Comprehensive Exam Syllabus

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1. Model Theory
    - Compactness theorem; proof using ultraproducts
    - Löwenheim-Skolem theorems
    - Diagrams
    - Types
    - Existence of κ-saturated structures
    - κ-categoricity
    - The Łoś-Vaught test
    - Elimination of quantifier arguments
    - Ehrenfeucht-Fraisse method
2. Commutative Algebra
    - Rings and modules
    - Radicals of ideals
    - Exact sequences of modules
    - Localization
    - Primary decomposition of ideals
    - Noetherian rings
    - Artin rings
    - Integral dependence
3. Category Theory
    - Categories
    - Functors
    - Natural transformations
    - Monomorphisms, epimorphisms
    - Categories of functors
    - Duality of categories
    - Representable functors
    - Yoneda's Lemma
    - Universal constructions: product, coproduct, limit
    - Adjoint functors
    - Equivalence of categories
    - Exactness/preservation of limits and colimits
    - Freyd's Adjoint Functor Theorem
    - Monads