

MR2016524 (2004k:03032) 03B40 68N18

Selinger, Peter (3-OTTW-MS)

Order-incompleteness and finite lambda reduction models. (English summary)

Theoret. Comput. Sci. **309** (2003), no. 1-3, 43–63.

Summary: “Many familiar models of the untyped lambda calculus are constructed by order-theoretic methods. This paper provides some basic new facts about ordered models of the lambda calculus. We show that in any partially ordered model that is complete for the theory of β - or $\beta\eta$ -conversion, the partial order is trivial on term denotations. Equivalently, the open and closed term algebras of the untyped lambda calculus cannot be non-trivially partially ordered. Our second result is a syntactical characterization, in terms of so-called generalized Mal’cev operators, of those lambda theories which cannot be induced by any non-trivially partially ordered model. We also consider a notion of finite models for the untyped lambda calculus, or more precisely, finite models of reduction. We demonstrate how such models can be used as practical tools for giving finitary proofs of term inequalities.”

References

1. S. Abramsky, A. Jung, Domain theory, in: S. Abramsky, D.M. Gabbay, T.S.E. Maibaum (Eds.), *Handbook of Logic in Computer Science*, Vol. 3, Clarendon Press, Oxford, 1994, pp. 1–168. [MR1365749](#)
2. H.P. Barendregt, *The Lambda Calculus, its Syntax and Semantics*, 2nd Edition, North-Holland, Amsterdam, 1984. [MR0774952](#)
3. G. Berry, Stable models of typed λ -calculi, in: *Proc. 5th Internat. Colloq. on Automata, Languages and Programming*, Lecture Notes in Computer Science, Vol. 62, Springer, Berlin, 1978, pp. 72–89. [MR0520840](#)
4. S. Bulman-Fleming, W. Taylor, Union-indecomposable varieties, *Colloq. Math.* **35** (1976) 189–199. [MR0404100](#)
5. P. Di Gianantonio, F. Honsell, S. Liani, G.D. Plotkin, Countable non-determinism and uncountable limits, in: *Proc. CONCUR ’94*, Lecture Notes in Computer Science, Vol. 836, Springer, Berlin, 1994, pp. 130–145. See also: Uncountable limits and the Lambda Calculus, *Nordic J. Comput.* **2**, 1995, pp. 127–146. [MR1322969](#)
6. J.-Y. Girard, The system F of variable types, fifteen years later, *Theoret. Comput. Sci.* **45** (1986) 159–192. [MR0867281](#)
7. J. Hagemann, A. Mitschke, On n -permutable congruences, *Algebra Universalis* **3** (1973) 8–12. [MR0330010](#)
8. F. Honsell, S. Ronchi Della Rocca, An approximation theorem for topological lambda models and the topological incompleteness of lambda calculus, *J. Comput. System Sci.* **45** (1) (1992) 49–75. [MR1170917](#)
9. M. Hyland, A syntactic characterization of the equality in some models for the lambda calculus, *J. London Math. Soc.* **12** (1976) 361–370. [MR0396232](#)
10. B. Jacobs, I. Margaria, M. Zacchi, Filter models with polymorphic types, *Theoret. Comput. Sci.* **95** (1992) 143–158. [MR1157801](#)
11. A.I. Mal’cev, K obščei teorii algebraičeskikh sistem, *Mat. Sb. (N. S.)* **35** (77) (1954) 3–20. [MR0065533](#)
12. G.D. Plotkin, A semantics for static type inference, *Inform. Comput.* **109** (1994)

- 256–299. [MR1269377](#)
13. G.D. Plotkin, On a question of H. Friedman, Inform. Comput. 126 (1) (1996) 74–77. [MR1389375](#)
 14. P. Selinger, Functionality, polymorphism, and concurrency: a mathematical investigation of programming paradigms, Ph.D. Thesis, University of Pennsylvania, 1997. [MR2696204](#)
 15. W. Taylor, Structures incompatible with varieties, Abstract 74T-A224, Notices Amer. Math. Soc. 21 (1974) A-529.
 16. C.P. Wadsworth, The relation between computational and denotational properties for Scott's D_∞ -models of the lambda-calculus, SIAM J. Comput. 5 (1976) 488–521. [MR0505308](#)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.