1. Develop a rigid body model of the CU Spacecraft Mockup, relating reaction wheel command torque $[Nm]$ to body rotational velocity $[\frac{rad}{s}]$. Use the Spin Module virtual instument to produce a sinusoidal torque input and measure the sinusoidal angular velocity output. Select a frequency and amplitude to produce rigid body motion with reasonable signal to noise ratio, so the sinusoidal signal amplitudes can be observed.

The torque applied by the reaction wheel is related to the angular momentum of the spacecraft mockup by Eqn. 1.

$$\tau_w = J_s \dot{\omega}_s \tag{1}$$

In the time domain, this is represented in Fig. 1, whereas Fig. 2 shows this model in the frequency domain.

Figure 1: Rigid body model in the time domain.

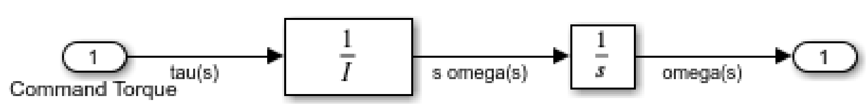

Figure 2: Rigid body model in the frequency domain.

These models give the following transfer function,

$$\begin{aligned} \frac{Y(s)}{U(s)} &= \frac{1}{sI_s} \\ &= H(j\omega) \\ &= \frac{B}{A} \end{aligned} \tag{2}$$

where $Y(s)$ is $\omega_s(s)$, $U(s)$ is $\tau(s)$, and A and B are the coefficients of the input and output signals, respectively, in the form of $u(t) = Asin(\omega t + \phi)$ and $y(t) = Bsin(\omega t + \theta)$. A and B can be found using the data from the Spin Module virtual instrument.

The Spin Module virtual instrument was configured to command a torque $\tau = 10mNm$ at a frequency of $\omega = 0.1Hz$. Because the rigid body model is in Nm, $A = 10E(-3)Nm$. Using the FFT performed by the virtual instrument, $B = 0.095\frac{rad}{s}$. With these coefficients, we find that the moment of inertia of the spacecraft mockup is $I_s = 0.1kg * m^2$.

2. Design a PD (unity feedback) control system to produce an overdamped step response with a 5% settling time of 3 seconds.

The PD unity feedback block model is shown below.

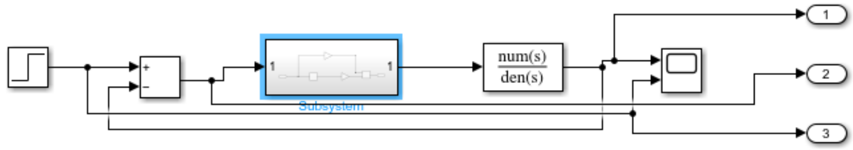

Figure 3: Rigid body model with PD unity feedback control.

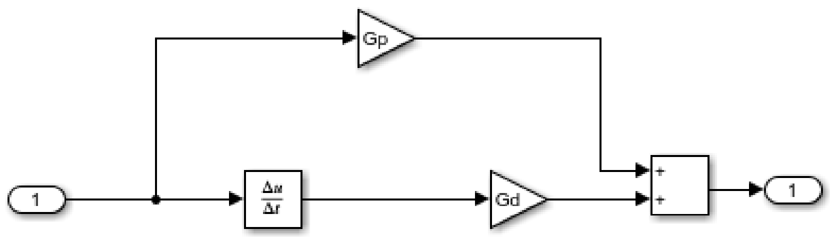

Figure 4: Rigid body model PD controller.

The associated transfer function is

$$\omega(s) = \frac{G_P}{s(G_D + I_s) + G_P}\tau_R(s) \tag{3}$$

By setting the denominator equal to 0, we obtain a pole at $s = \frac{-G_P}{G_D+I_s}$. From lecture 3, the 5% settling time is

$$t_{s,5} = \frac{-3}{Re(pole)} \tag{4}$$

where the pole has a negative real component. Rearranging, we find that the pole must be placed at -1. Selecting $G_P = 1$ arbitrarily, we get $G_D = 0.9$.

