Math 317: Homework 6b

NAME:

Section 3.2

Problem 1 (SA 3.2.1). Show that if B is obtained from A by performing one or more elementary row operations, then $\mathbf{R}(B) = \mathbf{R}(A)$.

Problem 2 (SA 3.2.5). Suppose $A = LU$, where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Give vectors that span $\mathbf{R}(A)$, $\mathbf{C}(A)$, and $\mathbf{N}(A)$.

Problem 3 (SA 3.2.6a). Construct a matrix whose column space contains $\begin{bmatrix}1\\1\\1\end{bmatrix}$ and $\begin{bmatrix}0\\1\\1\end{bmatrix}$ and whose null space contains $\begin{bmatrix}1\\0\\1\end{bmatrix}$ and $\begin{bmatrix}0\\1\\0\end{bmatrix}$, or explain why no such matrix exists.

Problem 4 (SA 3.2.10). Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Prove that

a. $\mathbf{N}(B) \subseteq \mathbf{N}(AB)$.

b. $\mathbf{C}(AB) \subseteq \mathbf{C}(A)$. [*Hint:* Use Proposition 2.1.]

c. $\mathbf{N}(B) = \mathbf{N}(AB)$ when A is $n \times n$ and nonsingular. [*Hint:* See the box on p. 12.]

d. $\mathbf{C}(AB) = \mathbf{C}(A)$ when B is $n \times n$ and nonsingular.

Problem 5 (SA 3.2.11). Let A be an $m \times n$ matrix. Prove that $\mathbf{N}(A^\top A) = \mathbf{N}(A)$.
[*Hint:* Use the previous problem (SA 3.2.10) and Exercise SA 2.5.15.]

The following problem is recommended but will not be graded.

Problem (SA 3.2.13). Let A be an $m \times n$ matrix with the property that $A^2 = A$.

a. Prove that $\mathbf{C}(A) = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x} = A\mathbf{x}\}$.

b. Prove that $\mathbf{N}(A) = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x} = \mathbf{u} - A\mathbf{u} \text{ for some } \mathbf{u} \in \mathbb{R}^n\}$.

c. Prove that $\mathbf{C}(A) \cap \mathbf{N}(A) = \{\mathbf{0}\}$.

d. Prove that $\mathbf{C}(A) + \mathbf{N}(A) = \mathbb{R}^n$.