x
- States – 4 energy storage elements, so 4 states
  - mass m_1 – $v_{m_1} = x_1$
  - mass m_2 – $v_{m_2} = x_2$
  - spring k_w – $f_{k_w} = x_3$
  - spring k_s – $f_{k_s} = x_4$

2)
$x_1^* = f_{m_1}$
$x_2^* = f_{m_2}$

$x_3^* = v_{k_w}$
$x_4^* = v_{k_s}$

3) $x_1^* = f(x_1, x_2, x_3, x_4) = F_{m_1}$



$F_{m_1} = F_\alpha - F_\beta$

$= (F_{V_s} + F_b) - (F_{V_w})$

$= -F_{V_w} + (F_{V_s} + F_b)$

$= F_{V_s} + F_b - F_{V_w}$

$= F_{V_s} + B v_b - F_{V_w}$

$= F_{V_s} + B[\dot{y} - \dot{x}] - F_{V_w}$

$= F_{V_s} + B[v_{m_2} - v_{m_1}] - F_{V_w}$

$x_1^* = F_{m_1} = x_4 + B[x_2 - x_1] - x_3$

$x_2^* = F_{m_2} = (-F_{V_s} - F_b)$

$= (-F_{V_s} - B v_b)$

$= (-F_{V_s} - B[\dot{y} - \dot{x}])$

$= (-F_{V_s} - B[v_{m_2} - v_{m_1}])$

$x_2^* = F_{m_2} = (-x_4 - B[x_2 - x_1])$

$x_3^* = v_{V_w} = \dot{x} - \dot{r}$

$= v_{m_1} - \dot{r}$

$x_3^* = v_{V_w} = x_1 - \dot{r}$

$x_4^* = v_{V_s} = \dot{y} - \dot{x}$

$= v_{m_2} - v_{m_1}$

$x_4^* = v_{V_s} = x_2 - x_1$

4) $\dot{x}_1 = \frac{1}{m_1} x_1^*$

$$\dot{x}_1 = \frac{1}{m_1}\left[x_3 - x_4 - \overbrace{B[x_2 - x_1])}^{pos}\right]$$

$$\dot{x}_2 = \frac{1}{m_2} x_2^*$$

$$\dot{x}_2 = \frac{1}{m_2}\left[(-x_4 - B[x_2 - x_1])\right]$$

$$\dot{x}_3 = K_w \, x_3^*$$

$$\dot{x}_3 = K_w[x_1 - \dot{r}]$$

$$\dot{x}_4 = K_s[x_2 - x_1]$$

$y_1 = \dot{x} = v_{m_1} = x_1$
$y_2 = \dot{y} = v_{m_2} = x_2$

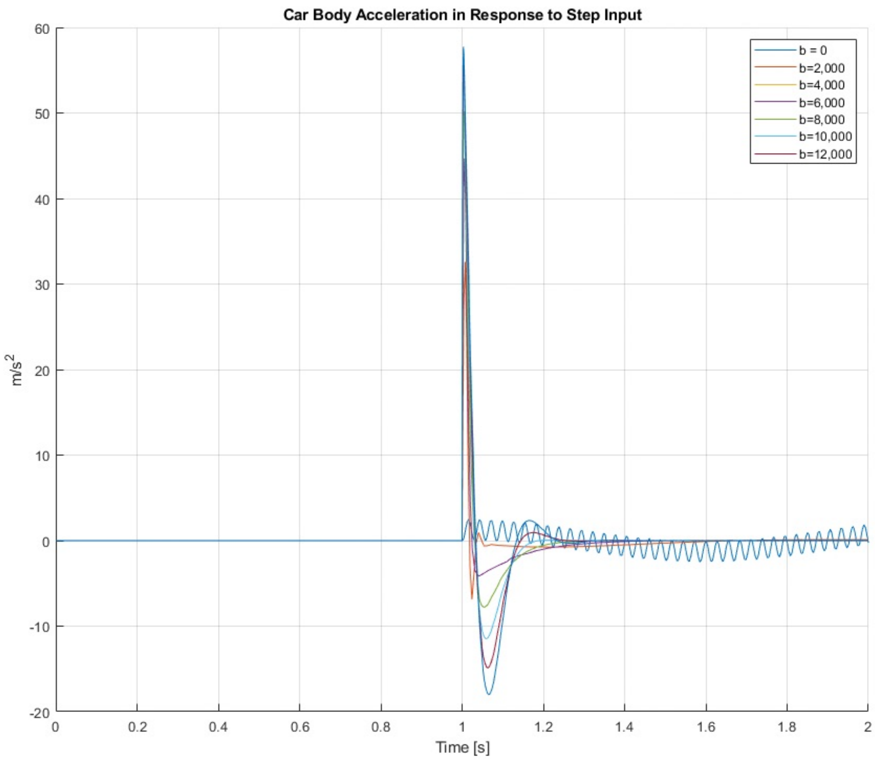
5)

