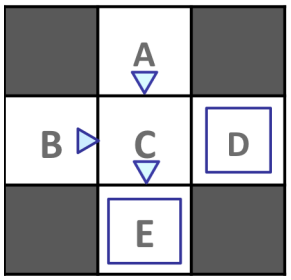
Name: _____ Student ID: _____

1. (5 points) (Model-Based RL: Grid)

Input Policy π



Assume: γ = 1

Observed Episodes (Training)

Episode 1

A, south, C, -1
C, south, E, -1
E, exit, x, +10

Episode 2

B, east, C, -1
C, south, D, -1
D, exit, x, -10

Episode 3

B, east, C, -1
C, south, E, -1
E, exit, x, +10

Episode 4

A, south, C, -1
C, south, E, -1
E, exit, x, +10

What model would be learned from the above observed episodes?

(a) T(A, south, C) = _____

(b) T(B, east, C) = _____

(c) T(C, south, E) = _____

(d) T(C, south, D) = _____

2. (22 points) (Model-Based RL: Cycle)

Consider an MDP with 3 states, A, B and C; and 2 actions Clockwise and Counterclockwise. We do not know the transition function or the reward function for the MDP, but instead, we are given samples of what an agent experiences when it interacts with the environment (although, we do know that we do not remain in the same state after taking an action). In this problem, we will first estimate the model (the transition function and the reward function), and then use the estimated model to find the optimal actions.

To find the optimal actions, model-based RL proceeds by computing the optimal V or Q value function with respect to the estimated T and R. This could be done with any of value iteration, policy iteration, or Q-value iteration. Last week you already solved some exercises that involved value iteration and policy iteration, so we will go with Q value iteration in this exercise.

Consider the following samples that the agent encountered.

Discount Factor, $\gamma = 0.5$

s	a	s'	T(s,a,s')	R(s,a,s')
A	Clockwise	B	M	N
A	Clockwise	C	O	P
A	Counterclockwise	B	0.400	0.000
A	Counterclockwise	C	0.600	-8.000
B	Clockwise	A	0.800	-3.000
B	Clockwise	C	0.200	0.000
B	Counterclockwise	A	0.800	-10.000
B	Counterclockwise	C	0.200	0.000
C	Clockwise	A	0.600	0.000
C	Clockwise	B	0.400	6.000
C	Counterclockwise	A	0.200	0.000
C	Counterclockwise	B	0.800	-8.000

Figure 1: Table for T(s,a,s') and R(s,a,s').

s	a	s'	r
A	Clockwise	B	0.0
A	Clockwise	B	0.0
A	Clockwise	B	0.0
A	Clockwise	C	-10.0
A	Clockwise	C	-10.0
A	Counterclockwise	C	-8.0
A	Counterclockwise	C	-8.0
A	Counterclockwise	B	0.0
A	Counterclockwise	B	0.0
A	Counterclockwise	C	-8.0

s	a	s'	r
B	Clockwise	A	-3.0
B	Clockwise	A	-3.0
B	Clockwise	A	-3.0
B	Clockwise	A	-3.0
B	Clockwise	C	0.0
B	Counterclockwise	A	-10.0
B	Counterclockwise	A	-10.0
B	Counterclockwise	A	-10.0
B	Counterclockwise	A	-10.0
B	Counterclockwise	C	0.0

s	a	s'	r
C	Clockwise	A	0.0
C	Clockwise	B	6.0
C	Clockwise	B	6.0
C	Clockwise	A	0.0
C	Clockwise	A	0.0
C	Counterclockwise	B	-8.0
C	Counterclockwise	B	-8.0
C	Counterclockwise	B	-8.0
C	Counterclockwise	A	0.0
C	Counterclockwise	B	-8.0

(a) We start by estimating the transition function, T(s,a,s') and reward function R(s,a,s') for this MDP. Fill in the missing values in the table for T(s,a,s') and R(s,a,s') shown in Figure 1.

M = _____ N = _____ O = _____ P = _____

(b) Now we will run Q-iteration using the estimated T and R functions. The values of Q_k(s,a), are given in the table below.

	A	B	C
Clockwise	-4.24	-3.76	0.72
Counterclockwise	-4.56	-9.36	-7.76

Fill in the values for Q_{k+1}(s, a).

i. Q(A, clockwise) = _____

ii. Q(A, Counterclockwise) = _____

iii. Q(B, clockwise) = _____

iv. Q(B, Counterclockwise) = _____

v. Q(C, clockwise) = _____

vi. Q(C, Counterclockwise) = _____

(c) Suppose Q-iteration converges to the following Q∗ function, Q∗(s,a).

