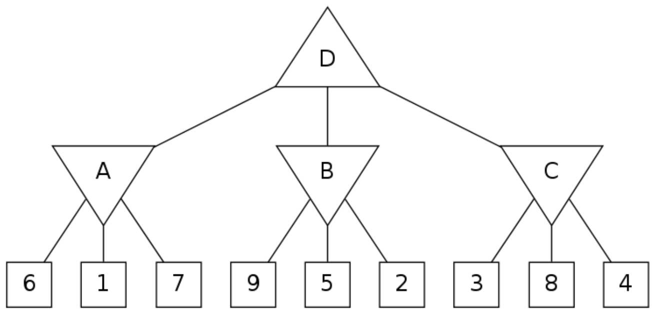
Name: ______________________________ Student ID: __________________

1. (7 points) Minimax

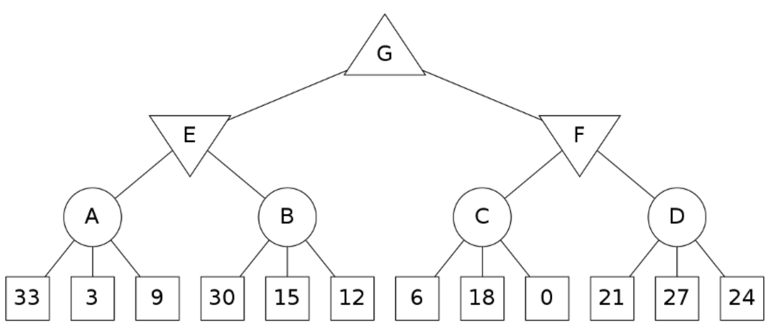
   Consider the zero-sum game tree shown below. Triangles that point up, such as at the top node (root), represent choices for the maximizing player; triangles that point down represent choices for the minimizing player. Outcome values for the maximizing player are listed for each leaf node, represented by the values in squares at the bottom of the tree. Assuming both players act optimally, carry out the minimax search algorithm. Enter the values for the letter nodes in the boxes below the tree.



   A: ____________ B: ____________ C: ____________ D: ____________

2. (8 points) Expectiminimax

   Consider the game tree shown below. As in the previous problem, triangles that point up, such as the top node (root), represent choices for the maximizing player; triangles that point down represent choices for the minimizing player. The circular nodes represent chance nodes in which each of the possible actions may be taken with equal probability. The square nodes at the bottom represent leaf nodes. Assuming both players act optimally, carry out the expectiminimax search algorithm. Enter the values for the letter nodes in the boxes below the tree.

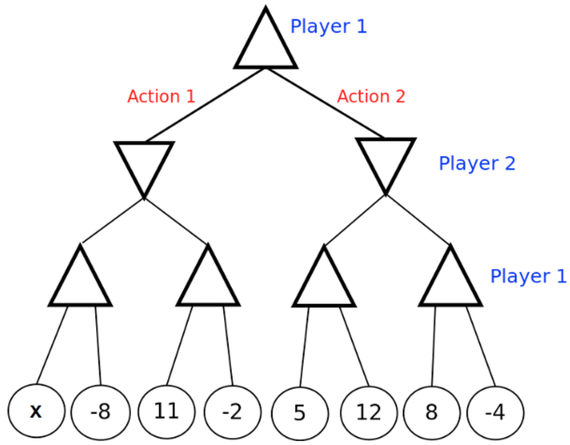


   A: ____________ B: ____________ C: ____________ D: ____________

   E: ____________ F: ____________ G: ____________

3. (12 points) Unknown Leaf Value

Consider the game tree in the figure below, where one of the leaves has an unknown payoff, x. Player 1 moves first, and attempts to maximize the value of the game.



Each of the next 3 questions asks you to write a constraint on x specifying the set of values it can take. Please specify your answer in one of the following forms:

- Write "All" if x can take on all values.
- Write "None" if x has no possible values.
- Use an inequality in the form "x<value", "x>value", or "value1<x< value2" to specify an interval of values. As an example, if you think x can take on all values larger than 16, you should enter "x>16".

