ASEN 5114	**Experiment 1**
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1. Find the parameters R_M, L_M, K_τ, and K_B from the motor specification sheet, noting units.

From the motor specification sheet:

$$\begin{aligned} R_M &= 19.2[\Omega] \\ L_M &= 1.9[mH] = 1.9x10^{-3}[H] \\ K_\tau &= 40.1[\frac{mNm}{A}] = 40.1x10^{-3}[\frac{Nm}{A}] \\ K_B &= 238[\frac{rpm}{V}] \end{aligned} \tag{1}$$

However, we defined K_B to be in units of $[\frac{V}{rad/s}]$, so K_B becomes:

$$K_B = \frac{1}{238} * \frac{60}{2\pi} = 40.1x10^{-3}[\frac{V}{rad/s}] \tag{2}$$

2. Simulate the system relating power amp voltage V_P to output angle θ_L using a single transfer function block in Simulink. Use a sinusoidal power amp input voltage (1V peak, 1Hz) and plot the corresponding input and output signals with appropriate units. does the output follow the input closely?

The transfer function relating input voltage V_P to output angle θ_L is

$$\begin{aligned} \theta_L(s) &= \frac{-1}{denominator} \\ &\text{where} \\ denominator &= s^3(\frac{J_{eq}L_M}{NK_\tau}) + s^2(\frac{J_{eq}R_M}{NK_\tau}) + s(NK_B) \end{aligned} \tag{3}$$

The Simulink model is shown in Fig. 1. The results are shown in Fig. 2. It's evident that θ_L does track well. The extrema of the output roughly coincide with the inflection points of the input signal, and the inflection points of the output roughly coincide with the extrema of the input. In the real world, this corresponds to the output reversing rotation direction when the polarity of the power amp voltage flips and the output beginning to slow down as the input voltage heads towards 0, respectively.

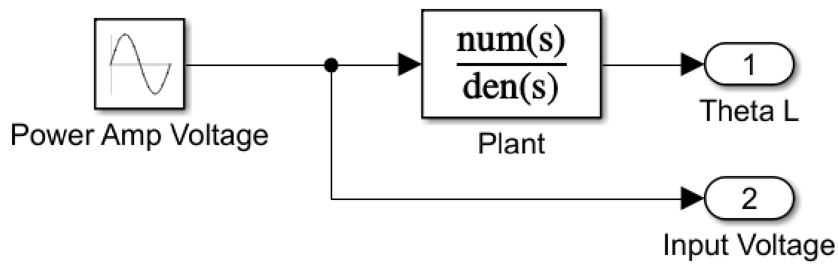

Figure 1: Open Loop System.

