CS 241: Spring 2022 **HW 8** **Due 20 April 11:59pm**

Name: ______________________________ Student ID: __________________

Instructions. Print out this assignment, fill in your answers in the space provided and upload your work to Gradescope. **Important**. You must write your answers on a printout of this pdf! Please do not upload additional pages or scratch work.

1. (4 points) An algorithm to find a "solution" to a propositional formula with n variables works by assigning, one at a time, all possible sets of truth values to the statement letters and finding an assignment that makes the propositional formula true. The unit of work for this algorithm is the examination of one set of truth values to determine whether the formula is true for that assignment. What is the worst case time complexity of this algorithm?

   ○ $\lg n$ ○ $n \lg n$ ○ n ○ $n!$ ○ 2^n

2. (4 points) Let $f(x)$ and $g(x)$ be functions. From which of the following statements could you conclude that $f = \Theta(g)$? (Select all that apply.)

   ○ There exists a constant C such that $f(n) \leq Cg(n)$ for all but finitely many n.

   ○ There exist constants C_1, C_2 such that $C_1 g(x) \leq C_2 f(x)$ for all x.

   ○ There exist constants C_1, C_2 such that $C_1 g(n) \leq f(n) \leq C_2 g(n)$ for all but finitely many n.

   ○ There exist $N \in \mathbb{N}$ and constants C_1, C_2 such that $C_1 g(n) \leq f(n) \leq C_2 g(n)$ for all $n \geq N$.

   ○ There exists an x such that $C_1 g(x) = C_2 f(x)$ for all constants C_1 and C_2.

3. (4 points) What is the (worst case) time complexity of the following `find` function, which searches for a key in a list. As usual, you should give an answer that is a function of n = the input size = length of input list `l`. You can assume the unit of work is the number of times the `findHelper` subroutine is called.

```
find(key, list) {

  findHelper(list, index) {

    if (length(list) == 0)
      return -1

    if (head(list) == key)
      return index

    findHelper(tail(list), index + 1)
  }

  findHelper(key, l, 0)
}
```

(Recall, `head(list)` returns the first element of `list` and `tail(list)` returns the rest of `list`—i.e., all elements except the first.)

- ◯ $O(\lg n)$
- ◯ $O(n \lg n)$
- ◯ $O(n)$
- ◯ $O(n!)$
- ◯ $O(2^n)$

4. (4 points) A Hawaiian favorite fast food is called "Loco Moco." It consists of a bed of rice under a meat patty with egg on top, the whole thing smothered in brown gravy. The rice can be white or brown, the egg can be fried, scrambled, or poached, the meat can be hamburger, Spam, sausage, bacon, turkey, hot dog, salmon, or mahi. Finally, you can request no gravy. How many different Loco Mocos can be ordered?

Answer. ____________

5. (4 points) Consider the menu for Kay's Quick Lunch shown below.

APPETIZERS	
Nachos	2.15
Salad	1.90
MAIN COURSES	
Hamburger	3.25
Cheeseburger	3.65
Fish Filet	3.15
BEVERAGES	
Tea	.70
Milk	.85
Cola	.75
Root Beer	.75

Figure 6.1.1 Kay's Quick Lunch menu.

(a) How many dinners at Kay's Quick Lunch consist of an optional appetizer, one main course, and an optional beverage?

Answer. _____________

(b) How many dinners at Kay's Quick Lunch consist of one appetizer, one main course, and an optional beverage?

Answer. _____________

(c) In how many ways can a diner choose one item from among the appetizers and main courses?

Answer. _____________

(d) In how many ways can a diner choose one item from among the appetizers and beverages at Kay's Quick Lunch?

Answer. _____________

6. (4 points) A committee is composed of Morgan, Tyler, Max, and Leslie. Two officers of the committee must be selected—a president and a secretary.
   (a) Assuming the roles of president and secretary cannot be filled by the same person, how many ways are there to select a president and secretary such that Tyler is either president or not an officer?

   Answer. ____________

   (b) Assuming the roles of president and secretary cannot be filled by the same person, how many selections are there in which Max is president or secretary?

   Answer. ____________

7. Consider the set S of all binary strings of length 8, $S = \{(x_1, \ldots, x_8) : x_i \in \{0, 1\}\}$.
   (a) How many begin with 111? *Answer.* ____________
   (b) How many contain exactly one 0? *Answer.* ____________
   (c) How many begin with 10 or have 0 as the third digit? *Answer.* ____________

8. In this exercise, we will assume a "hand" consists of 1 card drawn from a standard 52-card deck with flowers on the back and 1 card drawn from a standard 52-card deck with birds on the back. A standard deck has 13 cards from each of 4 suits (clubs, diamonds, hearts, spades). The 13 cards have face value 2 through jack, queen, king, or ace. Each face value is a "kind" of card. The jack, queen, and king are called "face cards."
   (a) How many hands consist of a pair of aces? *Answer.* ____________
   (b) How many hands contain all face cards? *Answer.* ____________
   (c) How many hands contain at least one king? *Answer.* ____________

9. (a) How many permutations of the characters in COMPUTER are there?

   ○ 8 ○ 2^8 ○ 8! ○ $3 \cdot 7!$ ○ $8 \cdot 7 / 2$

   (b) How many permutations of the characters in COMPUTER end in a vowel?

   ○ 8 ○ $8 \cdot 7!$ ○ $3 \cdot 7!$ ○ 8! - 8 ○ $8 \cdot 7 / 2$

10. Consider a group of 19 people comprised of 11 women and 8 men. Assume we can distinguish all individuals from one another (i.e., every person is unique).

    (a) In how many different ways can you seat these 19 people in a row?

    ◯ 19 ◯ 88 ◯ 19! ◯ 88! ◯ $8 \cdot 11!$

    (b) In how many different ways can you seat these 19 people if no two men sit together?

    ◯ $C(11, 2) \cdot 11! \cdot 8! \ / \ 2!$
    ◯ $C(12, 2) \cdot 12! \cdot 8! \ / \ 2!$
    ◯ $C(11, 8) \cdot 11! \cdot 8!$
    ◯ $C(12, 8) \cdot 11! \cdot 8!$
    ◯ $C(12, 8) \cdot 12! \cdot 8!$

    (c) In how many different ways can you seat these 19 people around a circular table. Assume we only care about relative positions of the people around the table; that is, the chairs, and all rotations of a given (relative) seating choice, are indistinguishable.

    ◯ 18! ◯ 19! ◯ 88! ◯ $2 \cdot 19!$ ◯ $2 \cdot 8! \cdot 11!$