Math 317: Homework 5

Problem 1 (SA 2.4.1a). Consider the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 3 & -1 \\ 3 & -3 & 0 \end{bmatrix}$ from Exercise 1.4.3a.

Find a product of elementary matrices $E = E_k \cdots E_2 E_1$ so that EA is in echelon form. Use the matrix E you've found to give constraint equations for $A\mathbf{x} = \mathbf{b}$ to be consistent.

Problem 2 (SA 2.4.3a). Find the LU decomposition of the matrix A of the previous exercise.

Problem 3 (SA 2.4.7). Show that the inverse of every elementary matrix is again an elementary matrix by giving a simple prescription for determining the inverse of each type of elementary matrix. (See the proof of Theorem 4.1 of Chapter 1.)

Problem 4 (SA 2.4.8). Prove or give a counterexample: Every invertible matrix can be written as a product of elementary matrices.

Problem 5 (SA 2.5.1beg). Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 3 \end{bmatrix}$.

Calculate each of the following expressions or explain why it is not defined.

b. $2A - B^\top$ e. $A^\top C$ g. $C^\top A^\top$

Problem 6 (SA 2.5.2abf). Let $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}$. Calculate the following matrices.

a. $\mathbf{a}\mathbf{a}^\top$ b. $\mathbf{a}^\top\mathbf{a}$ f. $\mathbf{a}^\top\mathbf{b}$

Problem 7 (SA 2.5.3b). Let $\mathbf{a} = (4, 3)$. Find the standard matrix for the projection $\text{proj}_{\mathbf{a}}$. [*Hint:* see Example 3 in the textbook.]

Problem 8 (SA 2.5.5). Suppose A and B are symmetric. Show that AB is symmetric if and only if $AB = BA$.

Problem 9 (SA 2.5.8). Suppose A is invertible. Check that $(A^{-1})^\top A^\top = I$ and $A^\top (A^{-1})^\top = I$ and deduce that $A^\top$ is likewise invertible with inverse $(A^{-1})^\top$.

Problem 10 (SA 2.5.15). Suppose A is an $m \times n$ matrix and $\mathbf{x} \in \mathbb{R}^n$ satisfies $(A^\top A)\mathbf{x} = 0$. Prove that $A\mathbf{x} = \mathbf{0}$. (Hint: What is $\|A\mathbf{x}\|$?)