(a) Assume Player 2 is a minimizing agent (and Player 1 knows this). For what values of x is Player 1 guaranteed to choose Action 1 for their first move?

Answer. ____________

(b) Assume Player 2 chooses actions at random with each action having equal probability (and Player 1 knows this). For what values of x is Player 1 guaranteed to choose Action 1?

Answer. ____________

(c) Denote the minimax value of the tree as the value of the root when Player 1 is the maximizer and Player 2 is the minimizer. Denote the expectimax value of the tree as the value of the root when Player 1 is the maximizer and Player 2 chooses actions at random (with equal probability). For what values of x is the minimax value of the tree worth more than the expectimax value of the tree?

Answer. ____________

(d) Is it possible to have a game, where the minimax value is strictly larger than the expectimax value? ☐ Yes ☐ No

4. (9 points) Alpha-Beta Pruning

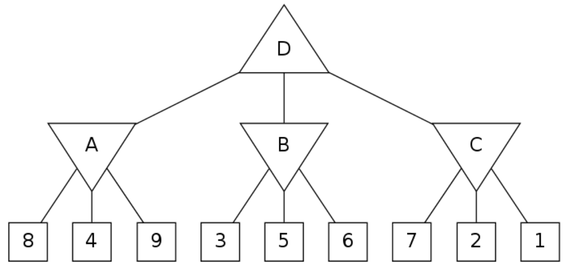


Figure 1: game tree

Consider the game tree shown in Figure 1. Triangles that point up, such as at the top node (root), represent choices for the maximizing player; triangles that point down represent choices for the minimizing player. Assuming both players act optimally, use alpha-beta pruning to find the value of the root node. The search goes from left to right; when choosing which child to visit first, choose the left-most unvisited child. In the blank spaces below, enter the values of the labeled nodes. Then, select the leaf nodes that don't get visited due to pruning.

Hint: Note that the value of a node where pruning occurs is not necessarily the maximum or minimum (depending on which node) of its children. When you prune on conditions $V > \beta$ or $V < \alpha$, assume that the value of the node is V.

A:____________ B: ____________ C: ____________ D: ____________

Check the boxes next to leaf nodes that are not visited due to pruning.

☐ 8 ☐ 4 ☐ 9 ☐ 3 ☐ 5 ☐ 6 ☐ 7 ☐ 2 ☐ 1

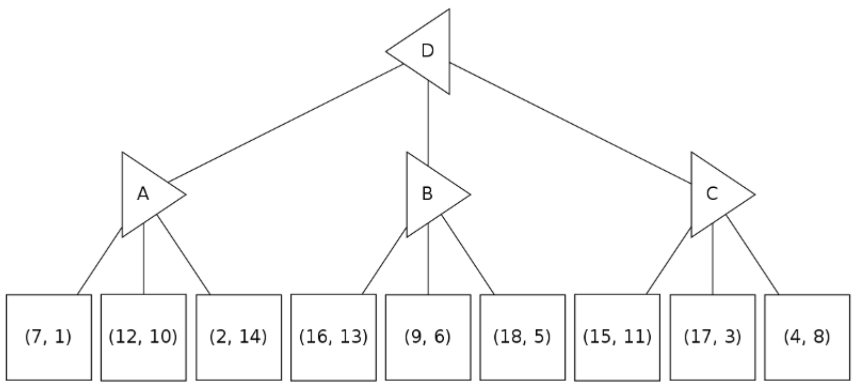
5. (8 points) Non-Zero-Sum Games

(a) The standard minimax algorithm calculates worst-case values in a zero-sum two player game, i.e. a game for which in all terminal states s, the utilities for players A (MAX) and B (MIN) obey $U_A(s) + U_B(s) = 0$. In this zero-sum setting, we know that $U_A(s) = -U_B(s)$, so we can think of player B as simply minimizing U_A.

