Math 317: Homework 2

NAME:

Problem 1 (SA 1.4.3af). For each of the following matrices A, determine its reduced echelon form and give a general solution to $A\mathbf{x} = \mathbf{0}$ in standard form.

$$\text{a.}\quad A = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 3 & -1 \\ 3 & -3 & 0 \end{bmatrix}, \qquad \text{f.}\quad A = \begin{bmatrix} 1 & 2 & 0 & -1 & -1 \\ -1 & -3 & 1 & 2 & 3 \\ 1 & -1 & 3 & 1 & 1 \\ 2 & -3 & 7 & 3 & 4 \end{bmatrix}.$$

Problem 2 (SA 1.4.4de). For the matrix A and vector $\mathbf{b}$ given, find the general solution of the equation $A\mathbf{x} = \mathbf{b}$ in standard form.

d.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

e.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 3 & 2 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 6 \\ 17 \end{bmatrix}$$

Problem 3 (SA 1.4.7a). One might need to find solutions of $A\mathbf{x} = \mathbf{b}$ for several different $\mathbf{b}$s, say $\mathbf{b}_1, \dots, \mathbf{b}_k$. In this situation, one can augment the matrix A with all the $\mathbf{b}$s simultaneously, forming the "multi-augmented" matrix $[A|\mathbf{b}_1\ \mathbf{b}_2\ \cdots\ \mathbf{b}_k]$. One can then read off the various solutions from the reduced echelon form of the multi-augmented matrix. Use this method to solve $A\mathbf{x} = \mathbf{b}_j$ for the given matrix A and vectors $\mathbf{b}_j$.

a.

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ -1 & 2 & 2 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

Problem 4 (SA 1.4.14). Let A be an $m \times n$ matrix, and let $\mathbf{b} \in \mathbb{R}^m$.

a. Show that if the vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$ are both solutions to $A\mathbf{x} = \mathbf{b}$, then $\mathbf{u} - \mathbf{v}$ is a solution to $A\mathbf{x} = \mathbf{0}$.

b. Suppose $\mathbf{u}$ is a solution to $A\mathbf{x} = \mathbf{0}$ and $\mathbf{p}$ is a solution to $A\mathbf{x} = \mathbf{b}$. Show that $\mathbf{u} + \mathbf{p}$ is a solution to $A\mathbf{x} = \mathbf{b}$. (*Hint:* Use Exercise 1.4.13.)

Problem 5 (SA 1.4.15a). Prove or give a counterexample: If A is an $m \times n$ matrix and $\mathbf{x} \in \mathbb{R}^n$ is a vector satisfying $A\mathbf{x} = \mathbf{0}$, then either every entry of A is 0 or $\mathbf{x} = \mathbf{0}$.