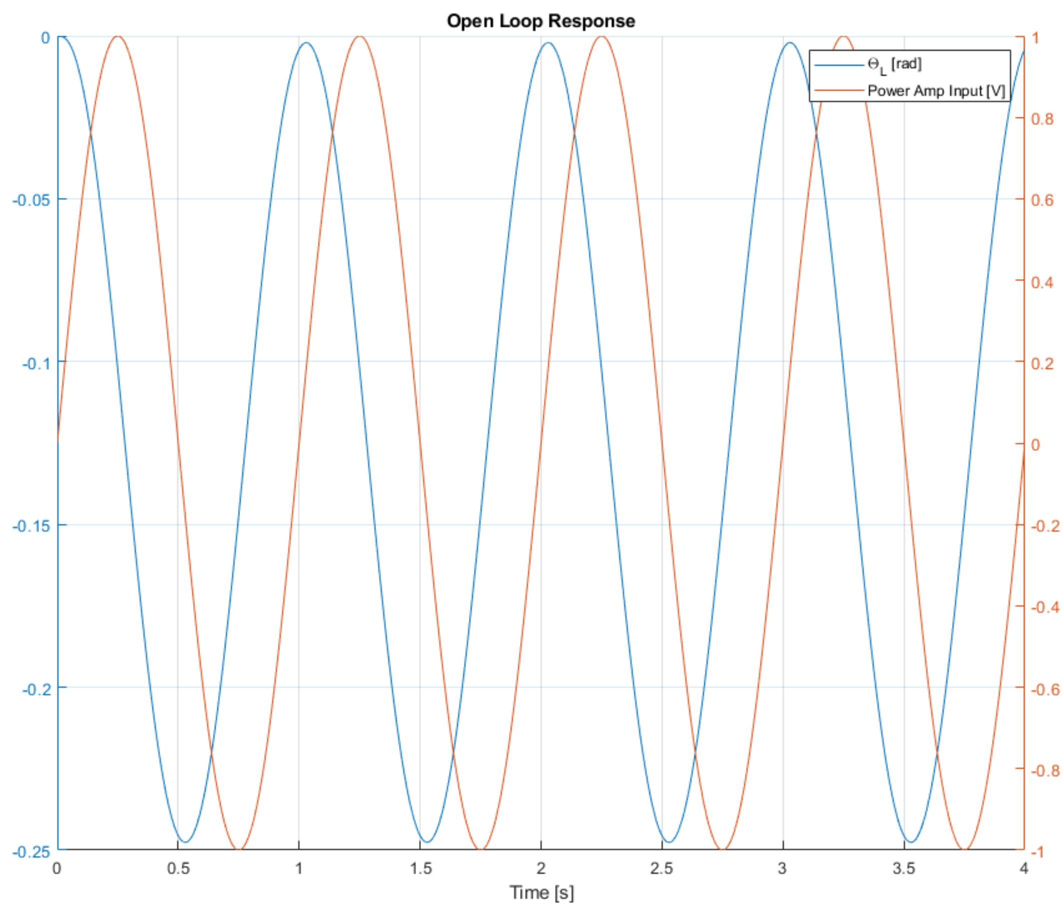

Figure 2: Open Loop System Behavior.

3. Derive the closed loop transfer function between reference angle θ_R and load shaft angle θ_L when the controller has a proportional-plus-derivative (PD) control law, i.e.

$$V_P = G_P(\theta_R - \theta_L) + G_D\frac{d}{dt}(\theta_R - \theta_L) \tag{4}$$

Determine the DC gain of the closed loop system.

First, we define

$$\theta_E = \theta_R - \theta_L \tag{5}$$

Then Eqn. 4 becomes

$$V_P = G_P\theta_E + G_D\frac{d}{dt}\theta_E \tag{6}$$

and performing a Laplace transform it becomes

$$V_P(s) = G_P\theta_E(s) + G_D s\theta_E(s) \tag{7}$$

Then, using the open loop transfer function relating θ_L and V_P in Eqn. 3, Eqn. **??** becomes

$$\theta_L(s) = -\frac{G_P\theta_E(s) + G_D s\theta_E(s)}{denominator} \tag{8}$$

Rearranging and using the definition of θ_E, the closed loop transfer function is

$$\begin{gathered}\theta_L(s) = -\frac{G_P + G_D s}{denominator}\theta_R(s) \\ \text{where} \\ denominator = s^3(\frac{J_{eq}L_M}{NK_\tau}) + s^2(\frac{J_{eq}R_M}{NK_\tau}) + s(NK_B - G_D) - G_P\end{gathered} \tag{9}$$

The DC gain is

$$\begin{aligned}\beta &= \lim_{s\to 0} -\frac{G_P + G_D s}{denominator} \\ \beta &= \frac{-G_P}{-G_P} = 1\end{aligned} \tag{10}$$

4. Construct a Simulink model of the control system including the PD control.

The Simulink model that represents the closed loop system is shown in Fig. 3, and the controller block is shown in Fig. 4.

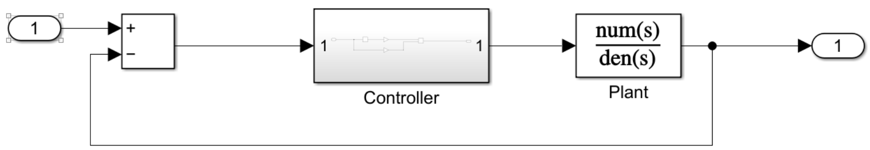

Figure 3: Closed Loop System.

