

Author's response: "Interval enforceable properties of finite groups"

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I would first like to thank the referee for many helpful comments and constructive criticisms. I believe the paper has been significantly improved by following most of the referee's suggestions.

Next I would like to address the referee's point that the paper would be improved if we had an example of a collection $\{L_1, \dots, L_n\}$ of lattices for which we believed:

- there is no finite group G having subgroups H_i with $\llbracket H_i, G \rrbracket \cong L_i$, and
- proving there is no such G will be easier than carrying out the programs described in other research.

I added some paragraphs to the introduction in an attempt to make it clearer that the paper is as much about cf-IE properties as it is about FLRP. Furthermore, rather than presenting a specific example of a collection of lattices, the paper presents a strategy which we hope will make it easier to produce a collection that makes solving the problem trivial (because of the parachute construction). That is, we wish to identify lattices $\{L_i\}$ that force a group to have a collection $\{P_i\}$ of properties that are (collectively) mutually exclusive.

Perhaps there are certain collections of group properties for which it is non-trivial to prove mutual exclusivity, but for the group properties I know of, and for all those that have been proved interval enforceable, mutual exclusivity is easy to check.

One goal of the paper is to motivate the study of such properties. Perhaps we will eventually be able to prove that, for every collection $\{P_i\}$ of cf-IE properties, there is a finite group having all of these properties.

Enumerated List of Suggestions. Below are the author's comments explaining how each point in the referee's enumerated list of suggestions has been addressed. (The page numbers refer to the page numbers of the original manuscript, as used in the referee report.)

- (1) The first two paragraphs of historical introduction have been shortened significantly, but not deleted entirely because the paper is not only about the FLRP. It is also about the study of group properties linked to subgroup lattice structure, and this topic has a long history which I believe should be at least mentioned.
- (2) The suggested references are now included.
- (3) The definition of interval enforceable has been made clearer.

- (4) Lines 14–16 on page 4, along with the whole paragraph to which they belonged, have been deleted.
- (5) It should now be clearer that the fraktur symbol is being defined for the first time on this line.
- (6) On line 50 of page 6, “the” has been inserted between “that” and “following.”
- (7) In footnote 7 on page 8, “discuss” has been changed to “discussed.”
- (8) The “tricky to follow” argument is, in fact, a little bit tricky on the first reading, but I am confident readers can follow it, perhaps with the assistance of a pencil. I believe I have presented it in the simplest possible way, and I don’t think it is too hard to follow, as long as one has a good picture of what is going on. Figure 2 is suggestive of the picture one should have in mind.
- (9) The expression unu^{-1} has been replaced with uwu^{-1} .
- (10) I first learned about the notation $\llbracket U_0, U \rrbracket_H$ from Borner’s paper, and my intention was to conform with what appears elsewhere in the literature. I don’t feel this change is crucial, and I would prefer to leave it as is. (In addition to my desire to conform with Borner’s use, I’d rather not have to change this in all of my notes, other papers, and slides about this topic.)
- (11) The notation \leq is standard. Given two algebras \mathbf{A} and \mathbf{B} , be they lattices, groups, or what have you, the notation $\mathbf{A} \leq \mathbf{B}$ means \mathbf{A} is a subalgebra of \mathbf{B} . Nonetheless, the referee’s suggestion to make this explicit has been heeded in the remarks following Lemma 3.5.
- (12) This has been fixed. It was actually a mistake in the argument, and I am grateful that the referee identified it. The point is that, in practice, we are never trying to identify a core-free interval enforceable property that is enforced by a one-element or two-element lattice. The new hypotheses of the theorem rule out the cases that were causing problems. Again, these are not the interesting cases. These hypotheses make the theorem a bit more cumbersome to state. On the other hand, after the change, it occurred to me that there is really only one equivalent statement here, not three as I had previously, so in that sense the theorem is more refined and less clumsy.
- (13) The two corollaries that now appear immediately after Dedekind’s Theorem provide the essential fact about modular elements.
- (14) Footnote 13 on page 19 has been deleted.

Other Changes. Some (not all) of the other changes I’ve made to the paper are as follows:

- (1) The paragraph beginning on line 34 page 2 was deleted.
- (2) Section 4, the last six pages of the paper, has been deleted. This was the section in which the (also now deleted) Lemma 3.5 had been used to demonstrate how effective this lemma could be for simplifying proofs of interval enforceability. It seemed the referee did not feel this added very much to the exposition, and I don't completely disagree. That section might be better suited to a different paper.