CS 241: Spring 2022 **HW 02** **Due 9 Feb 10:59pm**

Name: ____________________________ Student ID: ________________

(6pts) 1. Here's a logic puzzle by George J. Summers, called "Murder in the Family." Murder occurred one evening in the home of a father and mother and their son and daughter. One member of the family murdered another member, the third member witnessed the crime, and the fourth member was an accessory after the fact.

1. The accessory and the witness were of opposite sex.
2. The oldest member and the witness were of opposite sex.
3. The youngest member and the victim were of opposite sex.
4. The accessory was older than the victim.
5. The father was the oldest member.
6. The murderer was not the youngest member.

Which of the four—father, mother, son, or daughter—was the murderer? Solve this puzzle, and write a clear argument to establish that your answer is correct. Use grammatically correct English sentences (not a natural induction derivation tree).

(4pts) 2. (a) Show that the formulas $A \to B$ and $\neg A \vee B$ are logically equivalent by filling in the truth table below and showing that the two columns for $A \to B$ and $\neg A \vee B$ are identical (i.e., they have the same truth values for all possible truth values of A and B).

A	B	$\neg A$	$A \to B$	$\neg A \vee B$

(4pts) (b) Show that $A \to B$ and $\neg B \to \neg A$ are logically equivalent by constructing the appropriate truth table and showing that the columns for $A \to B$ and $\neg B \to \neg A$ are identical.

(4pts) 3. (a) Give a natural deduction proof of $A \wedge B$ from hypothesis $B \wedge A$.

(4pts) (b) Give a natural deduction proof of $A \vee B$ from hypothesis $B \vee A$.

(4pts) 4. (a) Give a natural deduction proof of $\neg(A \vee B)$ from hypothesis $\neg A \wedge \neg B$.

(2pts) (b) Explain how you would modify the diagram in your proof above to turn it into a proof of the implication $\neg A \wedge \neg B \to \neg(A \vee B)$.

(6pts) 5. Give a natural deduction proof of $\neg(A \wedge B)$ from the hypotheses $A \rightarrow C$ and $B \rightarrow \neg C$.

6. Recall the last exercise from Homework 1 (wc) involving kangaroos, frogs and hamsters.

(6pts) (a) Let A, B, and C denote "Alan likes kangaroos," "Betty likes frogs," and "Carl likes hamsters," respectively. Use these letters and the logical connectives ($\wedge$, $\vee$, $\rightarrow$, $\neg$) to express each of the following statements as a symbolic formula.

H1) Alan likes kangaroos, and either Betty likes frogs or Carl likes hamsters.

In symbols:

H2) If Betty likes frogs, then Alan doesn't like kangaroos.

In symbols:

H3) If Carl likes hamsters, then Betty likes frogs.

In symbols:

(4pts) (b) Write a natural deduction proof tree that starts with the three statements H1, H2, H3 as "axioms" and derives false $\perp$ (i.e., a contradiction).