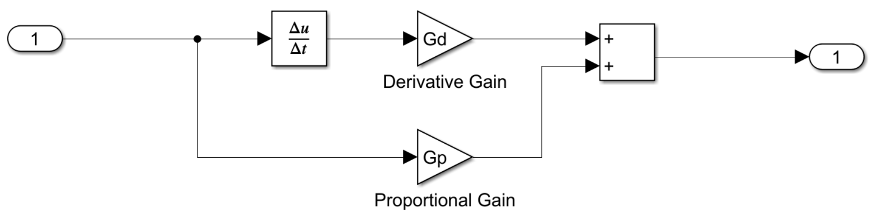

Figure 4: Closed Loop System Controller.

5. Plot the simulated step response (0.4 radian amplitude) of the closed loop system using a proportional gain of -10 $[V/rad]$ and a derivative gain of -0.1 $[\frac{V}{rad/s}]$. Compare the stead state tracking behavior to that predicted by the DC gain β.

The behavior of the system is plotted in Fig. 5. The final value of θ_L is equal to the value of the step input, which is in line with the DC gain $\beta = 1$.

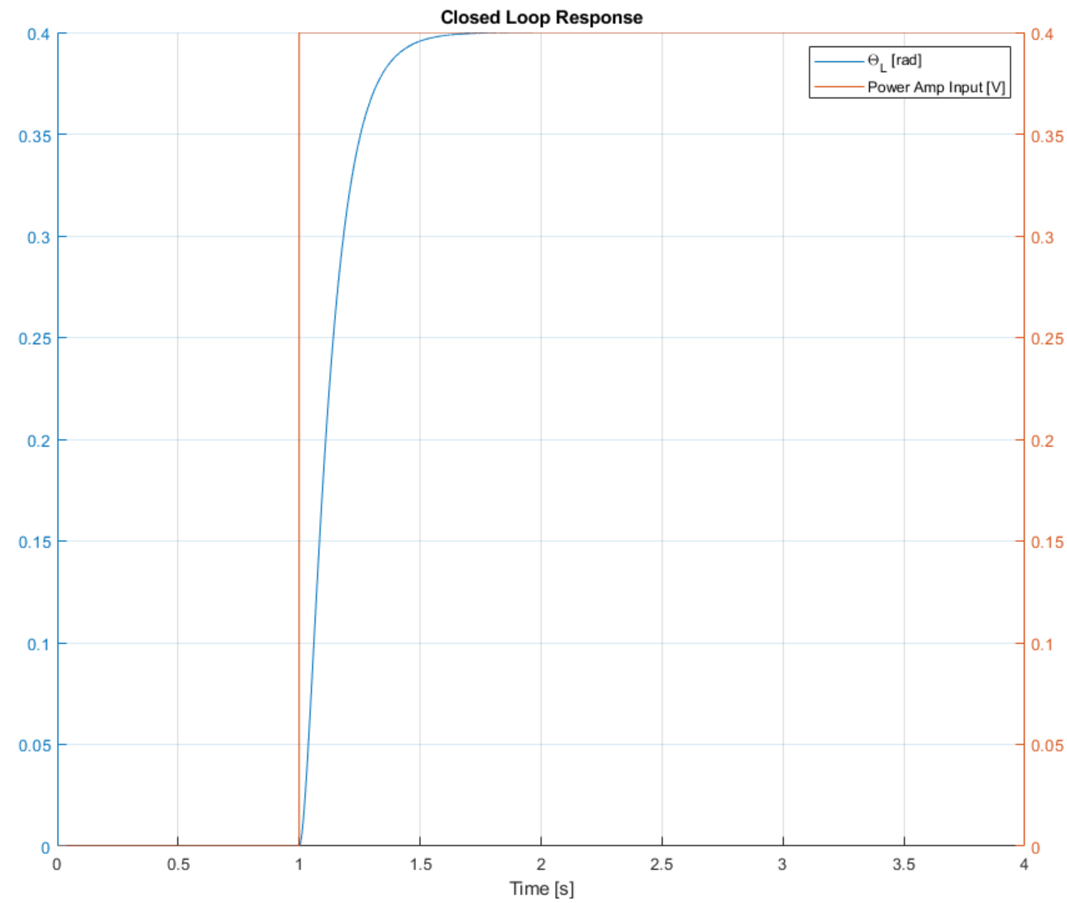

Figure 5: Closed Loop System Response to Step Input.