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

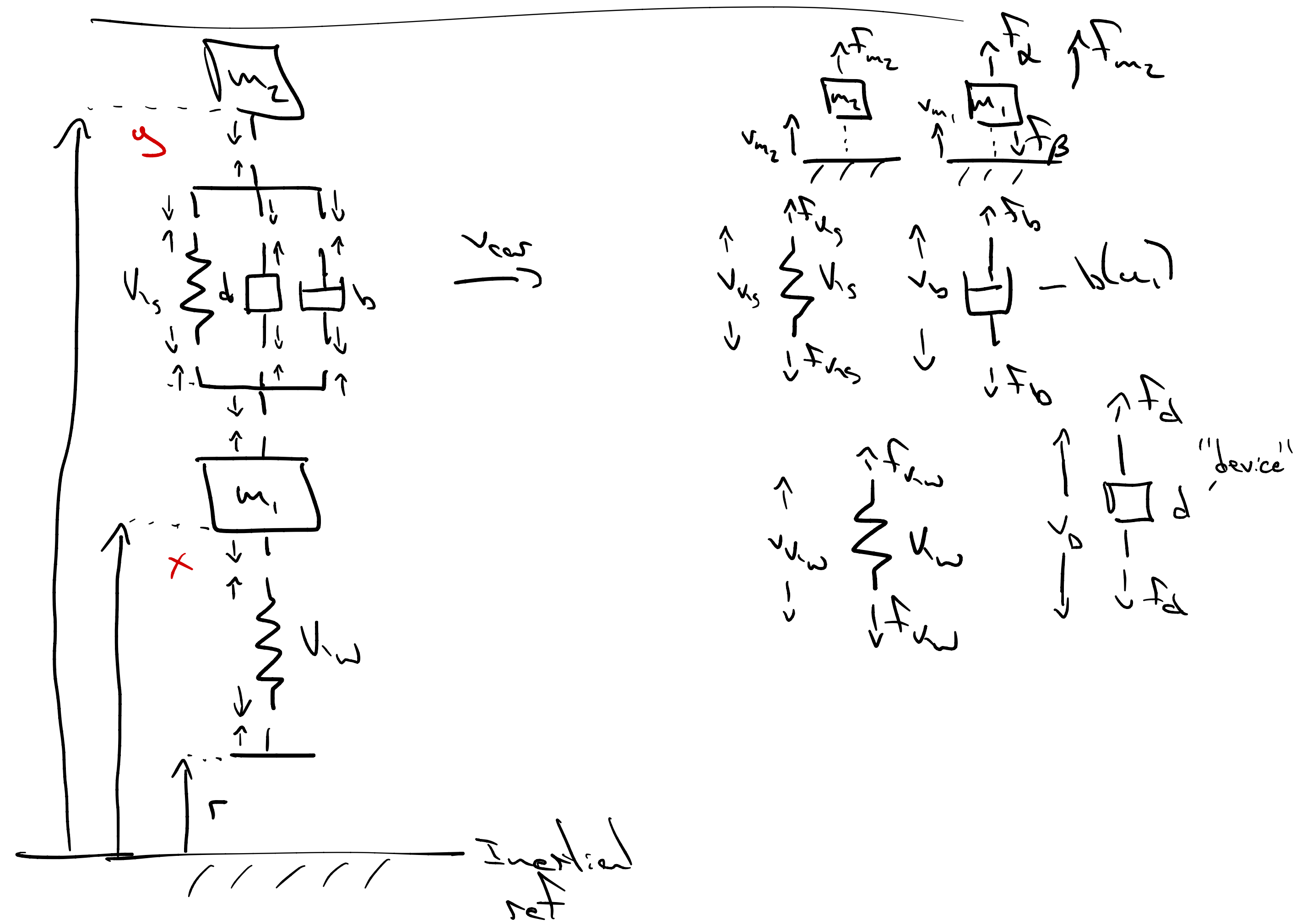
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -B/m_1 & B/m_1 & -1/m_1 & 1/m_1 \\ B/m_2 & -B/m_2 & \emptyset & -1/m_2 \\ K_w & \emptyset & \emptyset & \emptyset \\ -K_s & K_s & \emptyset & \emptyset \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \emptyset \\ \emptyset \\ -K_w \\ \emptyset \end{bmatrix} [\dot{r}]$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & \emptyset & \emptyset & \emptyset \\ \emptyset & 1 & \emptyset & \emptyset \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix} [\dot{r}]$$

