MR1637665 (2000a:08021) 08B05
Lipparini, Paolo (I-ROME2)
A characterization of varieties with a difference term. II. Neutral = meet semi-distributive. (English summary)
Canad. Math. Bull. **41** (1998), *no. 3,* 318–327.

In congruence modular varieties there is basically only one commutator operation on congruences of algebras, and it has good behavior. There are several ways to extend this commutator beyond modularity, but many desirable properties are lost in the process. A "small" extension, called the TC-commutator, is still very useful in tame congruence theory, but the structure of algebras that are abelian with respect to the TC-commutator can be very complicated. Alternatively, one can use a "big" commutator, the so-called linear commutator, which is more difficult to work with, but which ensures that the algebras that are abelian with respect to this commutator are well-behaved (subalgebras of reducts of modules over a ring). There are various other commutator concepts lying in between these two. A celebrated result of K. A. Kearnes and Á. Szendrei [Internat. J. Algebra Comput. **8** (1998), no. 4, 497–531 MR1663558] states that in varieties satisfying any nontrivial idempotent Mal′tsev condition, the linear commutator coincides with a symmetric version of the TC-commutator (implying some reasonably good behavior).

One of the good properties of the modular commutator is the existence of a so-called difference term that is a generalization of Mal′tsev's classical term characterizing congruence permutable varieties. In the present paper, the author proves that if a variety admits a weak version of the difference term with respect to the linear commutator, then this variety satisfies an idempotent Mal′tsev condition. From this, using the Kearnes-Szendrei theorem, and his earlier results, he obtains various characterizations of such varieties, in terms of Mal′tsev conditions and congruence identities. This property is also equivalent to the fact that the blocks of abelian congruences are affine.

A variety is called neutral with respect to a commutator if the commutator of any two congruences is their meet. This property implies the existence of a weak difference term, and therefore, by the results above, is independent of the choice of the commutator. Among several equivalent characterizations of neutrality, the author shows that this concept is equivalent to congruence meet-semidistributivity, and also to the property that M_3 does not occur as a sublattice in the congruence lattices of the algebras in the variety. His characterizations extend results of G. Czédli, R. McKenzie and D. Hobby.

{Part I has been reviewed [MR1411074].} *E. W. Kiss*