In this problem, you will consider the non-zero-sum generalization, in which the sum of the two players' utilities are not necessarily zero. The leaf utilities are now written as pairs (U_A, U_B). In this generalized setting, A seeks to maximize U_A, the first component, while B seeks to maximize U_B, the second component.

Consider the non-zero-sum game tree below. Note that left-pointing triangles (such as the root of the tree) correspond to player A, who maximizes the first component of the utility pair, whereas right-pointing triangles (nodes on the second layer) correspond to player B, who maximizes the second component of the utility pair.

Propagate the terminal utility pairs up the tree using the appropriate generalization of the minimax algorithm on this game tree. In case of ties, choose the leftmost child. Select the correct values for the letter nodes below the tree.



Your answer should be in the format (X,Y), where X is the value of Player A and Y is the value of Player B at a node (for example "(7,1)"). Note that the answer check is sensitive to formatting, so be sure to include the parenthesis and do not use any spaces in your answers.

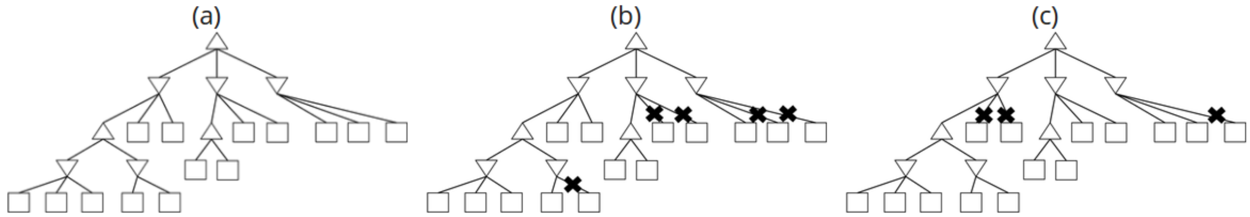
A: __________ B: __________ C: __________ D: __________

(b) In this problem, you will again consider the non-zero-sum generalization, in which the sum of the two players' utilities are not necessarily zero. The leaf utilities are now written as pairs (U_A, U_B). In this generalized setting, A seeks to maximize U_A, the first component, while B seeks to maximize U_B, the second component.

Assume that your generalization of the minimax algorithm calculates a value (U_A^*, U_B^*) for the root of the tree. Assume no utility value for A or for B appears more than once in the terminal nodes (this means there will be no need for tie-breaking). Which of the following statements are true?

- ☐ Assuming A and B both play optimally, player A's outcome is guaranteed to be exactly U_A^*.
- ☐ Assuming A and B both play optimally, player B's outcome is guaranteed to be exactly U_B^*.
- ☐ Assuming B plays sub-optimally (but A plays optimally), A's outcome is guaranteed to be at least U_A^*.

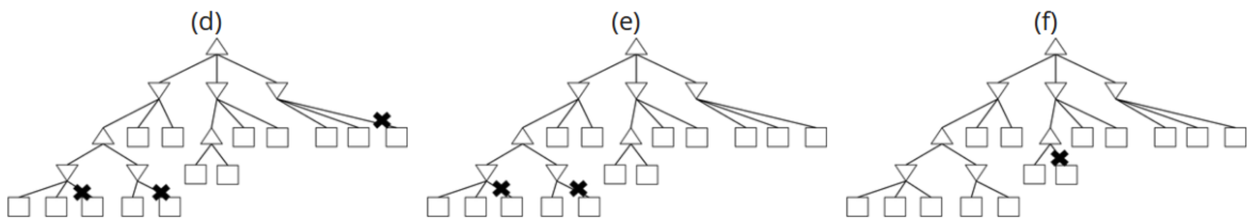
6. (10 points) (a) Assume we run α - β pruning, expanding successors from left to right, on a game with tree as shown in Figure (a) below.



Which of the following statements are true?