	A	B	C
Clockwise	-5.399	-4.573	-0.134
Counterclockwise	-5.755	-10.173	-8.769

What is the optimal action, either Clockwise or Counterclockwise, for each of the states?

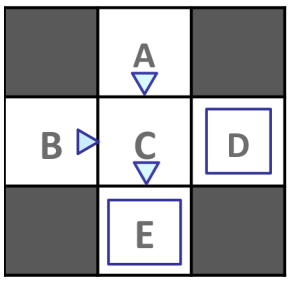
A: ☐ Clockwise ☐ Counterclockwise

B: ☐ Clockwise ☐ Counterclockwise

C: ☐ Clockwise ☐ Counterclockwise

3. (10 points) (Direct Evaluation)

Input Policy π



Assume: γ = 1

Observed Episodes (Training)

Episode 1

A, south, C, -1
C, south, E, -1
E, exit, x, +10

Episode 2

B, east, C, -1
C, south, D, -1
D, exit, x, -10

Episode 3

B, east, C, -1
C, south, E, -1
E, exit, x, +10

Episode 4

A, south, C, -1
C, south, E, -1
E, exit, x, +10

What are the estimates for the following quantities as obtained by direct evaluation:

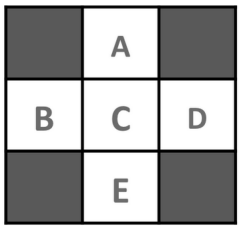
$\hat{V}^{\pi}$(A) = _____ $\hat{V}^{\pi}$(B) = _____ $\hat{V}^{\pi}$(C) = _____

$\hat{V}^{\pi}$(D) = _____ $\hat{V}^{\pi}$(E) = _____

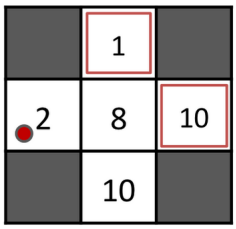
4. (10 points) (Temporal Difference Learning)

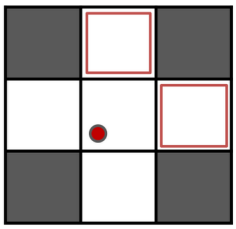
Consider the gridworld shown below. The left panel shows the name of each state A through E. The middle panel shows the current estimate of the value function V^π for each state. A transition is observed, that takes the agent from state B through taking action east into state C, and the agent receives a reward of -2. Assuming $\gamma = 1$, $\alpha = 1/2$, what are the value estimates after the TD learning update? (note: the value will change for one of the states only)

States



Observed Transition: B, east, C, -2





Assume: $\gamma = 1$, $\alpha = 1/2$

$$V^\pi(s) \leftarrow (1-\alpha)V^\pi(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^\pi(s')\right]$$

$\hat{V}^\pi$(A) = _____ $\hat{V}^\pi$(B) = _____ $\hat{V}^\pi$(C) = _____

$\hat{V}^\pi$(D) = _____ $\hat{V}^\pi$(E) = _____

5. (12 points) (Model-Free RL: Cycle)

Consider an MDP with 3 states, A, B and C; and 2 actions Clockwise and Counterclockwise. We do not know the transition function or the reward function for the MDP, but instead, we are given with samples of what an agent actually experiences when it interacts with the environment (although, we do know that we do not remain in the same state after taking an action). In this problem, instead of first estimating the transition and reward functions, we will directly estimate the Q function using Q-learning.

Assume, the discount factor, γ is 0.5 and the step size for Q-learning, α is 0.5.

Our current Q function, Q(s,a), is as follows.

	A	B	C
Clockwise	1.501	-0.451	2.73
Counterclockwise	3.153	-6.055	2.133

The agent encounters the following samples.

s	a	s'	r
A	Counterclockwise	C	8.0
C	Counterclockwise	A	0.0

Process the samples given above. Below fill in the Q-values after both samples have been accounted for.

Q(A, clockwise) = _____

Q(A, Counterclockwise) = _____

Q(B, clockwise) = _____

Q(B, Counterclockwise) = _____

Q(C, clockwise) = _____

Q(C, Counterclockwise) = _____

6. (5 points) (Q-Learning Properties) In general, for Q-Learning to converge to the optimal Q-values...

- ☐ It is necessary that every state-action pair is visited infinitely often.
- ☐ It is necessary that the learning rate α (weight given to new samples) is decreased to 0 over time.
- ☐ It is necessary that the discount γ is less than 1/2.
- ☐ It is necessary that actions get chosen according to argmax_a Q(s,a).

