Math 207 Section A, Quiz 8 Name: Solutions

To receive any credit, you must **show your work!**

1. Find bases for the four fundamental subspaces of the matrix

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 0 \end{bmatrix}$$ Recall $N(A) = \{ \underline{x} \in \mathbb{R}^3 \mid A\underline{x} = \underline{0} \}$

(a) $N(A)$: $A \xrightarrow{\frac{1}{2}R_2 \to R_2} \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_1 - 3R_2 \to R_1} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \end{bmatrix}$

To solve $Ax = 0$, let $x_3 = s$ be free variable.

Then $Ax = \begin{pmatrix} x_1 + 4x_3 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + 4s \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{l} x_1 = -4s \\ x_2 = 0 \end{array}$

So $x = s\begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix}$. A basis for $N(A)$ is $\left\{ \begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix} \right\}$.

(b) $N(A^T)$: $A^T = \begin{pmatrix} 1 & 0 \\ 3 & 2 \\ 4 & 0 \end{pmatrix} \xrightarrow{4R_1 - R_3 \to R_3} \begin{pmatrix} 1 & 0 \\ 3 & 2 \\ 0 & 0 \end{pmatrix} \xrightarrow{-3R_1 + R_2 \to R_2} \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{pmatrix}$

$\xrightarrow{\frac{1}{2}R_2 \to R_2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$. So $A^T\underline{x} = \underline{0}$ iff $\underline{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Therefore $N(A^T) = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$, which has basis $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$.

(Also acceptable answers: Basis = $\emptyset$ or "no basis".) ← (depends on conventions used.)

(c) $R(A)$:

$R(A) =$ "col space" of A or "range" of A $= \{ Ax \mid x \in \mathbb{R}^3 \}$.

$\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \end{bmatrix}$ as in part (a). This allows us to identify the 1ST two cols of A as basis vectors for the column space. So a basis for $R(A)$ is $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\}$.

(Note, however, that in this example the col space is all of $\mathbb{R}^2$, so any pair of linearly indep. vectors of length 2 is also a basis

(d) $R(A^T)$: $A^T = \begin{pmatrix} 1 & 0 \\ 3 & 2 \\ 4 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$ as in part (b), so a basis for the column space of A^T is $\left\{ \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \right\}$.

Remarks: Note that here we can only use the RREF of A^T to identify which columns of A^T serve as basis vectors for the column space of A^T. It is incorrect to conclude that a basis for $R(A^T)$ is $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$. Notice that the vector $\begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$ is not in span$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$, so certainly $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$ cannot be a basis for $R(A^T)$.

2. Let $T : R^3 \to R^3$ be a linear transformation such that

$$T(1,0,0) = (1,2,4), \quad T(0,1,0) = (3,2,1), \quad T(0,0,1) = (0,2,2).$$

Compute $T(1,0,3)$. (*Hint: First write* $(-1,0,3)$ *as a linear combination of basis vectors.*)

typo!

Write $\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, as suggested in the hint.

Then, since T is linear, $T\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} = 1 \cdot T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 0 \cdot T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 3 \cdot T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$= 1 \cdot \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + 0 \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + 3 \cdot \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ 10 \end{pmatrix}$$

Answer: $T(1,0,3) = \begin{pmatrix} 1 \\ 8 \\ 10 \end{pmatrix}$ or $(1, 8, 10)$

"On my honor as a student I, ______________________, have neither given nor received unauthorized aid on this quiz." (print name clearly)

Signature: ______________________ Date: ____________

Score: