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MR3145059 03B40 68N18 68Q05 68Q55

Carraro, A. [Carraro, Alberto] (F-PARIS7-PPS);
Salibra, A. [Salibra, Antonino] (I-VENE-AIS)

Ordered models of the lambda calculus. (English summary)

Log. Methods Comput. Sci. **9** (2013), *no. 4,* 4:21, 29 *pp.*

The introduction of this paper is a masterful overview of ordered lambda models and related problems. The technical contribution is also outstanding. Among other results, a question asked by Honsell and Plotkin in 2009 is answered.

The authors show their ability to solve deep problems and to tailor proof techniques to their needs at will. *Mariangiola Dezani-Ciancaglini*

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MR873596 (88a:08014) 08B15 03C05 06B15
Lampe, William A.
A property of the lattice of equational theories.
Algebra Universalis **23** (1986), *no. 1,* 61–69.

Let $L(\Sigma)$ denote the lattice of all equational theories that extend an equational theory Σ. The main result states that the lattice L of all equational theories in a fixed similarity type has the property that for each $c, z \in L$ and any family a_i $(i \in I)$ of elements in L, if $a_i \wedge c = z$ for each $i \in I$ and $\bigvee\{a_i : i \in I\} = 1$, then $c = z$. As a corollary it follows that no tight lattice is isomorphic to a lattice $L(\Sigma)$. In particular, the height 2 lattice M_n having n atoms is not isomorphic to an $L(\Sigma)$ for $n \geq 3$. *S. Comer*

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MR1257643 (95c:08009) 08B10 08A30 08B05
Lipparini, Paolo
Commutator theory without join-distributivity. **(English summary)**
Trans. Amer. Math. Soc. **346** (1994), *no. 1,* 177–202.

Commutator theory in varieties satisfying weaker properties than congruence modularity is investigated. The commutator $[\alpha,\beta]$ is defined as the smallest γ such that α centralizes β modulo γ, written as $C(\alpha,\beta;\gamma)$. In general, it is not true that $C(\alpha,\beta;\gamma)\wedge \gamma\le\gamma'\Rightarrow C(\alpha,\beta;\gamma')$; rather it is shown that this assumption implies left distributivity, i.e. $[\alpha+\beta,\gamma]=[\alpha,\gamma]+[\beta,\gamma]$. Left semidistributivity of the commutator always holds, i.e., $[\alpha_i,\beta]=\delta\Rightarrow[\bigvee\alpha_i,\beta]=\delta$.

A major tool in modular varieties is a difference term $d(x,y,z)$ satisfying $x\alpha y\Rightarrow d(x,y,y)=x[\alpha,\alpha]d(y,y,x)$. Many nonmodular varieties, amongst them inverse semigroups and all n-permutable varieties, are shown to possess a weak difference term satisfying only $x\alpha y\Rightarrow d(x,y,y)[\alpha,\alpha]x[\alpha,\alpha]d(y,y,x)$. Under the hypothesis of a weak difference term many of the reviewer's permutability results for modular varieties can be obtained, for instance $\alpha+\beta=(\alpha^{(n)}+\beta^{(n)})\circ\alpha\circ\beta\circ(\alpha^{(n)}+\beta^{(n)})$, from which some useful commutator equations, such as $[\alpha+\gamma,\alpha+\gamma]=\alpha+[\gamma,\gamma]$, follow.

As in the classical case, certain sublattices of congruence lattices reveal clues about the commutator. As an example, if M_3, the five-element modular nondistributive lattice, is a sublattice of $\mathrm{Con}(A)$, then the commutator of its top element must be below its bottom. Finally, a list is provided of lattices that cannot be sublattices of $\mathrm{Con}(A)$ if A has a weak difference term.

H. Peter Gumm

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MR1637665 (2000a:08021) 08B05
Lipparini, Paolo (I-ROME2)
A characterization of varieties with a difference term. II. Neutral = meet semi-distributive. (English summary)
Canad. Math. Bull. **41** (1998), *no. 3,* 318–327.

In congruence modular varieties there is basically only one commutator operation on congruences of algebras, and it has good behavior. There are several ways to extend this commutator beyond modularity, but many desirable properties are lost in the process. A "small" extension, called the TC-commutator, is still very useful in tame congruence theory, but the structure of algebras that are abelian with respect to the TC-commutator can be very complicated. Alternatively, one can use a "big" commutator, the so-called linear commutator, which is more difficult to work with, but which ensures that the algebras that are abelian with respect to this commutator are well-behaved (subalgebras of reducts of modules over a ring). There are various other commutator concepts lying in between these two. A celebrated result of K. A. Kearnes and Á. Szendrei [Internat. J. Algebra Comput. **8** (1998), no. 4, 497–531 MR1663558] states that in varieties satisfying any nontrivial idempotent Mal′tsev condition, the linear commutator coincides with a symmetric version of the TC-commutator (implying some reasonably good behavior).

