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A finite equational axiomatization of the functional algebras for the lambda calculus. (English summary)

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The authors use techniques of universal algebra to make an interesting original contribution to the theory of the untyped $\lambda\beta$-calculus. The main result is an explicit finite axiomatization of the valid equations between contexts. A context is a lambda term with some special variables called "holes". An equation between contexts is valid if every instance of it obtained by substituting arbitrary lambda terms for the holes, without renaming of bound variables, is provable in the $\lambda\beta$-calculus. So, for instance, denoting the hole by the Greek letter μ, we have that the equation $\lambda x.x\mu = \lambda y.y\mu$ is not valid, because on substituting the lambda term x for μ we obtain the unprovable equation $\lambda x.xx = \lambda y.yx$. An example of a valid equation, which is one of the proposed axioms, is $(\lambda xy.\xi)((\lambda y.\mu)z) = \lambda y.(\lambda x.\xi)((\lambda y.\mu)z)$, where μ, ξ are the holes, and the variable y is different from x and z. The proof takes place in the framework of the equational axiomatization LAA (lambda abstraction algebras) of the lambda calculus developed by D. Pigozzi and A. Salibra. Solving a problem raised by those authors, it is proved here that every model of LAA is a suitable expansion of a combinatory model of the lambda calculus. This was previously known only for the locally finite models. In the last part of the paper a completeness theorem for the infinite lambda calculus is given.

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