Referee's report on the paper "Interval enforceable properties of finite groups".

The paper at hand includes some new results on an interesting problem. However, the paper can and should be shortened considerably by streamlining some proofs and removing repetitive and somewhat irrelevant material from the exposition. The author should think carefully about what is essential in the paper and remove everything that is not.

A longstanding open question in finite group theory is "Is every finite lattice is isomorphic to an interval in the subgroup lattice of a finite group?". This question is of interest in universal algebra, as P. P. Pálfy and P.Pudlák showed that it has the same answer as the question "Is every finite lattice isomorphic to a congruence lattice of a finite algebra?". The consensus among group theorists is that the answer to these questions is "No". Among other things, the paper at hand contains the result that if the answer to these questions is "Yes", then, for every fiinite collection $\{L_1, \ldots, L_n\}$ of finite lattices, there is a finite group G and a collection $\{H_1, \ldots, H_n\}$ of core-free subgroups of G such that each L_i is isomorphic with the interval $[H_i, G]$ in the subgroup lattice of G. This is a nice observation. The paper would be improved if the author could give an example of such a collection of lattices for which he believes that

- there is no such finite group G, and
- proving there is no such G will be easier than carrying out any of the apparently difficult programs laid out in work (in various combinations) of M. Aschbacher, R. Baddeley, A. Lucchini and J. Shareshian.

The paper contains other results along the same line - namely, a list of questions having the same answer as the lattice representation questions described above. Again, while these results are nice, they would be nicer if they came along with a plan for the resolution of the original problem.

As noted above, I think that the exposition in the paper should be changed. After reading several of the proofs and convincing myself that they could be shortened and clarified considerably using the theory of modular elements in lattices, I arrived at the author's remarks on page 15, where he asserts that in fact results more general than some of his have already been proved using this theory. (As one of the references he gives when making this assertion is 100 pages long and the other is not easily available, perhaps he could have provided more details at this point.) The author explains why he chose to write the proofs the way he did, but I was unconvinced that an approach using modular elements would not be more efficient and clear. Modular elements in subgroup lattices are discussed at length in the book of R. Schmidt and are likely to be well understood by interested readers of this paper.

Along the same lines, while Lemma 3.5 is a nice observation, its only use in the paper occurs during the proof of Claim 4.2. This claim has an easy proof not requiring the lemma: Say N is a nontrivial normal subgroup of G contained in J_i. Then $KN = G$, as K is maximal and core-free. Now $KJ_i = KM_2 = G$ and $K \cap J_i = K \cap M_2 = H$, which is impossible as $J_i < M_2$.

Below are some minor comments about the exposition.

(1) I don't think the first two paragraphs of the introduction are really germane. The paper deals mostly with the lattice representation problem and closely related matters. I would remove these paragraphs or shorten them considerably. Let me point out that, if I understand the results in reference [30] correctly, the author has slightly misstated them, as one of the chains involved need not be maximal. However, the discussion of that reference is an excellent candidate for removal.
(2) Maybe the long list of references on line 19 of page 2 should include [3] and [23].
(3) I found the initial definition of interval enforceable on lines 8-9 of page 3 somewhat ambiguous. I could not tell if the class is meant to consist of *all* groups with property P.
(4) Lines 14-16 on page 4 are repetitive.
(5) The fraktur symbol appearing on line 12 of page 6 seems not to have been defined earlier.
(6) On line 50 of page 6, "the" should be inserted between "that" and "following".
(7) In footnote 7 on page 8, "discuss" should be "discussed".
(8) I found the paragraph starting on page 8 and ending on page 9 a little tricky to follow. Maybe this was due in part to the use of symbols in formulas that appear before these symbols are defined.
(9) I believe that in the formula on lines45-46 of page 9, unu^{-1} should be uwu^{-1}.
(10) I believe that the object defined as $[U_0, U]_H$ in (3.6) is usually called $[U_0, U]^H$. The second of these two notations was used for something else in (3.5). It might be a good idea to exchange these notations, as long as care can be taken to do it throughout the paper.

(11) I don't think the notation "$\leq$" for "is a sublattice of" is defined until after it is used in Lemma 3.5. I don't know that this is standard notation. When used in a paper involving subgroups, it is disturbing to the reader without a clear definition appearing before the first use.

(12) It is not clear to what "the theorem" on lines 43-44 of page 13 is referring.

(13) Should the author not accept my recommendation to use modular elements, it might be useful to state and prove a lemma saying that if N is a normal subgroup of G then the set of lattice theoretic complements to HN in $[H, G]$ is an antichain. This is pretty much the essential fact about modular elements that is used repeatedly in arguments of the type made in this paper and others.

(14) The content of footnote 13 on page 19 is likely to be well known to a reader of this paper.