3. Simulate this plant and control system response to a 0.4 Nm amplitude step input, including the command to the reaction wheel. Verify that the step response objectives are achieved in simulation, and note the maximum torque command values.

The output of the simulation is below.

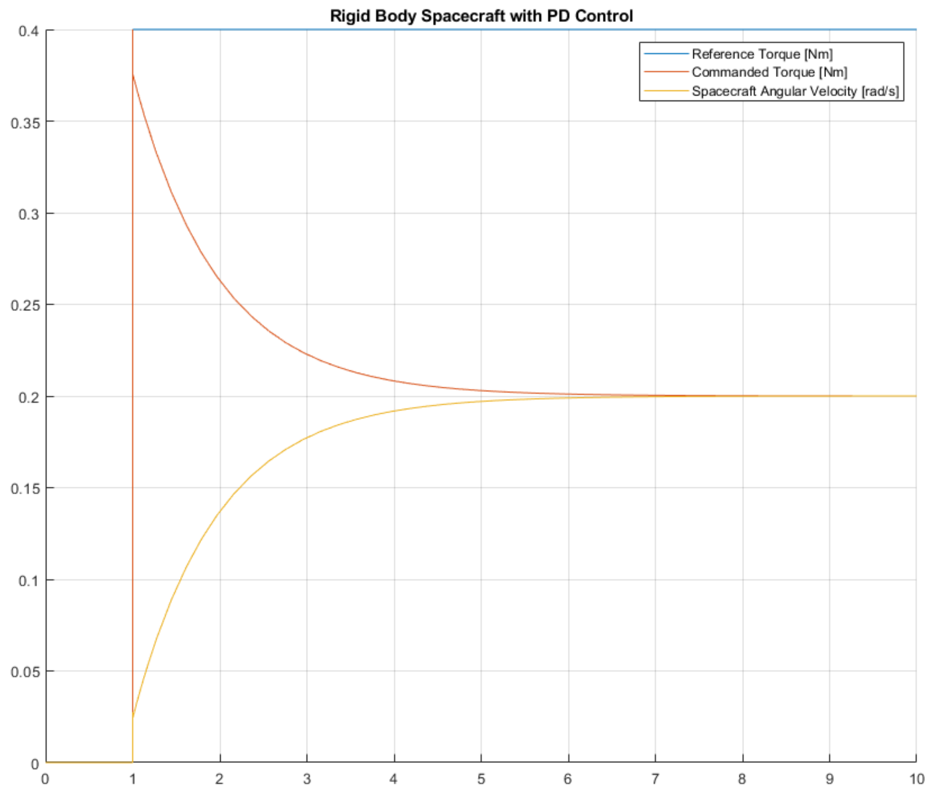

Figure 5: Rigid body model with PD control simulation.

Because the input to the system is in Nm and the output is in $\frac{rad}{s}$, the output will not perfectly match the input. In this case, the output reaches $\omega = 0.2[\frac{rad}{s}]$ and maintains that angular velocity until the simulation is stopped. However, it does reach 95% of that final value with 3 seconds!

MATLAB Code

```
%% Experiment 5
% William Watkins
% 17 February 2022

%% Clean up
clear all
close all
clc

%% Problem 2
I = 0.1;
Gd = 0.9;
Gp = 1;

%% Problem 3
simOut = sim('Rigid_Body_Spacecraft.slx');
output(1,:) = simOut.yout{1}.Values.Data';
tSpan(1,:) = simOut.yout{1}.Values.Time';
output(3,:) = simOut.yout{2}.Values.Data';
output(2,:) = simOut.yout{3}.Values.Data';

figure('Position', [200 200 1000 800]);
title('Rigid Body Spacecraft with PD Control')
hold on;
grid on;
plot(tSpan, output(2,:));
plot(tSpan, output(3,:));
plot(tSpan, output(1,:));
legend({'Reference Torque [Nm]','Commanded Torque [Nm]','Spacecraft Angular Velocity [rad/s
```