Teaching Philosophy, Mentoring, and Teaching Experience

William DeMeo

1. Introduction

After five years of teaching math and computer science in college classrooms, my motivation to teach continues to grow stronger. I have had the good fortune of working in math departments that place a heavy emphasis on teaching, and this experience has given me a deeper appreciation and respect for our responsibility to pass on knowledge to future generations. It has also taught me many lessons about how to teach well.

Since 2012 I have taught two math courses every semester, and have taken every opportunity available to attend teacher training sessions, read student evaluations, and solicit feedback from both students and peers. This has given me many occasions on which to reflect on my beliefs about education in general and effective teaching in particular. I have come to realize that my mission as a teacher revolves around three core principles, each of which has the main goal of engaging the student:

(1) **leadership,** to engage students by demonstrating a deep appreciation and passion for the subject;
(2) **communication,** to engage students by connecting with them at an appropriate level;
(3) **pedagogy,** to engage students by developing a set of best practices that stimulate them mentally and facilitate understanding.

The sections that follow consider each of these core principles in more detail and describe how they have come to form the core of my teaching philosophy.

2. Core Principles

2.1. **Leadership.** My passion for the subject being taught must be apparent to the students from the first day of class. This is essential for establishing credibility and, more importantly, for convincing the students that the subject is truly worthy of their attention. Such enthusiasm is contagious and helps motivate students to engage in active listening, participation, and learning. I have found that convincing students of my passion for whatever subject I teach is actually the easiest part of my job as an educator, and each semester my teaching evaluations reveal that my love of the subject is both obvious and contagious.

2.2. **Communication.** I strive to have an open, two-way dialog of friendly yet respectful communication in the classroom. It is important that all students—regardless of background, race, nationality, or gender—feel welcomed and encouraged to participate in a safe environment. Furthermore, it is important to let students know my door is always open for them to come to me with any issues they might encounter in class, about math or their academic and professional lives.

To accomplish these goals, I have developed a variety of strategies based on essential educational principles of active learning, diversity and sensitivity training, planning, presentation and assessment. Some of these techniques depend vitally on our awareness of student backgrounds, interests, and abilities, as we discuss in the next section.

2.3. **Learner diversity.** As we enjoy larger numbers of intelligent and eager learners from more and more diverse backgrounds, from minority groups, both domestic and international, we must be cognizant of both the learning style of individuals and the cultural diversity of the whole class. Using this as our guide, we can customize our instruction and assessment accordingly. Since a variety of learning styles are likely to be represented in any modern classroom, it is important to alternate frequently between instruction and assessment. Without frequent learner feedback and participation in every class, it is impossible to foster a comfortable atmosphere of communication and mutual understanding, and without that, we cannot assess the level of student engagement and we cannot know the degree to which students have processed the material covered.

3. Pedagogy

3.1. **Cycles and Iterations.** One effective method of teaching is to cycle through various modes of interaction with the students: presentation, participation, feedback and assessment. Each of these modes should itself vary throughout the class and the semester. Presentation mode, for example, can vary across a number of dimensions. First, the medium of presentation should alternate between speech/audible and written/graphical/visual to accommodate differences of learners.

Second, the level of detail at which material is presented should proceed along a spectrum from general to specific. This may depend on the subject matter, but I find it helpful to outline and motivate the entire course in the first lecture. Thereafter, roughly the first quarter of the course may be explained and motivated. Finally, each subsection of the first quarter is described and motivated, still using fairly general terms. Finally each section may be covered in detail.

Proceeding along such a path, from the general to the specific, reinforces material by reiterating the same idea at various levels of abstraction. While watching Bob Harper's lectures at CMU, I learned that this strategy has a name: "iterative deepening." Besides the obvious reinforcement effect that repetition has, another positive outcome of iterative deepening is that learners leave the course with a better understanding of how each topic fits into the greater scheme of the course. It is this broad view of the course, rather than the details covered in a particular section, that is likely to remain with the students long after the course has ended.

3.2. **The Learner's Role.** Another effective way to teach a classroom of diverse learners, is to help the students stay engaged by "making the subject their own." One can accomplish this by giving them some role in determining what they will learn and how they will learn it. If students are given the chance to arrive at an understanding of some problem or topic by relying on their own ingenuity and deriving their own methods for solving the problem, then, even if the problem remains unsolved, the student's appreciation of the problem or topic is typically much deeper than if they had listened passively to a lecture. This sort of learning can be facilitated in the classroom by providing just the right amount of guidance so that learners can reach correct conclusions. However, determining the appropriate level of guidance depends crucially on knowing the backgrounds and abilities of the learners.

