Name: ______________________________ Student ID: ________________

1. (14 points) PROBABILITY, PART I

   Below is a table listing the probabilities of three binary random variables. Fill in the correct values for each marginal or conditional probability below.

| $X_0$ | $X_1$ | $X_2$ | $P(X_0, X_1, X_2)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0.160 |
| 1 | 0 | 0 | 0.100 |
| 0 | 1 | 0 | 0.120 |
| 1 | 1 | 0 | 0.040 |
| 0 | 0 | 1 | 0.180 |
| 1 | 0 | 1 | 0.200 |
| 0 | 1 | 1 | 0.120 |
| 1 | 1 | 1 | 0.080 |

   $P(X_0 = 1, X_1 = 0, X_2 = 1) =$ ____________

   $P(X_0 = 0, X_1 = 1) =$ ____________

   $P(X_2 = 0) =$ ____________

   $P(X_1 = 0 \mid X_0 = 1) =$ ____________

   $P(X_0 = 1, X_1 = 0 \mid X_2 = 1) =$ ____________

   $P(X_0 = 1 \mid X_1 = 0, X_2 = 1) =$ ____________

2. (14 points) PROBABILITY, PART II

   Given are the prior distribution $P(X)$ and two conditional distributions $P(Y \mid X)$ and $P(Z \mid Y)$ shown below. Also, assume $Z$ is independent from $X$ given $Y$. All variables are binary (0-1 variables).

   Compute the following joint distributions based on the chain rule.

| $X$ | $P(X)$ |
|---|---|
| 0 | 0.500 |
| 1 | 0.500 |

| $Y$ | $X$ | $P(Y \mid X)$ |
|---|---|---|
| 0 | 0 | 0.600 |
| 1 | 0 | 0.400 |
| 0 | 1 | 0.900 |
| 1 | 1 | 0.100 |

| $Z$ | $Y$ | $P(Z \mid Y)$ |
|---|---|---|
| 0 | 0 | 0.100 |
| 1 | 0 | 0.900 |
| 0 | 1 | 0.700 |
| 1 | 1 | 0.300 |

   $P(X = 0, Y = 0) =$ ____________

   $P(X = 0, Y = 1) =$ ____________

   $P(X = 1, Y = 0) =$ ____________

   $P(X = 1, Y = 1) =$ ____________

   $P(X = 0, Y = 0, Z = 0) =$ ____________

   $P(X = 1, Y = 0, Z = 1) =$ ____________

   $P(X = 1, Y = 1, Z = 0) =$ ____________

   $P(X = 1, Y = 1, Z = 1) =$ ____________

3. (14 points) PROBABILITY, PART III

For each of the following four subparts, you are given three joint probability distribution tables. For each distribution, please identify if the given independence / conditional independence assumption is true or false.

For your convenience, we have also provided some marginal and conditional probability distribution tables that could assist you in solving this problem.

(a) $X$ is independent from $Y$.

| $X$ | $Y$ | $P(X,Y)$ |
|---|---|---|
| 0 | 0 | 0.240 |
| 1 | 0 | 0.160 |
| 0 | 1 | 0.360 |
| 1 | 1 | 0.240 |

| $X$ | $P(X)$ |
|---|---|
| 0 | 0.600 |
| 1 | 0.400 |

| $Y$ | $P(Y)$ |
|---|---|
| 0 | 0.400 |
| 1 | 0.600 |

☐ True ☐ False

(b) $X$ is independent from $Y$.

| $X$ | $Y$ | $P(X,Y)$ |
|---|---|---|
| 0 | 0 | 0.540 |
| 1 | 0 | 0.360 |
| 0 | 1 | 0.060 |
| 1 | 1 | 0.040 |

| $X$ | $P(X)$ |
|---|---|
| 0 | 0.600 |
| 1 | 0.400 |

| $X$ | $Y$ | $P(X\|Y)$ |
|---|---|---|
| 0 | 0 | 0.600 |
| 1 | 0 | 0.400 |
| 0 | 1 | 0.600 |
| 1 | 1 | 0.400 |

☐ True ☐ False

(c) $X$ is independent from $Y$ given $Z$.

