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Applying universal algebra to lambda calculus. (English summary)
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This important paper provides a detailed survey of recent work on λ calculus using universal algebra. Also, it shows that λ calculus and combinatory logic satisfy some unexpected algebraic properties. The Stone representation theorem is generalised to combinatory and λ abstraction algebras, showing that every such algebra can be decomposed as a weak product of indecomposable algebras. The semantics of λ calculus, in terms of directly indecomposable λ models, is shown to include all the main models, but is still incomplete. It follows from this that the continuous stable and strongly stable semantics are incomplete. Twenty-six open questions are discussed. *Martin W. Bunder*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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