Over the past few years I have experimented with this approach to great effect. When introducing a new topic, I often try to provide just an outline of the basic ideas, and a precise description of our objective and then ask students to think, to grow accustomed to

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seeking answers themselves, to not wait for more guidance or answers to come from someone else. Often I leave out basic terminology and help students develop what seems like the most natural definitions and notation that might help to reach the goal. We then embark upon a journey of discovering the theory together, by considering the class's collective contributions, and making the necessary adjustments. In some cases, the students do not need very much help to arrive at sensible and useful definitions. Before long our discussion can be honed into a more formal development of the important principles, theorems, and methods.

To give a simple example, I was recently impressed by a classroom of freshman and sophomore computer science majors who, with little guidance, had no trouble formulating set-theoretic definitions of the natural numbers, the integers, ordered pairs, and n-tuples, despite having had no prior exposure to set theory. The notions of equivalence relation and partial ordering also seemed quite natural to most of them. Thereafter, when the class participated in the lecture and as we solved example problems, employing the definitions that *they* "invented," the students seemed not only more energized that usual, but also more comfortable with the material—not surprisingly, since they had a hand in its development. Again, the goal is for students to *make the subject their own.*

This experience has given me a new perspective on effective teaching, and I have learned that with only minor adjustments in presentation style, some topics can be treated in a manner that helps students gain a deeper understanding than if they passively listen to a lecture. Furthermore, with very careful planning and time management, this approach doesn't necessarily require significantly more class time.

Providing students with the main ideas and objectives and encouraging them to develop their own methods and come to their own conclusions is not always appropriate for every topic at every level. However, there is strong evidence suggesting that, at least at the elementary school level, a hands-off approach can have quite miraculous outcomes.[1] At the college and graduate levels there is a standard curriculum that must be covered in a limited amount of time, and an entirely hands-off approach is infeasible and inappropriate. However, the benefits of class participation can be tremendous and must not be ignored at any level.

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[1]See, for example, http://www.wired.com/business/2013/10/free-thinkers/

4. MENTORING EXPERIENCE

Among my most gratifying accomplishments as an educator has been successfully engaging students in math and computer science research. The following subsections highlight some of these activities.

4.1. **Research Experiences for Undergraduates (REU).**

In the summer of 2016 at Iowa State, Cliff Bergman and I co-mentored a bright and precocious math major from University of Rochester named Charlotte Aten. It was quickly apparent that Charlotte was not satisfied with anything less that most general and elegant solution to a given problem. It was this taste for generality that attracted her to universal algebra and led her to Ames. It turned out, however, that even universal algebra was not general enough for Charlotte, so I suggested we take up category theory. This was a good fit and we spent much of the summer working through the basics of category theory up to the Yoneda Lemma. Consequently, Charlotte managed to accomplish one of her goals, which was to work out a generalized version of Cayley's Theorem for algebraic and relational structures.

4.2. **TypeFunc Research Group.**

At Iowa State I founded a research group called Type-Func (github.com/TypeFunc) which convened weekly during the academic year. In its first year, the group consisted of four undergraduate students—Paul Lubberstedt (math major), Hunter Praska (cs major), Hyeyoung Shin (cs major), and Joshua Thompson (math major)—and the goal was to generate interest in some approachable open research problems in areas related to algebra, logic, complexity and theoretical computer science, and to guide the students' reading and research activities in these areas.

There were some student presentations in spring 2015. Thompson presented proofs of some known results; he showed that the Law of Excluded Middle (LEM) is not refutable in Intuitionistic Propositional Logic, and he proved Diaconescu's Theorem (Choice implies LEM). Lubberstedt gave a presentation on the meaning of "judgment" in intuitionistic logic. I gave demonstrations of the Coq proof assistant, which motivated two of the students (Praska and Shin) to learn Coq by working through "Software Foundations" by Pierce, et al [PdAC+16]. Shin has since become an expert in Coq after working her way through most of the Software Foundations book, and then using the proof assistant to model a simple imperative programming language.

In late spring 2016 I taught the TypeFunc group about the Constraint Satisfaction Problem (csp) research I had been engaged in with Cliff Bergman. I explained the algebraic approach to the so called "csp dichotomy conjecture." The students seemed fascinated by this topic and, by the end of the semester, were very eager to attend a related workshop at Vanderbilt called "Open Problems in Universal Algebra" [Sha15].

