MATH 160 **Fall 2015**

EXAMINATION 2

RULES

- No books or notes or calculators allowed.
- No bathroom breaks until after you have completed and turned in your exam.
- Out of consideration for your classmates, do not make disturbing noises during the exam.
- **Phones must be turned off.**

Cheating will not be tolerated. If there are any indications that a student gave or received unauthorized aid on this test, the case will be referred to the ISU Office of Judicial Affairs.

When you finish the exam, please sign the following statement acknowledging that you understand this policy:

"On my honor as a student I, Solution Key (Print Name), have neither given nor received aid on this exam."

Signature: _____ Date: 2015/10/27

Part 1. Complete the table by blacking out letters corresponding to correct answers.

problem	answer choice				
1.	(a)	(b)	(c)	(d)	●
2.	(a)	(b)	(c)	(d)	●
3.	●	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	●
5.	(a)	(b)	(c)	●	(e)

1. If $g(x) = 2x^3 + 1$, then $g'(-2) =$

   (a) -12 (b) -11 (c) 0 (d) 12 (e) 24

2. If $y = x^{0.2}$, then $y' =$

   (a) $\ln(0.2)\, x^{0.2}$ (b) $x^{-0.8}$ (c) $0.2\, x^{-1.2}$ (d) $0.2 \ln x$ (e) $0.2\, x^{-0.8}$

3. If $f(x) = \frac{x^3}{x+1}$, then $f'(x) =$

   (a) $\frac{3x^2(x+1)-x^3}{(x+1)^2}$ (b) $\frac{x^3-3x^2(x+1)}{(x+1)^2}$ (c) $3x^2(x+1) + x^3$ (d) $3x^2 \ln(x+1)$

4. If $f(x) = \frac{1}{\sqrt[3]{x}}$, then $f'(x) =$

   (a) $-\frac{1}{3}x^{-2/3}$ (b) $\frac{2}{3}x^{-2/3}$ (c) $\frac{1}{3}x^{2/3}$ (d) $-3x^{-4}$ (e) $-\frac{1}{3}x^{-4/3}$

5. If $f(x) = \frac{1}{\sqrt[3]{x}}$, then $f''(1) =$

   (a) $-\frac{4}{9}$ (b) $-\frac{2}{9}$ (c) $\frac{2}{9}$ (d) $\frac{4}{9}$ (e) 12

- scratch -

1. $g'(x) = 6x^2 \qquad g'(-2) = 6\cdot(-2)^2 = \underline{\underline{24}}$

2. $y' = 0.2\,x^{0.2-1} = 0.2\,x^{-0.8}$

3. $f'(x) = \dfrac{3x^2(x+1) - x^3\cdot 1}{(x+1)^2}$

4. $f(x) = x^{-1/3} \qquad f'(x) = -\frac{1}{3}x^{-4/3}$

5. $f''(x) = \left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)x^{-7/3} = \frac{4}{9}x^{-7/3}$

$\therefore f''(1) = \frac{4}{9}$.

Part 2. Circle the best answer choices. If there is more than one correct answer choice, then **circle all correct choices.**

7. The derivative of $f(x)$ at $x = a$ is denoted $f'(a)$ and represents which of the following?

   (a) The average rate of change of $f(x)$ over a small interval to the right of a.

   (b) The instantaneous rate of change of $f(x)$ at the point $x = a$.

   (c) The limit of $f(x)$ as x approaches a.

   (d) The slope of the line tangent to the graph of $f(x)$ at the point $(a, f(a))$.

   (e) The slope of the line connecting the x-axis to the point $(a, f(a))$.

8. Let $f(x)$ be a differentiable function on everywhere - defined on $(-\infty, \infty)$. Which of the following statements are true?

   I. If $f(x)$ has a local maximum at $x = a$, then $f'(a) = 0$.

   II. If $f(x)$ has a local minimum at $x = a$, then $f'(a) = 0$.

   III. If $f'(a) = 0$, then $f(x)$ has either a local max or local min at $x = a$.

   (a) I and II only; (b) III only; (c) I, II, and III; (d) none of the above.

9. The graph of a function $f(x)$ is shown in Figure 2.20 below. The sign/value of the second derivative $f''(x)$ at the points a, b, and c, respectively, is

   (a) $+, 0, -$ (b) $+, 0, +$ (c) $-, 0, +$ (d) $-, 0, -$ (e) $+, +, -$

   (For example, select (a) $+, 0, -$ if you believe $f''(a) > 0$, $f''(b) = 0$, and $f''(c) < 0$.)