6. Repeat part 5, but using sinusoid reference signals at 0.4 rad amplitude, with frequencies of 0.2 Hz and 2 Hz. Comment on the relative tracking ability of the system at these two frequencies.

The results from the 0.2 Hz and 2 Hz input tests are graphed in Figs. 6 and 7, respectively. In the 0.2 Hz case, the output tracks the input well. The output comes within 0.87% of the amplitude of the input, and is only 0.13 rad out of phase with the input. Compare that with the 2 Hz case: the output does not track the 2 Hz input well at all. The output only reaches 43% of the input amplitude, and is 1.3 rad out of phase with the input - almost a quarter of a cycle!

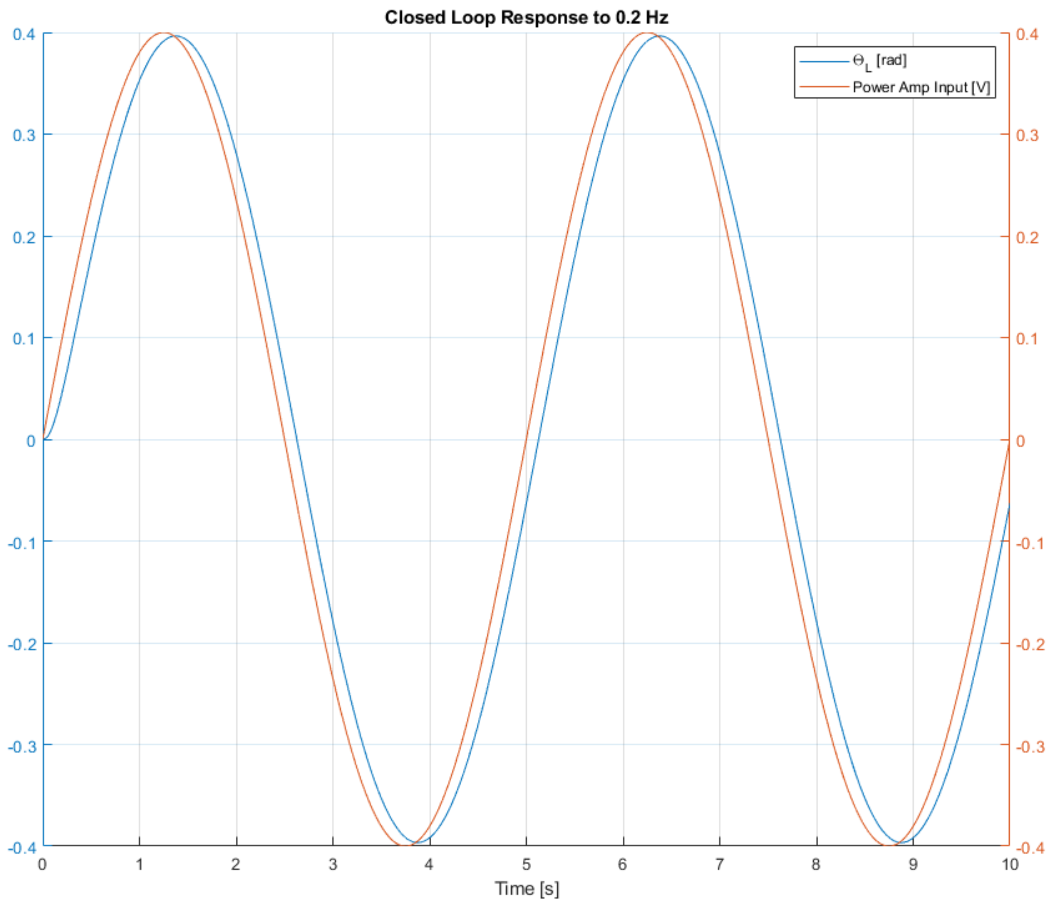

Figure 6: Closed Loop System Response to 0.2 Hz Input.