One of the good properties of the modular commutator is the existence of a so-called difference term that is a generalization of Mal′tsev's classical term characterizing congruence permutable varieties. In the present paper, the author proves that if a variety admits a weak version of the difference term with respect to the linear commutator, then this variety satisfies an idempotent Mal′tsev condition. From this, using the Kearnes-Szendrei theorem, and his earlier results, he obtains various characterizations of such varieties, in terms of Mal′tsev conditions and congruence identities. This property is also equivalent to the fact that the blocks of abelian congruences are affine.

A variety is called neutral with respect to a commutator if the commutator of any two congruences is their meet. This property implies the existence of a weak difference term, and therefore, by the results above, is independent of the choice of the commutator. Among several equivalent characterizations of neutrality, the author shows that this concept is equivalent to congruence meet-semidistributivity, and also to the property that M_3 does not occur as a sublattice in the congruence lattices of the algebras in the variety. His characterizations extend results of G. Czédli, R. McKenzie and D. Hobby.

{Part I has been reviewed [MR1411074].} *E. W. Kiss*

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MR2068680 (2005g:03017) 03B40 03C05 68N18

Lusin, Stefania (I-VENE-I); **Salibra, Antonino** (I-VENE-I)

The lattice of lambda theories. (English summary)

J. Logic Comput. **14** (2004), *no. 3,* 373–394.

The complete lattice λT of lambda theories consists of sets of equations between (untyped) closed lambda terms, closed under deduction in lambda calculus with free variables, in particular under the rule $M = N \Rightarrow \lambda x.M = \lambda x.N$. The set is ordered by inclusion and the intersection of a set of theories forms their meet and the deductive closure of the union their join. This lattice λT has been studied in the signature $\langle L, +, \cdot, 0, 1\rangle$, where $+, \cdot, 0, 1$ stand for join, intersection, the least and largest element, respectively. The relation $\mathcal{S} \subseteq \mathcal{T}$ can be defined by $\mathcal{S} \cdot \mathcal{T} = \mathcal{S}$. Results have been obtained using proof-theory (there is an element $\mathcal{H}$, equating all unsolvables, with a unique maximal extension $\mathcal{H}^*$ properly below the top 1), notions of reduction (there are $2^{\aleph_0}$ elements T with $\mathcal{H}^* \subseteq T \subseteq \mathcal{H}^*$), model-theory ($\mathcal{H}^*$ was discovered as the theory of Scott's D_∞ models of untyped lambda calculus) and even recursion-theory (λT is a dense ordering); these results are due to various authors [see H. P. Barendregt, *The lambda calculus*, Revised edition, North-Holland, Amsterdam, 1984; MR0774952]. The present paper studies λT using methods from universal algebra.

Lambda theories are not usual first-order equational algebraic theories. The reason is that there is an operator 'λx' having a binding effect on the expression following it. For this reason one has studied lambda abstraction algebras [see A. Salibra, Theoret. Comput. Sci. **249** (2000), no. 1, 197–240; MR1791956], in which these 'λx' belong to the algebraic signature and one axiomatizes the binding effect. This ensures that methods of universal algebra are applicable to λT.

The results of the paper are the following. 1. There are quasi-identities, i.e. statements of the form E_1 & ... & $E_n \rightarrow E$, with the $E_1, \dots, E_n, E$ all equations, that hold in λT, but not in arbitrary complete lattices. An example is $\mathcal{S} + \mathcal{T} = 1 \,\&\, \mathcal{S}\mathcal{G} = \mathcal{T}\mathcal{G} \rightarrow \mathcal{G} \subseteq \mathcal{S}$. 2. If an identity fails in some complete lattice, then it also fails in λT^+, where one has added to the set of lambda terms a finite set of constants. 3. There exists a $\mathcal{J} \in \lambda T$ such that the sublattice $[\mathcal{J}) = \{\mathcal{T} \in \lambda T \mid \mathcal{J} \subseteq \mathcal{T}\}$ satisfies $\mathcal{S}\mathcal{T} = \mathcal{S}\mathcal{G} \rightarrow \mathcal{S}\mathcal{T} = \mathcal{S}(\mathcal{T} + \mathcal{G})$. This theory $\mathcal{J}$ is axiomatized by (here $\Omega \equiv (\lambda a.aa)(\lambda a.aa)$ is the standard unsolvable term)

$$\Omega xx = x,\ \Omega xy = \Omega yx,\ \Omega x(\Omega yz) = \Omega(\Omega xy)z,$$

and is proved consistent by constructing a filter model using intersection types [see M. Coppo et al., in *Logic colloquium '82 (Florence, 1982)*, 241–262, North-Holland, Amsterdam, 1984; MR0762113].

Henk Barendregt

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MR2670239 (2011k:03035) 03B40 06E25 08A70
Manzonetto, Giulio (F-PARIS7-PPS); **Salibra, Antonino** (I-VENE2-C)
Applying universal algebra to lambda calculus. (English summary)
J. Logic Comput. **20** (2010), *no. 4,* 877–915.

