CS 241: Spring 2022 **HW 7** **Due 8 April 11:59pm**

Name: ______________________________ Student ID: __________________

Instructions. Print out this assignment, fill in your answers in the space provided and upload your work to Gradescope. **Important**. You must write your answers on a printout of this pdf! Please do not upload additional pages or scratch work.

1. (6 points) \$500 is invested in an account paying 10% interest compounded annually.

   (a) Write a recursive definition for $P(n)$, the amount in the account at the beginning of the n-th year.

   (b) After how many years will the account balance exceed \$700? (Prove your answer is correct.)

2. (6 points) The sequence $\{F(n) : n \in \mathbb{N}\}$ of *Fibonacci numbers* is defined as follows:

$F(0) = 0$, $F(1) = 1$, and $F(n) = F(n-1) + F(n-1)$, for $n > 1$.

(a) Prove the following property of the Fibonacci numbers directly from the definition: $F(n) = 5F(n-4) + 3F(n-5)$ for $n \geq 6$.

(b) Prove the following property of the Fibonacci numbers using the strong induction principle: $F(n) < 2^n$ for $n \geq 1$.

3. (4 points) In this exercise you will establish a useful formula for solving first order linear recurrence relations of the form $S(n) = cS(n-1) + g(n)$, where c is a constant and $g(n)$ is some function. (*Tip.* You may use this formula to solve other exercises in this assignment.)

   Use induction to prove that

$$S(n) = c^{n-1}S(1) + \sum_{i=1}^{n} c^{n-i} g(i). \tag{1}$$

   is a solution to the recurrence relation

$$S(n) = cS(n-1) + g(n) \tag{2}$$

   subject to the basis condition that $S(1)$ is known.

4. (4 points) Solve the following recurrence relation.
   $F(1) = 2$ and $F(n) = 2F(n-1) + 2^n$ for $n \geq 2$.

5. (4 points) Spam email is sent to 1000 e-mail addresses. After 1 second, each recipient machine broadcasts 10 new spam e-mails. The recipients of those 10 spam e-mails then sends 10 spam e-mails, and so on. How many emails are sent at the end of 20 seconds?

6. (4 points) Solve the recurrence relation subject to the initial conditions.

   - (recurrence relation) $F(n) = 6F(n-1) - 5F(n-2)$ for $n \geq 3$;
   - (initial conditions) $F(1) = 8$ and $F(2) = 16$.

7. (4 points) The following algorithm adds all the entries in a square $n \times n$ array A. Analyze the algorithm where the work unit is the addition operation.

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sum = 0
for i = 1 to n do
    for j = 1 to n do
        sum = sum + A(i, j)
    end for
end for
write("Total of all array elements is ", sum)
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8. (6 points) (a) Write the body of an iterative function to compute $n!$ for $n \geq 1$.

(b) Analyze this function where the work unit is the multiplication operation.

9. (6 points) (a) Write a recursive function to compute $n!$ for $n \geq 1$.

(b) Write a recurrence relation for the work done by this function.

(c) Solve the recurrence relation in part b.

(d) Compare your answer to part (c) with your answer to 8 (b).