Name: Solution Key

Part 1. Complete the table by blacking out letters corresponding to correct answers.

1.	●	(b)	(c)	(d)	(e)
2.	(a)	●	(c)	(d)	(e)
3.	(a)	(b)	●	(d)	(e)
4.	(a)	(b)	(c)	●	(e)
5.	●	(b)	(c)	(d)	(e)

In Problems 1–4 assume $f(x) = (x-1)^3$. (If the expression you are asked to compute is not defined or does not appear in the list of possible answers, choose "none of these.")

1. Find the value of $f(x)$ when $x = -1$.

(a) -8 (b) -4 (c) 0 (d) 8 (e) none of these

$f(-1) = (-1-1)^3 = (-2)^3 = -8$

2. What values of x give a $f(x)$ value of -1.

(a) -1 (b) 0 (c) 1 (d) 2 (e) none of these

$f(x) = -1 \iff (x-1)^3 = -1 \iff x-1 = \sqrt[3]{-1} \iff x-1=-1 \iff x=0.$

3. If $g(x) = \frac{1}{\sqrt{9-2x}}$, what is the value of $g(f(-1))$?

(a) $-1/\sqrt{7}$ (b) $1/\sqrt{17}$ (c) $1/5$ (d) $1/3$ (e) none of these

$f(-1) = -8$ so $g(f(-1)) = g(-8) = \frac{1}{\sqrt{9-2(-8)}} = \frac{1}{\sqrt{9-(-16)}} = \frac{1}{\sqrt{25}} = \underline{\underline{\frac{1}{5}}}$

4. Simplify the expression $\sqrt[3]{8r^6}\sqrt{s^4t^6}$. (Assume that r, s, and t are positive.)

(a) $2r^2s^6t^4$ (b) $8r^2s^2t^3$ (c) $2r^2s^6t^3$ (d) $2r^2s^2t^3$ (e) none of these

$\sqrt[3]{8r^6}\sqrt{s^4t^6} = 8^{1/3}(r^6)^{1/3}(s^4)^{1/2}(t^6)^{1/2} = 2 \cdot r^2 \cdot s^2 \cdot t^3$

5. Find the *domain* of the function

$$g(x) = \frac{\sqrt{x-6}}{x(x-8)}.$$

$g(x)$ is defined when $x-6 \geq 0$ and $x(x-8) \neq 0$.
i.e. $x \geq 6$ & $x \neq 8$ so $x \in [6,8) \cup (8,\infty)$

(a) $[6,8) \cup (8,\infty)$ (b) $[6,\infty)$ (c) $(8,\infty)$ (d) $(-\infty,0) \cup (0,8) \cup (8,\infty)$ (e) $(-\infty,\infty)$

Part 1. (cont.) Complete the table by blacking out letters matching correct answers.

6.	(a)	(b)	(c)	(d)	●
7.	●	●	(c)	●	(e)
8.	●	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	●	(e)
10.	(a)	(b)	(c)	●	(e)

6. Find the *range* of the function $f(x) = \sqrt{6 - 9x}$.

(a) $(-\infty, \infty)$ (b) $(-\infty, 0]$ (c) $[2/3, \infty)$ (d) $(-\infty, 2/3]$ (e) none of these

$f(x)$ is defined for $6-9x \geq 0 \iff 6 \geq 9x \iff 2/3 \geq x$, and as x ranges over values in $(-\infty, 2/3]$ the function takes on all values in $[0, \infty)$ = range.

7. Which function(s) has its domain identical with its range? (select all that apply)

(a) $f(x) = 1/x$ (b) $g(x) = x$ (c) $h(x) = x^2$ (d) $i(x) = \sqrt{x}$ (e) none of these

Domain$(1/x)$: $(-\infty,0)\cup(0,\infty)$
Range$(1/x)$: $(-\infty,0)\cup(0,\infty)$

Domain$(g) = (-\infty,\infty)$ = Range(g)

Domain$(x^2) = (-\infty,\infty)$ $\neq [0,\infty)$ = range(x^2)

Domain$(\sqrt{x}$ $= [0,\infty)$ = Range$(\sqrt{x}$

8. Simplify the expression $|\sqrt{3} - 1| + |8 + \sqrt{3}|$.

(a) $7 + 2\sqrt{3}$ (b) $2\sqrt{3}$ (c) $9 + 2\sqrt{3}$ (d) $-2\sqrt{3}$ (e) none of these

$\sqrt{3} - 1 > 0$ so $|\sqrt{3}-1| = \sqrt{3}-1$ So $|\sqrt{3}-1| + |8+\sqrt{3}| = \sqrt{3} - 1 + 8 + \sqrt{3}$