- ☐ There exists an assignment of utilities to the terminal nodes such that no pruning will be achieved (shown in Figure (a)).
- ☐ There exists an assignment of utilities to the terminal nodes such that the pruning shown in Figure (b) will be achieved.
- ☐ There exists an assignment of utilities to the terminal nodes such that the pruning shown in Figure (c) will be achieved.
- ☐ None of the above.

(b)



- ☐ There exists an assignment of utilities to the terminal nodes such that the pruning shown in Figure (d) will be achieved.
- ☐ There exists an assignment of utilities to the terminal nodes such that the pruning shown in Figure (e) will be achieved.
- ☐ There exists an assignment of utilities to the terminal nodes such that the pruning shown in Figure (f) will be achieved.
- ☐ None of the above.

7. (9 points) SUBOPTIMAL STRATEGIES

(a) Player MAX and player MIN are playing a zero-sum game with a finite number of possible moves. MAX calculates the minimax value of the root to be M. You may assume that at every turn, each player has at least 2 possible actions. You may also assume that a different sequence of moves will always lead to a different score (i.e., no two terminal nodes have the same score). Which of the following statements are true?

- ☐ Assume MIN is playing sub-optimally at every turn, but MAX does not know this. The outcome of the game could be larger than M (i.e. at least as good a MAX).
- ☐ Assume MIN is playing sub-optimally at every turn. If MAX plays according to the minimax strategy, the outcome of the game could be less than M.

(b) For this question, assume that MIN is playing randomly (with a uniform distribution) at every turn, and MAX knows this. Which of the following statements are true?

- ☐ There exists a policy for MAX such that MAX can guarantee a better outcome than M.
- ☐ There exists a policy for MAX such that MAX's expected outcome is better than M.
- ☐ To maximize his or her expected outcome, MAX should play according to the minimax strategy (i.e. the strategy that assumes MIN is playing optimally).

(c) Which of the following statements are true?

- ☐ Assume MIN is playing sub-optimally at every turn. MAX following the minimax policy will guarantee an outcome that is better than or equal to M.
- ☐ Assume MIN is playing sub-optimally at every turn, and MAX knows exactly how MIN will play. There exists a policy for MAX to guarantee an outcome that is better than or equal to M.

8. (9 points) RATIONALITY OF UTILITIES

(a) Consider a lottery L = [0.2 , A; 0.3, B; 0.4, C; 0.1, D], where the utility values of each of the outcomes are U(A) = 1, U(B) = 3, U(C) = 5, U(D) = 2. What is the utility of this lottery, U(L)?

Answer. ____________

(b) Consider a lottery L1 = [0.5, A; 0.5, L2], where U(A) = 4, and L2 = [0.5, X; 0.5, Y] is a lottery, and U(X) = 4, U(Y) = 8. What is the utility, U(L1), of the the first lottery?

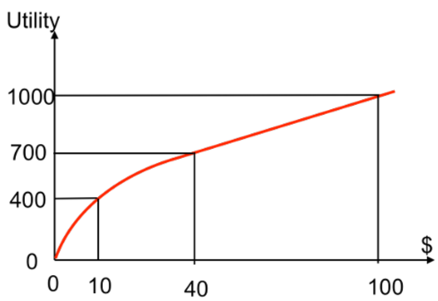
Answer. ____________

(c) Assume $A \succ B$, $B \succ L$, where $L = [0.5, C; 0.5, D]$, and $D \succ A$. Assuming rational preferences, which of the following statements are guaranteed to be true?

☐ $A \succ L$ ☐ $A \succ C$ ☐ $A \succ D$ ☐ $B \succ C$ ☐ $B \succ D$

9. (6 points) CERTAINTY EQUIVALENT VALUES

Consider the utility function shown below.



Under the above utility function, what is the certainty equivalent monetary value in dollars ($) of the lottery [0.6, $0 ; 0.4, $100]?

I.e., what is X such that $U(\$X) = U([0.6, \$0; 0.4, \$100])$?

[*Hint.* Keep in mind that $U([p, A; 1-p, B])$ is not equal to $U(pA + (1-p)B)$.]