This important paper provides a detailed survey of recent work on λ calculus using universal algebra. Also, it shows that λ calculus and combinatory logic satisfy some unexpected algebraic properties. The Stone representation theorem is generalised to combinatory and λ abstraction algebras, showing that every such algebra can be decomposed as a weak product of indecomposable algebras. The semantics of λ calculus, in terms of directly indecomposable λ models, is shown to include all the main models, but is still incomplete. It follows from this that the continuous stable and strongly stable semantics are incomplete. Twenty-six open questions are discussed. *Martin W. Bunder*

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MR1654292 (2000b:03050) 03B40 03G25
Pigozzi, Don [**Pigozzi, Don L.**] (1-IASU); **Salibra, Antonino** (I-VENE-AM)
Lambda abstraction algebras: coordinatizing models of Lambda calculus. (English summary)
Fund. Inform. **33** (1998), *no. 2,* 149–200.

Lambda abstraction algebras algebraize the untyped lambda calculus just as combinatory algebras do combinatory logic. Both these classes of algebras form a variety in the sense of universal algebra. This paper derives connections between lambda abstraction algebras, subclasses of these that are "functional", "locally finite" and "rich", and combinatory algebras, lambda algebras and lambda models.

The main result of the paper is a stronger version of the functional representation theorem for locally finite lambda abstraction algebras, the algebraic analogue of the completeness theorem of lambda calculus. *Martin W. Bunder*

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MR1321662 (96b:68123) 68Q65 03B40 03G25
Pigozzi, Don [**Pigozzi, Don L.**] (1-IASU); **Salibra, Antonino** (I-VENE-AM)
Lambda abstraction algebras: representation theorems. (English summary)
Selected papers of AMAST '93 (Enschede, 1993).
Theoret. Comput. Sci. **140** (1995), *no. 1,* 5–52.

Summary: "Lambda abstraction algebras (LAAs) are designed to algebraize the untyped lambda calculus in the same way cylindric and polyadic algebras algebraize the first-order predicate logic. Like combinatory algebras they can be defined by true identities and thus form a variety in the sense of universal algebra, but they differ from combinatory algebras in several important respects. The most natural LAAs are obtained by coordinatizing environment models of the lambda calculus. This gives rise to two classes of LAAs of functions of finite arity: functional LAAs (FLA) and point-relativized functional LAAs (RFA). It is shown that RFA is a variety and is the smallest variety including FLA.

"Dimension-complemented LAAs constitute the widest class of LAAs that can be represented as an algebra of functions and are known to have a natural intrinsic characterization. We prove that every dimension-complemented LAA is isomorphic to a member of RFA. This is the crucial step in showing that RFA is a variety."

{For the collection containing this paper see MR1321661}

Martin W. Bunder

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MR1659467 (2000b:03051) 03B40 68N18
Salibra, Antonino (I-VENE-I);
Goldblatt, Robert [**Goldblatt, Robert Ian**] (NZ-VCTR-SMC)

A finite equational axiomatization of the functional algebras for the lambda calculus. (English summary)

Inform. and Comput. **148** (1999), *no. 1,* 71–130.

The authors use techniques of universal algebra to make an interesting original contribution to the theory of the untyped $\lambda\beta$-calculus. The main result is an explicit finite axiomatization of the valid equations between contexts. A context is a lambda term with some special variables called "holes". An equation between contexts is valid if every instance of it obtained by substituting arbitrary lambda terms for the holes, without renaming of bound variables, is provable in the $\lambda\beta$-calculus. So, for instance, denoting the hole by the Greek letter μ, we have that the equation $\lambda x.x\mu = \lambda y.y\mu$ is not valid, because on substituting the lambda term x for μ we obtain the unprovable equation $\lambda x.xx = \lambda y.yx$. An example of a valid equation, which is one of the proposed axioms, is $(\lambda xy.\xi)((\lambda y.\mu)z) = \lambda y.(\lambda x.\xi)((\lambda y.\mu)z)$, where μ, ξ are the holes, and the variable y is different from x and z. The proof takes place in the framework of the equational axiomatization LAA (lambda abstraction algebras) of the lambda calculus developed by D. Pigozzi and A. Salibra. Solving a problem raised by those authors, it is proved here that every model of LAA is a suitable expansion of a combinatory model of the lambda calculus. This was previously known only for the locally finite models. In the last part of the paper a completeness theorem for the infinite lambda calculus is given.

Alessandro Berarducci

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MR2016524 (2004k:03032) 03B40 68N18
Selinger, Peter (3-OTTW-MS)
Order-incompleteness and finite lambda reduction models. (English summary)
Theoret. Comput. Sci. **309** (2003), *no. 1-3,* 43–63.

Summary: "Many familiar models of the untyped lambda calculus are constructed by order-theoretic methods. This paper provides some basic new facts about ordered models of the lambda calculus. We show that in any partially ordered model that is complete for the theory of β- or $\beta\eta$-conversion, the partial order is trivial on term denotations. Equivalently, the open and closed term algebras of the untyped lambda calculus cannot be non-trivially partially ordered. Our second result is a syntactical characterization, in terms of so-called generalized Mal′cev operators, of those lambda theories which cannot be induced by any non-trivially partially ordered model. We also consider a notion of finite models for the untyped lambda calculus, or more precisely, finite models of reduction. We demonstrate how such models can be used as practical tools for giving finitary proofs of term inequalities."

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