$8+\sqrt{3} > 0$ so $|8+\sqrt{3}| = 8+\sqrt{3}$ $= 2\sqrt{3} + 7$

9. The equation of the line that passes through the points $(2, 6)$ and $(3, 13)$ is

(a) $y = \frac{1}{7}x + \frac{8}{7}$ (b) $y = -\frac{1}{7}x + \frac{20}{7}$ (c) $y = -7x + 20$ (d) $y = 7x - 8$ (e) none of these

Slope: $\frac{\text{rise}}{\text{run}} = \frac{13-6}{3-2} = \frac{7}{1} = 7.$ $y = 7x + b$. Plug in a point on the line to determine b. e.g. $6 = 7(2) + b$

so $b = -8$

10. Find the set of all x values where the function $f(x)$ is continuous.

$$f(x) = \frac{x^2 - 4}{x - 2}$$ is continuous where $x - 2 \neq 0$ i.e. $x \neq 2$.

(a) $(-\infty, \infty)$ (b) $(-\infty, -2)$ (c) $(2, \infty)$ (d) $(-\infty, 2) \cup (2, \infty)$ (e) none of these

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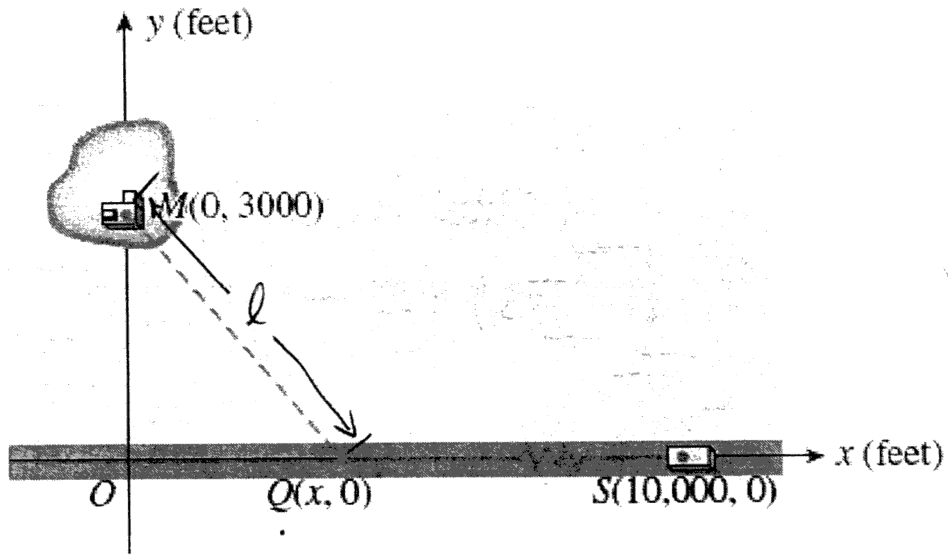
Part 2.

11. In the following diagram, S represents the position of a power relay station located on a straight coastal highway, and M shows the location of a marine biology experimental station on an island. A cable is to be laid connecting the relay station with the experimental station. The cost of running the cable on land is \$3/running foot and the cost of running cable under water is \$5/running foot.



i. (4pts) Find an expression in terms of x that gives the total cost of laying the cable.

$\ell = \sqrt{x^2+3000^2}$ $\quad$ $C(x) = 3(10{,}000-x)+5\ell$

Answer: $C(x) =$ $3(10000-x)+5\sqrt{x^2+3000^2}$

ii. (3pts) What is the total cost when $x = 4{,}000$?

$C(4000) = 3(10000-4000)+5\sqrt{4000^2+3000^2}$

$= 3\cdot 6000 + 5\sqrt{16{,}000{,}000+9{,}000{,}000}$

$= 18000 + 5\cdot\sqrt{25000000}$

Answer: $C(4000) =$ \$43,000

$= 18000 + 5\cdot 5000$

$= 18000 + 25000$

12. (15pts) Evaluate each limit. If the limit does not exist, write DNE. (Show your work!)

i.

$$\lim_{x \to 2} \frac{2x^2+1}{x^2+5x-4}$$

Plug in $x=2$

$$\frac{2(4)+1}{4+10-4} = \frac{9}{10}$$

Answer: 9/10

ii.

$$\lim_{x \to -1} \frac{x^2-4x}{x^2-3x-4} = \lim_{x \to -1} \frac{x(x-4)}{(x-4)(x+1)}$$

$$= \lim_{x \to -1} \frac{x}{x+1} \quad \left(\longrightarrow \frac{-1}{0}\right) \text{ DNE}$$

Answer: DNE

iii.

$$\lim_{x \to -2} \frac{x^2+4x+4}{x^4-16} = \lim_{x \to -2} \frac{(x+2)^2}{(x^2-4)(x^2+4)}$$

$$= \lim_{x \to -2} \frac{(x+2)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}(x^2+4)} = \frac{0}{(-4)(8)} = 0.$$

Answer: 0

iv.