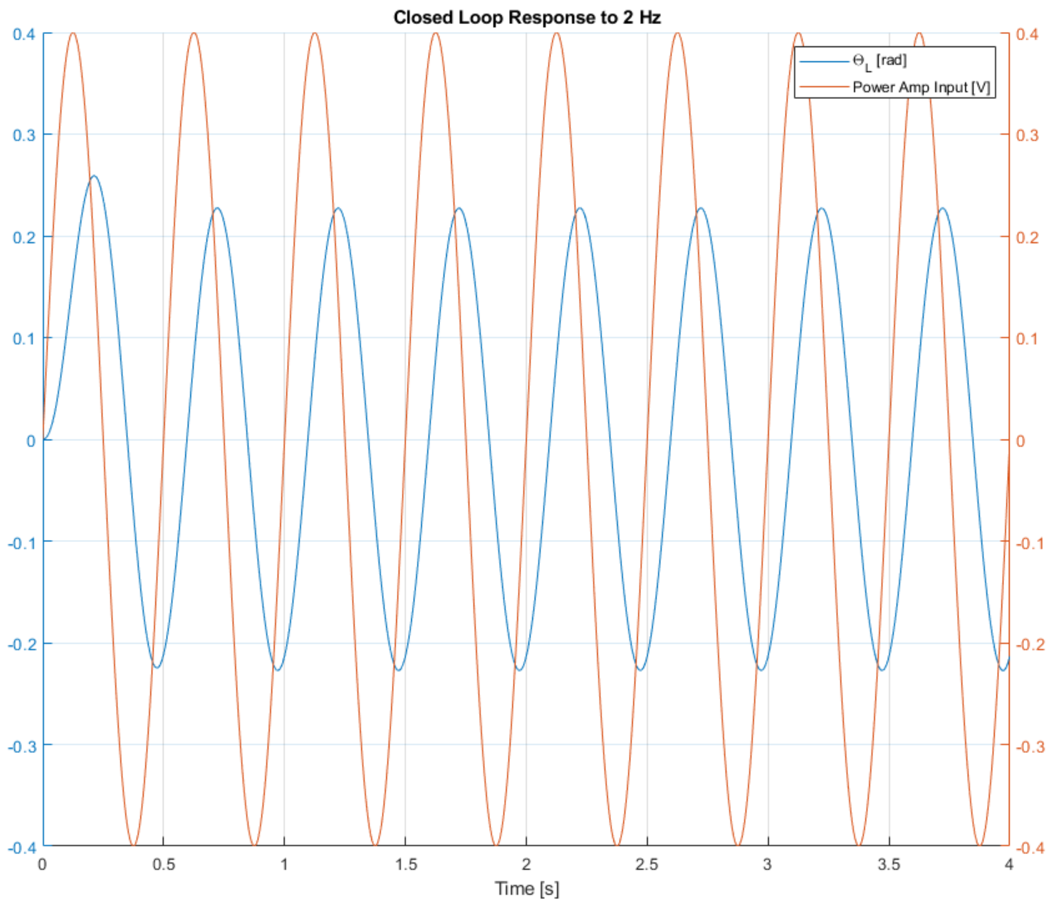

Figure 7: Closed Loop System Response to 2 Hz Input.