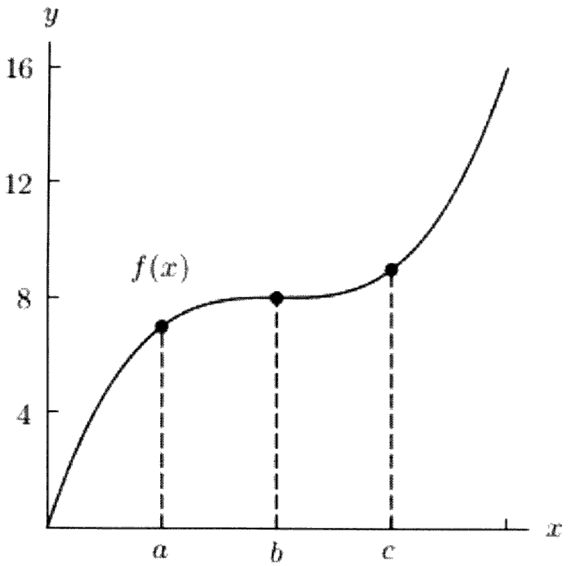
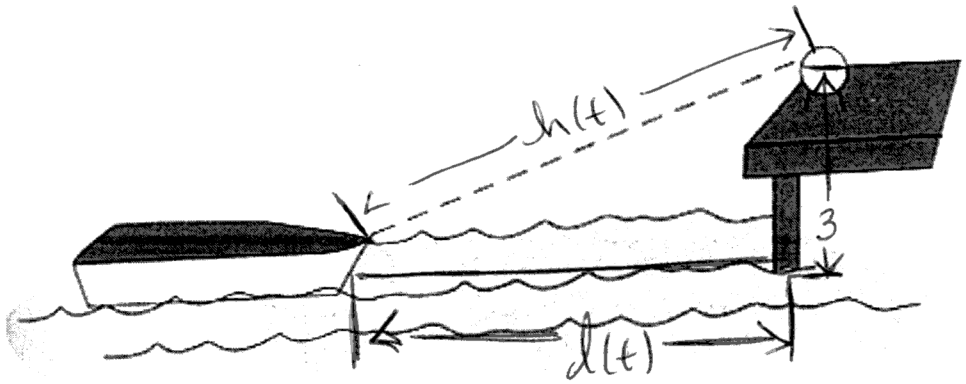


Figure 2.20

10. A boat is pulled toward a dock by means of a rope wound on a drum that is located 3 meters above the bow of the boat. If the rope is being pulled in at the rate of 8 meters per minute, how fast is the boat approaching the dock at the time t_1 when it is 4 meters from the dock?



(*Hints:* define functions $d(t)$ and $h(t)$ representing distances at time t from the boat to the dock and drum, respectively; relate $d(t)$ and $h(t)$ using the Pythagorean Theorem; differentiate implicitly, then plug in the given data and solve for $d'(t_1)$.)

$h^2(t) = d^2(t) + 3^2$ (Pythagorean thm)

Diff. Implicitly,

$2h(t)h'(t) = 2d(t)d'(t)$

So $d'(t) = \dfrac{h(t)h'(t)}{d(t)}$

$d(t_1) = 4 \Rightarrow h(t_1) = \sqrt{4^2+3^2} = 5$ ($h'(t_1) = 8$ m/min was given in Prob. Statement)

So $d'(t_1) = \dfrac{h(t_1)h'(t_1)}{d(t_1)} = \dfrac{5 \cdot 8}{4} = \underline{10}$

Answer: $d'(t_1) =$ 10 meters/min

11. Consider the function $f(x) = 170 + 8x^3 + x^4$. Find the intervals of increase/decrease, local extrema, and the intervals where f is concave up/down. Indicate your answers by placing check marks in the appropriate places below.

Show your work in the space provided. (no work ⇒ no credit)

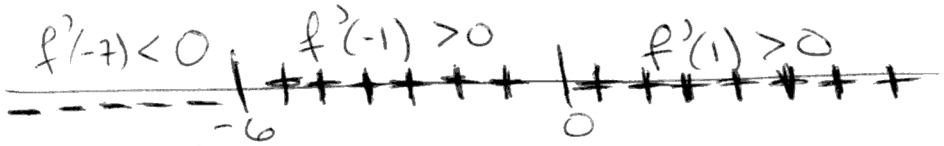
$f'(x) = 24x^2 + 4x^3 = 4x^2(6+x)$

So $f'(x) = 0$ when $x = 0$ or $x = -6$.

$f''(x) = 48x + 12x^2 = 12x(4+x)$

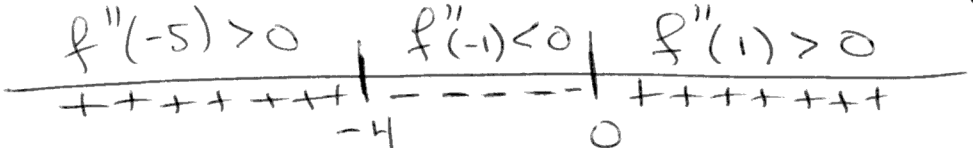
So $f''(x) = 0$ when $x = 0$ or $x = -4$.

f':



∴ f decreasing on $(-\infty, -6)$, increasing on $(-6, 0) \cup (0, \infty)$

f'':



∴ f concave up on $(-\infty, -4) \cup (0, \infty)$; down on $(-4, 0)$.

(a) On what interval(s) is $f(x)$ increasing? (Select all that apply.)

___ $(-\infty, -6)$ ___ $(-\infty, -4)$ ✓ $(-6, 0)$ ✓ $(0, \infty)$ ___ $(-\infty, 4)$

(b) On what interval(s) is $f(x)$ decreasing? (Select all that apply.)

✓ $(-\infty, -6)$ ___ $(-\infty, -4)$ ___ $(-6, 0)$ ___ $(0, \infty)$ ___ $(-\infty, 4)$

(c) For what x value(s) does $f(x)$ have a local maximum? (Select all that apply.)

___ -6 ___ -4 ___ 0 ___ 6 ✓ none of these

(d) For what x value(s) does $f(x)$ have a local minimum? (Select all that apply.)

✓ -6 ___ -4 ___ 0 ___ 6 ___ none of these

(e) On what interval(s) is the graph of $f(x)$ concave up? (Select all that apply.)

___ $(-\infty, 0)$ ✓ $(-\infty, -4)$ ___ $(-4, 0)$ ✓ $(0, \infty)$ ___ $(-6, \infty)$

(f) On what interval(s) is the graph of $f(x)$ concave down? (Select all that apply.)

___ $(-\infty, -6)$ ___ $(-\infty, -4)$ ✓ $(-4, 0)$ ___ $(0, \infty)$ ___ $(-6, \infty)$