The response to a unit step input can be seen below. The acceleration of y is plotted, as that is what humans define "comfort" as relating to. A comfortable damping coefficient is one that minimizes the acceleration & doesn't rapidly oscillate. So, a damping coefficient b between 6,000 & 8,000 would be most desireable.


Car Body Acceleration in Response to Step Input
m/s^2
Time [s]
b = 0
b=2,000
b=4,000
b=6,000
b=8,000
b=10,000
b=12,000
60
50
40
30
20
10
0
-10
-20
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

2)



Topology

$$f_\beta = f_{k_w}$$

$$f_\alpha = f_{k_s} + f_d + f_b$$

$$f_{m_2} = -f_{k_s} - f_b - f_d$$

$$f_{m_1} = f_\alpha - f_\beta$$

$$v_{k_w} = \dot{x} - \dot{r}$$

$$v_{m_1} = \dot{x}$$

$$v_{k_s} = v_D = \dot{y} - \dot{x} = v_b$$

$$v_{m_2} = \dot{y}$$

Element

$$\dot{v}_{m_2} = \frac{1}{m_2} f_{m_2}$$

$$\dot{v}_{m_1} = \frac{1}{m_1} f_{m_1}$$

$$\dot{f}_{k_w} = k_w \, v_{k_w}$$

$$\dot{f}_{k_s} = k_s \, v_{k_s}$$

$$f_b = b(u_1) \cdot v_b$$

$$f_d = u_2$$

Inputs

r

Outputs

y, x

$\cancel{y_2} = \dot{y} = v_{m_2} = x_2$
$y_1 = \dot{x} = v_{m_1} = x_1$

SSM Method

1)
- Inputs – r
- Outputs – y, x
- States – 4 energy storage elements, so 4 states
  - mass m_1 – $v_{m_1} = x_1$
  - mass m_2 – $v_{m_2} = x_2$
  - spring V_{iw} – $f_{V_{iw}} = x_3$
  - spring V_s – $f_{V_s} = x_4$

2)

$\dot{x}_1 = f_{m_1}$
$\dot{x}_2 = f_{m_2}$

$\dot{x}_3 = v_{V_{iw}}$
$\dot{x}_4 = v_{V_s}$

3) $\dot{x}_1 = g(x_1, x_2, x_3, x_4) = f_{m_1}$

$$f_{m_1} = f_\alpha - f_\beta$$
$$= (f_{V_s} + f_b + f_d) - f_{V_{iw}}$$
$$= f_{V_s} + b(u_1) v_b + f_d - f_{V_{iw}}$$
$$= f_{V_s} + b(u_1)[\dot{y} - \dot{x}] + f_d - f_{V_{iw}}$$
$$= f_{V_s} + b(u_1)[v_{m_2} - v_{m_1}] + f_d - f_{V_{iw}}$$