MATLAB Code

```
%% Experiment 1
% William Watkins
% 16 January 2022

%% Constants/Properties/Problem 1
Jconv = 1/0.00034171718982094; %[g*cm^2 / lb*in^2]
Jc = 1*10^-7; % [kg*m^2 / g*cm^2]
Rm = 19.2;       % [Ohms]
Lm = 0.0019;     % [Henrys]
Ktau = 40.1*10^-3;   % [Nm/A]
Kb = 1/238 / (2*pi/60); % [V/(rad/s)]
rBig = 2.51; % [in]
rSmall = 0.79; % [in]
Ngearhead = 10;
Ngears = rBig/rSmall;
Jm = 12.5 * Jc; % [kg*m^2]
Jgearhead = 0.6 * Jc; % [kg*m^2]
N = Ngearhead * Ngears;

%% Problem 2
Jtm = Jm; % [kg*m^2]
rhoAl = 0.097; % [lb/in^2]
h = 0.199; % [in]
wBeam = 0.504; % [in]
dBeam = 10; % [in]
sSquare = 1.75; % [in]
Vsq = sSquare^2 * h; % [in^3]
Vbeam = h * wBeam * dBeam; % [in^3]
mSquare = Vsq * rhoAl; % [lb]
mBeam = Vbeam * rhoAl; % [lb]
Jbeam = (1/12) * mBeam * (dBeam^2 + wBeam^2); % [lb*in^2]
Jsquare = (1/12) * mSquare * 2 * sSquare^2; % [lb*in^2]
z = dBeam/2 + sSquare/2; % [in]
JPA = Jbeam + mBeam*z^2; % [lb*in^2]
Jl = JPA + Jsquare; % [lb*in^2]
Jl = Jl * Jconv * Jc; % [kg*m^2]
Jeq = Jl + N^2 * Jtm; % [kg*m^2]
s3 = (Jeq/(N*Ktau))*Lm;
s2 = (Jeq/(N*Ktau))*Rm;
s1 = N*Kb;

simOut1 = sim('Experiment_1_Model.slx');
thetaL1(1,:) = simOut1.yout{1}.Values.Data';
tSpan1(1,:) = simOut1.yout{1}.Values.Time';
thetaL1(2,:) = simOut1.yout{2}.Values.Data';
tSpan1(2,:) = simOut1.yout{2}.Values.Time';
figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
```

```
plot(tSpan1(1,:),thetaL1(1,:))
yyaxis right
plot(tSpan1(2,:),thetaL1(2,:))
legend({'\Theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Open Loop Response')
xlabel('Time [s]')

%% Question 4/5
Gp = -10; % [V/rad]
Gd = -0.1; % [V/(rad/s)]

simOut5 = sim('PD_Control_Step.slx');
thetaL5(1,:) = simOut5.yout{1}.Values.Data';
tSpan5(1,:) = simOut5.yout{1}.Values.Time';
thetaL5(2,:) = simOut5.yout{2}.Values.Data';
tSpan5(2,:) = simOut5.yout{2}.Values.Time';
figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
plot(tSpan5(1,:),thetaL5(1,:))
yyaxis right
plot(tSpan5(2,:),thetaL5(2,:))
legend({'\Theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Closed Loop Response')
xlabel('Time [s]')

%% Question 6
simOut6a = sim('PD_Control_Sine02.slx');
thetaL6a(1,:) = simOut6a.yout{1}.Values.Data';
tSpan6a(1,:) = simOut6a.yout{1}.Values.Time';
thetaL6a(2,:) = simOut6a.yout{2}.Values.Data';
tSpan6a(2,:) = simOut6a.yout{2}.Values.Time';
figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
plot(tSpan6a(1,:),thetaL6a(1,:))
yyaxis right
plot(tSpan6a(2,:),thetaL6a(2,:))
legend({'\Theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Closed Loop Response to 0.2 Hz')
xlabel('Time [s]')

simOut6b = sim('PD_Control_Sine2.slx');
thetaL6b(1,:) = simOut6b.yout{1}.Values.Data';
tSpan6b(1,:) = simOut6b.yout{1}.Values.Time';
thetaL6b(2,:) = simOut6b.yout{2}.Values.Data';
tSpan6b(2,:) = simOut6b.yout{2}.Values.Time';
figure('Position', [200 200 1000 800]);
hold on;
grid on;
yyaxis left
plot(tSpan6b(1,:),thetaL6b(1,:))
```

```
ylim([-0.4 0.4])
yyaxis right
plot(tSpan6b(2,:),thetaL6b(2,:))
ylim([-0.4 0.4])
legend({'\Theta_L [rad]','Power Amp Input [V]'},'Location','northeast')
title('Closed Loop Response to 2 Hz')
xlabel('Time [s]')
```