CS 241: Spring 2022 **HW 01 (written component)** **Due 2 Feb 10:59pm**

Name: ______________________________ Student ID: __________________

1. Express each of the following statements in the form P $\vee$ Q, or P $\wedge$ $\neg$ Q. Be sure to state what you're taking the variables P and Q to mean in each case.

   Example. The number x is greater than 0, but y is not equal to x.

   Solution. Let P be $x > 0$. Let Q be $y = x$. Answer: P $\wedge$ $\neg$ Q

(3pts) (a) At least one of the numbers x or y is 0.

(3pts) (b) x belongs to A but not B.

(3pts) (c) x does not belong to both A and B^c. (B^c denotes the complement of B)

(10pts) 2. Let A and B be subsets of some domain Ω. Prove that $(A \backslash B) \cup (B \backslash A) = (A \cup B) \backslash (A \cap B)$ by showing that the expression on the left is a subset of that on the right and vice-versa.

(10pts) 3. Two sets A and B are said to be **disjoint** if they have no element in common; that is, $A \cap B = \emptyset$. Prove that if A and B are disjoint, $C \subseteq A$, and $D \subseteq B$, then C and D are disjoint.

(10pts) 4. Let A, B, and C be subsets of some domain Ω. Give a *calculational proof* of the identity $A \setminus (B \cup C) = (A \setminus B) \setminus C$. You may use the following facts, but be sure to mention when you use one.

(fact 1) $C \setminus D = C \cap D^c$

(fact 2) $(B \cup C)^c = B^c \cap C^c$.

(fact 3) $(A \cap B) \cap C = A \cap (B \cap C)$.

(0pts) 5. Let I and J be sets and consider $\{A_{i,j} : i \in I, j \in J\}$ (that is, a family of sets indexed by I and J). Prove that

$$\bigcup_{i\in I} \bigcap_{j\in J} A_{i,j} \subseteq \bigcap_{j\in J} \bigcup_{i\in I} A_{i,j}.$$

Also, find a family $\{A_{i,j} : i \in I, j \in J\}$ where the reverse inclusion does *not* hold.

(10pts) 6. Here's a logic puzzle by George J. Summers, called "Murder in the Family." Murder occurred one evening in the home of a father and mother and their son and daughter. One member of the family murdered another member, the third member witnessed the crime, and the fourth member was an accessory after the fact.

a. The accessory and the witness were of opposite sex.
b. The oldest member and the witness were of opposite sex.
c. The youngest member and the victim were of opposite sex.
d. The accessory was older than the victim.
e. The father was the oldest member.
f. The murderer was not the youngest member.

Define the mnemonics variables: f $\equiv$ father, m $\equiv$ mother, s $\equiv$ son, d $\equiv$ daughter. Let ♂ and ♀ denote the sets of all men and women, respectively (so f $\in$ ♂, s $\in$ ♂, m $\in$ ♀, and d $\in$ ♀). Let A, M, V, and W denote the (singleton) sets of accessories, murderers, victims, and witnesses (respectively), and let O and Y denote the (singleton) sets of the oldest and youngest family members.

Notice that only the son or daughter can be the youngest, and only the mother or father can be the oldest. So we have s $\in$ Y or d $\in$ Y, and f $\in$ O or m $\in$ O.

With these conventions, the first clue in the list above can be written as follows:

a. $[(A \subseteq ♂) \wedge (W \subseteq ♀)] \bigvee [(A \subseteq ♀) \wedge (W \subseteq ♂)]$

In other words, either the accessory is male and the witness female, or vice-versa. Represent the other five clues in a similar manner.

(10pts) 7. Consider the following three hypotheses:

1. Alan likes kangaroos, and either Betty likes frogs or Carl likes hamsters.

2. If Betty likes frogs, then Alan doesn't like kangaroos.

3. If Carl likes hamsters, then Betty likes frogs.

Write a clear argument to show that these three hypotheses are contradictory.