4.3. **Mentoring at the University of Colorado, Boulder.**

During the Spring 2018 semester I co-lead (with Keith Kearnes) a reading course based on the book *Algebraic Theories*, by Jiri Adamek [ARV11]. This course was attended by a very talented graduate student named Paul Lassard. I also had the opportunity to serve on a number of Ph.D. prelim exam committees (for Jordan DuBeau, Ali Latfi, Athena Sparks, and Michael Wheeler), one Ph.D. thesis defense committee (for Jeffrey Shriner), and one honors thesis defense committee (for Zetong Xue).

4.4. **Undergraduate Thesis Advising at Iowa State University.** At Iowa State, I advised and collaborated with Joshua Thompson, one of our brightest math majors, and a student in the Honors College. Thompson worked with me on the project of determining which algebraic structures have "absorbing subuniverses," and he (along with Praska and Shin, of the TypeFunc group) accompanied Cliff Bergman and me to the Shanks Workshop on "Open Problems in Universal Algebra" at Vanderbilt in 2015 [Sha15]. Thompson has said this was a "life-changing experience."

Josh Thompson's senior thesis research was a great success and he solved the open problem that I had presented him with. Specifically, Thompson identified all proper absorbing subuniverses of the 4-element algebras that Bergman and I had been studying as part of our algebraic CSP project.

4.5. **Undergraduate Thesis Advising at University of South Carolina.** I advised and collaborated with Matthew Corley, a talented undergraduate student in the Honors College, who worked with me on a project involving nonabelian harmonic analysis of musical signals. Professor Reginald Bain (Director of the Experimental Music Studio in USC's Music Department) joined this project as a co-mentor. I guided Corley though the process of developing a formal project proposal, research schedule and budget, which we submitted through the university's grant office. The proposal earned Corley a prestigious *Magellan Scholarship Grant* to support his work on the project. I gave a preliminary report on this collaboration in a talk entitled "What does a nonabelian group sound like?" at the MAA session *At the Intersection of Mathematics and the Arts*, which took place at the Joint Math Meetings in Baltimore, January, 2014.[2]

4.6. **Pi Mu Epsilon and High School Math Contests.** For two years at University of South Carolina I served as a faculty mentor for Pi Mu Epsilon and the Gamecock Math Club, and served on the High School Math Contest Committee charged with creating the written portion of the South Carolina High School Math Contest exam (2013, 2014).

4.7. **Mentoring at the University of Hawaii.** As a graduate student, I had the opportunity to serve as mathematics mentor to the undergraduates of the Bio-Math Program—a joint program with students from the Biology, Mathematics, and Zoology Departments. These students worked on the project of cataloging sounds of marine life, and especially fish species, with the goal of creating a comprehensive library of audio samples that could be used by zoologists for their research. Since I have a great deal of experience in digital signal processing and dsp software, my primary roles were teaching the students some signal processing theory and helping them get up to speed with the software tools required to achieve their goals.

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[2]An abstract for the MAA talk is available at:
http://www.ams.org/amsmtgs/2160_abstracts/1096-c5-2578.pdf
The project web page resides at http://soundmath.github.io/GroupSound.

5. CURRICULUM DEVELOPMENT

5.1. **Lean in the Classroom.** I was lucky enough to teach a graduate course in Model Theory during the Spring 2018 semester at University of Colorado, Boulder. This year I am teaching Abstract Algebra in the Fall of 2018, and Discrete Math (which serves as an "introduction to proofs" class) in both the Fall and Spring semesters of the 2018/19 academic year. My experience with these courses has convinced me that the *Lean Theorem Proving Language* has an important role to play in the teaching logic and proof to undergraduates.

In the Fall 2018, I presented logic to my first- and second-year undergraduate students using Gentzen style natural deduction almost exclusively. (Of course, the students were also exposed to classical logic and truth tables, and I taught them the not-so-subtle differences between classical and constructive reasoning principles.) Because natural deduction involves only a very small set of concrete derivation rules, and essentially no axioms, it is ideal for getting the students up and proving with first-order logic. The students seem more confident and satisfied with their proofs than the students in previous courses, when the treatment was more traditional, based entirely on set theory, truth tables, and proof sequences.

Another benefit of using Gentzen style natural deduction was that it enabled the students to better appreciate *soundness* and *completeness*. It seems easier to explain these concepts to students who have some experience building proof derivation trees (that are purely syntactic) on the one hand, while on the other hand having some experience with truth values and what it means for a formula to be true in a particular model.

Evidently, Paul Taylor has witnessed a similar phenomenon, as he descirbes in [Tay99], where he refers to "proof boxes" (essentially natural deduction diagrams).