| $X$ | $Y$ | $Z$ | $P(X, Y, Z)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0.280 |
| 1 | 0 | 0 | 0.070 |
| 0 | 1 | 0 | 0.210 |
| 1 | 1 | 0 | 0.140 |
| 0 | 0 | 1 | 0.060 |
| 1 | 0 | 1 | 0.060 |
| 0 | 1 | 1 | 0.030 |
| 1 | 1 | 1 | 0.150 |

| $X$ | $Z$ | $P(X\|Z)$ |
|---|---|---|
| 0 | 0 | 0.700 |
| 1 | 0 | 0.300 |
| 0 | 1 | 0.300 |
| 1 | 1 | 0.700 |

| $Y$ | $Z$ | $P(Y\|Z)$ |
|---|---|---|
| 0 | 0 | 0.500 |
| 1 | 0 | 0.500 |
| 0 | 1 | 0.400 |
| 1 | 1 | 0.600 |

| $X$ | $Y$ | $Z$ | $P(X, Y\|Z)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0.400 |
| 1 | 0 | 0 | 0.100 |
| 0 | 1 | 0 | 0.300 |
| 1 | 1 | 0 | 0.200 |
| 0 | 0 | 1 | 0.200 |
| 1 | 0 | 1 | 0.200 |
| 0 | 1 | 1 | 0.100 |
| 1 | 1 | 1 | 0.500 |

☐ True ☐ False

(d) $X$ is independent from $Y$ given $Z$.

| $X$ | $Y$ | $Z$ | $P(X, Y, Z)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0.140 |
| 1 | 0 | 0 | 0.140 |
| 0 | 1 | 0 | 0.060 |
| 1 | 1 | 0 | 0.060 |
| 0 | 0 | 1 | 0.048 |
| 1 | 0 | 1 | 0.192 |
| 0 | 1 | 1 | 0.072 |
| 1 | 1 | 1 | 0.288 |

| $X$ | $Z$ | $P(X\|Z)$ |
|---|---|---|
| 0 | 0 | 0.500 |
| 1 | 0 | 0.500 |
| 0 | 1 | 0.200 |
| 1 | 1 | 0.800 |

| $Y$ | $Z$ | $P(Y\|Z)$ |
|---|---|---|
| 0 | 0 | 0.700 |
| 1 | 0 | 0.300 |
| 0 | 1 | 0.400 |
| 1 | 1 | 0.600 |

| $X$ | $Y$ | $Z$ | $P(X, Y\|Z)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0.350 |
| 1 | 0 | 0 | 0.350 |
| 0 | 1 | 0 | 0.150 |
| 1 | 1 | 0 | 0.150 |
| 0 | 0 | 1 | 0.080 |
| 1 | 0 | 1 | 0.320 |
| 0 | 1 | 1 | 0.120 |
| 1 | 1 | 1 | 0.480 |

☐ True ☐ False

4. (16 points) Chain Rule

Select all expressions that are equivalent to the specified probability using the given independence assumptions.

(a) Given no independence assumptions, $P(A, B \mid C) =$

- ☐ $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- ☐ $\frac{P(B,C|A)P(A)}{P(B,C)}$
- ☐ $P(A \mid B, C)P(B \mid C)$
- ☐ $\frac{P(A|C)P(B,C)}{P(C)}$

(b) Given that A is independent of B given C, $P(A, B \mid C) =$

- ☐ $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- ☐ $\frac{P(B,C|A)P(A)}{P(B,C)}$
- ☐ $P(A \mid B, C)P(B \mid C)$
- ☐ $\frac{P(A|C)P(B,C)}{P(C)}$

(c) Given no independence assumptions, $P(A \mid B, C) =$

- ☐ $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- ☐ $\frac{P(B,C|A)P(A)}{P(B,C)}$
- ☐ $\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$
- ☐ $\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

(d) Given that A is independent of B given C, $P(A \mid B, C) =$

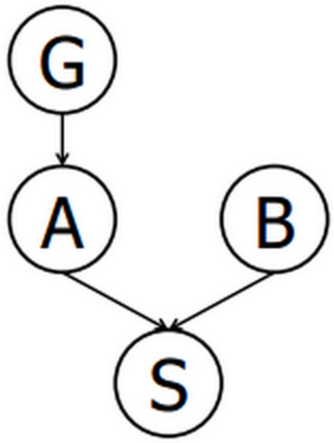
- ☐ $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- ☐ $\frac{P(B,C|A)P(A)}{P(B,C)}$
- ☐ $\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$
- ☐ $\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

5. (16 points) BAYES' NETS AND PROBABILITY

Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding probability tables for this situation are shown below.

