CSCI 150 Discrete Mathematics
Test I example

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(1 point) Draw the cutest cat you can make.

(1 point) Sign your drawing by **writing** your Name:_____

Problem 1: As simple as you can make it please

(a) **(1 point)**: What is the 417^{th} term in the following sum:

$$\sum_{i=0}^{2018}(3i+2)$$

(b) **(1 point)** Find

$$\prod_{i=1}^{0}(1+\frac{1}{i})$$

(c) **(1 point)** Is it possible to construct a graph with 5 nodes and the following set of degrees $\{1,2,3,4,3\}$? If Yes, give one. If No, briefly explain why.

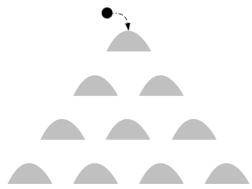
(d) **(1 point)** What is the coefficient of z^3y^4 in the following product? *Hint:* Replace $1+z$ by x first and use a well-known theorem that we studied.

$$\underbrace{(1+z+y)(1+z+y)\dots(1+z+y)}_{10 \text{ times}}$$

(e) **(1 point)** Amy has 127 books (she reads a lot!). In how many ways can she stack them?

Problem 2: A bumpy way

For a given n, we make a structure of bumps by placing $2n$ rows of bumps as shown in the figure below for $n = 2$ ($2n = 4$ and thus 4 rows).



(a) **(2 points)** In terms of n, how many bumps are there in total? (your answer should be general for any n and in the simplest possible form you can make)

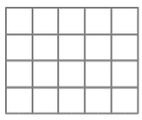
When a ball is dropped as shown above, it moves either Left or Right upon hitting a bump. Let M be the set of paths that bring the ball on its way down to the exact **middle** of the last row, which is possible since the last row has $2n$ bumps (an even number).

(b) **(2 points)** Establish a one-to-one correspondence (bijection) between the set M and a special set of binary strings that you must **clearly define**.

(c) **(2 points)** Find $|M|$.

(d) **(2 points)** How many paths don't bring the ball to the middle of the last row of bumps?

Problem 3: Proof by contradiction
(3 points) Prove by contradiction that a 4×5 rectangle cannot be covered by quatrominos: *Hint*: Use the coloring idea from the mouse trap example discussed in class, then identify two types of quatrominos based on color.



a quatromino

Problem 4: Counting stuff...

(a) **(3 points)** In a store, you find 3 kinds of postcards. You have 8 friends and you decide to send each a postcard. In how many ways can you do that?

(b) **(3 points)** In how many ways can we make a 3-letter word (not necessarily found in the dictionary) if the 3 letters must appear in alphabetical order, and the 3-letters are either all the same or all different?

(c) **(3 points)** A kid has 10 markers of different colors. Each marker has a cap of the same color, but he lost all the caps. His mother found only 7 of them, and asked him to close the markers to prevent them from drying. In how many ways can he do that? (Note: it's not my son!)

(d) **(3 points)** We would like to build a tower by stacking 10 cubes. Cubes come in 3 colors: red, green, and blue. The only requirement is that from base to top, we have to see all three colors in that order: red, green, blue. How many possible towers can we make? *Hint*: Think about an equation involving 3 variables.