$$\lim_{y \to \infty} \frac{1-3y^2}{2y^2+5y} = \lim_{y \to \infty} \frac{y^2(1/y^2-3)}{y^2(2+5/y)} = -\frac{3}{2}$$

Answer: -3/2

v.

$$\lim_{x \to \infty} (x - \sqrt{x^2+6x}) \quad (\longrightarrow \infty - \infty)$$

$$= \lim_{x \to \infty} \frac{(x-\sqrt{x^2+6x})(x+\sqrt{x^2+6x})}{x+\sqrt{x^2+6x}}$$

Answer: -3

$$= \lim_{x \to \infty} \frac{x^2-(x^2+6x)}{x+\sqrt{x^2+6x}} = \lim_{x \to \infty} \frac{-6x}{x+\sqrt{x^2(1+6/x)}} = \lim_{x \to \infty} \frac{x(-6)}{x(1+\sqrt{1+6/x})}$$

$$= \lim \frac{-6}{1+\sqrt{1+6/x}} = \frac{-6}{1+1} = -3.$$

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13. (4pts) For what value of k will the function f be continuous on $(-\infty, \infty)$? (You must justify your answer and show your work in order to receive credit on this problem.)

$$f(x) = \begin{cases} \frac{x^2-16}{x+4}, & \text{if } x \neq -4, \\ k, & \text{if } x = -4. \end{cases}$$

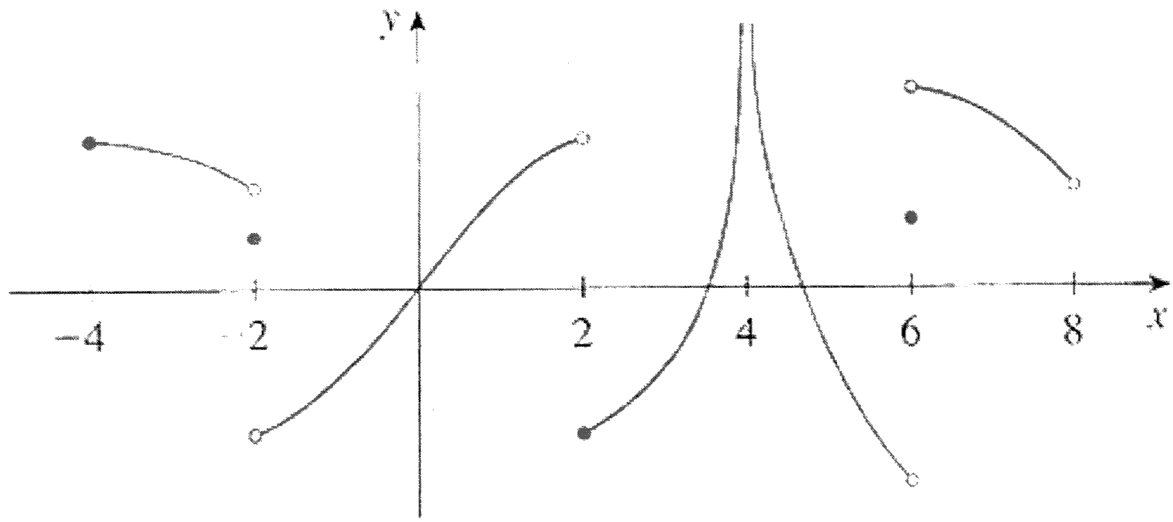
$$f(x) = \frac{(x-4)(x+4)}{x+4} \quad \text{when } x \neq -4.$$

This has a discontinuity at $x=-4$, but it is "removable", in the sense that the limit ~~exi~~ as $x \to -4$ exists and is equal to -8, so if we define $f(-4) = -8$, then

Answer: $k =$ -8

$\lim_{x \to -4} f(x) = -8 = f(-4)$, so f is continuous at $x=-4$ (as well as every other $x \in (-\infty, \infty)$).

14. (4pts) Consider the graph of a function shown below. Identify the domain of the function and the set of values at which the function is continuous. (Circle letters next to correct answers.)



The **domain** of the function:

(a) $[-4, 8)$

(b) $[-4, 4) \cup (4, 8)$

(c) $(-\infty, -4) \cup [8, \infty)$

(d) $[-4, -2) \cup (-2, 2) \cup [2, 6) \cup (6, 8)$

(e) $[-4, -2) \cup (-2, 2) \cup (2, 4) \cup (4, 6) \cup (6, 8)$

x values where the function is **continuous**:

(a) $[-4, 8)$

(b) $[-4, 4) \cup (4, 8)$

(c) $(-\infty, -4) \cup [8, \infty)$

(d) $[-4, -2) \cup (-2, 2) \cup [2, 6) \cup (6, 8)$

(e) $[-4, -2) \cup (-2, 2) \cup (2, 4) \cup (4, 6) \cup (6, 8)$