**CS 241: Spring 2022** **HW 4** **Due 25 Feb 11:59pm 11:59pm**

Name: ______________________________ Student ID: ________________

**Instructions**. Print out this assignment, fill in your answers in the space provided and upload your work to Gradescope. **Important**. You must write your answers on a printout of this pdf! Please do not upload additional pages or scratch work.

1. (4 points) Given two statements A and B, we say that A is *stronger* than B if A → B, while A is *weaker* than B if B → A. We say A and B are *equivalent* if each implies the other; that is, A → B ∧ B → A. (We sometimes denote the latter by A ↔ B.)

   Consider the following induction principle.

   **(A) Principle of Mathematical Induction**. Let $S(n)$ be a predicate whose domain is the set of natural numbers. Suppose there exists $m$ such that $S(m)$ holds and $\forall n \geq m$ if $S(n)$ holds then so does $S(n+1)$. Then $S(n)$ holds for every natural number $n \geq m$.

   We state this principle formally as follows:

   $$\exists m(S(m) \wedge \forall n(n \geq m \rightarrow S(n) \rightarrow S(n+1))) \rightarrow \forall n(n \geq m \rightarrow S(n)). \tag{1}$$

   Consider the following induction principle which is closer to the one appearing on page 90 of our textbook.[1]

   **(B) Principle of Mathematical Induction**. Let $S(n)$ be a predicate whose domain is the set of natural numbers. Suppose $S(0)$ holds and suppose that whenever $S(n)$ holds then so does $S(n+1)$. Then $S(n)$ holds for every natural number $n$.

   (a) Express induction principle (B) as a logical formula (just like we did in (1) for (A)).

   (b) Is induction principle (A) stronger, weaker, or equivalent to induction principle (B)?

   ◯ (A) is stronger than (B).
   ◯ (B) is stronger than (A).
   ◯ (A) and (B) are equivalent.

[1] We modified it slightly to allow for the case $n = 0$.

(c) (Extra Credit.) For up to 3 bonus points, prove your answer to part (b) is correct.

2. (6 points) Using induction, prove that the equations below are true for all natural numbers $n \in \mathbb{N}$.

   (a) $0 \cdot 1 + 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = n(n+1)(n+2)/3$

   (b) $0^3 + 1^3 + 2^3 + \cdots + n^3 = \left[n(n+1)/2\right]^2$

3. (6 points) Use induction to verify the inequality.

(a) $2n + 1 \leq 2^n$, for all $n \geq 3$, where $n \in \mathbb{N}$.

(b) $(1 + x)^n \geq 1 + nx$, for all $x \geq -1$ and $n \in \mathbb{N}$.

4. (4 points) Show that any amount of postage greater than or equal to 12 cents can be achieved by using only 3-cent and 7-cent stamps.