

Constraint Satisfaction Problems, Graph Theory, and Universal Algebra

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What is a CSP?

Informally, a **C**onstraint **S**atisfaction **P**roblem consists of

- a list of variables ranging over a finite domain and
- a set of constraints on those variables.

Problem: can we assign values to all the variables so that all of the constraints are satisfied?

Examples

A system of linear equations is a CSP

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

Also, a system of nonlinear equations is a CSP

$$\begin{array}{rcl} a_{11}x_1^2x_3 + a_{12}x_2x_3x_7 & + \cdots + a_{1n}x_4x_n^3 & = b_1 \\ a_{21}x_2x_5 + a_{22}x_2 & + \cdots + a_{2n}x_4^3 & = b_2 \\ & \vdots & \\ a_{m1}x_3x_5x_8 + a_{m2}x_2 & + \cdots + a_{mn}x_n & = b_m \end{array}$$