Answer. ____________

10. (14 points) Preferences and Utilities

Our Pacman board now has food pellets of 3 different sizes - pellet $P_1$of radius 1, P_2 of radius 2 and P_3 of radius 3. In different moods, Pacman has different preferences among these pellets. In each of the following questions, you are given Pacman's preference for the different pellets. From among the options pick the utility functions that are consistent with Pacman's preferences, where each utility function $U(r)$ is given as a function of the pellet radius r, and is defined over non-negative values of r.

(a) $P_1 \sim P_2 \sim P_3$

☐ $U(r) = 0$ ☐ $U(r) = 3$ ☐ $U(r) = r$ ☐ $U(r) = 2r + 4$ ☐ $U(r) = -r$
☐ $U(r) = r^2$ ☐ $U(r) = -r^2$ ☐ $U(r) = \sqrt{r}$ ☐ $U(r) = -\sqrt{r}$
☐ Irrational preferences!

(b) $P_1 \prec P_2 \prec P_3$

☐ $U(r) = 0$ ☐ $U(r) = 3$ ☐ $U(r) = r$ ☐ $U(r) = 2r + 4$ ☐ $U(r) = -r$
☐ $U(r) = r^2$ ☐ $U(r) = -r^2$ ☐ $U(r) = \sqrt{r}$ ☐ $U(r) = -\sqrt{r}$
☐ Irrational preferences!

(c) $P_1 \succ P_2 \succ P_3$

☐ $U(r) = 0$ ☐ $U(r) = 3$ ☐ $U(r) = r$ ☐ $U(r) = 2r + 4$ ☐ $U(r) = -r$
☐ $U(r) = r^2$ ☐ $U(r) = -r^2$ ☐ $U(r) = \sqrt{r}$ ☐ $U(r) = -\sqrt{r}$
☐ Irrational preferences!

(d) $(P_1 \prec P_2 \prec P_3)$ and $(P_2 \prec$ (50-50 lottery among P_1 and P_3))

☐ $U(r) = 0$ ☐ $U(r) = 3$ ☐ $U(r) = r$ ☐ $U(r) = 2r + 4$ ☐ $U(r) = -r$
☐ $U(r) = r^2$ ☐ $U(r) = -r^2$ ☐ $U(r) = \sqrt{r}$ ☐ $U(r) = -\sqrt{r}$
☐ Irrational preferences!

(e) $P_1 \succ P_2 \succ P_3$ and $P_2 \succ$ (50-50 lottery among P_1 and P_3)

☐ $U(r) = 0$ ☐ $U(r) = 3$ ☐ $U(r) = r$ ☐ $U(r) = 2r + 4$ ☐ $U(r) = -r$
☐ $U(r) = r^2$ ☐ $U(r) = -r^2$ ☐ $U(r) = \sqrt{r}$ ☐ $U(r) = -\sqrt{r}$
☐ Irrational preferences!

(f) $P_1 \prec P_2 \prec P_3$ and (50-50 lottery among P_2 & P_3) $\prec$ (50-50 lottery among P_1 & P_2)

☐ $U(r) = 0$ ☐ $U(r) = 3$ ☐ $U(r) = r$ ☐ $U(r) = 2r + 4$ ☐ $U(r) = -r$
☐ $U(r) = r^2$ ☐ $U(r) = -r^2$ ☐ $U(r) = \sqrt{r}$ ☐ $U(r) = -\sqrt{r}$
☐ Irrational preferences!

(g) Which of the following would be a utility function for a risk-seeking preference? That is, for which utility(s) would Pacman prefer entering a lottery for a random food pellet, with expected size s, over receiving a pellet of size s?

☐ $U(r) = 0$ ☐ $U(r) = 3$ ☐ $U(r) = r$ ☐ $U(r) = 2r + 4$ ☐ $U(r) = -r$
☐ $U(r) = r^2$ ☐ $U(r) = -r^2$ ☐ $U(r) = \sqrt{r}$ ☐ $U(r) = -\sqrt{r}$