7. (12 points) (Exploration and Exploitation)

(a) For each of the following action-selection methods, indicate which option describes it best.

i. With probability p, select argmax$_a$ Q(s,a). With probability 1-p, select a random action. p = 0.99.

- ☐ Mostly exploration
- ☐ Mostly exploitation
- ☐ Mix of both

ii. Select action a with probability

$$P(a \mid s) = \frac{e^{Q(s,a)/\tau}}{\sum_{a'} e^{Q(s,a')/\tau}}$$

where τ is a temperature parameter that is decreased over time.

- ☐ Mostly exploration
- ☐ Mostly exploitation
- ☐ Mix of both

iii. Always select a random action.

- ☐ Mostly exploration
- ☐ Mostly exploitation
- ☐ Mix of both

iv. Keep track of a count, K_{sa}, for each state-action tuple, (s,a), of the number of times that tuple has been seen and select argmax$_a$ (Q(s,a) - K_{sa}).

- ☐ Mostly exploration
- ☐ Mostly exploitation
- ☐ Mix of both

(b) Which of the above method(s) would be advisable to use when doing Q-Learning?

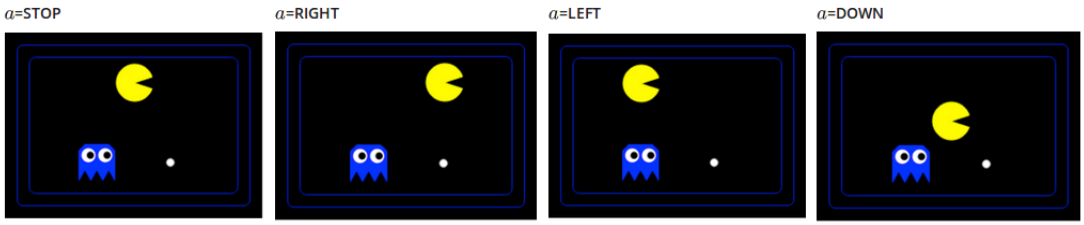
☐ A ☐ B ☐ C ☐ D

8. (6 points) (Feature-Based Representation: Actions)

A Pacman agent is using a feature-based representation to estimate the Q(s, a) value of taking an action in a state, and the features the agent uses are:

f_0 = 1/(Manhattan distance to closest food + 1)
f_1 = 1/(Manhattan distance to closest ghost + 1)

The images below show the result of taking actions STOP, RIGHT, LEFT, and DOWN from a state A. The feature vectors for each action are shown below each image. For example, the feature representation f(s=A, a=STOP) = [1/4, 1/4].



The agent picks the action according to argmax_a Q(s,a) = w^T f(s,a) = w_0 f_0(s,a) + w_1 f_1(s,a), where the features f_i(s,a) are as defined above, and w is a weight vector.

(a) Using the weight vector w = [0.2, 0.5], which action, of the ones shown above, would the agent take from state A?

- ☐ STOP
- ☐ RIGHT
- ☐ LEFT
- ☐ DOWN

(b) Using the weight vector w = [0.2, -1], which action, of the ones shown above, would the agent take from state A?

- ☐ STOP
- ☐ RIGHT
- ☐ LEFT
- ☐ DOWN

9. (18 points) (Feature-Based Representation: Update)

Consider the following feature based representation of the Q-function:

$Q(s,a) = w_1\ f_1(s,a) + w_2\ f_2(s,a)$ with

$f_1(s,a) = 1$ / (Manhattan distance to nearest dot after executing action a in state s)

$f_2(s,a) =$ (Manhattan distance to nearest ghost after executing action a in state s)

(a) Assume $w_1 = 1$, $w_2 = 10$. For the state s shown below, find the following quantities. Assume that the red and blue ghosts are both sitting on top of a dot.

Q(s, West) = _____ Q(s, South) = _____

Based on this approximate Q-function, which action would be chosen:

☐ West ☐ South

(b) Assume Pac-Man moves West. This results in the state s' shown below. Pac-Man receives reward 9 (10 for eating a dot and -1 living penalty).

Q(s', West) = _____ Q(s', East) = _____

What is the sample value (assuming $\gamma = 1$)?

sample = $r + \gamma \max\{Q(s', a') : a' \in \text{Actions}\}$ = _____

(c) Now let's compute the update to the weights. Let $\alpha = 0.5$.

difference = $r + \gamma \max\{Q(s',a') : a' \in \text{Actions}\} - Q(s,a)$ = _____

$w_1 \leftarrow w_1 + \alpha$ (difference) $f_1(s,a)$ = _____

$w_2 \leftarrow w_2 + \alpha$ (difference) $f_2(s, a)$ = _____