

Name: _____

Student ID: _____

Instructions. Print out this assignment, fill in your answers in the space provided and upload your work to Gradescope. **Important.** You must write your answers on a printout of this pdf! Please do not upload additional pages or scratch work.

1. (10 points) \$500 is invested in an account paying 10% interest compounded annually.
 - (a) Write a recursive definition for $P(n)$, the amount in the account at the beginning of the n -th year.

 - (b) After how many years will the account balance exceed \$700? (Prove your answer is correct.)

2. (10 points) The sequence $\{F(n) : n \in \mathbb{N}\}$ of *Fibonacci numbers* is defined as follows:

$F(0) = 0$, $F(1) = 1$, and $F(n) = F(n-1) + F(n-2)$, for $n > 1$.

(a) Prove the following property of the Fibonacci numbers directly from the definition:

$$F(n) = 5F(n-4) + 3F(n-5) \text{ for } n \geq 6.$$

(b) Prove the following property of the Fibonacci numbers using the strong induction principle: $F(n) < 2^n$ for $n \geq 1$.

3. (6 points) In this exercise you will establish a useful formula for solving first order linear recurrence relations of the form $S(n) = cS(n-1) + g(n)$, where c is a constant and $g(n)$ is some function. (*Tip.* You may use this formula to solve other exercises in this assignment.)

Use induction to prove that

$$S(n) = c^{n-1}S(1) + \sum_{i=1}^n c^{n-i}g(i). \quad (1)$$

is a solution to the recurrence relation

$$S(n) = cS(n-1) + g(n) \quad (2)$$

subject to the basis condition that $S(1)$ is known.

4. (10 points) Solve the following recurrence relation.

$$F(1) = 2 \text{ and } F(n) = 2F(n-1) + 2^n \text{ for } n \geq 2.$$

5. (6 points) Spam email is sent to 1000 e-mail addresses. After 1 second, each recipient machine broadcasts 10 new spam e-mails. The recipients of those 10 spam e-mails then sends 10 spam e-mails, and so on. How many emails are sent at the end of 20 seconds?

6. (6 points) Solve the recurrence relation subject to the initial conditions.

- (recurrence relation) $F(n) = 6F(n-1) - 5F(n-2)$ for $n \geq 3$;
- (initial conditions) $F(1) = 8$ and $F(2) = 16$.

7. (6 points) The following algorithm adds all the entries in a square $n \times n$ array A . Analyze the algorithm where the work unit is the addition operation.

```
sum = 0
for  $i = 1$  to  $n$  do
  for  $j = 1$  to  $n$  do
    sum = sum +  $A(i, j)$ 
  end for
end for
write("Total of all array elements is ", sum)
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8. (10 points) (a) Write the body of an iterative function to compute $n!$ for $n \geq 1$.

(b) Analyze this function where the work unit is the multiplication operation.

9. (10 points) (a) Write a recursive function to compute $n!$ for $n \geq 1$.

(b) Write a recurrence relation for the work done by this function.

(c) Solve the recurrence relation in part b.

(d) Compare your answer to part (c) with your answer to 8 (b).