Math 317: Homework 10

NAME:

Exercise (SA 4.3.5a). Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, and consider the basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ for $\mathbb{R}^2$. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation whose standard matrix is $[T]_{\mathcal{E}} = \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$. Find the matrix $[T]_{\mathcal{B}}$.

Exercise (SA 4.3.7). Suppose the standard matrix for a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is $[T]_{\mathcal{E}} = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 3 \\ 1 & -1 & 1 \end{bmatrix}$. Find the matrix representation $[T]_{\mathcal{B}}$ of T with respect to the basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

Hint: For the first two exercises, apply the change-of-basis formula given in Proposition 3.2 on page 215 of the textbook.

Exercise (SA 4.3.16). Let $V = \text{Span}((1, 0, 2, 1), (0, 1, -1, 1)) \leq \mathbb{R}^4$. Use the change-of-basis formula to find the standard matrix for $\text{proj}_V : \mathbb{R}^4 \to \mathbb{R}^4$. (*Hint:* this is similar to Example 9 on page 216.)

Exercise (SA 4.3.19). Let $\mathbf{e}_1$, $\mathbf{e}_2$ denote the standard basis, as usual. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$T(\mathbf{e}_1) = 8\mathbf{e}_1 - 4\mathbf{e}_2 \quad \text{and} \quad T(\mathbf{e}_2) = 9\mathbf{e}_1 - 4\mathbf{e}_2.$$

1. Give the standard matrix for T.
2. Let $\mathbf{v}_1 = 3\mathbf{e}_1 - 2\mathbf{e}_2$ and $\mathbf{v}_2 = -\mathbf{e}_1 + \mathbf{e}_2$. Calculate the matrix for T with respect to the basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$.
3. Is T diagonalizable? Give your reasoning. (Hint: See part d of Exercise 18.)