| $G$ | $P(G)$ |
|---|---|
| $g$ | 0.10 |
| $\neg g$ | 0.90 |

| $G$ | $A$ | $P(A \mid G)$ |
|---|---|---|
| $g$ | $a$ | 1.00 |
| $g$ | $\neg a$ | 0.00 |
| $\neg g$ | $a$ | 0.10 |
| $\neg g$ | $\neg a$ | 0.90 |




| $B$ | $P(B)$ |
|---|---|
| $b$ | 0.40 |
| $\neg b$ | 0.60 |

| $A$ | $B$ | $S$ | $P(S \mid A, B)$ |
|---|---|---|---|
| $a$ | $b$ | $s$ | 1.00 |
| $a$ | $b$ | $\neg s$ | 0.00 |
| $a$ | $\neg b$ | $s$ | 0.90 |
| $a$ | $\neg b$ | $\neg s$ | 0.10 |
| $\neg a$ | $b$ | $s$ | 0.80 |
| $\neg a$ | $b$ | $\neg s$ | 0.20 |
| $\neg a$ | $\neg b$ | $s$ | 0.10 |
| $\neg a$ | $\neg b$ | $\neg s$ | 0.90 |

(a) P(g, a, b, s) = ____________

(b) Probability patient has disease A = ____________

(c) Prob. patient has disease A given they have disease B = ____________

(d) Prob. patient has disease A given they have symptom S and disease B = ____________

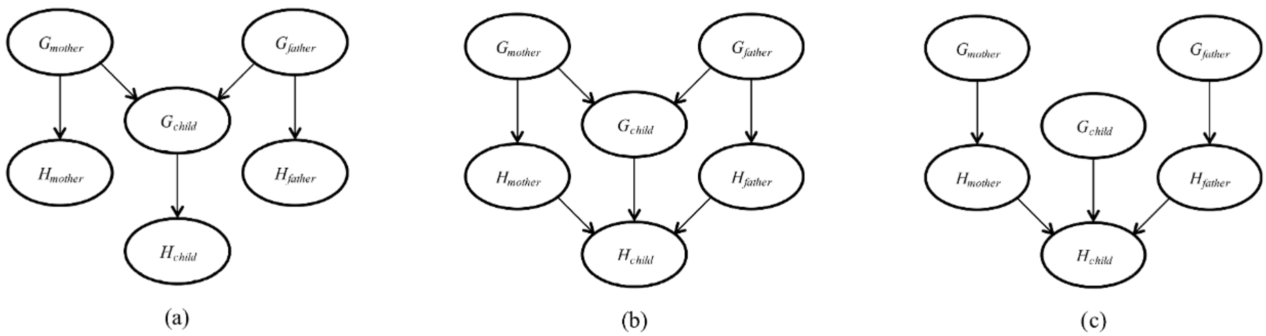
(e) Prob. patient has disease carrying variation G given they have disease A = ____________

(f) Prob. patient has disease carrying variation G given they have disease B = ____________

6. (14 points) Bayes' Nets Independence

Let $H_x$ be a random variable denoting the handedness of an individual $x$, with possible values $l$ or $r$. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene $G_x$, also with values $l$ or $r$, and perhaps actual handedness turns out mostly the same (with some probability $s$) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability $m$ of a random mutation flipping the handedness.

The following images are possible models involving the genes $G$ and handednesses $H$.



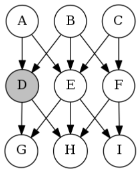

i. Which of the three networks above claim that
$P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})$?
☐ (a) ☐ (b) ☐ (c)

ii. Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?
☐ (a) ☐ (b) ☐ (c)

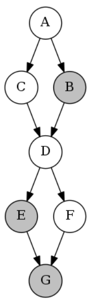
iii. Which of the three networks is the best description of the hypothesis?
☐ (a) ☐ (b) ☐ (c)

7. (12 points) D-SEPARATION.

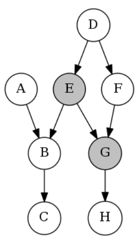
Given several graphical models, shown below, each associated with an independence (or conditional independence) assertion; specify whether the assertion is true or false.

i. 

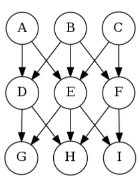
It is guaranteed that $G$ is independent of $H$ given $D$. ☐ True ☐ False

ii. 

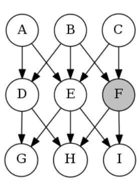
It is guaranteed that $A$ is independent of $D$ given $E, B, G$. ☐ True ☐ False

iii. 

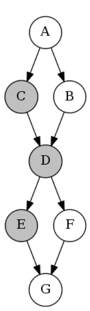
It is guaranteed that $H$ is independent of $B$ given $G, E$. ☐ True ☐ False

iv. 

It is guaranteed that $A$ is independent of $C$. ☐ True ☐ False

v. 

It is guaranteed that $D$ is independent of $C$ given $F$. ☐ True ☐ False

vi. 

It is guaranteed that $G$ is independent of $B$ given $C, E, D$. ☐ True ☐ False