Lecture 2 CS241 ①

- Sets "Naive" set theory
  - construction of sets
  - set builder notation
  - set relations & operations
  - proving set containment
  - proving set equality
  - Venn diagrams
  - properties of sets.

Type theory

- Propositions

Sets

This exists! ← Axiom!

$\emptyset$ = the empty set.
(has "size" 0)

$\{ \quad \} = \emptyset$

$\boxed{suc(n) = n+1}$

$\{\emptyset\}$ ← not empty

has size 1.

$\emptyset \neq \{\emptyset\}$

Later

nat : Type

→ 0 : nat

→ suc : nat → nat

(Later)

$\mathbb{N} = \{0, 1, 2, \ldots\}$

$0 = \emptyset$, $1 = \{\emptyset\}$,

$2 = \{\emptyset, \{\emptyset\}\}$, ...

$\{3, 7\} \neq \{3, \{7\}\}$

$2 = \{0, 1\}$

Set of all nats > 3.

"such that"

"belongs to"

$\{4, 5, 6, \ldots\} = \{x \mid x \in \mathbb{N} \text{ and } x > 3\}$

"Set builder notation"

"is an element of"

$\{x \mid x$ is even and
x is divisible by 4
and $x > 100\}$

$\emptyset = \{\ \}$

$2 = \{0, 1\} = \{\emptyset, \{\emptyset\}\}$

(not "set")

Relations on the collection of all sets

There is no set of all sets!

→ Let $X = \{x \mid x \notin x\}$

$x \in A$

$x \notin A$

x does not belong to A

Does X belong to itself?

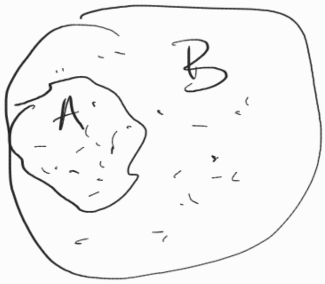
- If $X \in X$ then it satisfies so $X \notin X$
- If $X \notin X$ then $X \in X$!

If A, B are sets then

$A \subseteq B$ means every element of A is also an element of B.

"subset" relation

i.e. For all x, if $x \in A$ then $x \in B$.



$A = B$ means

$A \subseteq B$ and $B \subseteq A$

To prove $A=B$ it's usually easiest to do it in 2 steps.

$$\left.\begin{array}{l} \to 1.\ A \subseteq B \\ \to 2.\ B \subseteq A \end{array}\right\} \Longleftrightarrow A = B$$

Examples

Examples

1. Let $A=\{1,2\}$ $\quad B=\{1,2,5\}$

$A\subseteq B$ if $x\in A$ then $x=1$ or $x=2$

In both cases $x\in B$.

$$A\subseteq B \iff \boxed{\text{For all } x,\ \text{if } x\in A \text{ then } x\in B.}$$

2. More generally to prove $A\subseteq B$.

Fix some $x\in A$. Show $x\in B$.

EX $\quad X=\{x \mid x^2+x-2=0\}$ $\qquad$ Let $\mathbb{Z}=\{\dots,-1,0,1,2,\dots\}$

Show $X\subseteq\mathbb{Z}$.

Proof. Let $x\in X$. Show $x\in\mathbb{Z}$.

$\Rightarrow x=-2$ or $x=1$ so $x\in\mathbb{Z}$.

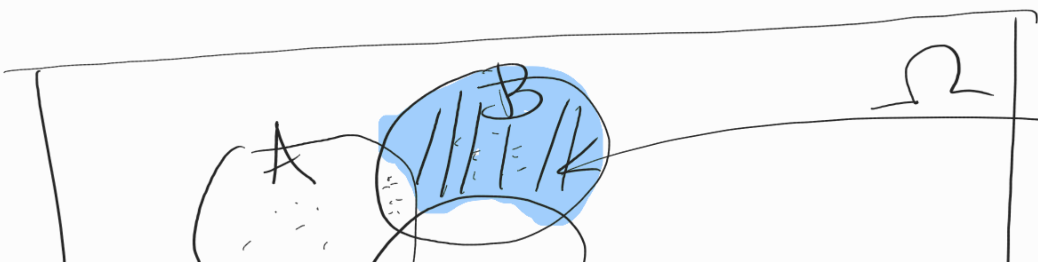
$$x^2+x-2=(x+2)(x-1)=0$$

iff $x=-2$ or $x=1$

Graphical representation of sets

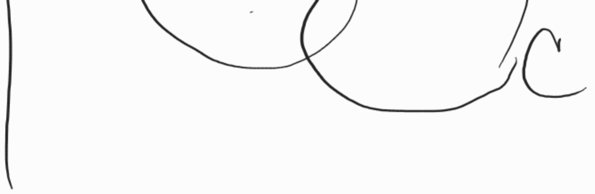
(Venn diagrams)

Ω (big omega) = the whole world



Elements that belong to B

A or C.

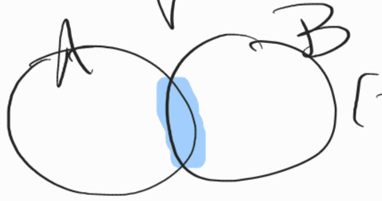


but no A o…

$A \subseteq B \qquad A = B.$

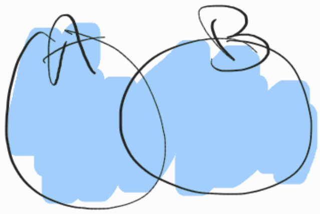
Relations

Operations on sets



$A \cap B$ = "A intersect B"

$= \{x \mid x \in A \text{ and } x \in B\}.$



$A \cup B$ = "A union B"

$= \{x \mid x \in A \text{ or } x \in B \text{ (or both)}\}$

$\boxed{\overline{A} = \{x \mid x \notin A\}}$

"A complement"

$A - B = \{x \mid x \in A \text{ and } x \notin B\}.$