$$\dot{x}_1 = f_{m_1} = x_4 + b(u_1)[x_2 - x_1] + u_2 - x_3$$

$$x_2^* = F_{m_2} = (-F_{v_s} - F_b - F_d)$$

$$= -F_{v_s} - b(u_1) v_b - F_d$$

$$= -F_{v_s} - b(u_1)[\dot{y} - \dot{x}] - F_d$$

$$= -F_{v_s} - b(u_1)[v_{m_2} - v_{m_1}] - F_d$$

$$x_2^* = F_{m_2} = -x_4 - b(u_1)[x_2 - x_1] - u_2$$

$$x_3^* = v_{v_w} = \dot{x} - \dot{r}$$

$$= v_{m_1} - \dot{r}$$

$$x_3^* = v_{v_w} = x_1 - \dot{r}$$

$$x_4^* = v_{v_s} = \dot{y} - \dot{x}$$

$$= v_{m_2} - v_{m_1}$$

$$x_4^* = v_{v_s} = x_2 - x_1$$

4) $\dot{x}_1 = \frac{1}{m_1} x_1^*$

$$\dot{x}_1 = \frac{1}{m_1}[x_4 + b(u_1)[x_2 - x_1] + u_2 - x_3]$$

$$\dot{x}_2 = \frac{1}{m_2} x_2^*$$

$$\dot{x}_2 = \frac{1}{m_2}[-x_4 - b(u_1)[x_2 - x_1] - u_2]$$

$$\dot{x}_3 = k_w x_3^*$$

$$\dot{x}_3 = k_w[x_1 - \dot{r}]$$

$$\dot{x}_4 = k_s[x_2 - x_1]$$

$$y_1 = \dot{x} = v_{m_1} = x_1$$
$$y_2 = \dot{y} = v_{m_2} = x_2$$

5)

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

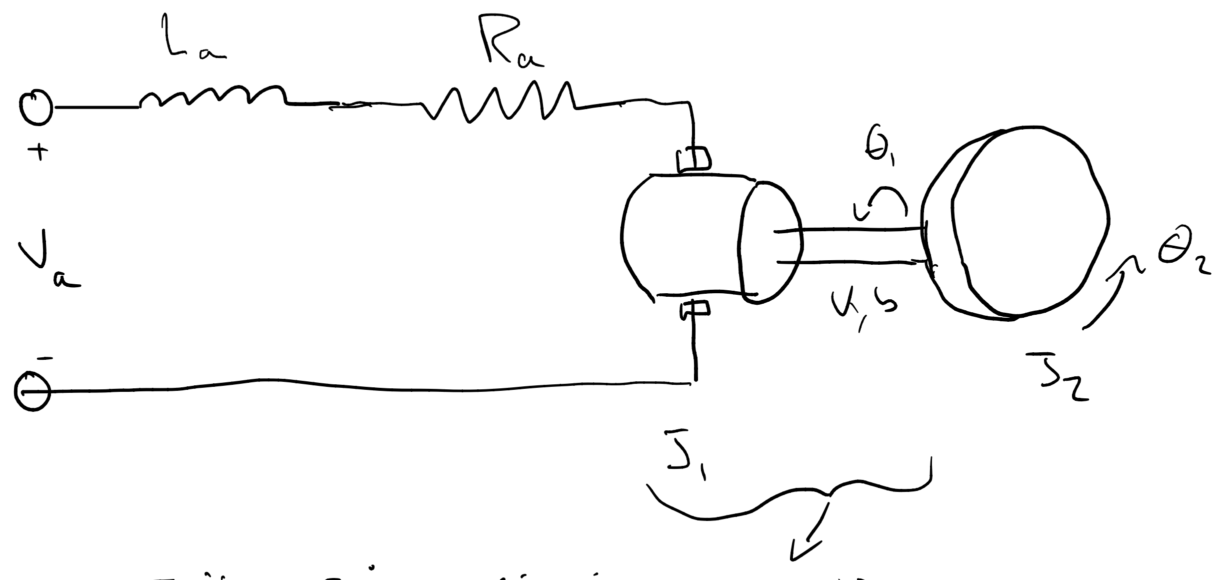
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -b(u_2)/m_1 & b(u_2)/m_1 & -1/m_1 & 1/m_1 \\ b(u_2)/m_2 & -b(u_2)/m_2 & 0 & -1/m_2 \\ V_{hw} & 0 & 0 & 0 \\ -V_{hs} & V_{hs} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 1/m_1 & 0 \\ 0 & 0 & 0 \\ -V_{hw} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{r} \\ u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [\dot{r}]$$

b) The resulting system is <u>not</u> linear. While the input u_2 does not make the system nonlinear, using a changable damping coefficient that is tied to the inputs of the system does make it non-linear.

c) Yes, replacing the springs and shock absorber with u_2 is possible. However, it would require very large forces to do so, and even with hardware available today that would be difficult to implement without some sort of delay between the input & the output.

3)



$$\tau_m = J_1\ddot{\theta}_1 + B\dot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + K_1(\theta_1 - \theta_2)$$

$$\tau_m = K_t I_m = J_1\ddot{\theta}_1 + B\dot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + K_1(\theta_1 - \theta_2)$$

SSM

Inputs — Output

V_a — θ_1

States:

$x_1 = \theta_1$

$x_2 = \theta_2$

$x_3 = I_L$

Really not sure how to do this. I know you can rewrite inductor as a spring & resistor as a damper, but how can you connect those to the motor?