> "I have seen logic introduced to first year undergraduates both in the form of truth-assignments and using proof boxes, and firmly believe that the box method is preferable... it is a formal version of the way mathematicians actually reason, even if they claim to use Boolean algebra when asked."

About half of the students in my undergraduate class were computer science majors, and most of them have had some prior programming experience. So, I decided to introduce Lean in the classroom in order to keep the students more engaged and to further develop their logical agility and reinforce their understanding of the reciprocal roles played by the introduction and elimination rules of natural deduction. At the same time, I did not want to alienate any students who had no experience with, and possibly some aversion to, computer programming. As a compromise I introduced Lean in the classroom via team homework assignments. As it turns out, their Lean proofs were among their best proofs! They became quite skilled at constructing formal proofs in Lean, and their confidence and understanding of the analogous paper-and-pencil proofs seemed much stronger after having worked with the proof assistant.

Other educators have reported similar successful outcomes with proof assistants in a classroom setting [Nip12, Pie09]. Kevin Buzzard at Imperial College London has the ambitious goal of formalizing the entire undergraduate mathematics curriculum in Lean [Buz18]. These circumstances point to a future where computer-aided theorem proving tools will be routinely used, resulting in better teaching, deeper student understanding, and ultimately future generations of scientists who are more productive and reliable.

5.2. **Colorado Lean User's Group.** During my first year as a postdoc at CU Boulder, I started the *Colorado Lean User's Group* with the goal of encouraging graduate students in math and computer science to learn how to conduct their work in the Lean proof assistant. This effort centered around weekly hour-long meetings in the CU *Programming Languages and Verification Lab* conference room with about five graduate students. We used the hour to learn about and discuss Lean. By the end of the year, some of us had gained a fair amount of expertise in Lean and we are now using the language to support our research projects.

5.3. **Short Course on Mathematical Foundations of Computing (MFC).** During my time as a visiting assistant professor at University of Hawaii, I started the "Hawaii MFC Circle," the Hawaii Chapter of TypeFunc (see Section 4.2). With help from some colleagues—Dusko Pavlovic in Information and Computer Science and Bjørn Kjos-Hanssen in Math—I developed a short course on the Mathematical Foundations of Computing Science that took place in April of 2017. Modelled on the successful European summer schools in theoretical computer science, the course covered a subset of the following topics: Lambda Calculus, Type Theory, Functional Programming and Dependent Types, Automated Theorem Proving in Coq..

The enthusiastic response to the initial proposal from graduate students in the Math and Computer Science Departments—and even one student at the Institute for Astronomy—surpassed my expectations. Within the first 24 hours, over a dozen students expressed a desire to take the course, and some even said they had been waiting for this kind of offering since entering the phd program. In the end we taught about 10 students for three weeks in April. Our Github team repository, including course materials, can be found at github.com/TypeFunc.

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University of Colorado, Boulder (as Burnett Meyer Instructor)

Math 2001: Discrete Mathematics	Spring 2019
Math 2001: Discrete Mathematics	Fall 2018
Math 3140: Abstract Algebra	Fall 2018
Math 6000: Model Theory (graduate course)	Spring 2018
Math 2130: Linear Algebra	Spring 2018
Math 2130: Linear Algebra	Fall 2017

University of Hawaii (as Visiting Assistant Professor)

Math 215: Applied Calculus	Spring 2017
Math 480: Senior Seminar	Spring 2017
Math 244: Calculus IV	Fall 2016
Math 321: Introduction to Advanced Math	Fall 2016

Iowa State University (as Postdoctoral Associate)

Math 317: Linear Algebra	Spring 2016
Math 317: Linear Algebra	Fall 2015
Math 160: Survey of Calculus	Fall 2015
Math 207: Elementary Linear Algebra	Spring 2015
Math 165: Calculus I	Spring 2015
Math 301: Abstract Algebra	Fall 2014
Math 165: Calculus I	Fall 2014

University of South Carolina (as Visiting Assistant Professor)

Math 700: Linear Algebra (graduate course)	Spring 2014
Math 141: Calculus I	Spring 2014
Math 374: Discrete Structures	Fall 2013
Math 122: Calculus for Business and Social Sciences	Fall 2013
Math 374: Discrete Structures	Spring 2013
Math 122: Calculus for Business and Social Sciences	Spring 2013
Math 241: Vector Calculus	Fall 2012
Math 122: Calculus for Business and Social Sciences	Fall 2012

University of Hawaii (as Graduate Student Instructor)

Math 371: Probability Theory	Summer 2011
Math 215: Applied Calculus I	Summer 2009
Math 100: Mathematical Reasoning	Summer 2010